

# Using Schemas to Teach Students to Solve Word Problems



# Sarah R. Powell, Ph.D.

Associate Professor  
The University of Texas at Austin



[srpowell@utexas.edu](mailto:srpowell@utexas.edu)



[@sarahpowellphd](https://twitter.com/sarahpowellphd)





Introduce yourself.

Describe your role as an educator.

Describe the mathematics you support.



# Using Schemas to Teach Students to Solve Word Problems

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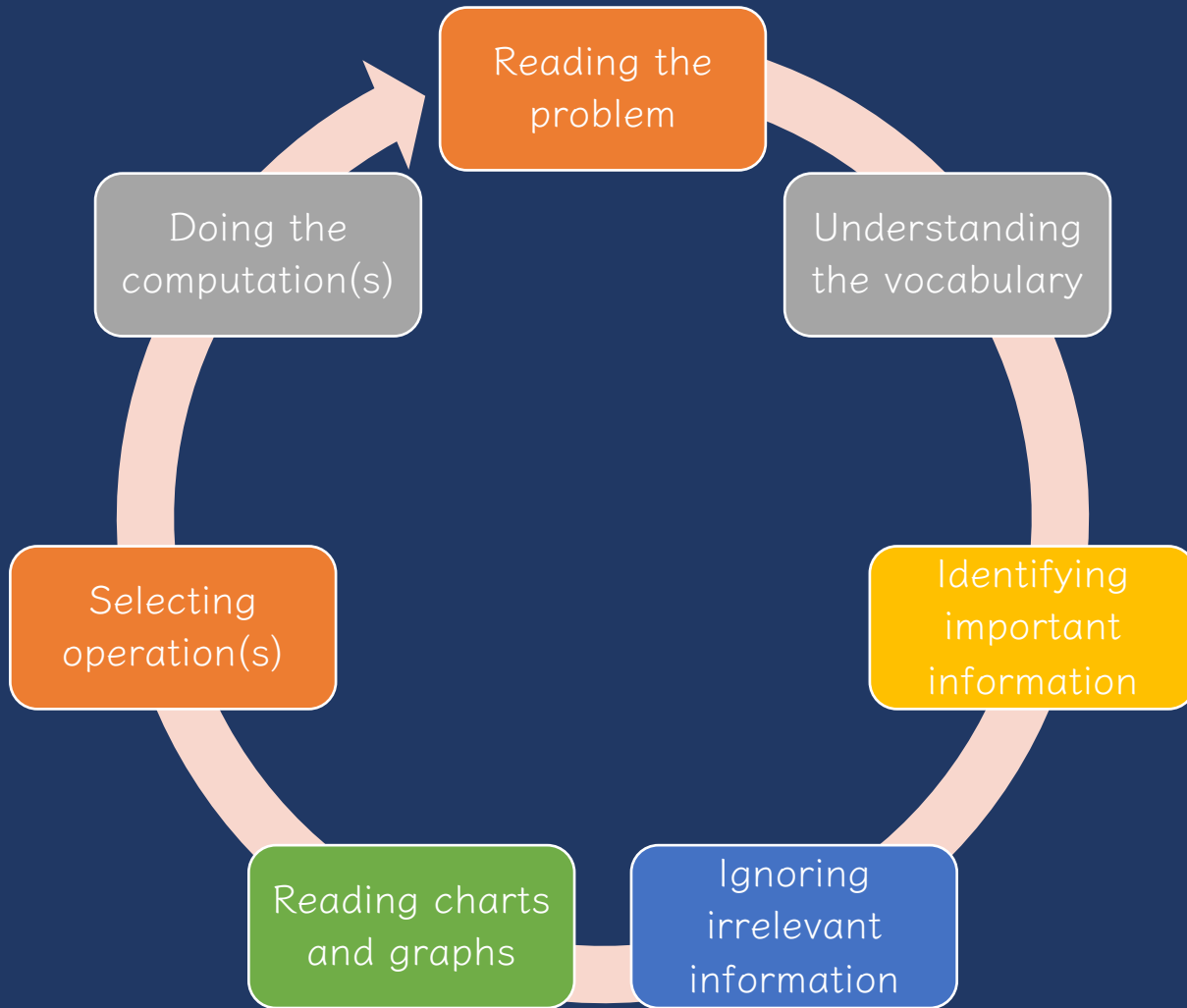
@sarahpowellphd

www.sarahpowellphd.com

## Word-Problem Solving

## Teaching Problem Solving





# Ineffective Strategies





# 1. Keywords tied to operations



Lincoln had 8 pencils **fewer** than Roscoe. If Roscoe had 18 pencils, how many pencils did Lincoln have?

Lincoln had 8 pencils **fewer** than Roscoe. If Lincoln had 18 pencils, how many pencils did Roscoe have?



### Key Words Used in Math Word Problems

Addition Words	Subtraction Words
<ul style="list-style-type: none"> <li>add</li> <li>all together or altogether</li> <li>and</li> <li>both</li> <li>combined</li> <li>how many in all</li> <li>how much</li> <li>in all</li> <li>increased by</li> <li>plus</li> <li>sum</li> <li>together</li> <li>total</li> </ul>	<ul style="list-style-type: none"> <li>change</li> <li>decreased by</li> <li>difference</li> <li>fewer or fewer than</li> <li>how many are left (or have left)</li> <li>how many did not have</li> <li>how many</li> <li>how much taller, heavier, less or less</li> <li>lost</li> <li>minus</li> <li>need to</li> <li>reduce</li> <li>remain</li> <li>subtract</li> <li>take away</li> </ul>
	
Multiplication Words	Division Words
<ul style="list-style-type: none"> <li>x by (dimension)</li> <li>double</li> <li>each group</li> <li>every</li> <li>factor of</li> <li>increased by</li> <li>multiplied by</li> <li>x of</li> <li>x product</li> <li>x times</li> <li>x triple</li> </ul>	<ul style="list-style-type: none"> <li>as much</li> <li>cut up</li> <li>each group</li> <li>equal share</li> <li>half (or other parts)</li> <li>how many parts</li> <li>per</li> <li>percent</li> <li>quotient</li> <li>ratio of</li> <li>separated</li> <li>share some</li> </ul>
	

### OPERATION cue words

ADDITION	SUBTRACTION
<ul style="list-style-type: none"> <li>and</li> <li>total</li> <li>join</li> <li>more than</li> <li>in all</li> <li>sum</li> <li>altogether</li> <li>increased</li> </ul>	<ul style="list-style-type: none"> <li>less than</li> <li>decreased</li> <li>remaining</li> <li>left</li> <li>fewer</li> <li>take away</li> <li>difference</li> <li>minus</li> </ul>
MULTIPLICATION	DIVISION
<ul style="list-style-type: none"> <li>product</li> <li>times</li> <li>as many as</li> <li>of</li> <li>by</li> <li>equal groups</li> </ul>	<ul style="list-style-type: none"> <li>quotient</li> <li>each</li> <li>broken into</li> <li>distributed evenly</li> <li>parts</li> </ul>

### KEY WORDS

ADDITION	MULTIPLICATION
<ul style="list-style-type: none"> <li>-sum</li> <li>-total</li> <li>-more than</li> <li>-plus</li> </ul>	<ul style="list-style-type: none"> <li>-product</li> <li>-double</li> <li>-area</li> <li>-times</li> <li>-per</li> <li>-every</li> <li>-each</li> <li>-by</li> </ul>
SUBTRACTION	DIVISION
<ul style="list-style-type: none"> <li>-difference</li> <li>-remain</li> <li>-left</li> <li>-less than</li> <li>-minus</li> <li>-how many more</li> <li>-fewer than</li> <li>-decrease</li> <li>-give away</li> <li>-reduce</li> <li>-discount</li> <li>-how many more</li> </ul>	<ul style="list-style-type: none"> <li>-quotient</li> <li>-divide by</li> <li>-into</li> <li>-split</li> <li>-out of</li> <li>-shared</li> <li>-per</li> <li>-every</li> <li>-each</li> <li>-evenly</li> <li>-equal groups</li> <li>-half</li> </ul>

### Math POSTER

### Division

Taking a total and sharing it

### Addition

Putting two or more things/amounts together.

Keywords: Total, Altogether, In all, Sum, more than, added to, plus, join

### Key Words for All Operations

Addition	Subtraction
<ul style="list-style-type: none"> <li>Sum</li> <li>Total</li> <li>Plus</li> <li>In all</li> <li>And</li> <li>Join</li> <li>Altogether</li> <li>Perimeter</li> <li>Together</li> </ul>	<ul style="list-style-type: none"> <li>Fewer</li> <li>Less</li> <li>Exceed</li> <li>Remain</li> <li>Are not</li> <li>Minus</li> <li>Difference</li> <li>How many more</li> <li>Take away</li> <li>Left over</li> </ul>

When they say... They mean...

