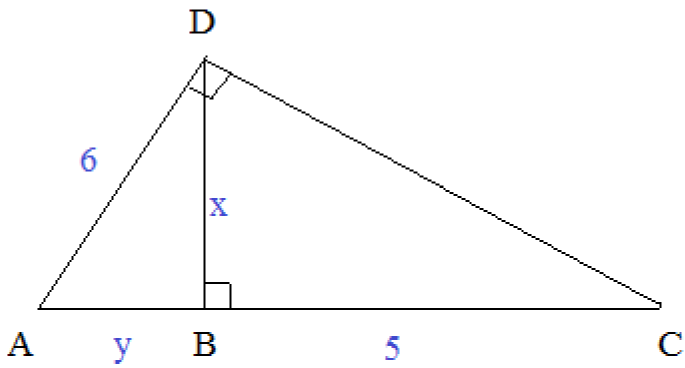
Geometry: Similarity, Ratio, and Proportion Questions

(...and, Solutions)



Includes similarity concepts, algebra, proofs, applications, and more.

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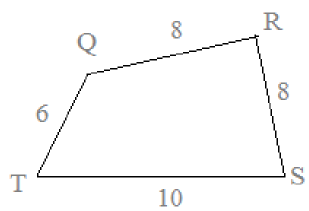
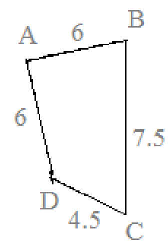
Similarity, Area, and Volume

Two polygons are similar if the corresponding angles are congruent and the corresponding sides are proportional.

Example: Write a similarity statement for the following (similar) quadrilaterals.





The corresponding sides are proportional, and the angles are congruent...

In order, the similarity statement is

QRST ∼ DABC

(Note: QRST ∼ ABCD is not an accurate statement)

All squares are proportional.

All circles are proportional.

All squares and circles have the same shapes.

(All equilateral triangles and regular polygons are proportional)

Perimeter, Area, and Volume ratios

If polygon A is similar to polygon B, and the ratio is a:b or $\frac{a}{b}$

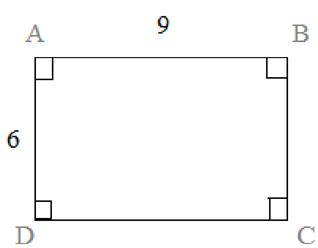
the ratio of the corresponding sides is a : b,

the ratio of the perimeters is a : b

the ratio of the areas is $a^2 : b^2$

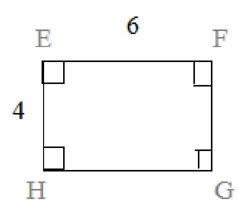
and, the ratio of the volumes is $a^3 : b^3$

Example: □ABCD ∼ □EFGH



Perimeter: 30

Area: 54



Perimeter: 20

Area: 24

All (corresponding) angles are congruent.

Corresponding sides have a ratio of 3 : 2

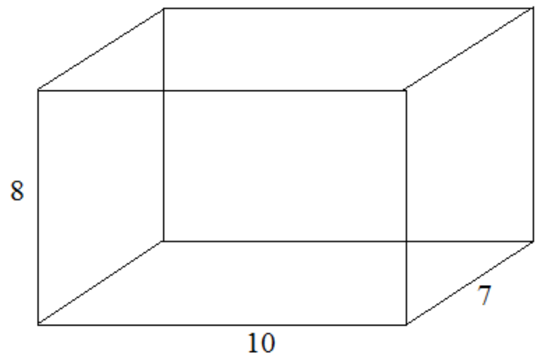
ratio of perimeters = 30 : 24 or

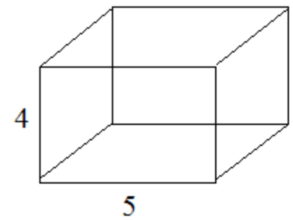
ratio of perimeters is 30 : 20 (or, 3:2)

ratio of areas is 54 : 24 (or, 9:4)

$3^2 : 2^2$

Example: The following prisms are similar. Find the similarity ratio of the *small solid to the large solid*. Then, determine the ratio of the areas and volume.





The similarity ratio will be the ratio of any corresponding sides....

4 : 8 or 5 : 10 -----> the ratio of small to large is 1 : 2 (the ratio of *large to small* is 2 : 1)

The ratio of the (surface) areas is 1 : 4

The ratio of the volumes is 1 : 8

To verify, simply find the areas and volumes!

Example: Find the similarity ratio of 2 (similar) prisms with surface areas 121 square feet and 225 square feet.

Since the ratio of area is the 'similarity squared', we can square root the above areas.

The similarity ratio -- ratio of sides -- of the small to the big prism is 11 : 15

Example: The volume of 2 similar solids is 125 inches^3 and 343 inches^3 .
If the surface area of the larger solid is 250 inches^2 , what is the surface area of the smaller solid?

First, we need to find the similarity ratio.

Ratio of volumes: 125 : 343

Ratio of the sides: $\sqrt[3]{125\ \text{inches}^3} : \sqrt[3]{343\ \text{inches}^3}$

5 inches : 7 inches

Second, find the ratio of the areas...

