Noise bucket effect for impulse noise in OFDM

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Impulse noise is a significant problem in some orthogonal frequency division multiplexing (OFDM) applications. It has been observed in practice that the degradation caused by impulse noise depends only on the total energy of the noise during each OFDM symbol, not on the structure of the noise. This 'noise bucket' effect is explained by showing that even for a small number of impulses per symbol the noise distribution at the input of the receiver decision device is close to Gaussian. This is because of the spreading effect of the discrete Fourier transform.

Introduction: One of the advantages of orthogonal frequency division multiplexing (OFDM) compared to single carrier systems is its robustness against impulse noise. However impulse noise is still a serious problem in OFDM based systems including digital video broadcasting (DVB) [1, 2] and several techniques to mitigate the effects of impulse noise have been proposed [3, 4].

There are many different sources of impulse noise such as car ignitions, high voltage cables and domestic electrical appliances such as hair dryers and microwave ovens, thus it is difficult to model the details of impulse noise accurately. However it has recently been observed that in practice the system degradation caused by impulse noise in DVB systems depends only on the total noise energy within one OFDM symbol period, not on the detailed distribution of the noise energy within the symbol. This has been described as an extension of the 'noise bucket' effect [2, 5]. In this Letter, we present a theoretical explanation for this practical observation and show that it is true for a wide range of OFDM systems and forms of impulse noise.

In OFDM, decisions about the transmitted data are based on the signals at the output of the receiver discrete Fourier transform (DFT). We show that even when the noise at the receiver is impulsive, the noise after the DFT is approximately Gaussian. This is a result of the spreading effect of the DFT. Thus irrespective of the detailed structure of the impulse noise, the error rate which is observed is approximately the same as the error rate that would result for Gaussian noise with the same total energy per symbol.

Impulse noise and its effects in OFDM: Impulse noise can potentially affect a number of receiver functions. For example, it can cause the input amplifier to overload or disrupt the automatic gain control. These effects are not considered in this Letter; here only the effect of impulse noise on the baseband digital section of the receiver is considered.

The received time domain baseband OFDM signal is mathematically expressed as

$$r(k) = x(k) = n(k) + i(k) \tag{1}$$

where $x(k)$ is the wanted OFDM signal, $n(k)$ is additive white Gaussian noise (AWGN) with zero mean and variance $E\{|n(k)|^2\} = \sigma_n^2$ and $i(k)$ is the impulse noise. $E\{\cdot\}$ is the expectation operator. The statistical properties of $i(k)$ depend on both the form of impulse noise at the receiver input and on the filtering properties of the receiver front end. A number of impulse noise models based on theoretical analysis [6] or experimental data [7] have been presented in previous literature. However very recent research by the BBC, which measured a variety of impulse noise sources of practical importance in OFDM applications, concluded that impulse noise can be modelled as gated Gaussian noise [2].

The transmitted data is recovered from the received signal, $r(k)$ by first discarding the cyclic prefix and then performing an N point DFT on the remaining sequence $\{r(k)\}_{n=0}^{N-1}$. That is

$$R(l) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} r(k) \exp\left(\frac{-j2\pi lk}{N}\right) = X(l) + N(l) + I(l) \tag{2}$$

where $I(l)$ is given by

$$I(l) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} i(k) \exp\left(\frac{-j2\pi lk}{N}\right), \quad \text{for } l = 0, 1, \ldots, N-1 \tag{3}$$

From (3) it can be seen that the impulse noise component on each output of the receiver DFT depends on *all* of the components of $i(k)$, the impulse noise at the input to the receiver DFT. If the number of non-zero components of $i(k)$ is large enough for the central limit theorem to apply, then clearly the real and imaginary components of $I(l)$ have Gaussian distributions. In this Letter we show that even when the number of non-zero components is less than this, the distributions are very close to Gaussian and the symbol error rate (SER) is very close to that which would result from Gaussian noise with the same total noise energy per OFDM symbol.