### Math Key Words

Addition	Subtraction	Multiplication	Division
<ul style="list-style-type: none"> <li>plus</li> <li>sum</li> <li>add</li> <li>total</li> <li>all together</li> <li>increase</li> <li>more</li> <li>combine</li> </ul>	<ul style="list-style-type: none"> <li>subtract</li> <li>minus</li> <li>difference</li> <li>left</li> <li>left over</li> <li>decrease</li> <li>take away</li> <li>fewer</li> </ul>	<ul style="list-style-type: none"> <li>times</li> <li>product</li> <li>factor</li> <li>double</li> <li>groups</li> <li>each</li> <li>area</li> <li>rows</li> </ul>	<ul style="list-style-type: none"> <li>quotient</li> <li>split</li> <li>share</li> <li>divide</li> <li>separate</li> <li>each</li> <li>average</li> <li>equal groups</li> </ul>

### Problem Solving Key Words

Addition	Subtraction
<ul style="list-style-type: none"> <li>add</li> <li>together</li> </ul>	<ul style="list-style-type: none"> <li>are not</li> <li>decrease</li> <li>difference</li> <li>fewer, larger, shorter</li> <li>left</li> <li>less than</li> <li>minus</li> <li>remain</li> <li>take away</li> </ul>

### Math Key Words

Addition	Subtraction	Multiplication	Division
<ul style="list-style-type: none"> <li>plus</li> <li>sum</li> <li>add</li> <li>total</li> <li>all together</li> <li>increase</li> <li>more</li> <li>combine</li> </ul>	<ul style="list-style-type: none"> <li>subtract</li> <li>minus</li> <li>difference</li> <li>left</li> <li>left over</li> <li>decrease</li> <li>take away</li> <li>fewer</li> </ul>	<ul style="list-style-type: none"> <li>times</li> <li>product</li> <li>factor</li> <li>double</li> <li>groups</li> <li>each</li> <li>area</li> <li>rows</li> </ul>	<ul style="list-style-type: none"> <li>quotient</li> <li>split</li> <li>share</li> <li>divide</li> <li>separate</li> <li>each</li> <li>average</li> <li>equal groups</li> </ul>

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<ul style="list-style-type: none"> <li>plus</li> <li>sum</li> <li>add</li> <li>total</li> <li>all together</li> <li>increase</li> <li>more</li> <li>combine</li> </ul>	<ul style="list-style-type: none"> <li>subtract</li> <li>minus</li> <li>difference</li> <li>left</li> <li>left over</li> <li>decrease</li> <li>take away</li> <li>fewer</li> </ul>	<ul style="list-style-type: none"> <li>times</li> <li>product</li> <li>factor</li> <li>double</li> <li>groups</li> <li>each</li> <li>area</li> <li>rows</li> </ul>	<ul style="list-style-type: none"> <li>quotient</li> <li>split</li> <li>share</li> <li>divide</li> <li>separate</li> <li>each</li> <li>average</li> <li>equal groups</li> </ul>

### key words

<ul style="list-style-type: none"> <li>addition</li> <li>sum</li> <li>both</li> <li>more than</li> </ul>	<ul style="list-style-type: none"> <li>in all</li> <li>together</li> <li>perimeter</li> </ul>	<ul style="list-style-type: none"> <li>total</li> <li>plus</li> <li>add</li> </ul>
<ul style="list-style-type: none"> <li>subtract</li> <li>main</li> <li>less than</li> </ul>	<ul style="list-style-type: none"> <li>decrease</li> <li>take away</li> <li>how many more...</li> </ul>	<ul style="list-style-type: none"> <li>fewer</li> <li>minus</li> </ul>

combined

triple factor product

multiply each per in all multiple area double times

average

division split quotient equal groups divide half shared equity each

distribute

### Math Operation - Key Words

Addition	Subtraction	Multiplication	Division
<ul style="list-style-type: none"> <li>add</li> <li>altogether</li> <li>and</li> <li>both</li> <li>in all</li> <li>sum</li> <li>total</li> <li>increase</li> </ul>	<ul style="list-style-type: none"> <li>difference</li> <li>fewer than</li> <li>gave/take away</li> <li>decreased by</li> <li>how many more</li> <li>show much longer/smaller/shorter</li> <li>minus</li> <li>remaining</li> </ul>	<ul style="list-style-type: none"> <li>area</li> <li>product</li> <li>Each</li> <li>by - of - per</li> <li>Times</li> <li>double, twice, triple</li> <li>total</li> <li>increase</li> </ul>	<ul style="list-style-type: none"> <li>quotient</li> <li>divide</li> <li>into</li> <li>equal parts/share equally</li> <li>per</li> <li>amount of each</li> </ul>

### Math Key Words

Addition	Subtraction	Multiplication	Division
<ul style="list-style-type: none"> <li>plus</li> <li>sum</li> <li>add</li> <li>total</li> <li>all together</li> <li>increase</li> <li>more</li> <li>combine</li> </ul>	<ul style="list-style-type: none"> <li>subtract</li> <li>minus</li> <li>difference</li> <li>left</li> <li>left over</li> <li>decrease</li> <li>take away</li> <li>fewer</li> </ul>	<ul style="list-style-type: none"> <li>times</li> <li>product</li> <li>factor</li> <li>double</li> <li>groups</li> <li>each</li> <li>area</li> <li>rows</li> </ul>	<ul style="list-style-type: none"> <li>quotient</li> <li>split</li> <li>share</li> <li>divide</li> <li>separate</li> <li>each</li> <li>average</li> <li>equal groups</li> </ul>



# Math Words Poster Set

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*Description of Single-Step Word Problems (n = 132)*

Schema	Occurrence of schema		Any keyword		Schema-specific keywords <sup>a</sup>		Multiple keywords <sup>a</sup>		Keyword(s) led to correct solution <sup>a</sup>	
	n	%	n	%	n	%	n	%	n	%
Total	27	20.5	26	96.3	23	88.5	5	19.2	21	80.8
Difference	17	12.9	17	100.0	14	82.4	2	11.8	12	70.6
Change	11	8.3	7	63.6	5	71.4	5	71.4	2	28.6
Equal groups	29	22.0	26	89.7	22	84.6	18	69.2	8	30.8
Comparison	10	7.6	9	90.0	9	100.0	4	44.4	5	55.6
Ratios or proportions	29	22.0	23	79.3	9	39.1	9	39.1	6	26.1
Product of measures	9	6.8	9	100.0	8	88.9	1	11.1	5	55.6

<sup>a</sup>When a problem featured a keyword.





*Description of Multi-Step Word Problems (n = 84)*

Schema	Occurrence of schema <sup>a</sup>		Any keyword		Keyword(s) led to correct solution <sup>b</sup>	
	<i>n</i>	%	<i>n</i>	%	<i>n</i>	%
Total	40	47.6	39	97.5	3	7.7
Difference	11	13.1	11	100.0	1	9.1
Change	21	23.8	19	95.0	1	5.3
Equal groups	49	58.3	48	98.0	1	2.1
Comparison	7	8.3	7	100.0	0	0.0
Ratios or proportions	22	25.0	16	76.2	1	6.3
Product of measures	7	8.3	7	100.0	2	28.6

<sup>a</sup>Sum across schemas does not equal 100 because each word problem featured more than one schema.

<sup>b</sup>When a problem featured a keyword.



Mr. Rivera's taxable income is \$20 each hour before taxes are taken out. Mr. Rivera worked a total of 40 hours each week for 50 weeks.

What is the dollar amount, to the nearest dollar, taken out for taxes based on Mr. Rivera's taxable income?

Jessica rented 1 video game and 3 movies for a total of \$11.50.

- The video game cost \$4.75 to rent.
- The movies cost the same amount each to rent.

What amount, in dollars, did Jessica pay to rent each movie?

The temperature of a substance decreased by  $24^{\circ}\text{C}$  per minute for 3 minutes. What was the overall change of the temperature of the substance?



Keywords are important to identify and understand

Keywords are the mathematical vocabulary that help an students understand what the story is about and what they need to do

Talk about keywords  
("What does *more than* tell you about?")