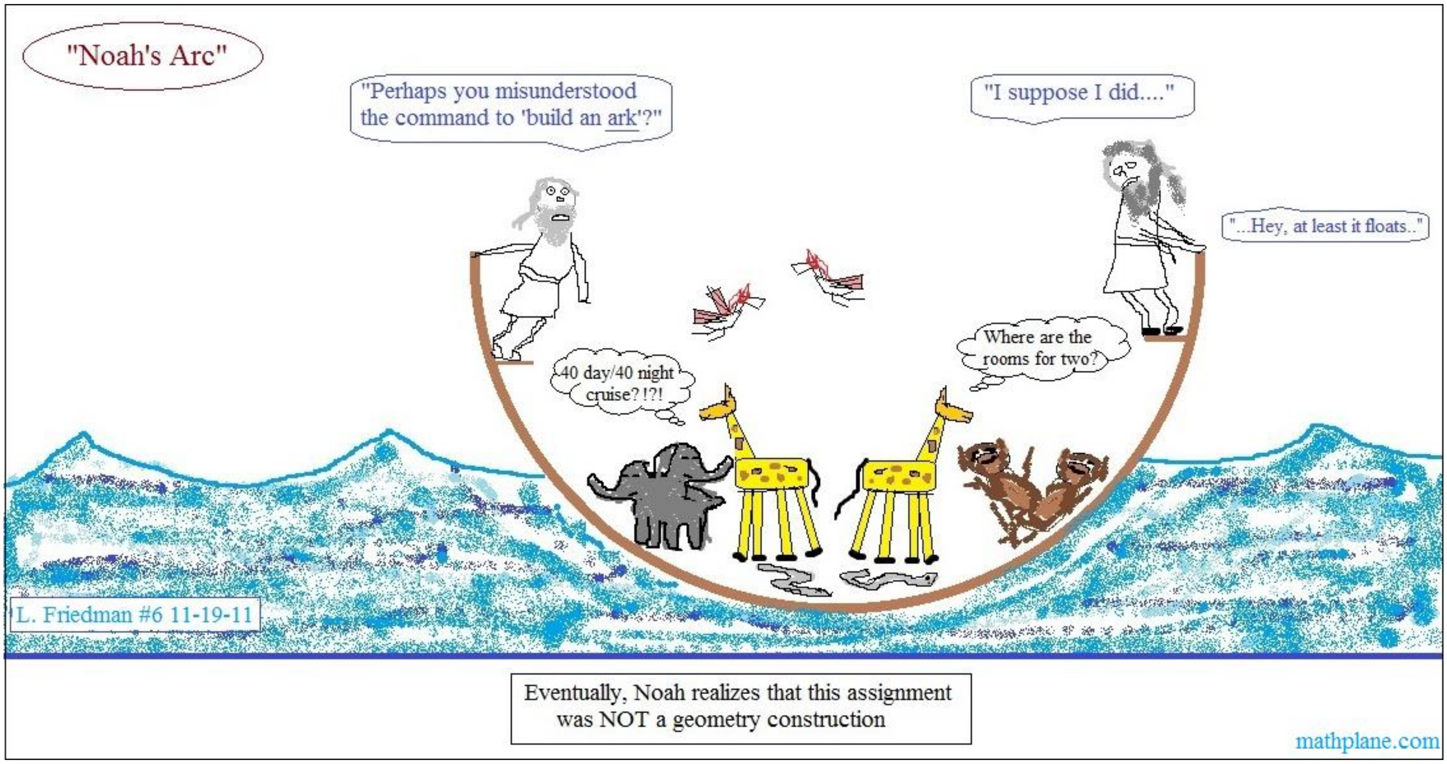
$(5\ \text{inches})^2 : (7\ \text{inches})^2$

$25\ \text{inches}^2 : 49\ \text{inches}^2$

Finally, use the area ratios to determine the actual values...

$$\frac{25\ \text{inches}^2}{49\ \text{inches}^2} = \frac{\text{smaller solid area}}{250\ \text{inches}^2}$$

approximately 127.55 square inches


"Noah's Arc"
"Perhaps you misunderstood the command to 'build an ark'?"
"I suppose I did...."
"...Hey, at least it floats.."
Where are the rooms for two?
40 day/40 night cruise? !?!
L. Friedman #6 11-19-11
Eventually, Noah realizes that this assignment was NOT a geometry construction
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QUESTIONS -→

Geometry: Similarity, Ratio, and Proportion

1) To estimate the height of a tree, Joe stands in the shadow of the tree so that their shadows end at the same point. Joe is 6'3" tall and his shadow is 16 feet. If he is 64 feet from the tree, what is the height of the tree?

2) Find the ratio of x to y:

$$2x = 3y$$

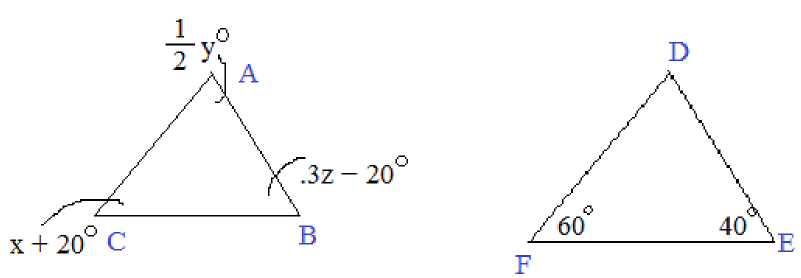
3) Each side of square A is 6 cm.
Each side of square B is 9 cm.

What is the ratio of the *perimeters* of A to B?

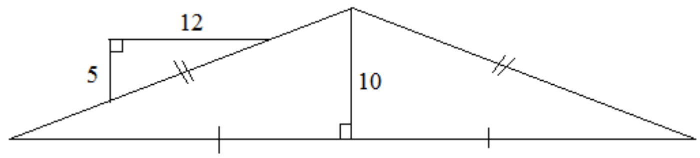
What is the ratio of the *areas* of the 'squares'?

4) $\triangle ABC \sim \triangle DEF$

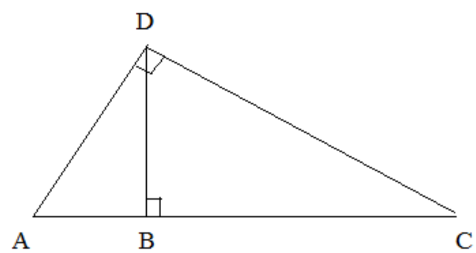
What is $\frac{x + y + z}{2}$?



5) The roof of a house has a slope 5/12.
Based on the diagram, what is the length of the house?



