Math 4315 PDE's

Solve

$$u_x - u_y = u \qquad u(x,0) = x e^x$$

~~Note~~ CE

$$x_s = 1 \Rightarrow x = s + a(r)$$

$$y_s = -1 \qquad y = -s + b(r)$$

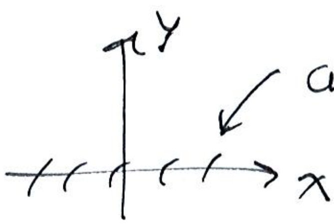
$$u_s = u \qquad u = c(r) e^s$$

so $x + y = a + b = A(r)$

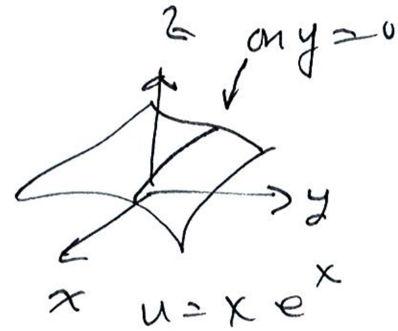
$$u = c(r) e^{x - a(r)} = c(r) e^{-a(r)} e^x = e^x B(r)$$

Sol^n: $u = e^x f(x+y)$

Now B.C.



on $y = 0$ $u = x e^x$



$$u(x,0) = e^x f(x) = x e^x$$

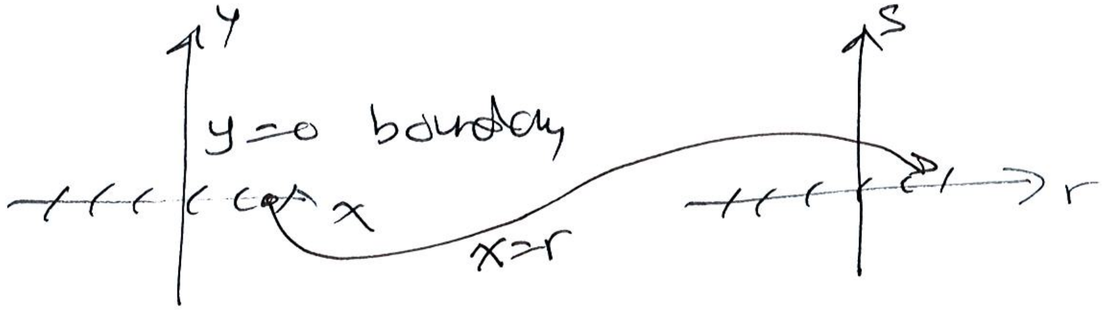
$$\Rightarrow \quad f(x) = x \quad \text{so} \quad f(x+y) = x + y$$