*But, do not tie a keyword to a specific operation!*





## 2. Presenting problems by operation



Name: \_\_\_\_\_

Date: \_\_\_\_\_

## Addition Word Problems

Solve the word problems. Show your work.

1. Noah had 12 books. He got 5 more books. How many books did Noah have in all?
2. Bonnie found 8 rocks on her front yard and 7 rocks in her backyard. How many rocks did she find in all?
3. Edward had 5 toy cars. He got 3 more toy cars. How many toy cars did Edward have in all?
4. Mariela collected 11 feathers. She found 3 more feathers. How many feathers did she have in all?
5. LaMonte made 14 cookies. He made 7 more cookies. How many cookies did LaMonte have in all?

## Division Word Problems

1. Zookeeper Al had 567 bananas. He gave an equal number of bananas to 9 monkeys in the zoo and 567 bananas. How many bananas did each monkey get? And how many are left over?
2. Betty has 427 oranges. She wants to pack them up equally in 23 boxes. How many oranges will she have in each box and how much does she have left over?
3. Mr. King has 1376 pages of paper. He wants to give 32 pages to each student. How many extra pages will he have left over?
4. Mr. King has 1376 pages of scrap paper. He instead wants to give 32 pages to each student. Will there be enough scrap paper for 40 students. How much more scrap paper does he need?





# Effective Strategies



Teach an attack strategy

Teach about schemas



## Attack Strategy

### SOLVE

- Study the problem.
- Organize the facts.
- Line up the plan.
- Verify the plan with computation.
- Examine the answer.

### R-CUBES

- Read the problem.
- Circle key numbers.
- Underline the question.
- Box action words.
- Evaluate steps.
- Solve and check.

### UPS✓

**U**NDERSTAND  
Read and explain.

**P**LAN  
How will you solve the problem?

**S**OLVE  
Set up and do the math!

✓**C**HECK  
Does your answer make sense?



# RIDE

**R**ead the problem.

**I**dentify the relevant information.

**D**etermine the operation and unit for the answer.

**E**nter the correct numbers and calculate, then check the answer.

# RIDGES

Read the problem.

I know statement.

Draw a picture.

Goal statement.

Equation development.

Solve the equation.



## STAR

**S**top and read the problem carefully.

**T**hink about your plan and the strategy you will use.

**A**ct. Follow your plan and solve the problem.

**R**eview your answer.

## RICE

**R**ead and record the problem.

**I**llustrate your thinking.

**C**ompute.

**E**xplain your thinking.



## SUPER

Slowly read the story problem twice.

Underline the question and circle the numbers you need.

Picture it. Draw the scenario to show what is happening.

Explain the problem with a number sentence.

Rewrite the answer in a sentence.

## SHINES

Slowly and carefully read the problem.

Highlight or underline key information.

Identify the question by drawing a circle around it.

Now solve the problem. Show your work.

Examine your work for precision, accuracy, and clarity.

Share your answer by writing a sentence.



## SOLVE

**S**tudy the problem.

**O**rganize the facts.

**L**ine up the plan.

**V**erify the plan with computation.

**E**xamine the answer.

## R-CUBES

**R**ead the problem.

**C**ircle key numbers.

**U**nderline the question.

**B**ox action words.

**E**valuate steps.

**S**olve and check.



# UPS✓

## UNDERSTAND

Read and explain.

## PLAN

How will you solve the problem?

## SOLVE

Set up and do the math!

## ✓CHECK

Does your answer make sense?

Created by: Sarah Powell (srpowell@austin.utexas.edu)







Share your favorite attack strategy.



Teach an attack strategy

Teach about schemas



Total

Difference

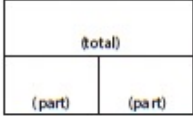




Change

Equal Groups

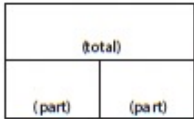

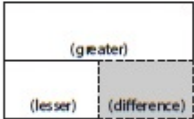
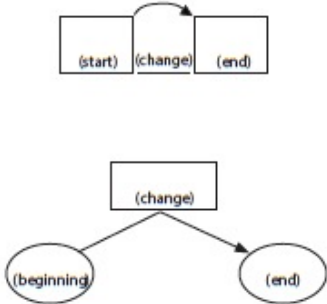
Comparison

Ratios/Proportions



Schema and Definition	Equations and Graphic Organizers	Examples	Variations
<p>Total (Combine; Part-part- whole) Parts combined for a sum</p>	<p><math>P1 + P2 = T</math> (part + part = total)</p> 	<p>Sum unknown: Lyle has 11 red apples and 18 green apples. How many apples does Lyle have altogether?</p> <p>Part unknown: Lyle has 29 red and green apples. If 11 of the apples are red, how many green apples does Lyle have?</p>	<p>More than two parts: Lyle has 34 apples. Of the apples, 11 are red, 18 are green, and the rest are yellow. How many yellow apples does Lyle have?</p>
<p>Difference (Compare) Sets compared for a difference</p>	<p><math>B - s = D</math> (bigger - smaller = difference)</p>  <p><math>G - L = D</math> (greater - less = difference)</p> 	<p>Difference unknown: Sasha wrote 85 words in her essay, and Tabitha wrote 110 words. How many fewer words did Sasha write than Tabitha?</p> <p>Bigger/greater unknown: Tabitha wrote 25 more words than Sasha. If Sasha wrote 85 words, how many words did Tabitha write?</p> <p>Smaller/lesser unknown: Tabitha wrote 110 words in her essay. Sasha wrote 25 words fewer than Tabitha. How many words did Sasha write?</p>	<p>(None)</p>
<p>Change (Join; Separate) An amount that increases or decreases</p>	<p><math>ST +/- C = E</math> (start +/- change = end)</p>  	<p>End (increase) unknown: Jorge had \$52. Then, he earned \$16 babysitting. How much money does Jorge have now?</p> <p>Change (increase) unknown: Jorge had \$52. Then, he earned some money babysitting. Now, Jorge has \$68. How much did Jorge earn babysitting?</p> <p>Start (increase) unknown: Jorge has some money, and then he earned \$16 for babysitting. Now, Jorge has \$68. How much money did he have to start with?</p> <p>End (decrease) unknown: Jorge had \$52. Then, he spent \$29 at the ballpark. How much money does Jorge have now?</p> <p>Change (decrease) unknown: Jorge had \$52 but spent some money when he went to the ballpark. Now, Jorge has \$23. How much did Jorge spend at the ballpark?</p> <p>Start (decrease) unknown: Jorge had some money. Then, he spent \$29 at the ballpark and has \$23 left. How much money did Jorge have before going to the ballpark?</p>	<p>Multiple changes: Jorge had \$78. He stopped and bought a pair of shoes for \$42 and then he spent \$12 at the grocery. How much money does Jorge have now?</p>



Schema and Definition	Equations and Graphic Organizers	Examples	Variations
<p>Total (Combine; Part-part-whole) Parts combined for a sum</p>	<p><math>P1 + P2 = T</math> (part + part = total)</p> 	<p>Sum unknown: Lyle has 11 red apples and 18 green apples. How many apples does Lyle have altogether?</p> <p>Part unknown: Lyle has 29 red and green apples. If 11 of the apples are red, how many green apples does Lyle have?</p>	<p>More than two parts: Lyle has 34 apples. Of the apples, 11 are red, 18 are green, and the rest are yellow. How many yellow apples does Lyle have?</p>
<p>Difference (Compare) Sets compared for a difference</p>	<p><math>B - s = D</math> (bigger - smaller = difference)</p>  <p><math>G - L = D</math> (greater - less = difference)</p> 	<p>Difference unknown: Sasha wrote 85 words in her essay, and Tabitha wrote 110 words. How many fewer words did Sasha write than Tabitha?</p> <p>Bigger/greater unknown: Tabitha wrote 25 more words than Sasha. If Sasha wrote 85 words, how many words did Tabitha write?</p> <p>Smaller/lesser unknown: Tabitha wrote 110 words in her essay. Sasha wrote 25 words fewer than Tabitha. How many words did Sasha write?</p>	<p>(None)</p>
<p>Change (Join; Separate) An amount that increases or decreases</p>	<p><math>ST \pm C = E</math> (start +/- change = end)</p> 	<p>End (increase) unknown: Jorge had \$52. Then, he earned \$16 babysitting. How much money does Jorge have now?</p> <p>Change (increase) unknown: Jorge had \$52. Then, he earned some money babysitting. Now, Jorge has \$68. How much did Jorge earn babysitting?</p> <p>Start (increase) unknown: Jorge has some money, and then he earned \$16 for babysitting. Now, Jorge has \$68. How much money did he have to start with?</p> <p>End (decrease) unknown: Jorge had \$52. Then, he spent \$29 at the ballpark. How much money does Jorge have now?</p> <p>Change (decrease) unknown: Jorge had \$52 but spent some money when he went to the ballpark. Now, Jorge has \$23. How much did Jorge spend at the ballpark?</p> <p>Start (decrease) unknown: Jorge had some money. Then, he spent \$29 at the ballpark and has \$23 left. How much money did Jorge have before going to the ballpark?</p>	<p>Multiple changes: Jorge had \$78. He stopped and bought a pair of shoes for \$42 and then he spent \$12 at the grocery. How much money does Jorge have now?</p>



Total

Part-part-whole  
Combine

**Parts** put together into a **total**

Daniela saw **3** canoes and **8** kayaks.  
How many boats did Daniela see?