6) Find the length $\overline{DB}$ and $\overline{AB}$



$\overline{DB} \perp \overline{AC}$

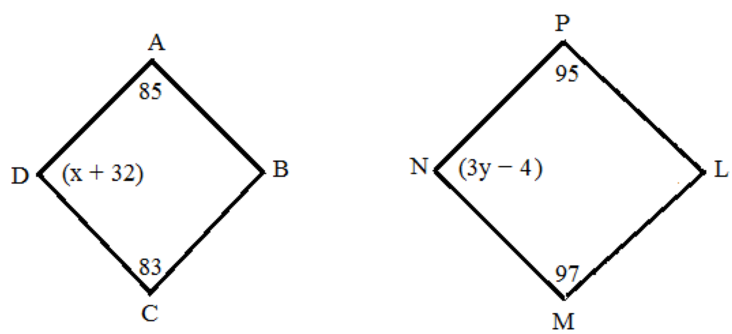
$\overline{AD} \perp \overline{CD}$

$\overline{BC} = 5$

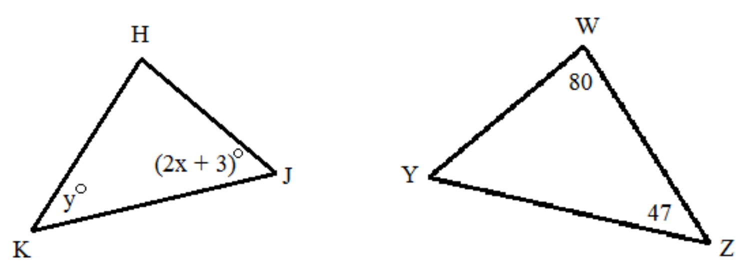
$\overline{AD} = 6$

7) Find x and y:

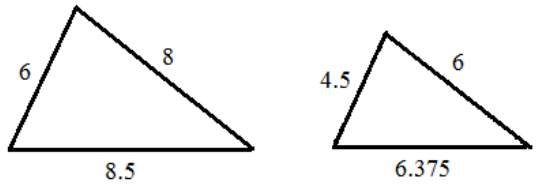
a) ABCD ~ LMNP

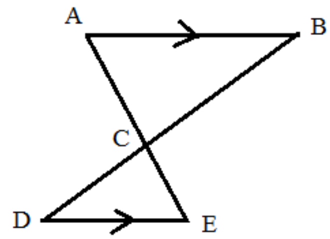


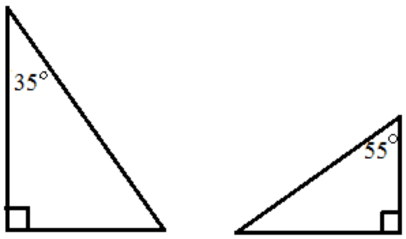
b) △HJK ~ △WYZ

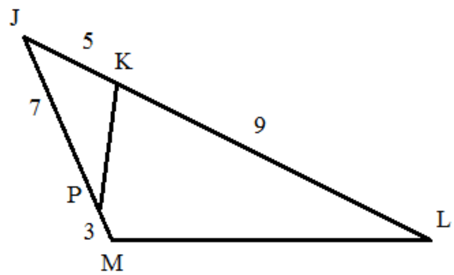


8) Determine if the following triangles are similar.
(Justify your answer)









9) $\triangle CDE \sim \triangle MNP$

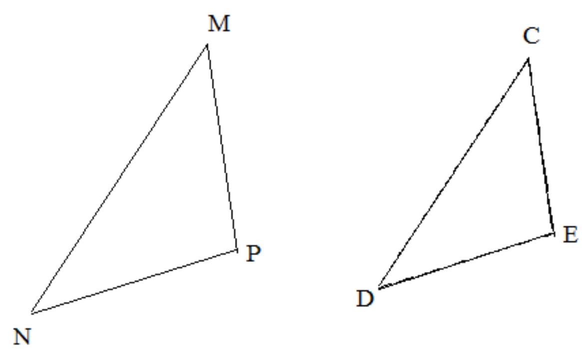
$\angle N = 40$

$\angle D = 3x + 5y$

$\angle P = 106 - x$

$\angle C = 4x + 2y$

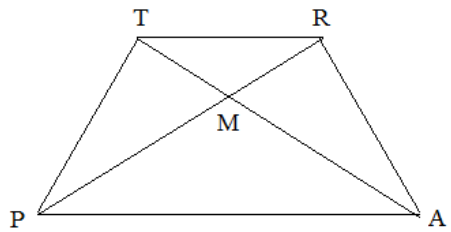
What is x and y?



PROOFS:

10) Given: TRAP is trapezoid with bases $\overline{TR}$ and $\overline{PA}$

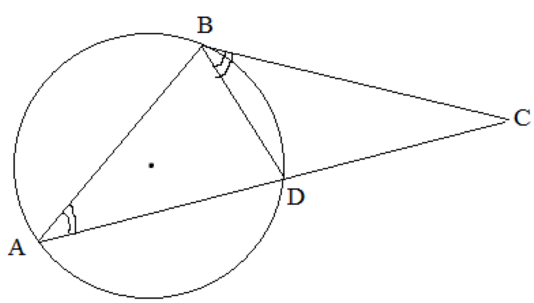
Prove: $\triangle TRM \sim \triangle APM$



Statements	Reasons

11) Given: $\angle CBD \cong \angle A$

Prove: $\triangle CDB \sim \triangle CBA$

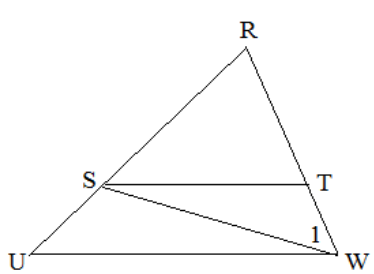


Statements	Reasons

12) Given: $\frac{\overline{RU}}{\overline{SW}} = \frac{\overline{RW}}{\overline{RT}}$

$\angle R = \angle 1$

Prove: $\overline{ST} \parallel \overline{UW}$



Statements	Reasons

13) Find the similarity ratio of 2 prisms with surface areas 144 feet^2 and 100 feet^2

14) The lateral area of 2 similar paint cans is 441 square cm and 961 square cm.
If the volume of the small can is 1200 cubic cm, what is the volume of the large can?

15) The length of a scale model car is 9". If the length of the actual car is 16 feet.
What is the ratio of the car to its scale model?

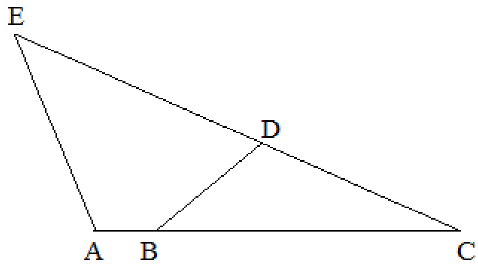
16) The diameter of a sphere is 10 feet. If you double the length of the diameter, how much does the surface area increase? How much does the volume increase?

