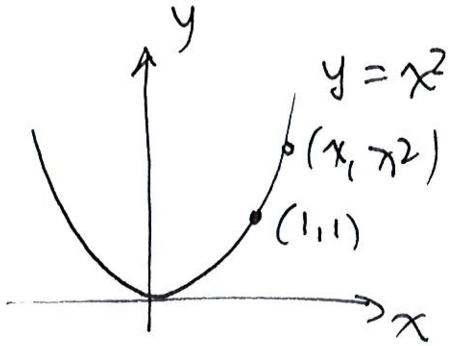
Math 1496 Calc 1



last class we asked about
the tangent to $y = x^2$ at (1,1)

Since (1,1) is on the curve

then $y = mx + b$ (straight line) gives $1 = 1 \cdot m + b$

so $b = 1 - m$

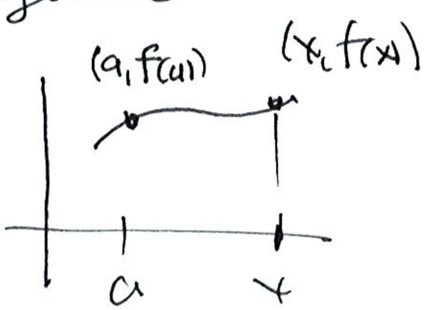
& line $y = mx + 1 - m$

For the slope m we consider a pt (x, x^2)

and considered $\frac{\Delta y}{\Delta x} = \frac{x^2 - 1}{x - 1}$

$$\lim_{x \to 1} \frac{x^2-1}{x-1} = \lim_{x \to 1} \frac{\cancel{(x-1)}(x+1)}{\cancel{(x-1)}} = \lim_{x \to 1} x+1 = 2$$

In general



$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a} = f'(a)$$

derivative at a pt.

ex 1 $f(x) = 3x+1$ find $f'(2)$

$f(2) = 3(2)+1 = 7$

$$\lim_{x\to 2} \frac{f(x)-f(2)}{x-2} = \lim_{x\to 2} \frac{3x+1-7}{x-2} = \lim_{x\to 2} \frac{3x-6}{x-2}$$

$$= \lim_{x\to 2} \frac{3\cancel{(x-2)}}{\cancel{x-2}} = 3 \quad \text{so} \quad f'(2) = 3$$

ex 2 $f(x) = \sqrt{x}$ find $f'(1)$?

$$\lim_{x\to 1} \frac{f(x)-f(1)}{x-1} = \lim_{x\to 1} \frac{\sqrt{x}-1}{x-1}$$

$$= \lim_{x\to 1} \frac{\sqrt{x}-1}{x-1} \cdot \frac{\sqrt{x}+1}{\sqrt{x}+1} = \lim_{x\to 1} \frac{\cancel{x-1}}{\cancel{(x-1)}(\sqrt{x}+1)}$$

$$= \lim_{x\to 1} \frac{1}{\sqrt{x}+1} = \frac{1}{1+1} = 1/2$$

Ex 3 $\quad f(x) = \begin{cases} 3x-3 & x \geq 2 \\ x+1 & x < 2 \end{cases}$

Cont^s ? $\quad \lim_{x \to 2^-} x+1 = 3$

$\lim_{x \to 2^+} 3x-3 = 6-3 = 3$

limit exist $\lim_{x \to 2} f = 3$

$f(2) = 3(2)-3 = 3$ same so cont^s

Deriv.? $\quad \lim_{x \to 2} \frac{f(x)-f(2)}{x-2}$ Now use diff branches $f(2) = 3$

$\lim_{x \to 2^-} \frac{x+1-3}{x-2} = \lim_{x \to 2^-} \frac{x-2}{x-2} = 1$

$\lim_{x \to 2^+} \frac{3x-3-3}{x-2} = \lim_{x \to 2^+} \frac{3\cancel{(x-2)}}{\cancel{(x-2)}} = 3$

different

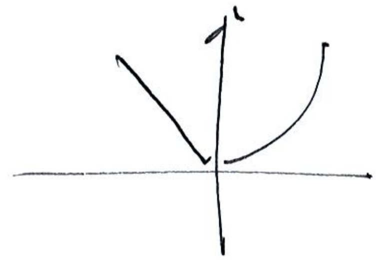
If (1) $\lim_{x \to a^-} \frac{f(x)-f(a)}{x-a} = \lim_{x \to a^+} \frac{f(x)-f(a)}{x-a}$

we say f is "differentiable" at $x=a$

so in previous example

f is not differentiable at $x=2$

Ex $f(x) = \begin{cases} -x & x \le 0 \\ x^2 & x > 0 \end{cases}$



$f(0) = 0$

$$\lim_{x \to 0^-} \frac{-x - 0}{x - 0} = -1$$

$$\lim_{x \to 0^+} \frac{x^2 - 0}{x - 0} = \lim_{x \to 0^+} x = 0 \neq -1$$

so not diff at $x = 0$