Total

Daniela saw **11** boats. If **3** of the boats  
were canoes, how many were kayaks?

Part

Daniela saw **11** boats. **8** of the boats  
were kayaks, how many were canoes?

Part



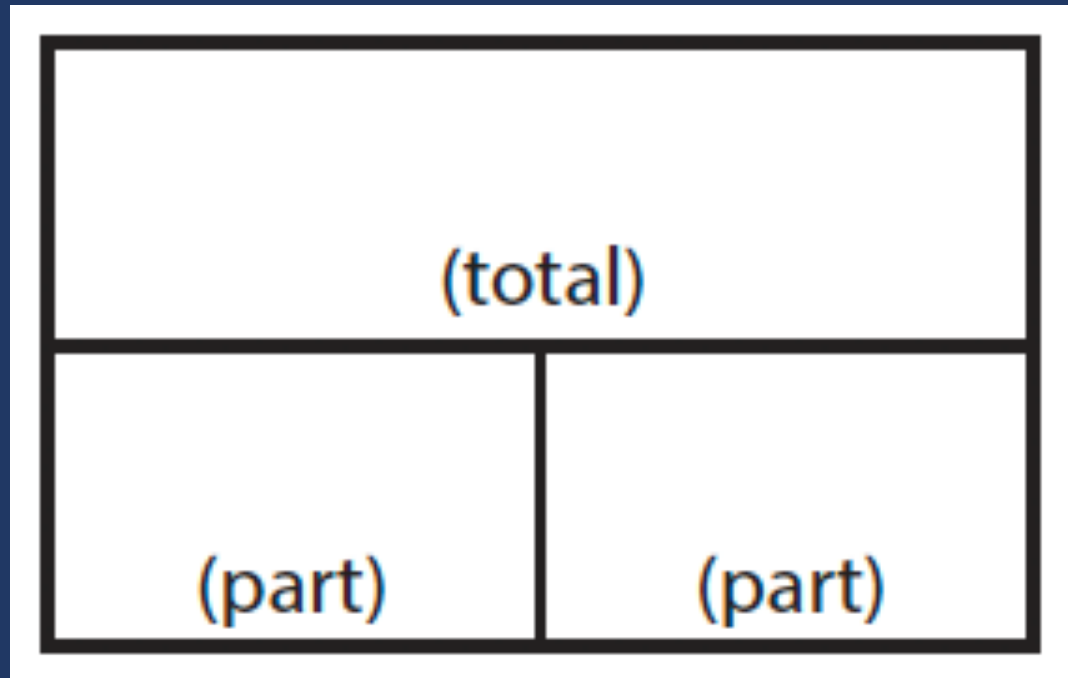
Total

“Are parts put together for a total?”



Total

$$P1 + P2 = T$$





# Total

## Additive Word Problems

A.  
Ali delivered 12 boxes of cookies on Friday and 25 boxes of cookies on Saturday. How many boxes of cookies did Ali deliver?

B.  
In March and April, it rained a total of 11.4 inches. If it rained 3.9 inches in March, how many inches did it rain in April?

C.  
Audrey has 162 wooden beads and 95 glass beads. What is the difference between Audrey's wooden beads and glass beads?

D.  
Damian's dog eats  $9\frac{1}{2}$  cups of dog food each week. Monte's dog eats  $4\frac{1}{4}$  cups less each week than Damian's dog. How much does Monte's dog eat in a week?



Total



Share a Total problem.



# Difference Compare

**Greater** and **lesser** amounts compared for a **difference**

Adrianna has **10** pencils. Tracy has **4** pencils. How many more pencils does Adrianna have?

Difference

Adrianna has **6** more pencils than Tracy. If Tracy has **4** pencils, how many does Adrianna have?

Greater amount

Tracy has **6** fewer pencils than Adrianna. Adrianna has **10** pencils. How many pencils does Tracy have?

Lesser amount



Total

“Are parts put together for a total?”

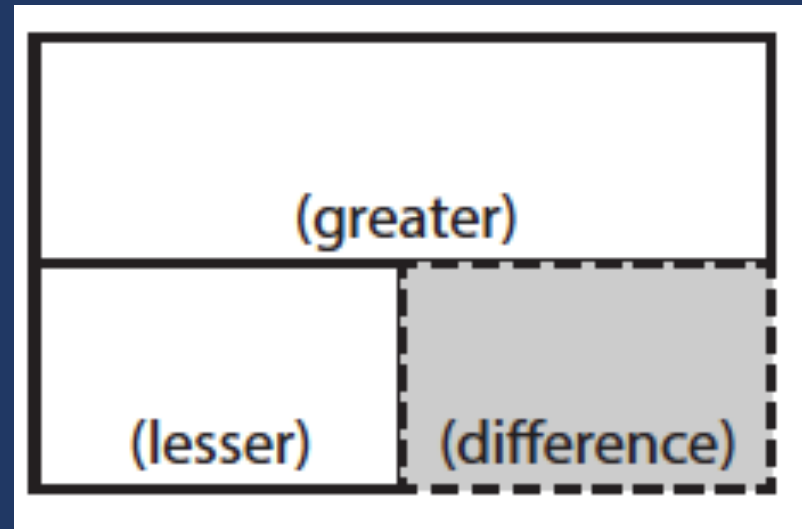
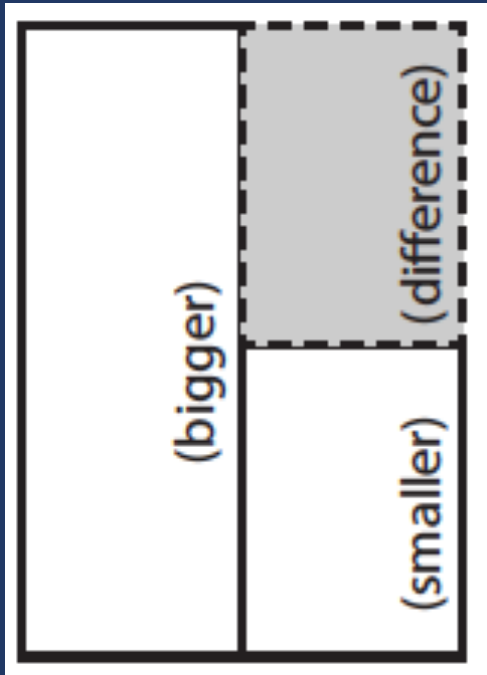
Difference

“Are amounts compared for a difference?”



# Difference

$$G - L = D$$



# Difference

## Additive Word Problems

A.  
Ali delivered 12 boxes of cookies on Friday and 25 boxes of cookies on Saturday. How many boxes of cookies did Ali deliver?

B.  
In March and April, it rained a total of 11.4 inches. If it rained 3.9 inches in March, how many inches did it rain in April?

C.  
Audrey has 162 wooden beads and 95 glass beads. What is the difference between Audrey's wooden beads and glass beads?

D.  
Damian's dog eats  $9\frac{1}{2}$  cups of dog food each week. Monte's dog eats  $4\frac{1}{4}$  cups less each week than Damian's dog. How much does Monte's dog eat in a week?



# Difference



Share a Difference problem.



# Change

Join

An amount that **increases** or **decreases**

Nickole had **6** notebooks. Then, she bought **3** notebooks. How many notebooks does Nickole have now?

End amount

Nickole had **6** notebooks. Then, she bought a few more notebooks. Now, Nickole has **9** notebooks. How many notebooks did she buy?

Change amount

Nickole had some notebooks. Then, she bought **3** notebooks. Now, Nickole has **9** notebooks. How many notebooks did she have to start with?

Start amount





# Change

Separate

An amount that increases or **decreases**

Samantha baked **20** cookies. Then, she ate **3** of the cookies. How many cookies does Samantha have now?