(No-Choice Theorem)

17) Given: ∠DBC ≅ ∠E

Prove: ∠A ≅ ∠BDC

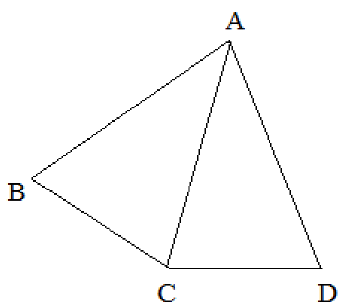
Statements	Reasons

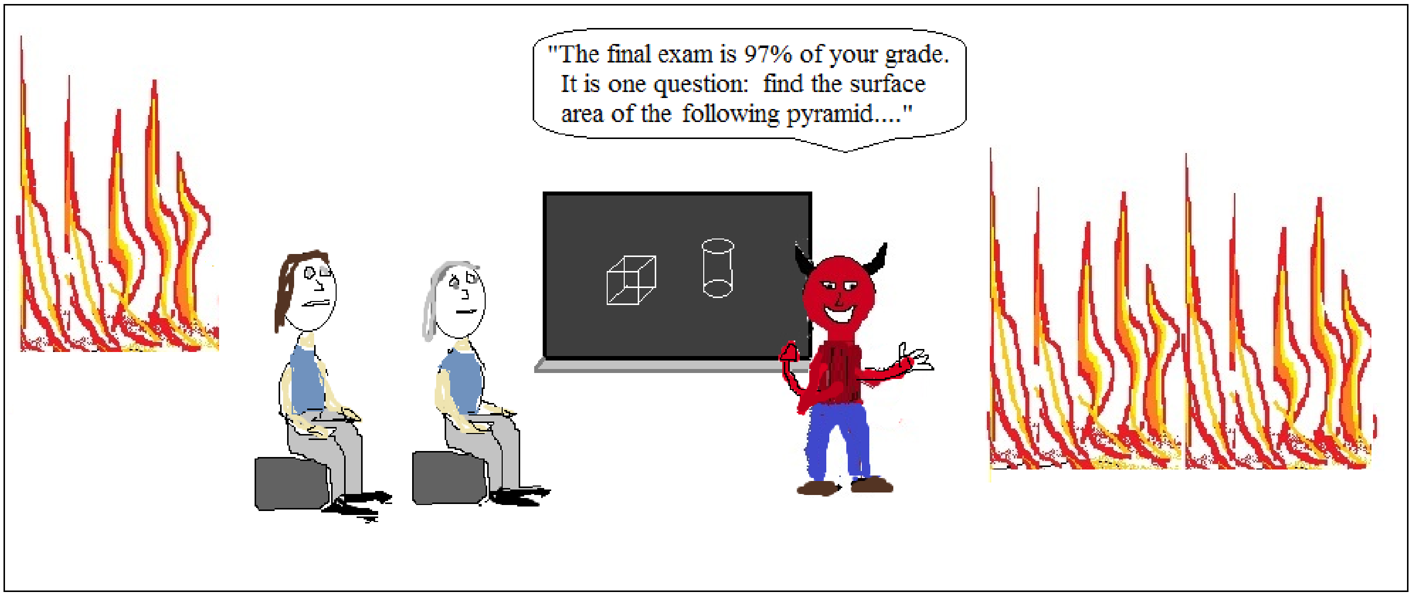


18) Given: ∠ABC ≅ ∠ACD

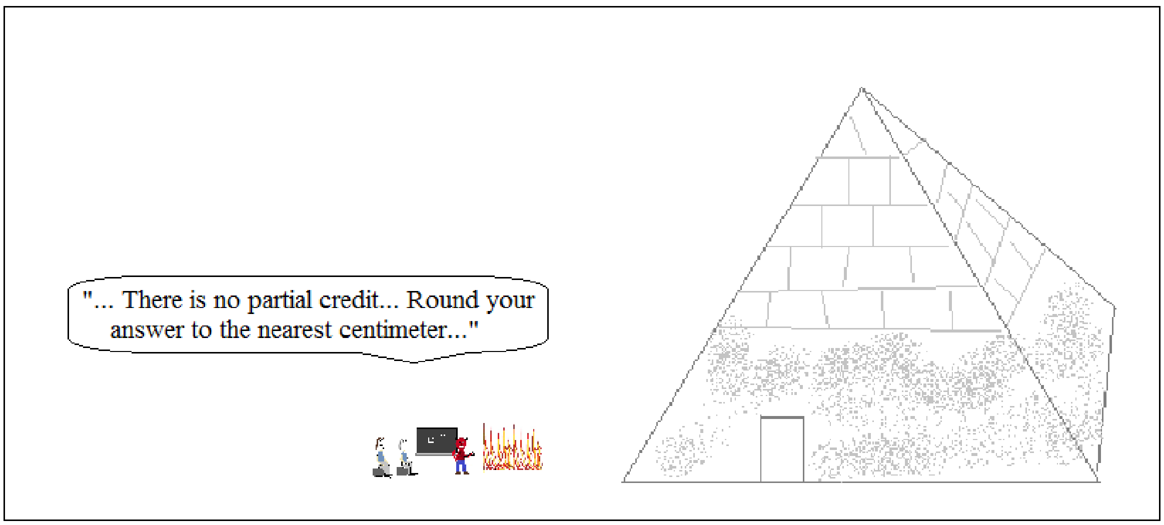
∠ACB ≅ ∠D

Are the triangles congruent?





Math in Hell



In its 1000 year history, no one ever passed Mr. Devlin's Geometry class.

