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Contraction Preconditioner in FD EM Modeling and its Parallelization

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SUMMARY

We introduce two novel preconditioning methods for 3D finite difference (FD) frequency domain electromagnetic modeling. The first preconditioner is based on the layered earth Green's function, the second one is based on a special transformation of the original system of FD equations into a system with a contraction operator. This transformation extends to the FD modeling the approach originally developed for the integral equation modeling method. We also examine the spectral properties of the two preconditioned methods.

For numerical study of the developed methods, we have designed a complex marine 3D geoelectrical model. The numerical experiments confirm the results of theoretical analysis: an iterative solver with the contraction preconditioner converges faster or at near the same speed as with the Green's function preconditioner. We apply these two solvers to controlled source modeling and demonstrate that both approaches are very effective, and memory saving. We have also developed a parallel version of the algorithm and studied the scalability of shared and distributed memory parallelization. We conclude that given ambiguity in defining a parallelization scheme, parallelization over sources should be exploited in the first place, while shared memory parallelization for a particular source should be exploited in the second place.

Keywords: forward modeling, frequency domain, finite difference, preconditioning, parallelization

INTRODUCTION

Fast solutions of large algebraic systems arising in 3D frequency domain electromagnetic modeling are known to be of major importance in geophysical applications. Several direct and iterative methods were developed and tested over decades, however, the direct methods require large memory allocations making the iterative methods more attractive.

It is well known that the chosen preconditioner significantly dictates the performance of the iterative solvers. When choosing a preconditioner, several aspects must be considered: low algorithmic complexity, spectral properties of the preconditioned system, robustness for a wide range of frequencies and high-contrast models, good scalability, and small auxiliary memory required. Yavich and Zhdanov (2016) presented a novel preconditioning approach based on a FD contraction operator, which satisfied to several of the above requirements.

In this paper, we extend this study and illustrate the performance of the method with complex marine controlled source modeling and shared and distributed memory parallelization.

NUMERICAL MODELING APPROACH

In this note, we consider electromagnetic modeling in 3D heterogeneous isotropic media in the frequency domain. The secondary (anomalous) electric field, $\boldsymbol{E}_a(x,y,z)$ satisfies the following system of partial differential equations,

$$curl\ curl\ \boldsymbol{E}_a - i\omega\mu_0\sigma\boldsymbol{E}_a = i\omega\mu_0\sigma_a\boldsymbol{E}_b, \qquad (1)$$

where ω is the source angular frequency, μ_0 is the magnetic permeability of the free space, $\sigma(x,y,z)$ and $\sigma_a(x,y,z)$ are the total and anomalous conductivities respectively, and $\boldsymbol{E}_b(x,y,z)$ is the background response due to background conductivity model $\sigma_b(z)$. The equations are solved for $\boldsymbol{E}_a(x,y,z)$ numerically in some bounded domain and completed with zero Dirichlet boundary conditions. We also assume that the following double inequality holds,

$$\alpha\sigma_b(z) \le \sigma(x,y,z) \le \beta\sigma_b(z), \quad 0 < \alpha \le 1 \le \beta,$$

which controls the contrast of anomalous conductivity with respect to the background.

Given a nonuniform computational grid with $N_x \times N_y \times N_z$ cells, we apply the conventional edge-based FD discretization (Yee, 1966; Weiss and Newman, 2002; Yavich and Zhdanov, 2016). Introducing the diagonal matrices of discrete conductivities for the total, background, and anomalous models, $\boldsymbol{\Sigma} = \boldsymbol{\Sigma}_b + \boldsymbol{\Sigma}_a$, as well as the respective discrete electric fields, $\boldsymbol{e} = \boldsymbol{e}_a +$

$\boldsymbol{e}_b$, one can write a discrete form of equation (1) as follows:

$$\boldsymbol{A}\boldsymbol{e}_a = i\omega\mu_0\boldsymbol{\Sigma}_a\boldsymbol{e}_b, \qquad (2)$$

where $\boldsymbol{A}$ is a square system matrix corresponding to the total conductivity. For layered background models, $\boldsymbol{e}_b$ can be computed by averaging its analytical counterpart. We denote the size of this system as n, $\sim 3N_xN_yN_z$.