End amount

Samantha baked **20** cookies. Then, she ate some of the cookies. Now, she has **17** cookies. How many cookies did Samantha eat?

Change amount

Samantha baked some cookies. She ate **3** of the cookies and has **17** cookies left. How many cookies did Samantha bake?

Start amount



Total

“Are parts put together for a total?”

Difference

“Are amounts compared for a difference?”

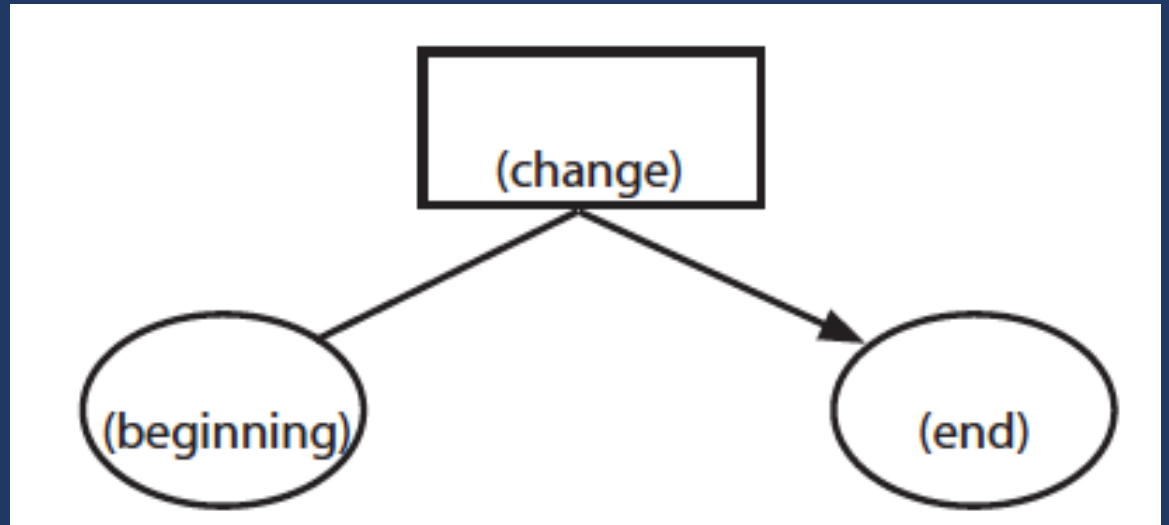
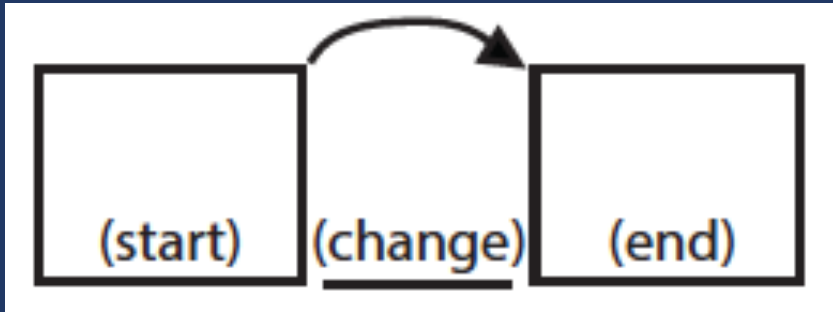
Change

“Does an amount increase or decrease?”



# Change

$$ST \quad + / - \quad C \quad = \quad E$$



# Change

## Additive Word Problems

E.

A plant was  $3\frac{3}{4}$  inches tall at the beginning of June. By the end of July, the plant was  $9\frac{1}{8}$  inches tall. How many inches did the plant grow in 2 months?

F.

Martina has some money in her bank account. Then, she spent \$135.69 and has a balance of -\$24.80. How much money did Martina have to begin with?

G.

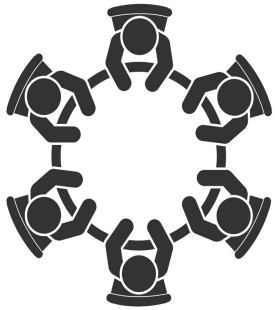
Sam mows lawns and made \$560 last week. She made \$95 on Monday, \$135 on Tuesday, and \$70 on Wednesday. How much did Sam make on Thursday and Friday?

H.

Hui saved \$70 in January. In February, she spent \$64 of the money she saved. She saved \$92 more in March. How much has Hui saved by the end of March?



# Change



Share a Change problem.





# Schema Check!



# Change

Pablo goes to a stamp show where he can share, buy, and sell stamps.

## 26. Part A

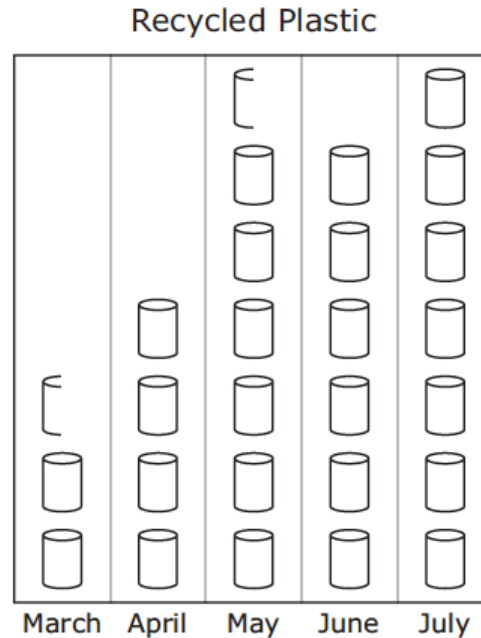
The first day, Pablo starts with 744 stamps. He buys 27 stamps from his friend. He then sells 139 stamps.


What is the total number of stamps that Pablo has after the first day of the stamp show?



# Difference

The graph below shows the number of pounds of plastic the Keller family recycled for five months.



Each  means 20 pounds.

Based on the graph, how many more pounds of plastic did the family recycle in July than in April?



# Total

Mr. Conley delivers packages. The bar graph shows the total number of packages he delivered on five days last week.



## 10. Part A

What is the total number of packages Mr. Conley delivered on Monday and Tuesday?

- Ⓐ 300
- Ⓑ 340
- Ⓒ 350
- Ⓓ 360

## Additive Word Problems

E.

A plant was  $3 \frac{3}{4}$  inches tall at the beginning of June. By the end of July, the plant was  $9 \frac{1}{8}$  inches tall. How many inches did the plant grow in 2 months?

F.

Martina has some money in her bank account. Then, she spent \$135.69 and has a balance of -\$24.80. How much money did Martina have to begin with?


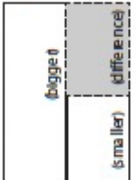
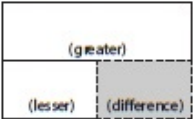

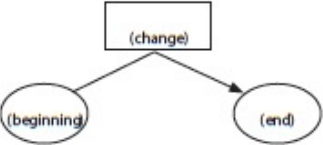
G.

Sam mows lawns and made \$560 last week. She made \$95 on Monday, \$135 on Tuesday, and \$70 on Wednesday. How much did Sam make on Thursday and Friday?

H.

Hui saved \$70 in January. In February, she spent \$64 of the money she saved. She saved \$92 more in March. How much has Hui saved by the end of March?