LanceAF #39 7-1-12
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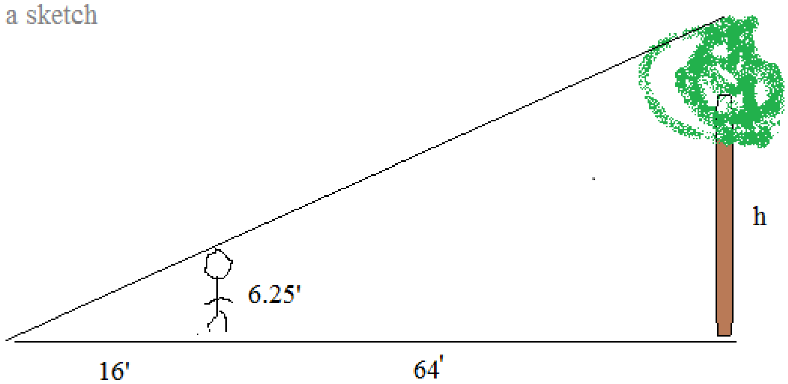
SOLUTIONS -→

Geometry: Similarity, Ratio, and Proportion

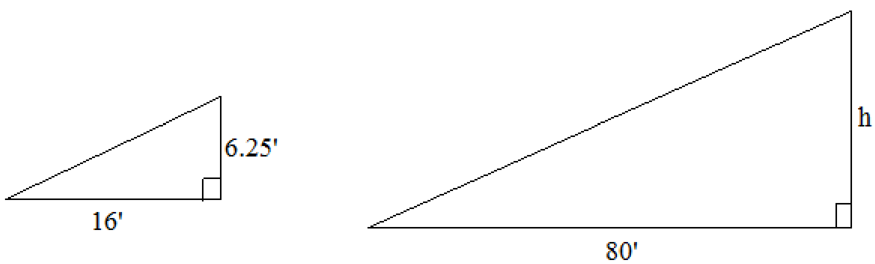
SOLUTIONS

1) To estimate the height of a tree, Joe stands in the shadow of the tree so that their shadows end at the same point. Joe is 6'3" tall and his shadow is 16 feet. If he is 64 feet from the tree, what is the height of the tree?

Step 1: Draw a sketch



Step 2: Construct the triangles/proportions



Step 3: Solve

$$\frac{6.25}{h} = \frac{16'}{80'} \quad \frac{\text{person}}{\text{tree}}$$

$$\frac{6.25}{h} = \frac{1}{5}$$

$h = 31.25'$

2) Find the ratio of x to y:

$2x = 3y$

$$\frac{2x}{y} = 3$$

$$\frac{x}{y} = \frac{3}{2}$$

3:2

3) Each side of square A is 6 cm.
Each side of square B is 9 cm.

What is the ratio of the *perimeters* of A to B?

Perimeter ratio is identical to sides ratio.

6:9

perimeter of A: 24
perimeter of B: 36

24:36 or 6:9
or 2:3

What is the ratio of the *areas* of the 'squares'?

Area ratio is ratio of the "squares"

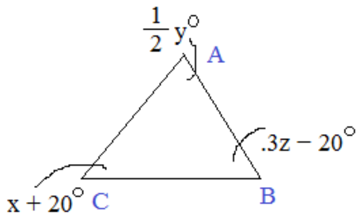
area of A: 36
area of B: 81

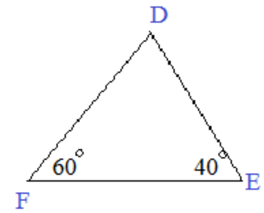
$6^2 : 9^2$

36:81 or 4:9

4) $\triangle ABC \sim \triangle DEF$

What is $\dfrac{x + y + z}{2}$?





D + E + F = 180 (sum of interior angles of triangle = 180 degrees)

D + 40 + 60 = 180 D = 80 degrees

Since triangles are similar, corresponding angles are congruent.

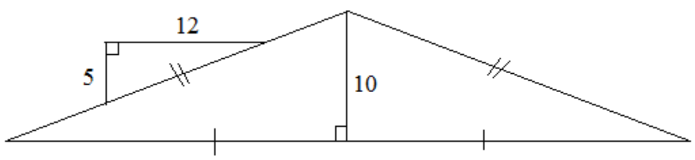
A = D $\frac{1}{2}y = 80$ y = 160

B = E .3z - 20 = 40 z = 200

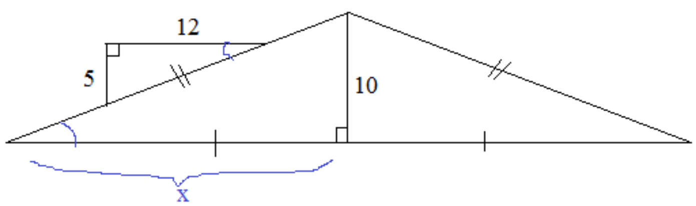
C = F x + 20 = 60 x = 40

$\dfrac{40 + 160 + 200}{2} = 200$

5) The roof of a house has a slope 5/12.
Based on the diagram, what is the length of the house?



Since the left side of the small triangle is vertical and the altitude of the house triangle is vertical, we know that the base of the house and the 12 segment are parallel...
If parallel lines cut by transversal, then alternate interior angles are congruent..

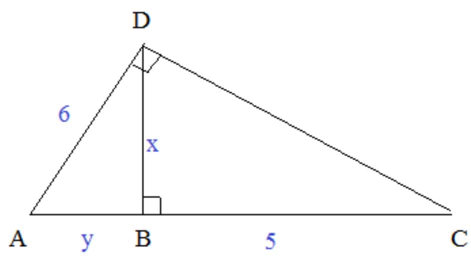


Because of AA similarity, the small triangle is similar to the left (and right) house triangle.

$\dfrac{5}{10} = \dfrac{12}{x}$ x = 24

therefore, length of house is 48

6) Find the length $\overline{DB}$ and $\overline{AB}$



$\overline{DB} \perp \overline{AC}$

$\overline{AD} \perp \overline{CD}$

$\overline{BC} = 5$

$\overline{AD} = 6$

$x^2 + y^2 = 36$ (Pythagorean Theorem)

$\dfrac{y}{x} = \dfrac{x}{5}$ $\dfrac{\text{"left/small leg"}}{\text{"bottom/large leg"}}$ Similar triangles

$x^2 = 5y$

$5y + y^2 = 36$

$y^2 + 5y - 36 = 0$

$(y + 9)(y - 4) = 0$

y = 4 (but, not -9 --- distance cannot be negative!)

