The Mahgoub Transform of Derived Function Demonstrated by Heaviside Function

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Abstract-We have demonstrated that the Mahgoub transform of derived function can be demonstrated by an infinite arrangement or Heaviside function. Identified with this theme, the advanced thoughtcan be likewise connected to additional changes.

Keywords-Heaviside function, Mahgoub transform.

I. INTRODUCTION

Integral transform methods have been investigated to deal with various issues in the differential equations with starting or limit conditions. Laplace, Sumudu Kamal ,and Elzaki transforms are such run of the mill things [5]. Among these, the Mahgoub transform strategy is an astonishing and basic instrument, and this expect a vocation to handle explicitly starting worth issues unaccompanied by first choosing a arrangement .The Mahgoub transform of derived function have been looked into from multiple points of view to solve differential equations. The foremost points are[1]

$$ꝳ[ƛ'(\mathcal{T})] \quad = Þ\mathfrak{f}(Þ) - Þ\, ƛ(0)$$

$$ꝳ[\, ƛ''(\mathcal{T})] \;= Þ^2\mathfrak{f}(Þ) \;-\; Þ^2\, ƛ(0) - \; Þƛ'(0)$$

In this article, we might want to propose the new methodology of $ꝳ[ƛ'(\mathcal{T})]$ by modifying the decision capacity of differential structure in mix by segments. The got outcome is $ꝳ[ƛ'(\mathcal{T})]$can be spoken to by an infinite arrangement or Heaviside work.

2.Fundamental records-On the off chance that $ƛ(\mathcal{T})$ is a capacity defined for all $\mathcal{T} \geq 0$, its Mahgoub transform of $ƛ(\mathcal{T})$ set up with $e^{-Þ\mathcal{T}}$, $0\ to\ \infty$. It is an element of Þ, and is defined by $\mathfrak{f}(Þ)$; consequently

$$ꝳ[ƛ(\mathcal{T})] \;=\; \mathfrak{f}(Þ) = \; Þ\int_0^\infty ƛ(\mathcal{T})\, e^{-Þ\mathcal{T}} d\mathcal{T}\, , \qquad \mathcal{T} \geq 0.$$

given the basic of $ƛ(\mathcal{T})$ exists. In the above condition, if the kernel be changed to$e^{-s\mathcal{T}} / \frac{1}{u} e^{\frac{-\mathcal{T}}{u}} \,/\, u e^{\frac{-\mathcal{T}}{u}}$, we call Laplace/Sumudu/Elzaki transform, individually.

Remark : The peruse can be perused progressively about the Mahgoub transform in [1-4].

3 Main Results

We should need to suggest $ꝳ[ƛ']$ can be spoken as an limitless arrangement of $Þ^k$ by evolving the possibility of capacity of differential structure in coordination by segment, and allot with the shown by Heaviside function of it

.**Th. 3.1** The Mahgoub transform of the first derived basis of $ƛ(\mathcal{T})$ gratify

$$ꝳ\{ƛ'\} = \sum_{k=1}^{m^*} ƛ^k\,(0) + ꝳ\left(ƛ^{(m^*+1)}\right)$$

for $ƛ^k$ is the k -th derivative of a presented with basis $ƛ(\mathcal{T})$. As $n \to \infty$,

$$ꝳ\{ƛ'\} = \sum_{k=1}^{n} ƛ^k\,(0).$$

The over head statemet holds if $ƛ(\mathcal{T})$ and $ƛ'(\mathcal{T})$ are consistent for all $\mathcal{T} \geq 0$ and amuse the increase condition.

Proof: The announcement by the mathematical prelude. For $k = 1$, by integration by segment,

$$ꝳ\{ƛ'\} = Þ\int_0^\infty e^{-Þ\mathcal{T}}\, ƛ'(\mathcal{T}) d\mathcal{T}$$

$$= Þ\left[\left[-\tfrac{1}{Þ} e^{-Þ\mathcal{T}} ƛ'(\mathcal{T})\right]_0^\infty + \tfrac{1}{Þ}\int_0^\infty e^{-Þ\mathcal{T}}\, ƛ''(\mathcal{T}) d\mathcal{T}\right]$$

$$= ƛ'(0) + ꝳ\{ƛ''\}$$

holds. Next, we guess that

$$ꝳ\{ƛ'\} = \sum_{k=1}^{m^*} ƛ^k (0) + ꝳ\left(ƛ^{(m^*+1)}\right), \qquad (■)$$

$$ꝳ\{ƛ'\} = \sum_{k=1}^{m^*+1} ƛ^k (0) + ꝳ\left(ƛ^{(m^*+2)}\right).$$

In (■),

$$ꝳ\left\{ƛ^{(m^*+1)}\right\} = Þ \int_0^\infty e^{-Þ\mathcal{T}} ƛ^{(m^*+1)}(\mathcal{T})d\mathcal{T}$$