Schema and Definition	Equations and Graphic Organizers	Examples	Variations
<p>Total (Combine; Part-part-whole) Parts combined for a sum</p>	<p><math>P1 + P2 = T</math> (part + part = total)</p> 	<p>Sum unknown: Lyle has 11 red apples and 18 green apples. How many apples does Lyle have altogether?</p> <p>Part unknown: Lyle has 29 red and green apples. If 11 of the apples are red, how many green apples does Lyle have?</p>	<p>More than two parts: Lyle has 34 apples. Of the apples, 11 are red, 18 are green, and the rest are yellow. How many yellow apples does Lyle have?</p>
<p>Difference (Compare) Sets compared for a difference</p>	<p><math>B - s = D</math> (bigger - smaller = difference)</p>  <p><math>G - L = D</math> (greater - less = difference)</p> 	<p>Difference unknown: Sasha wrote 85 words in her essay, and Tabitha wrote 110 words. How many fewer words did Sasha write than Tabitha?</p> <p>Bigger/greater unknown: Tabitha wrote 25 more words than Sasha. If Sasha wrote 85 words, how many words did Tabitha write?</p> <p>Smaller/lesser unknown: Tabitha wrote 110 words in her essay. Sasha wrote 25 words fewer than Tabitha. How many words did Sasha write?</p>	<p>(None)</p>
<p>Change (Join; Separate) An amount that increases or decreases</p>	<p><math>ST \pm C = E</math> (start +/- change = end)</p>  	<p>End (increase) unknown: Jorge had \$52. Then, he earned \$16 babysitting. How much money does Jorge have now?</p> <p>End (decrease) unknown: Jorge had \$52. Then, he spent \$29 at the ballpark. How much money does Jorge have now?</p> <p>Change (increase) unknown: Jorge had \$52. Then, he earned some money babysitting. Now, Jorge has \$68. How much did Jorge earn babysitting?</p> <p>Change (decrease) unknown: Jorge had \$52 but spent some money when he went to the ballpark. Now, Jorge has \$23. How much did Jorge spend at the ballpark?</p> <p>Start (increase) unknown: Jorge has some money, and then he earned \$16 for babysitting. Now, Jorge has \$68. How much money did he have to start with?</p> <p>Start (decrease) unknown: Jorge had some money. Then, he spent \$29 at the ballpark and has \$23 left. How much money did Jorge have before going to the ballpark?</p>	<p>Multiple changes: Jorge had \$78. He stopped and bought a pair of shoes for \$42 and then he spent \$12 at the grocery. How much money does Jorge have now?</p>



Total

Difference

Change

Equal Groups

Comparison

Ratios/Proportions



Schema and Definition	Graphic Organizers	Examples			Variations		
<p><b>Equal Groups (Vary)</b> A number of equal sets or units</p>		<p><b>Product unknown:</b> Maria bought 5 cartons of eggs with 12 eggs in each carton. How many eggs did Maria buy?</p>	<p><b>Groups unknown:</b> Maria bought 60 eggs. The eggs were sold in cartons with 12 eggs each. How many cartons of eggs did Maria buy?</p>	<p><b>Number unknown:</b> Maria bought 5 cartons of eggs for a total of 60 eggs. How many eggs were in each carton?</p>	<p><b>With rate:</b> Maria bought 5 cartons of eggs. Each carton cost \$2.95. How much did Maria spend on eggs?</p>		
<p><b>Comparison</b> One set as a multiple or part of another set</p>		<p><b>Product unknown:</b> Malik picked 7 flowers. Danica picked 3 times as many flowers. How many flowers did Danica pick?</p>	<p><b>Set unknown:</b> Danica picked 3 times as many flowers as Malik. If Danica picked 21 flowers, how many flowers did Malik pick?</p>	<p><b>Times unknown:</b> Malik picked 7 flowers. Danica picked 21 flowers. How many times more flowers did Danica pick?</p>	<p><b>With fraction:</b> Malik picked 25 red and yellow flowers. If 1/5 of the flowers were yellow, how many were red?</p>		
<p><b>Proportions</b></p>		<p><b>Subject unknown:</b> Sally typed 56 words in 2 minutes. How many words could Sally type in 7 minutes?</p>	<p><b>Object unknown:</b> Sally typed 56 words in 2 minutes. How many minutes would it take Sally to type 192 words?</p>	<p><b>Base unknown:</b> Justin baked cookies and brownies. The ratio of cookies to brownies was 3:5. If he baked 15 cookies, how many brownies did he bake?</p>	<p><b>Compared unknown:</b> Justin baked cookies and brownies. The ratio of cookies to brownies was 3:5. If he baked 25 brownies, how many cookies did he bake?</p>	<p><b>Ratio unknown:</b> Justin baked 15 cookies and 25 brownies. What's the ratio of cookies to brownies?</p>	<p><b>With percentage:</b> Watson received an 80% on his science quiz. If the test had 40 questions, how many questions did Watson answer correctly?</p> <p><b>With unit rate:</b> Paula bought 5 boxes of markers. She spent \$9.75. What is the price of one box of markers?</p>



# Equal Groups

Array  
Vary

**Groups** multiplied by **number in each group** for a **product**

Toni has **2** boxes of crayons. There are **12** crayons in each box. How many crayons does Toni have altogether?

Product

Toni has **24** crayons. They want to place them equally into **2** boxes. How many crayons will Toni place in each box?

Number in each group

Toni has **24** crayons. They put them into boxes with **12** crayons each. How many boxes did Toni use?

Groups



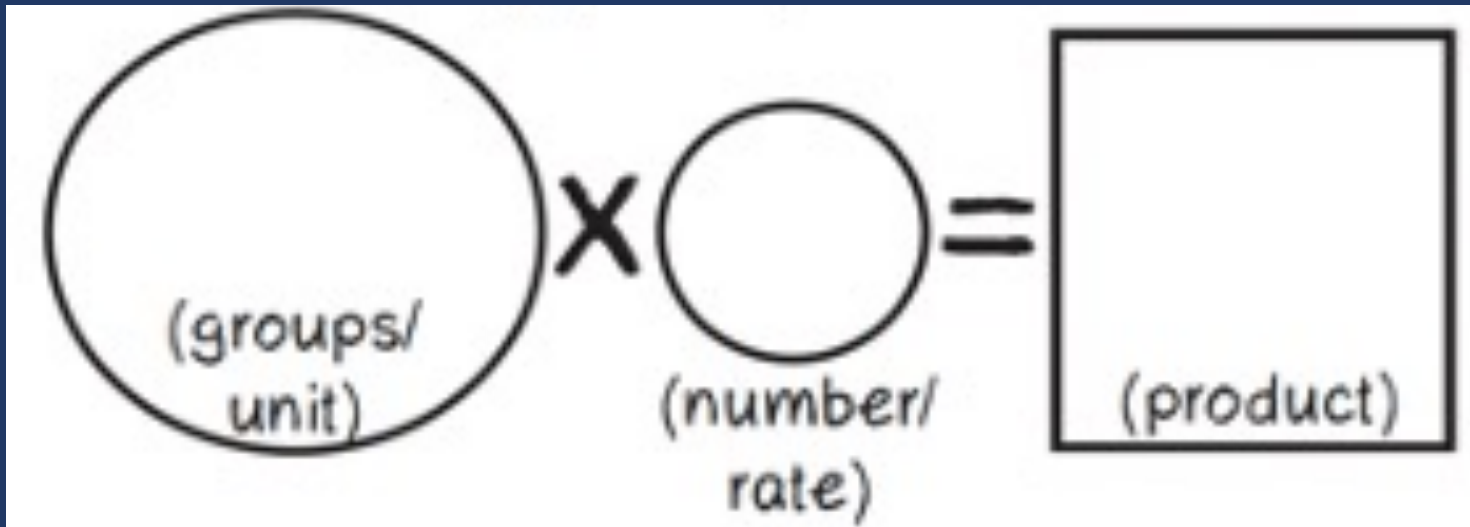
# Equal Groups

“Are there groups with an equal number in each group?”



# Equal Groups

$$GR \times N(E) = P$$





# Equal Groups

## Multiplicative Word Problems

A.  
Lola baked 6 pies. For each pie, Lola used 5 apples. How many apples did Lola use?

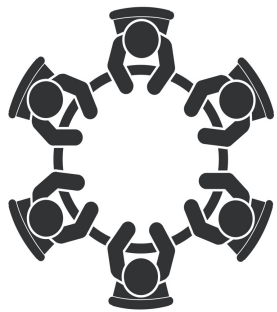
B.  
Zachary has 3 feet of string. He makes bracelets, and each bracelet needs  $5 \frac{1}{4}$  inches of string. How many bracelets could Zachary make?

C.  
Enrique has 2 times as many pencils as Ava. Ava has 6 pencils. How many pencils does Enrique have?

D.  
Susan has 7 times as many books as Mo. Mo has 18 books. How many books Susan has?



# Equal Groups



Share an Equal Groups problem.



# Comparison

**Set** multiplied by a number of **times**  
for a **product**

Brooke ran **6** minutes. Shaleeni ran **4**  
times longer than Brooke. How many  
minutes did Shaleeni run?

Set

Number of  
times

Product



## Equal Groups

“Are there groups with an equal number in each group?”

