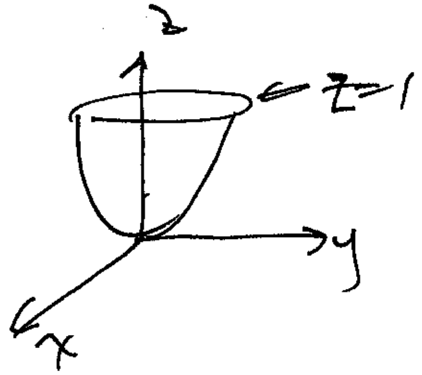
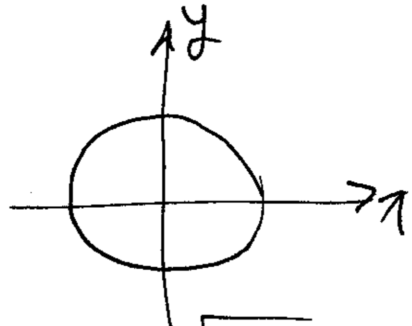
Math 2471 — Calc 3

Last Class





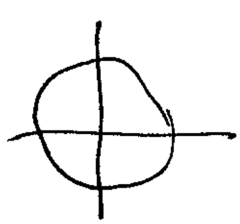
$$\iint_R f(x,y)\,dA = \int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \{1-(x^2+y^2)\}\,dy\,dx$$

Maybe we can do this an easier way!

Recall from Calc 2 polar coords

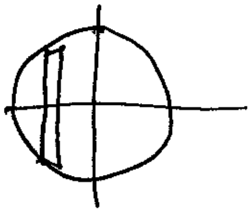
$$x = r\cos\theta \qquad\qquad x^2+y^2=r^2$$
$$\text{or}$$
$$y = r\sin\theta \qquad\qquad \tan\theta = y/x$$

When we integrate over a circle (this is R)

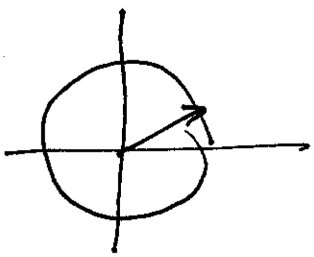


bottom curve → top curve

left pt → right pt

if  the rectangle "sweeps" out the region

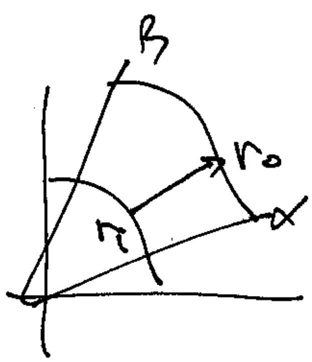
so with polar coords



$r: 0 \to 1$ then $\theta: 0 \to 2\pi$

this will also sweep out the region

so for double integral

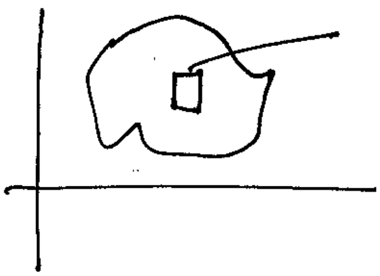


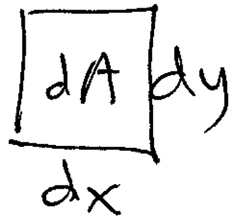
r_i - inner curve

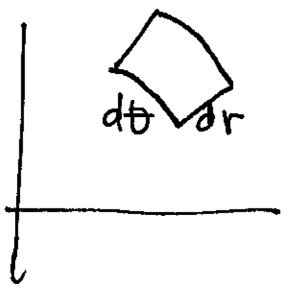
r_o - outer curve

$$\int_{\alpha}^{\beta} \int_{r_i}^{r_o} f(\cancel{x}r\cos\theta, r\sin\theta)\, dA$$