$$= Þ\left[\left[\left[-\frac{1}{Þ}e^{-Þ\mathcal{T}}ƛ^{(m^*+1)}\right]_0^\infty + \frac{1}{Þ}\int_0^\infty e^{-Þ\mathcal{T}} ƛ^{(m^*+2)}(\mathcal{T})d\mathcal{T}\right]\right]$$

$$= ƛ^{(m^*+1)}(0) + ꝳ\left\{ƛ^{m^*+2}\right\}$$

Consequently, from (■),

$$ꝳ\{ƛ'\} = \sum_{k=1}^{m^*} ƛ^{(k)} (0) + \left[ƛ^{(m^*+1)}(0) + ꝳ\left(ƛ^{(m^*+2)}\right)\right]$$

$$= \sum_{k=1}^{m^*+1} ƛ^{(k)} (0) + ꝳ\left(ƛ^{(m^*+2)}\right). \qquad (A)$$

In such a way, if the Eq.(A) influence for$k$, it influence for $k+1$. Therefore, by mathematical prelude, the Eq.(A) is genuine for complete natural figure$n$.

Th. 3.2

$$ꝳ\{ƛ'\} = Þe^{-Þn} ƛ(n) - Þƛ(0) + Þ^2 \int_0^n e^{-Þ\mathcal{T}} ƛ(\mathcal{T})d\mathcal{T} + Þ\, ꝳ[ƛ'(\mathcal{T})\, \mathfrak{u}(\mathcal{T}-n)]$$

for complete n and $\mathfrak{u}$ is the unit step .

Proof. Using mathematical prelude. For $n = 1$,

$$ꝳ\{ƛ'\} = Þ \int_0^\infty e^{-Þ\mathcal{T}} ƛ'(\mathcal{T})d\mathcal{T}$$

$$= Þ\left[\left[e^{-Þ\mathcal{T}}ƛ(\mathcal{T})\right]_0^1 + Þ\int_0^1 e^{-Þ\mathcal{T}}ƛ(\mathcal{T})d\mathcal{T} + \int_1^\infty e^{-Þ\mathcal{T}} ƛ'(\mathcal{T})\, d\mathcal{T}\right]$$

$$= [Þe^{-Þ}ƛ(1) - Þƛ(0)] + Þ^2 \int_0^1 e^{-Þ\mathcal{T}} ƛ(\mathcal{T})d\mathcal{T} + Þ\, ꝳ[ƛ'(\mathcal{T})\mathfrak{u}(\mathcal{T}-1)].$$

Alongside, we accept that the equality influence if$n = k^{**}$ i.e.,

$$ꝳ\{ƛ'\} = \left[Þe^{-Þk^{**}} ƛ(k^{**}) - Þƛ(0)\right] + Þ^2 \int_0^{k^{**}} e^{-Þ\mathcal{T}}ƛ(\mathcal{T})d\mathcal{T} + Þꝳ[ƛ'(\mathcal{T})\mathfrak{u}(\mathcal{T}-k^{**})]. \qquad (■■)$$

Give us a chance to demonstrate that

$$ꝳ\{ƛ'\} = \left[Þe^{-Þ(k^{**}+1)}ƛ(k^{**}+1) - Þƛ(0)\right] + Þ^2 \int_0^{k^{**}+1} e^{-Þ\mathcal{T}}ƛ(\mathcal{T})d\mathcal{T} + Þꝳ[ƛ'(\mathcal{T})\mathfrak{u}(\mathcal{T}-k^{**}-1)].$$

From (■■),

$$ꝳ\{ƛ'\} = \left[Þ\, e^{-Þk^{**}}ƛ(k^{**}) - Þƛ(0)\right] + Þ^2 \int_0^{k^{**}} e^{-Þ\mathcal{T}}ƛ(\mathcal{T})d\mathcal{T} + Þ\int_{k^{**}}^\infty e^{-Þ\mathcal{T}}\, ƛ'(\mathcal{T})\, d\mathcal{T}\, . \qquad (■■■)$$