Fig. 1 shows the complementary cumulative distribution function (CCDF) against absolute real of $I(l)$ for varying numbers of impulses Ψ per OFDM symbol. The mean power of the impulses was set to unity. Impulse occurrences are distributed randomly over time T, the OFDM symbol period. $N = 64$. As Ψ increases, the simulated results closely agree with the theoretical Gaussian curve. The CCDF of the time domain impulse noise is also shown for the case of $\Psi = 8$. This is clearly not Gaussian.

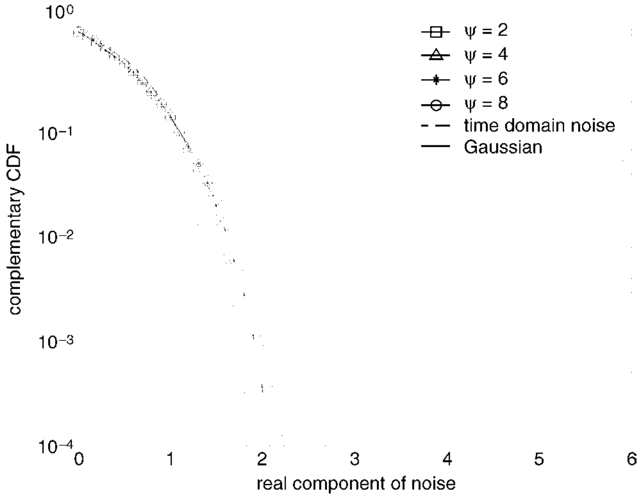

Fig. 1 *CCDF against absolute real component of noise for various Ψ*

Fig. 2 shows the results for three different impulse noise models. These are randomly distributed impulses with uniform amplitude distribution, randomly distributed impulses of equal amplitude and the gated Gaussian model [2]. $\Psi = 8$. Again simulated results are very close to the theoretical Gaussian curve. At the OFDM receiver, because of the spreading effect of the DFT, the frequency domain impulse interference becomes Gaussian largely independent of the precise time domain structure of the noise.

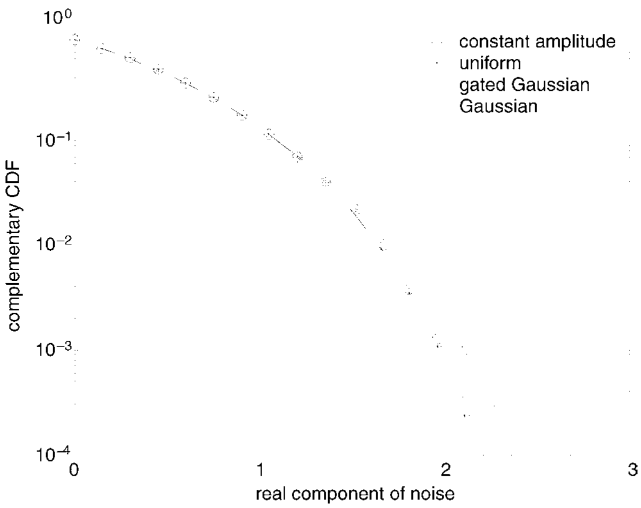

Fig. 2 *CCDF against absolute real component of noise*

Error performance: Impulse noise can cause significant system degradation. A simple expression for the SER performance is given using the Gaussian approximation.

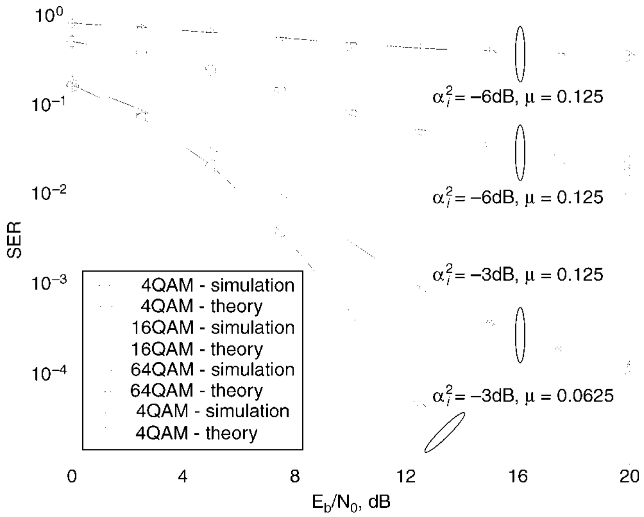

Fig. 3 *Symbol error rate against E_b/N_0*

Using the SER expression for M-ary square QAM signal in the presence of AWGN given in [8], the symbol error probability can be written as