Let $\boldsymbol{A}_b$ be the FD system matrix, corresponding to the background conductivity model. Importantly, this matrix can be implicitly factorized, and the action of the inverse matrix can be efficiently computed (see the next Section). As a result, it can be used as a preconditioner to (2):

$$\boldsymbol{A}_b^{-1}\boldsymbol{A}\boldsymbol{e}_a = i\omega\mu_0\boldsymbol{A}_b^{-1}\boldsymbol{\Sigma}_a\boldsymbol{e}_b \qquad (3)$$

We will refer $\boldsymbol{A}_b^{-1}$ as the Green's function (GF) preconditioner. The complexity of applying the GF preconditioner is $O(n^{4/3})$, and auxiliary memory required is near $3n$ only. Applying the analysis presented in Yavich and Zhdanov (2016), the spectral condition number (ratio of the largest eigenvalue to the smallest) of the GF preconditioned system can be estimated as follows,

$$cond(\boldsymbol{A}_b^{-1}\boldsymbol{A}) \leq {}^{\beta}/_{\alpha}. \qquad (4)$$

This result implies that convergence of an iterative solver applied to (3) has minor or no dependence on the grid size and cells aspect ratio, as well as the frequency, while it degrades on models with high-contrast bodies.

To minimize the impact of high-contrast bodies, another preconditioner could be constructed. Let us define the modified FD Green's operator according to the following formula:

$$\mathcal{G}_b^M = 2i\omega\mu_0\boldsymbol{\Sigma}_b^{\frac{1}{2}}\boldsymbol{A}_b^{-1}\boldsymbol{\Sigma}_b^{\frac{1}{2}} + \boldsymbol{I}. \qquad (5)$$

Using this operator, equation (3) can be written in an equivalent form as follows

$$\hat{\boldsymbol{e}}_a = \mathcal{G}_b^M\,\boldsymbol{K}_2\boldsymbol{K}_1^{-1}\hat{\boldsymbol{e}}_a + i\omega\mu_0\boldsymbol{\Sigma}_b^{\frac{1}{2}}\boldsymbol{A}_b^{-1}\boldsymbol{\Sigma}_a\boldsymbol{e}_b, \qquad (6)$$

where $\hat{\mathbf{e}}_a = \mathbf{K}_1\mathbf{e}_a$, and $\mathbf{K}_1$, $\mathbf{K}_2$ are diagonal matrices,

$$\boldsymbol{K}_1 = \tfrac{1}{2}(\boldsymbol{\Sigma}+\boldsymbol{\Sigma}_b)\boldsymbol{\Sigma}_b^{-\frac{1}{2}}, \quad \boldsymbol{K}_2 = \tfrac{1}{2}(\boldsymbol{\Sigma}-\boldsymbol{\Sigma}_b)\boldsymbol{\Sigma}_b^{-\frac{1}{2}}. \qquad (7)$$

By introducing a new operator,

$$\boldsymbol{C} = \mathcal{G}_b^M\,\boldsymbol{K}_2\boldsymbol{K}_1^{-1}, \qquad (8)$$

we rewrite system (6) as follows:

$$(\boldsymbol{I}-\boldsymbol{C})\hat{\boldsymbol{e}}_a = i\omega\mu_0\boldsymbol{\Sigma}_b^{\frac{1}{2}}\boldsymbol{A}_b^{-1}\boldsymbol{\Sigma}_a\boldsymbol{e}_b. \qquad (9)$$

We will refer this system a contraction operator (CO) preconditioned system, since it was shown in Yavich and Zhdanov (2016) that $\|\boldsymbol{C}\| < 1$. Moreover, it can be proved that this preconditioned system has a smaller or equal spectral condition number than that of the GF preconditioner, implying faster or equal convergence of the iterative solvers,

$$cond(\boldsymbol{I}-\boldsymbol{C}) \leq \max\{{}^{1}/_{\alpha}, \beta\}. \qquad (10)$$

The complexity of applying the CO preconditioner is $O(n^{4/3})$ as well. In the next sections we will discuss parallelization of the preconditioners and present some numerical examples.

PARALLELIZATION

Multi-frequency and multi-source simulations of the electromagnetic response are often encountered in the geophysical applications, especially in marine studies. Distribution of different frequencies and sources across several computer nodes on a distributed-memory system is the de-facto standard parallelization strategy. On the other hand, computation of responses due to a single source is natural to parallelize for shared memory architecture. This second level of parallelization depends on the actual modeling algorithm used. It is discussed below for the preconditioners presented earlier.