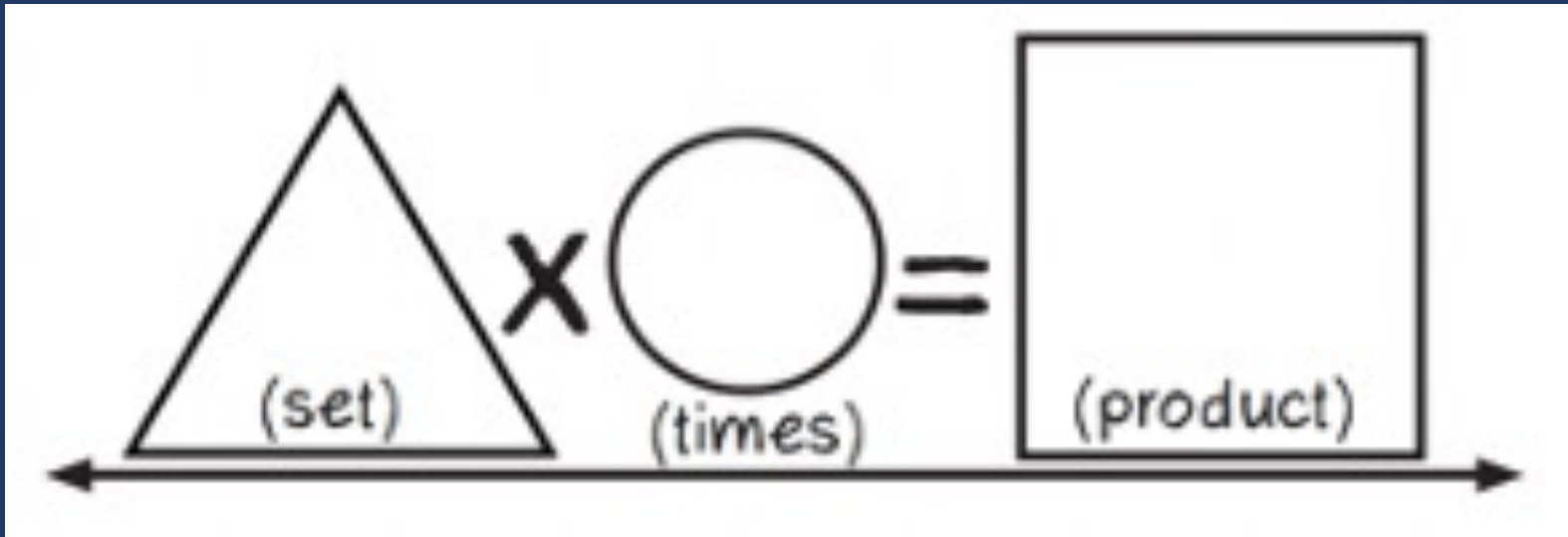
## Comparison

“Is a set compared a number of times?”



# Comparison

$$S \times T = P$$



# Comparison

## Multiplicative Word Problems

A.  
Lola baked 6 pies. For each pie, Lola used 5 apples. How many apples did Lola use?

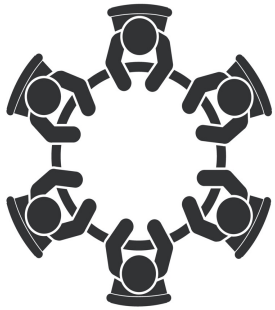
B.  
Zachary has 3 feet of string. He makes bracelets, and each bracelet needs  $5 \frac{1}{4}$  inches of string. How many bracelets could Zachary make?

C.  
Enrique has 2 times as many pencils as Ava. Ava has 6 pencils. How many pencils does Enrique have?

D.  
Susan has 7 times as many books as Mo. Mo has 18 books. How many books Susan has?



# Comparison



Share a Comparison problem.



# Ratios/Proportions

Description of **relationships** among quantities

Melissa baked cookies and brownies. The ratio of cookies to brownies was **3:5**. If she baked **25** brownies, how many cookies did she bake?

Emma typed **56** words in **2** minutes. At this rate, how many words could Emma type in **7** minutes?





## Equal Groups

“Are there groups with an equal number in each group?”

## Comparison

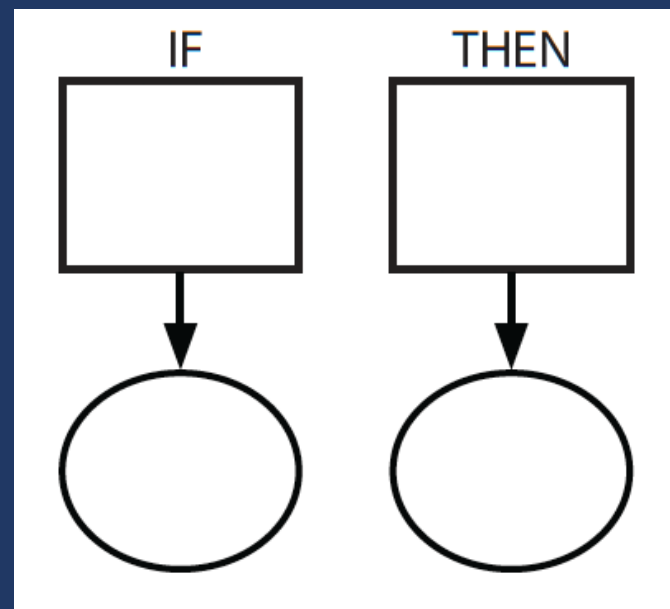
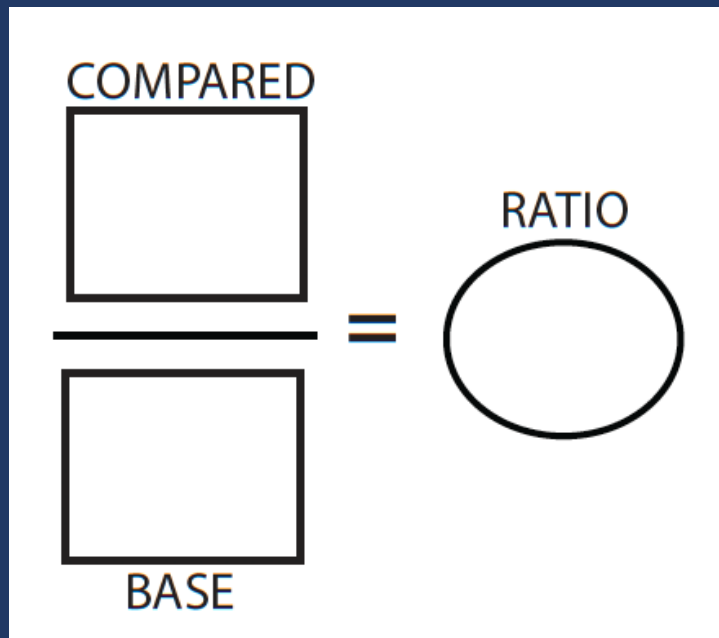
“Is a set compared a number of times?”

## Ratios/Proportions

“Are there relationships among quantities - if this, then this?”



# Ratios/Proportions





# Schema Check!



# Equal Groups

Mr. Kowolski ordered 35 boxes of granola bars. Each box contained 24 granola bars.

What is the total number of granola bars Mr. Kowolski ordered?

# Ratios/Proportions

A company makes 625 cell phone cases each day. How many cell phone cases does the company make in 31 days?



# Comparison

Danielle's full-grown dog weighs 10 times as much as her puppy. The puppy weighs 9 pounds.

Enter the number of pounds the full-grown dog weighs.

Schema and Definition	Graphic Organizers	Examples			Variations			
<b>Equal Groups (Vary)</b> A number of equal sets or units		<b>Product unknown:</b> Maria bought 5 cartons of eggs with 12 eggs in each carton. How many eggs did Maria buy?	<b>Groups unknown:</b> Maria bought 60 eggs. The eggs were sold in cartons with 12 eggs each. How many cartons of eggs did Maria buy?	<b>Number unknown:</b> Maria bought 5 cartons of eggs for a total of 60 eggs. How many eggs were in each carton?	<b>With rate:</b> Maria bought 5 cartons of eggs. Each carton cost \$2.95. How much did Maria spend on eggs?			
<b>Comparison</b> One set as a multiple or part of another set		<b>Product unknown:</b> Malik picked 7 flowers. Danica picked 3 times as many flowers. How many flowers did Danica pick?	<b>Set unknown:</b> Danica picked 3 times as many flowers as Malik. If Danica picked 21 flowers, how many flowers did Malik pick?	<b>Times unknown:</b> Malik picked 7 flowers. Danica picked 21 flowers. How many times more flowers did Danica pick?	<b>With fraction:</b> Malik picked 25 red and yellow flowers. If 1/5 of the flowers were yellow, how many were red?			
<b>Proportions</b>		<b>Subject unknown:</b> Sally typed 56 words in 2 minutes. How many words could Sally type in 7 minutes?	<b>Object unknown:</b> Sally typed 56 words in 2 minutes. How many minutes would it take Sally to type 192 words?	<b>Base unknown:</b> Justin baked cookies and brownies. The ratio of cookies to brownies was 3:5. If he baked 15 cookies, how many brownies did he bake?	<b>Compared unknown:</b> Justin baked cookies and brownies. The ratio of cookies to brownies was 3:5. If he baked 25 brownies, how many cookies did he bake?	<b>Ratio unknown:</b> Justin baked 15 cookies and 25 brownies. What's the ratio of cookies to brownies?	<b>With percentage:</b> Watson received an 80% on his science quiz. If the test had 40 questions, how many questions did Watson answer correctly?	<b>With unit rate:</b> Paula bought 5 boxes of markers. She spent \$9.75. What is the price of one box of markers?