$$P_s = 1 - \left[1 - 2\left(1 - \frac{1}{\sqrt{M}}\right)Q\left(\sqrt{\frac{3}{(M-1)}\lambda}\right)\right] \tag{4}$$

where λ denotes the average signal-to-noise ratio (SNR) per subcarrier, and the complementary error function erfc($\cdot$) is related to $Q(x)$ as

$$Q(x) = \frac{1}{2}\operatorname{erfc}\left(\frac{x}{\sqrt{2}}\right) \tag{5}$$

Equation (5) is also valid for evaluating SER in the case of impulse noise interference. However λ must be modified as the effective SNR per subcarrier as seen by the receiver decision device, i.e. taking into account the noise energy due to impulse interference as well. In a frequency selective fading channel *each* subcarrier will have a different λ value. In that case the average SER is calculated from (4) for all N subcarriers and then averaged as

$$\bar{P}_s = \frac{1}{N}\sum_{l=1}^{N} P_s(l) \tag{6}$$

where $P_s(l)$ is the lth subcarrier SER. Fig. 3 shows the SER against E_b/N_0 using a gated Gaussian impulse noise model [2] for 4, 16 and 64 QAM. μ defines the percentage of impulse noise occurrences within an OFDM symbol period, and σ_i^2 is its variance. The total noise power is then $\sigma^2 = \mu\sigma_i^2 + \sigma_n^2$. Two cases for 4 QAM are considered by varying μ with σ_i^2 held constant. Note the close agreement between simulation and theoretical results. Further simulations, although not presented, showed the Gaussian approximation is valid in most cases with different values of μ and σ_i^2. Although the degradation depends only on the total energy of the impulse noise, the effectiveness of impulse mitigation techniques does depend on the structure of the noise [3].

Conclusion: It has been shown that the 'noise bucket' concept applies in quantifying the performance of OFDM impaired by impulse noise. The decision noise becomes approximately Gaussian even with a small number of impulse occurrences. The theoretical and simulation results for system SER agree closely.

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References

1 Wu, Y.: 'Performance comparison of ATSC 8-VSB and DVB-T COFDM transmission systems for digital television terrestrial broadcasting', *IEEE Trans. Consumer Electron.*, 1999, **45**, (3), pp. 916–924

2 Lago-Fernández, J., and Salter, J.: 'Modelling impulsive interference in DVB-T: statistical analysis, test waveforms and receiver performance', BBC R&D White Paper. [Online]. Available: http://www.bbc.co.uk/rd/pubs/whp/whp080.html

3 Armstrong, J., and Suraweera, H.A.: 'Impulse noise mitigation for OFDM using decision directed noise estimation'. Proc. IEEE ISSSTA 2004, Sydney, Australia, Aug./Sept. 2004 (accepted for publication)

4 Zhidkov, S.V.: 'Impulse noise suppression in OFDM based communication systems', *IEEE Trans. Consumer Electron.*, 2003, **49**, (4), pp. 944–948

5 Poole, R.: 'DVB-T transmission, reception and measurement', 2002, DTG Monograph

6 Ghosh, M.: 'Analysis of the effect of impulse noise on multicarrier and single carrier QAM systems', *IEEE Trans. Commun.*, 1996, **44**, (2), pp. 145–147

7 Lind, L.F., and Mufti, N.A.: 'Efficient method for modelling impulse noise in a communication system', *Electron. Lett.*, 1996, **32**, (16), pp. 1440–1441

8 Proakis, G.J.: 'Digital communications' (McGraw-Hill, New York, USA, 1995, 3rd edn.)