$$\text{so} \quad \boxed{u(x,y) = e^x (x+y)}$$

Alternatively

As we transform from $(x,y) \to (r,s)$

BS



we choose $s=0$ as new boundary and connect the 2 via $x=r$

so B.C $u(x,0) = xe^{x}$

$\Rightarrow$ when $s=0$ $x=r$, $y=0$, $u=re^{r}$

Now we solve

$x_s = 1$
$y_s = 1$
$u_s = u$

subject to

$s=0$
$x=r$
$y=0$
$u=re^{r}$

$x_s = 1 \Rightarrow x = s + a(r)$ $\quad s=0$ $x=r \Rightarrow a(r)=r$

so $\boxed{x = s + r}$

$y_s = 1 \Rightarrow y = s + b(r)$ $\quad s=0, y=0 \Rightarrow b(r)=0$

$\boxed{y = s}$

$u_s = u \Rightarrow u = c(r)e^{s}$

$s=0 \quad u = re^{r}$ so $re^{r} = c(r)$

so $\quad u = re^{r}.e^{s}$

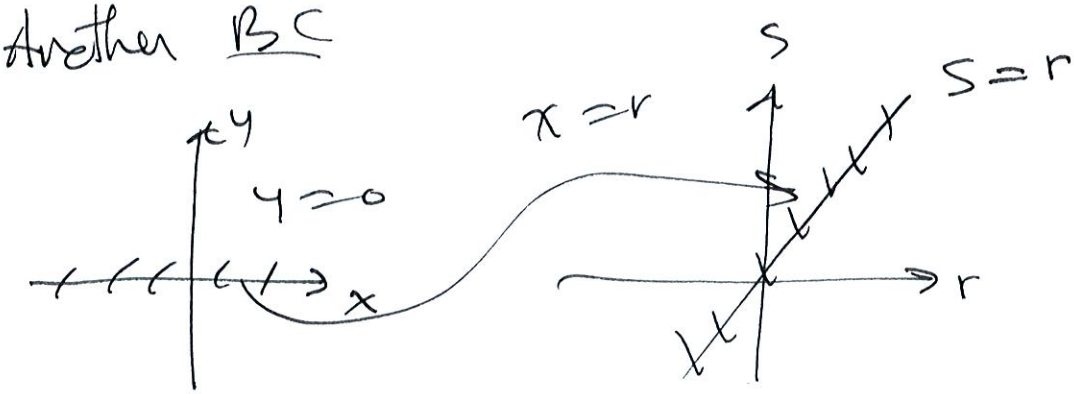
Now the sol^n is given parametrically

$x = s+r, \quad y=-s, \quad u = re^{r+s}$

$r = x+y$

$u = (x+y)e^{x}$ (same sol^n)

Another BC



instead on $s=r$

$x=r, \; y=0, \; u=re^{r}$

Solve when at $s=0$

$x_s = 1 \quad x = r$

$y_s = -1 \quad y = 0$

$u_s = u \quad u = re^{r}$

$s=r \quad x=r$

$x_s = 1 \Rightarrow x = s + a(r) \qquad r = r + a(r) \Rightarrow a(r) = 0$

$\boxed{x = s}$

$y_s = -1 \quad y = -s + b(r) \qquad s = r \quad y = 0 \Rightarrow 0 = -r + b(r)$

$b(r) = r$

$\boxed{y = -s + r}$

$u_s = u \Rightarrow u = c(r)\, e^{s} \qquad u = re^{r} \quad s = r$

so $re^{r} = c(r)\, e^{r} \Rightarrow c(r) = r$

$\boxed{u = re^{s}}$

$r = y + s = x + y$

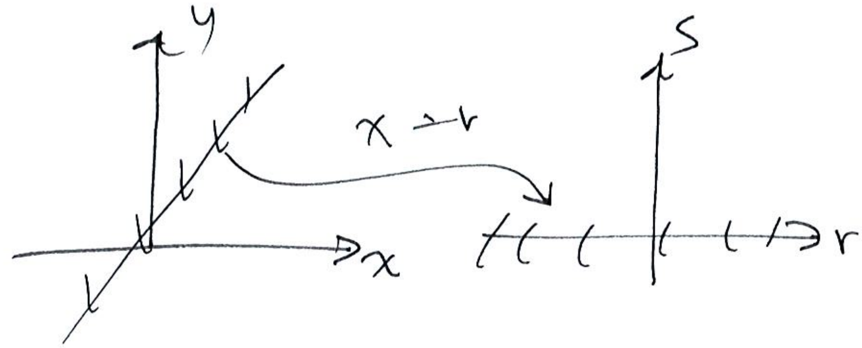
so $u = (x+y)e^{x}$ same sol^n

Ex2 $\quad xu_x - yu_y = x \qquad u(x,x) = x + x^4$

$x_s = x$

$y_s = -y$

$u_s = x$



$x_s = x \quad x = a(r)e^{s}$

$y_s = -y \quad y = b(r)e^{-s}$

$u_s = x = a(r)e^{s}$

$u = a(r)e^{s} + c(r)$

$xy = a(r)\,b(r) = A(r)$

Sol^n

$u = x + f(xy)$

BC

$u = x + x^4$ when $y = x$

$x + x^4 = x + f(x^2)$

$f(x^2) = x^4 \qquad f(\lambda) = \lambda^2$

$u = x + (xy)^2$

$s = 0$

$x = r,\ y = r,\ u = r + r^4$

$x_s = x \quad x = a(r)e^{s}$

$s = 0 \quad x = r \quad a(r) = r \qquad \boxed{x = re^{s}}$

$y_s = -y \Rightarrow y = b(r)e^{-s}$

$s = 0 \quad y = r \quad b(r) = r \qquad \boxed{y = re^{-s}}$

$u_s = x = re^{s} \qquad s = 0$

$u = re^{s} + c(r) \qquad u = r + r^4 \Rightarrow c = r^4$

$\boxed{u = re^{s} + r^4}$

$xy = r^2 \qquad (xy)^2 = r^4$

$u = x + (xy)^2$

same