but what is dA







$d\theta$ – a change in angle

for the length $s = r\theta$

So $ds = r\,d\theta$

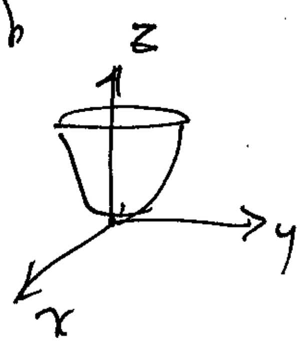
So

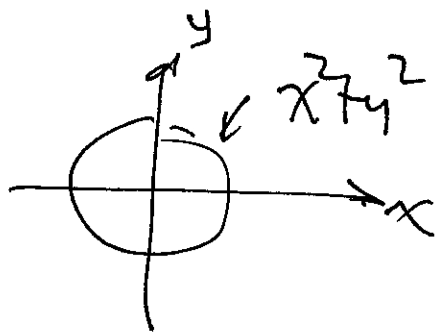
$$dA = dr\,ds = dr \cdot r\,d\theta$$
$$= r\,dr\,d\theta$$

important

So
$$\int_{\alpha}^{\beta} \int_{r_i}^{r_o} f(r\cos\theta, r\sin\theta)\, r\,dr\,d\theta$$

Previous prob





$$V = \int_{\theta=0}^{2\pi}\int_{r=0}^{1}(1-r^2)\,r\,dr\,d\theta$$

$r - r^3$

$$= \int_0^{2\pi} \frac{r^2}{2} - \frac{r^4}{4}\Big|_0^1 \, d\theta = \int_0^{2\pi} \frac{1}{4}\, d\theta = \frac{\theta}{4}\Big|_0^{2\pi} = \frac{2\pi}{4} = \pi/2$$

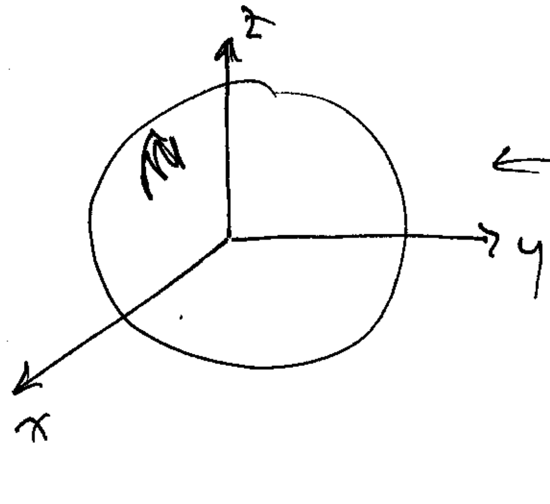
A much easier calculation!

Ex 2 Find the volume of the sphere

$x^2+y^2+z^2 = 9$

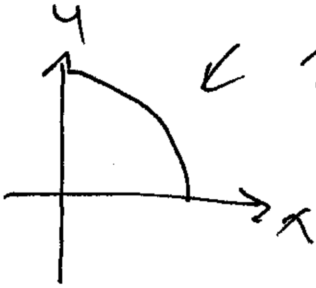
$V = \frac{4}{3}\pi r^3$ Here $r=3$ so $V = \frac{4}{3}\pi \cdot 3^3$

$= 4\pi \cdot 9 = 36\pi$



← due to the symmetry we calculate in 1st octant & mult by 8

$$V = 8\iint_R \sqrt{9-x^2-y^2}\,dA$$



$\leftarrow$ $x^2+y^2=9$

$r=3$

$$8\int_0^{\pi/2}\int_{r=0}^{3}\sqrt{9-r^2}\,r\,dr\,d\theta$$

$u = 9-r^2$

$du = -2r\,dr$

$$\int_{r=0}^{3}\sqrt{9-r^2}\,r\,dr = \int_{u=9}^{0}\sqrt{u}\left(-\frac{du}{2}\right)$$

$r=0 \quad u=9$

$r=3 \quad u=0$

$$= \frac{1}{2}\int_0^9 \sqrt{u}\,du = \frac{1}{\cancel{2}}\,\frac{\cancel{2}u^{3/2}}{3}\Big|_0^9 = \frac{9^{3/2}}{3} = 9$$

So

$$8\int_0^{\pi/2} 9\,d\theta = 8\cdot 9\,\theta\Big|_0^{\pi/2} = 8\cdot 9\cdot \pi/2 = 4\cdot 9\pi = 36\pi$$