Total

Difference

Change

Equal Groups

Comparison

Ratios/Proportions





Teach an attack strategy

Teach about schemas





# Pirate Math Equation Quest

About

Research

Individual

Small Group

STAAR

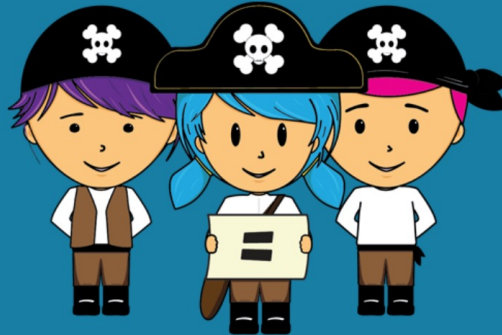
Videos

## Welcome to Pirate Math Equation Quest!

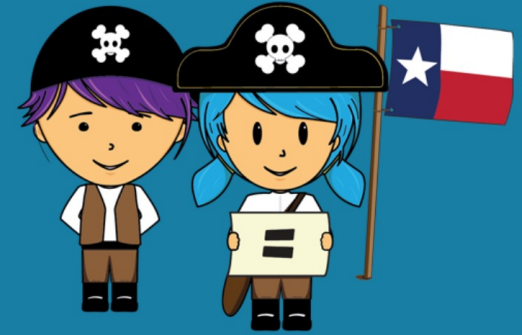
### Individual Word-Problem Intervention



### Small-Group Word-Problem Intervention



### Small-Group Word-Problem Intervention for STAAR



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## Intensive Intervention in Mathematics Course Content

NCII, through a collaboration with the University of Connecticut, developed a set of course content focused on developing educators' skills in designing and delivering intensive mathematics instruction. This content is designed to support faculty and professional development providers with instructing pre-service and in-service educators who are developing and/or refining their implementation of intensive mathematics intervention.

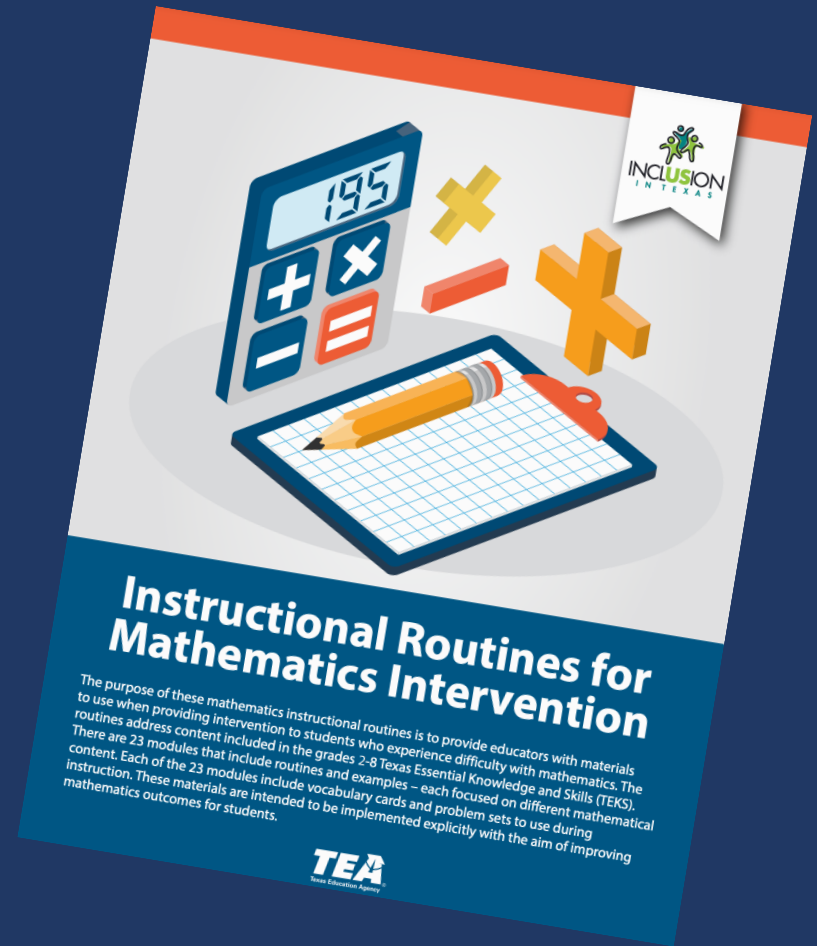
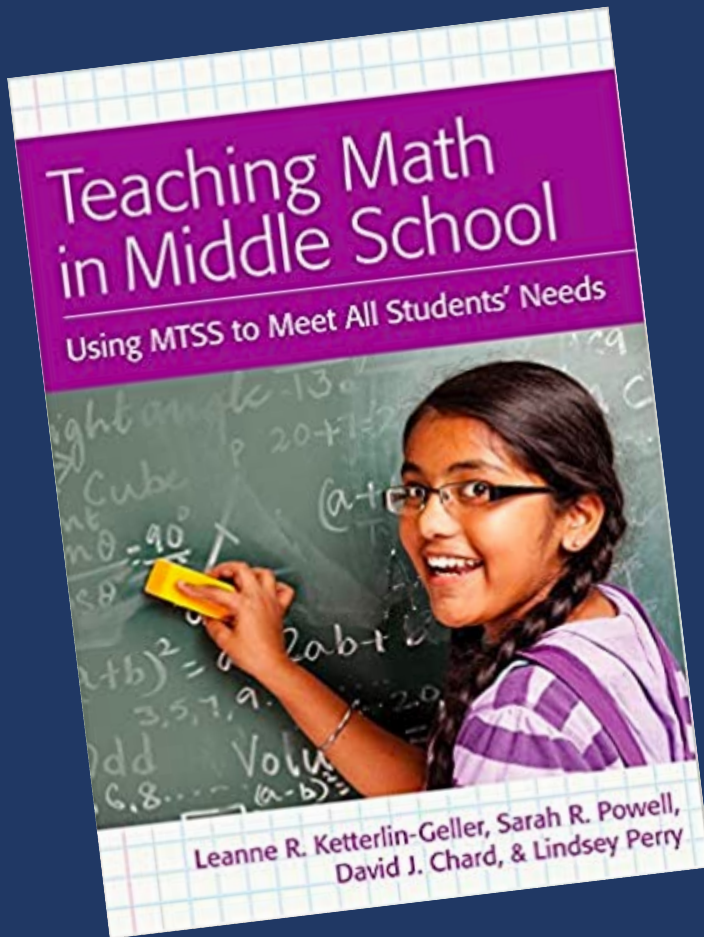
Intensive instruction was recently identified as a [high-leverage practice in special education](#), and DBI is a research based approach to delivering intensive instruction across content areas (NCII, 2013). This course provides learners with an opportunity to extend their understanding of intensive instruction through in-depth exposure to DBI in mathematics, complete with exemplars from actual classroom teachers.

NCII, through a collaboration with the University of Connecticut and the [National Center on Leadership in Intensive Intervention](#) and with support from the [CEEDAR Center](#), developed course content focused on enhancing educators' skills in intensive mathematics intervention. The course includes eight modules that can support faculty and professional development providers with instructing pre-service and in-service educators who are learning to implement intensive mathematics intervention through data-based individualization (DBI). The content in this course complements concepts covered in the [Features of Explicit Instruction Course](#) and so we suggest that users complete both courses.



### MODULE 5: INTENSIVE MATHEMATICS INTERVENTION: INSTRUCTIONAL STRATEGIES





# Sarah R. Powell, Ph.D.

Associate Professor  
The University of Texas at Austin



[srpowell@utexas.edu](mailto:srpowell@utexas.edu)



[@sarahpowellphd](https://twitter.com/sarahpowellphd)