Here,

$$Þ\int_{k^{**}}^\infty e^{-Þ\mathcal{T}}\, ƛ'(\mathcal{T})d\mathcal{T} = Þ\int_{k^{**}}^{k^{**}+1} e^{-Þ\mathcal{T}}\, ƛ'(\mathcal{T})d\mathcal{T} + Þ\int_{k.^{**}+1}^\infty e^{-Þ\mathcal{T}}\, ƛ'(\mathcal{T})d\mathcal{T}$$

$$= Þ\left[e^{-Þ\mathcal{T}}\, ƛ(\mathcal{T})\right]_{k^{**}}^{k^{**}+1} + Þ^2 \int_{k^{**}}^{k^{**}+1} e^{-Þ\mathcal{T}}\, ƛ(\mathcal{T})d\mathcal{T} + Þꝳ[ƛ'(\mathcal{T})\mathfrak{u}(\mathcal{T}-k^{**}-1)]$$

$$= \left[Þ\, e^{-Þ(k^{**}+1)}\, ƛ(k^{**}+1) - Þe^{-Þk^{**}}ƛ(k^{**})\right] + Þ^2 \int_{k^{**}}^{k^{**}+1} e^{-Þ\mathcal{T}}\, ƛ(\mathcal{T})dt + Þꝳ[ƛ'(\mathcal{T})\mathfrak{u}(\mathcal{T}-k^{**}-1)]. \qquad (■■■■)$$

Substituting(■■■■) to(■■■),

$$\mathfrak{m}\{ \lambda' \} = \left[\text{Þ}e^{-\text{Þ}k^{**}} \lambda(k^{**}) - \text{Þ}\lambda(0) \right] + \text{Þ}^2 \int_0^{k^{**}} e^{-\text{Þ}\mathcal{T}} \lambda(\mathcal{T}) d\mathcal{T}$$
$$+ \text{Þ}e^{-\text{Þ}(k^{**}+1)} \lambda(k^{**}+1) - \text{Þ}e^{-\text{Þ}k^{**}} \lambda(k^{**})$$
$$+ \text{Þ}^2 \int_{k^{**}}^{k^{**}+1} e^{-\text{Þ}\mathcal{T}} \lambda(\mathcal{T}) d\mathcal{T}$$
$$+ \text{Þ}\mathfrak{m}[\lambda'(\mathcal{T})\ \mathfrak{u}(\mathcal{T} - k^{**} - 1)]$$

$$= \left[\text{Þ}e^{-\text{Þ}(k^{**}+1)} \lambda(k^{**}+1) - \text{Þ}\lambda(0) \right]$$
$$+ \text{Þ}^2 \int_0^{k^{**}+1} e^{-\text{Þ}\mathcal{T}} \lambda(\mathcal{T}) d\mathcal{T}$$
$$+ \text{Þ}\mathfrak{m}[\lambda'(\mathcal{T}) \mathfrak{u}(\mathcal{T} - k^{**} - 1)].$$

The legitimacy of the balance for complete natural figure $n$succeeds by mathematical prelude. Obviously Theorem 3.2 are

$$\mathfrak{m}\{ \lambda' \} = \text{Þ}e^{-n\text{Þ}} \lambda(n) - \text{Þ}\lambda(0) + \text{Þ}^2 \int_0^n e^{-\text{Þ}\mathcal{T}} \lambda(\mathcal{T}) d\mathcal{T}$$

for $\mathcal{T} < n$. With the proposed thought, we can apply on other integral transforms.

II. CONCLUSION

In this paper, we introduce Mahgoub transform for derived function demonstrated by Heaviside function. Proposed method is successfully implemented by this interesting transform, we can conclude that Mahgoub transform considered as a nice refinement in existing numerical techniques.

III. REFERENCES

[1]. Mahgoub,M.:The New Integral transform Mahgoub transform. Advances in Theoretical and Applied Mathematics.11(4), 451-458(2016).

[2]. Khandelwal ,Y., Singh ,S.,Khandelwal,R.:Solution of fractional ordinary differential equation by Mahgoub transform.International Journal of Creative Research Thoughts. 6(1),1494-1499(2018).

[3]. Khandelwal ,Y., Umar ,B.A.,Kumawat,P.:Solution of the Blasius equation by using Adomain Mahgoub transform. International Journal of Mathematics Trends and Technology.56(5), 303–306 (2018).

[4]. Khandelwal ,R.,Khandelwal ,Y., Chanchal ,P.,:Mahgoub deterioration method and its application in solving duo-combination of nonlinear PDE's. Mathematical Journal of Interdisciplinary Sciences. 7(1),37–44(2018).https://doi.org/10.15415/mjis.2018.71006

[5]. Cho,I., Kim,H.:The Laplace transform of derivative expressed by Heaviside function.Appl. Mat. Sci.7(90),4455 – 4460(2013).http://dx.doi.org/10.12988/ams.2013.36301