The calculation of the act of inverse matrix $\boldsymbol{A}_b^{-1}$ on a given vector is the most computationally complex step in application of these preconditioners. This calculation is based on the discrete separation of the variables (Martikainen et al. 2003; Zaslavsky et al. 2011) and involves the following operations: $3N_z$ applications of the 2D discrete Fourier transforms of $N_x \times N_y$ arrays, N_xN_y solutions of banded linear systems with $O(N_z)$ unknowns, and diagonal scaling. Implementation of the CO preconditioner adds to these operations four diagonal scalings and two vector additions.

When the preconditioners are incorporated into an iterative solver (we considered BiCGStab solver), another set of operations has to be parallelized: scalar-vector multiplications, vector additions, vector inner products and norms. All of the mentioned operations are easily parallelized on shared memory: the $3N_z$ Fourier transforms as well as N_xN_y solutions of banded linear systems were distributed within available threads. Parallelization of the vector operations and diagonal scaling is straightforward. We implemented them using OpenMP (OMP).

NUMERICAL EXAMPLES

To illustrate performance of the discussed preconditioning algorithms, we designed a marine geoelectrical model (Figure 1). The model involved a 2D host medium (200 m deep sea of 0.3 Ω·m, four faulted sedimentary layers of 1 to 16 Ω·m, and 100 Ω·m basement) and 3D hydrocarbon deposit (≈ 5×8×0.2 km^3) of 200 Ω·m.

We considered a marine setup with a streamer towed in the x-direction at a depth of 10 m below the sea surface. The source frequency was set to 0.25 Hz. An array of uniformly distributed 48 receivers had an offset range from 2 to 14 km. The source and receivers were point electrical dipoles oriented along the x-axis.

At each source position a separate $172 \times 96 \times 83$ nonuniform computational grid was generated with the smallest cell size of 137×138×73 m^3 resulting in near 4 million discrete unknowns.

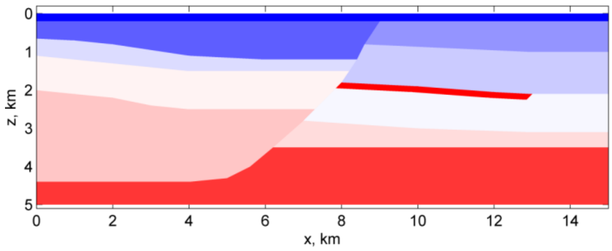

Figure 1. Marine resistivity model involving a host medium and 3D hydrocarbon deposit. The host medium is homogeneous in the y-direction.

Table 1 presents serial performance of the BiCGStab iterative solver leveraged by GF and CO preconditioners. Iteration count, N_{it}, and execution time, t, in seconds required to reach tolerance of 1e-8 are shown for a serial run. We observed that the use of the CO preconditioner gave us a 2.5 time speedup versus GF preconditioner both with respect to iteration count and execution time. We checked that memory allocated by the CO preconditioner was near 350 Mb. This confirms that the approach is very memory economical.

Table 1. Iteration count, N_{it}, and CPU time, t, in seconds of the serial BiCGStab iterative solver needed to reach tolerance 1e-8, leveraged by GF and CO preconditioners

GF		CO	
N_{it}	t, s	N_{it}	t, s
78	445	31	180

We studied data sensitivity to the deposit assuming the streamer was towed along 15 km observation line. We normalized response by response of the model with no deposit. Amplitude and phase of the normalized response are plotted in Figure 2. We observe a good sensitivity of data to the deposit: 48 % for amplitude, 32 ° for phase.

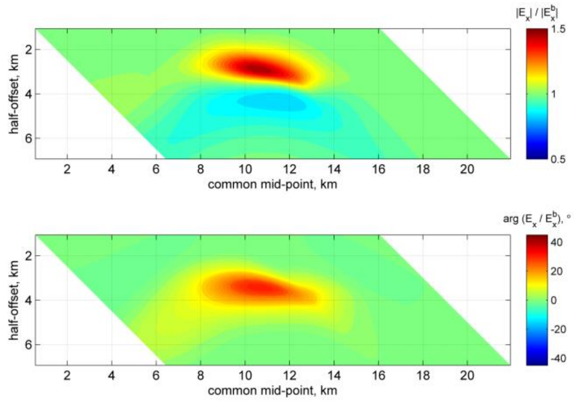

Figure 2. Normalized inline response amplitude (upper) and phase (lower).