Since y = 4,

$x = \sqrt{20} = 2\sqrt{5}$

$\overline{DB} = 2\sqrt{5}$

$\overline{AB} = 4$

7) Find x and y:

ABCD ∼ LMNP

since A = L

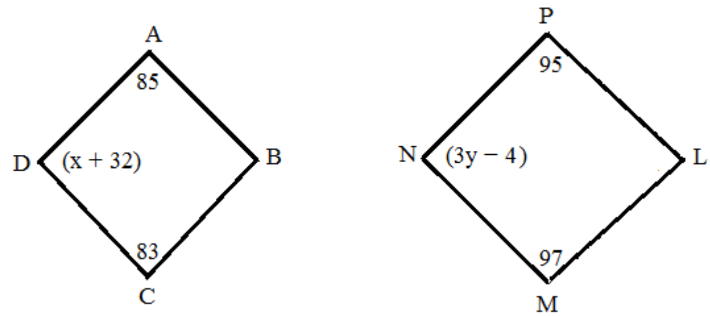
95 + 85 + 97 + (3y - 4) = 360

y = 29

B = M = 97

85 + 97 + 83 + (x + 32) = 360

x = 63



△HJK ∼ △WYZ

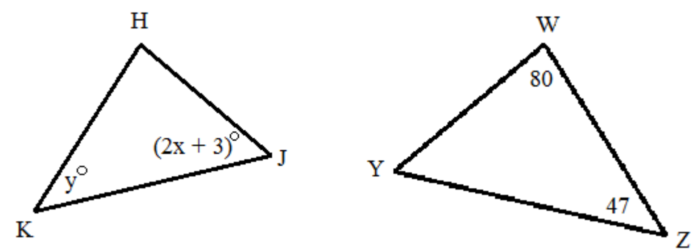
H = W = 80

J = Y = 53

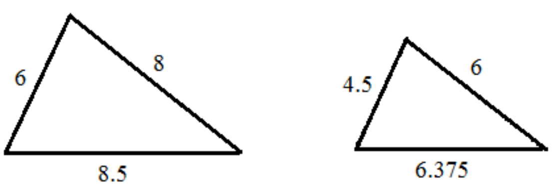
so, x = 25

K = Z

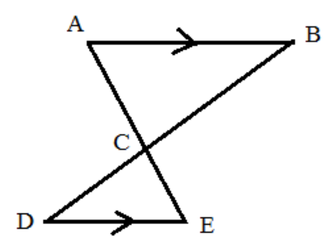
so, y = 47



8) Determine if the following triangles are similar.
(Justify your answer)

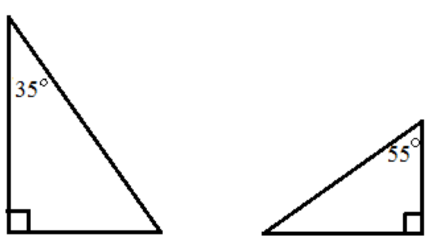


Side-Side-Side
(The ratios of corresponding sides are the same)



Angle-Angle
(vertical angles, and alternate interior angles, so the triangles have 3 corresponding angles that are congruent

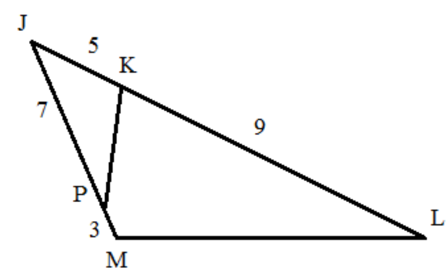
△CAB ∼ △CED



Angle-Angle
(If 2 angles are congruent, then triangles are similar)

NOTE: They *might be* congruent. But, a pair of corresponding sides must be congruent)

Also, "no choice theorem": If 2 pairs of angles are congruent, then the third angles are congruent.



Side-Angle-Side △KJP ∼ △MJL

$\frac{JK}{JM} = \frac{JP}{JL} = \frac{1}{2}$ $\angle J = \angle J$

9) $\triangle CDE \sim \triangle MNP$

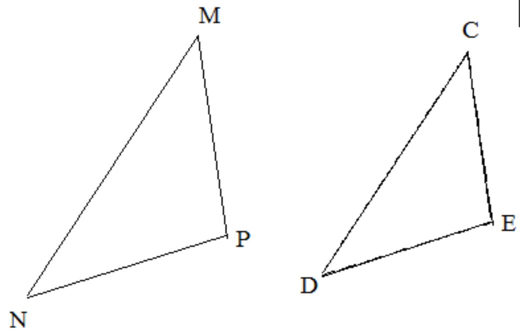
$\angle N = 40$

$\angle D = 3x + 5y$

$\angle P = 106 - x$

$\angle C = 4x + 2y$

What is x and y?



SOLUTIONS

Since triangles are similar, corresponding angles are congruent.

N = D $\quad 40 = 3x + 5y$

Sum of interior angles of triangle is 180.

$M + N + P = 180$
$M + 40 + (106 - x) = 180$

$M = 34 + x$

Then, C = M

$4x + 2y = 34 + x$

$40 = 3x + 5y$

$34 = 3x + 2y$

$6 = 3y$

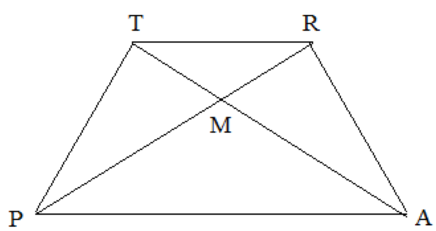
$y = 2$

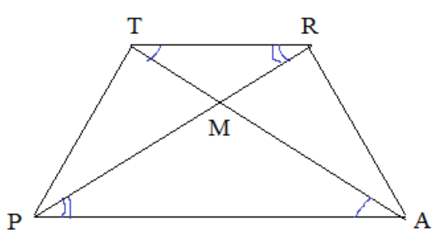
$x = 10$
$y = 2$

PROOFS:

10) Given: TRAP is trapezoid with bases $\overline{TR}$ and $\overline{PA}$

Prove: $\triangle TRM \sim \triangle APM$

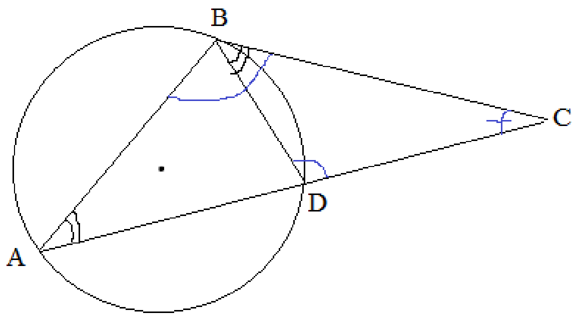




Statements	Reasons
1. TRAP is trapezoid with bases TR and PA	1. Given
2. $\overline{TR} \parallel \overline{PA}$	2. Definition of Trapezoid
3. $\angle TRP \cong \angle APR$	3. If parallel lines cut by transversal, then alternate interior angles congruent
4. $\angle RTA \cong \angle PAT$	4. Same
5. $\triangle TRM \sim \triangle APM$	5. AA (Angle Angle) similarity

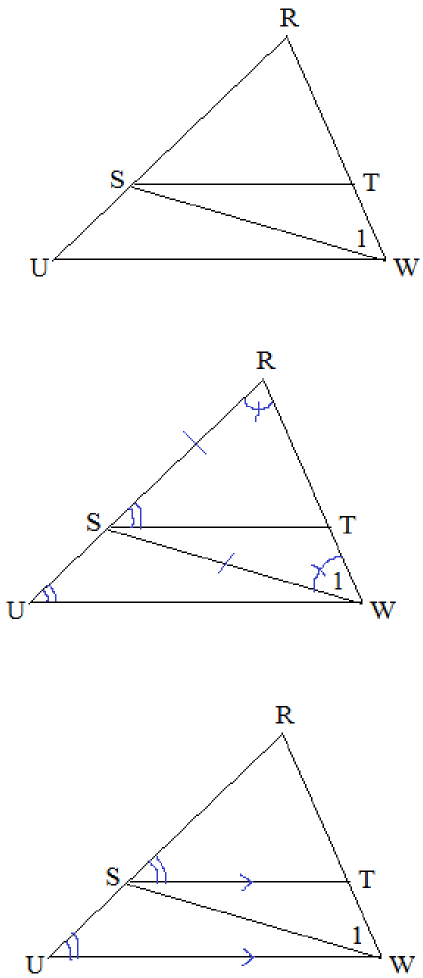
11) Given: $\angle CBD \cong \angle A$

Prove: $\triangle CDB \sim \triangle CBA$



Statements	Reasons
1. $\angle CBD \cong \angle A$	1. Given
2. $\angle C \cong \angle C$	2. Reflexive Property
3. $\triangle CDB \sim \triangle CBA$	3. AA (Angle-Angle) similarity theorem

12)



Given: $\dfrac{\overline{RU}}{\overline{SW}} = \dfrac{\overline{RW}}{\overline{RT}}$

$\angle R = \angle 1$

Prove: $\overline{ST} \parallel \overline{UW}$

Statements	Reasons
1. $\dfrac{RU}{SW} = \dfrac{RW}{RT}$	1. Given
2. $\angle R = \angle 1$	2. Given
3. $\overline{SR} \cong \overline{SW}$	3. If congruent angles, then opposite sides are congruent.
4. $\dfrac{RU}{SR} = \dfrac{RW}{RT}$	4. Substitution
5. $\angle R = \angle R$	5. Reflexive Property
6. $\triangle RST \sim \triangle RUW$	6. SAS similarity (2 corresponding sides are similar and included angle congruent)
7. $\angle RST = \angle RUW$	7. If triangles similar, then corresponding angles congruent
8. $\overline{ST} \parallel \overline{UW}$	8. If corresponding angles are congruent, then lines are parallel

13) Find the similarity ratio of 2 prisms with surface areas 144 $feet^2$ and 100 $feet^2$

If similarity ratio is $\frac{a}{b}$ then ratio of areas is $\frac{a^2}{b^2}$

In this case the areas are given: $\frac{144\ feet^2}{100\ feet^2}$ so, the similarity ratio is the square roots

12 feet : 10 feet

6 : 5

or $\frac{6}{5}$

14) The lateral area of 2 similar paint cans is 441 square cm and 961 square cm.
If the volume of the small can is 1200 cubic cm, what is the volume of the large can?

First, find the similarity ratio...

ratio of areas: $\frac{441\ cm^2}{961\ cm^2}$ ratio of sides (similarity ratio) $\sqrt{\frac{441\ cm^2}{961\ cm^2}} = \frac{21\ cm}{31\ cm} = \frac{21}{31}$

Second, find the ratio of the volumes...

ratio of volumes: $\left(\frac{21}{31}\right)^3 = \frac{9261}{29791}$

Finally, use ratio of volumes to find volume of large can... $\frac{9261}{29791} = \frac{1200\ cm^3}{\text{large can}}$

large can is approx. 3861 cubic centimeters

15) The length of a scale model car is 9". If the length of the actual car is 16 feet.
What is the ratio of the car to its scale model?

First, convert to identical units!

scale model is 9"

actual car is 16' or 192"

Then, express the ratio in the correct order...

"The ratio of the car to its scale model" is 192" : 9"

or, 64 : 3

16) The diameter of a sphere is 10 feet. If you double the length of the diameter, how much does the surface area increase? How much does the volume increase?

All spheres (and circles) are similar...

If we double the diameter, the surface area should be 4x

and, the volume should be 8x

$SA = 4\pi(\text{radius})^2$

10 feet: $SA = 100\pi$ square feet

20 feet: $SA = 400\pi$ square feet

$\text{Volume} = \frac{4}{3}\pi(\text{radius})^3$

10 feet: $V = \frac{500}{3}\pi$ cubic feet

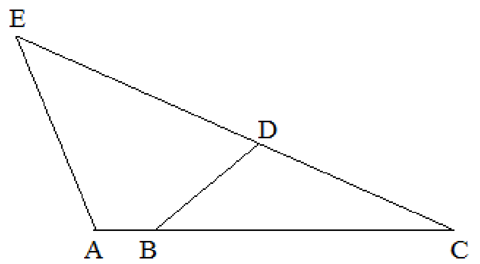
20 feet: $V = \frac{4000}{3}\pi$ cubic feet

(No-Choice Theorem)