We tested scalability of our parallelization of the two preconditioned solvers. The used compute node was equipped with dual ten-core Intel Xeon CPU E5-2670 v2 running at 2.50 GHz with 64 GB of memory. Figure 3 shows speedup received when using from 1 to 20 threads mapped to separate cores. The obtained speedup of 12 times for 20 threads for both GF and CO preconditioners is appropriate since grid dimensions were not divisible by the number of threads, which is typical for realistic modeling.

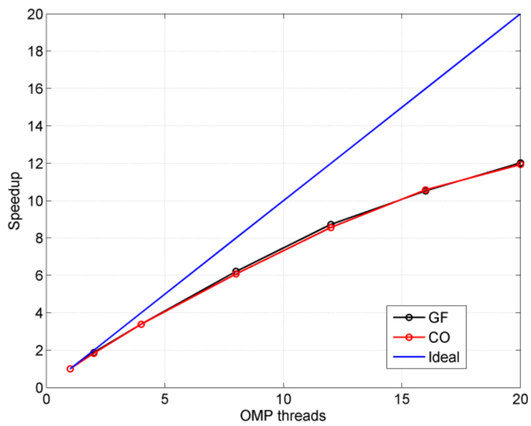

Figure 3. Speedup versus OpenMP threads used for GF and CO preconditioned BiCGStab.

We also checked speedup received by MPI parallelization over sources. Each MPI process was modeling one or several sources. In this test, we considered 20 sources from marine modeling discussed earlier. Figure 4 shows speedup of the CO preconditioned solver obtained by using from 1 to 20 MPI processes running on a single compute node. The speedup is quite high, 17.5 times. This result was expected since small amount of data is communicated in this case. Notice that execution of 20 solvers in parallel on a single machine was possible due to low memory consumption of our approach.

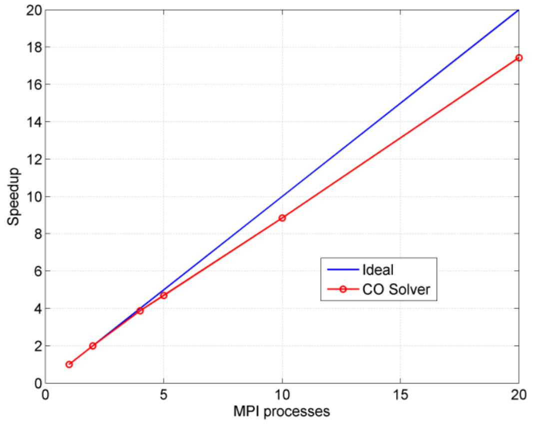

Figure 4. Speedup in modeling of 20 sources vs. number of MPI processes used, pinned to separate cores of a single compute node.

A test of hybrid MPI/OpenMP parallelization is shown in Figure 5. Given 20 sources and a twenty-core compute node, we studied which combination results in a shorter wall-clock time. Evidently, the MPI programing model is preferable to the OpenMP one, which is in agreement with the results, presented above.

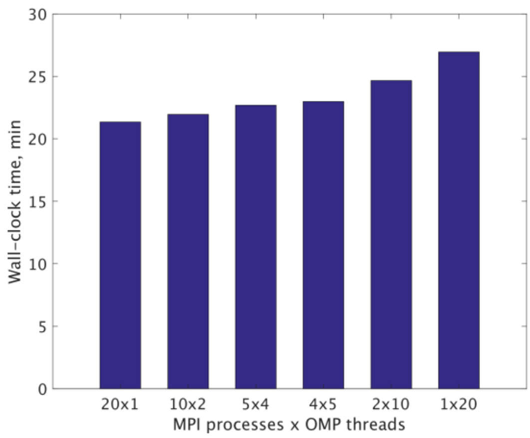

Figure 5. Wall-clock time of twenty source modeling vs. different MPI/OpenMP combinations within a single compute node.

We conclude that given ambiguity in defining a parallelization scheme, parallelization over sources should be exploited in the first place, while shared memory parallelization for a particular source should be exploited in the second place.

CONCLUSION

We presented two novel preconditioning methods for 3D finite difference frequency domain modeling. The first preconditioner is based on the layered earth Green's function, the second one is based on a special transformation to an algebraic system with a contraction operator. Their spectral properties were analysed and confirmed by numerical experiments: an iterative solver with the CO preconditioner converged faster or at the same speed as the GF preconditioner. We applied the two solvers to marine controlled source modeling and demonstrated that these approaches are practical, fast, and very memory economical. We also tested scalability of shared and distributed memory parallelization and obtained a decent speedup.

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