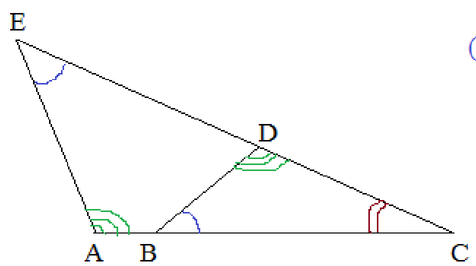
SOLUTIONS

17) Given: ∠DBC ≅ ∠E

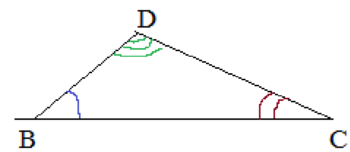
Prove: ∠A ≅ ∠BDC

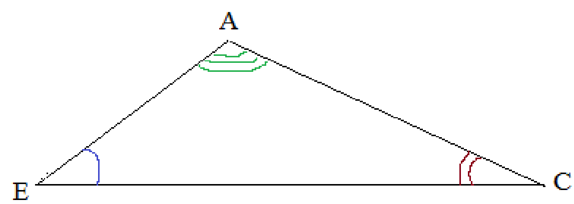


Statements	Reasons
1. ∠DBC ≅ ∠E	1. Given
2. ∠C ≅ ∠C	2. Reflexive Property
3. ∠A ≅ ∠BDC	3. No-Choice Theorem (If 2 angles of one triangle are congruent to 2 angles of another triangle, then the 3rd angles of both triangles are congruent)



Note: The angles are congruent.
So, the triangles are *similar*.
(We need at least one pair of congruent sides for congruent triangles)





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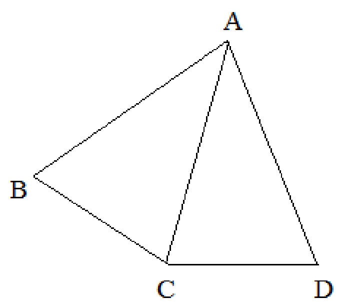
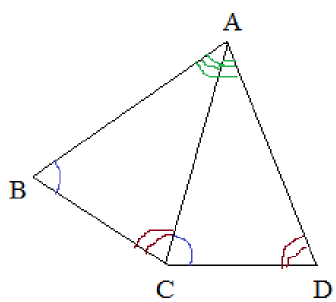
18) Given: ∠ABC ≅ ∠ACD
∠ACB ≅ ∠D

Are the triangles congruent?





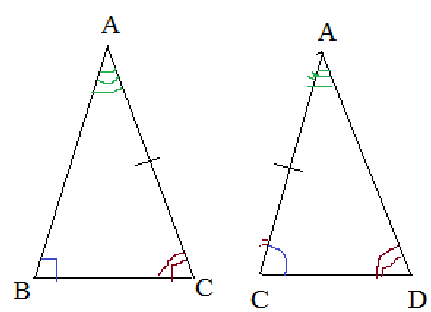
Since two angles are congruent, the 3rd angles must be congruent (no-choice theorem)

We have angle-angle-angle...

(Similar Triangles)

BUT, the triangles may or may not be congruent...

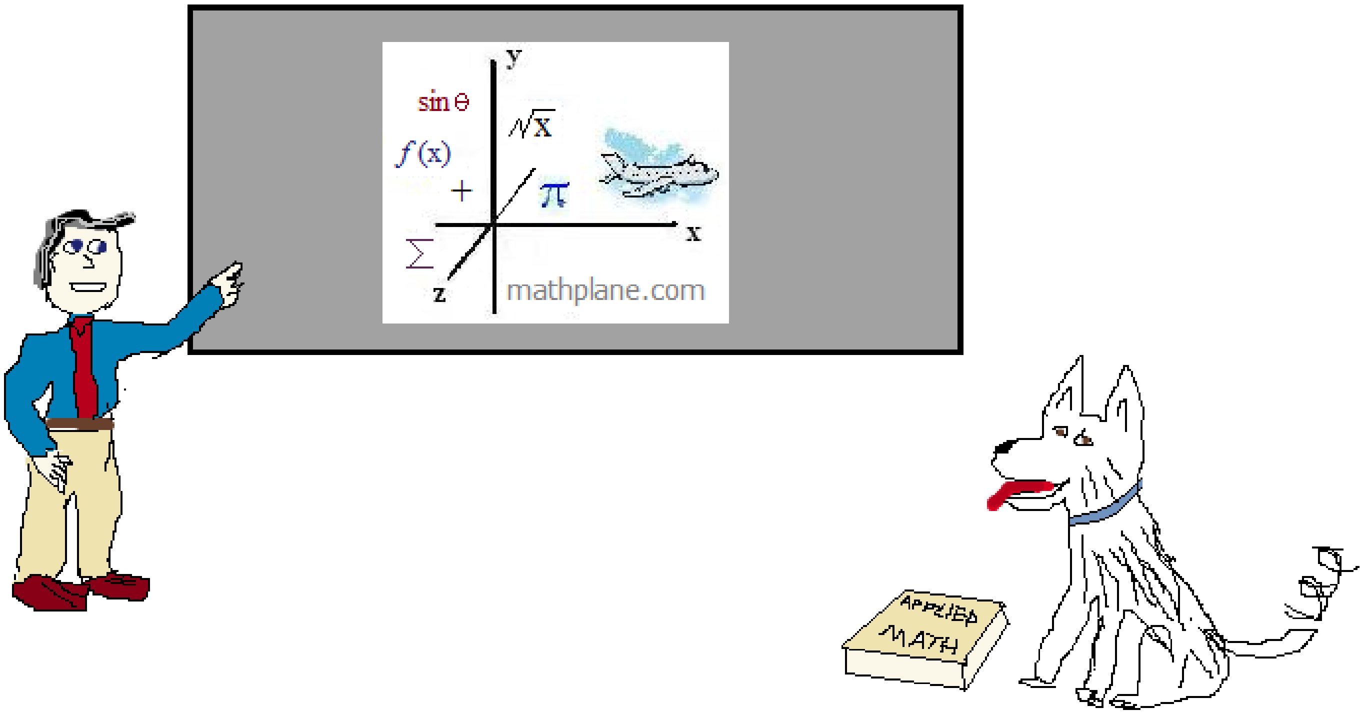
NOTE: $\overline{AC}$ in △ABC does not correspond to $\overline{AC}$ in △ACD



Thanks for visiting. (Hope it helped!)

If you have questions, suggestions, or requests, let us know.

Cheers



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