A Parallel Elicitation-Free Protocol for Allocating Indivisible Goods

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Resource Allocation

- Allocating indivisible goods among multiple agent is an important problem in AI.
- Drawbacks of centralized methods(combinatorial auction):
  - Elicitation process and winner determination procedure can be expensive.
  - Agents may be reluctant to reveal their full preferences.
  - In some real world situation, monetary side payments are impossible or unwelcome.
- [*Bouveret and Lang*, *IJCAI*, 2011] studied a sequential elicitation-free protocol.

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Our Method

Generalization the sequential protocol:

- A parallel policy (i.e., an agent selection policy) is given before allocation begins.
- At every stage, some agents are selected to report their favorite objects among remains.
- If an object is reported by more than one agent, then they draw lots to get it.

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Definition

- m indivisible objects $\mathcal{O} = \{o_1, \ldots, o_m\}$, and n agents $\mathcal{N} = \{1, 2, \ldots, n\}$;
- $\succ_i$: agent i's ordinal preference (strict total order) over $\mathcal{O}$;
- *profile* R: $R = \langle \succ_1, \ldots, \succ_n \rangle$;

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Assumption

- Agents have additive utilities;
- Scoring function maps the rank to the utility value(same for all agents);
- Full independence (all profiles are equally probable).
- Two specific scoring functions:
  - Borda: $g_B(k) = m - k + 1$
  - lexicographic: $g_L(k) = 2^{m-k}$.

Social Welfare

- Social welfare: $sw_F(\pi) = F(u_1(\pi), \ldots, u_n(\pi))$
  - $u_i(\pi)$: agent i's expected utility over all profiles.
- Two specific social welfare functions:
  - utilitarian criterion $F_u(u_1, \ldots, u_n) = \sum_{i=1}^{n} u_i$.
  - egalitarian criterion $F_e(u_1, \ldots, u_n) = \min\{u_i | 1 \leq i \leq n\}$.

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Parallel Policy

- a parallel policy is a function ϖ: $(\mathit{Reporting\_set} \times \mathit{loser\_set})^* \rightarrow \mathit{Reporting\_set}$.
- Given a finite sequence $\sigma = \langle \mathcal{N}_1, \mathcal{N'}_1 \rangle, \ldots, \langle \mathcal{N}_k, \mathcal{N'}_k \rangle$, ϖ designates the set of agents reporting at stage $k + 1$.
- An allocation history can be induced by ϖ, which is nondeterministic.

Two Parallel Policies

Two specific parallel policies:

- All-reporting policy: all the agents report at every stage.
- Loser-reporting policy: all the agents losing some lot at the current stage report at the next stage

Example

Let $m = 5$, $n = 3$, All-reporting policy. Suppose $R = \langle \succ_1, \succ_2, \succ_3 \rangle$ s.t.

- $\succ_1 = o_1 \succ o_2 \succ o_3 \succ o_4 \succ o_5$,
- $\succ_2 = o_4 \succ o_2 \succ o_5 \succ o_1 \succ o_3$,
- $\succ_3 = o_1 \succ o_3 \succ o_5 \succ o_4 \succ o_2$.

stage	1	2	3	total
*agent*1	$\frac{1}{2}o_1$	$\frac{1}{2}o_2$	$\frac{1}{3}o_5$	$\frac{1}{2}o_1 + \frac{1}{2}o_2 + \frac{1}{3}o_5$
*agent*2	o_4	$\frac{1}{2}o_2$	$\frac{1}{3}o_5$	$o_4 + \frac{1}{2}o_2 + \frac{1}{3}o_5$
*agent*3	$\frac{1}{2}o_1$	o_3	$\frac{1}{3}o_5$	$\frac{1}{2}o_1 + o_3 + \frac{1}{3}o_5$

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Optimal sequence

- [*Bouveret and Lang*, *IJCAI*, 2011] conjectured that finding an optimal sequential policy is NP-hard.
- [*Kalinowski et al.*, *IJCAI*, 2013] proved that the alternating policy (i.e., 1212 . . .) is optimal for two agents under Borda scoring function(utilitarian criterion).
- However, the problem of finding optimal sequence for more than two agents, or under other scoring functions, is still open.
- In another hand, parallel policy ϖ_A (i.e., all–reporting) is very natural and simple.

Comparison

Table: Utilitarian Social Welfare of ϖ_{π^*} and ϖ_A under g_B

	$n=2$			$n=3$			$n=4$		
m	π^*	sw_{π^*}	sw_A	π^*	sw_{π^*}	sw_A	π^*	sw_{π^*}	sw_A
4	22	12.292	12.292	31	13.083	13.297	4	13.583	13.885
5	221	18.625	18.625	32	20.033	20.382	41	20.800	21.351
6	222	26.396	26.396	33	28.622	28.840	42	29.600	30.377
7	2221	35.396	35.396	331	38.511	38.864			
8	2222	45.820	45.820	332	49.936	50.381			
9	22221	57.487	57.487						
10	22222	70.569	70.569						

- Conjecture 1: Under utilitarian criterion and convex scoring functions, $sw(\varpi_{\pi^*}) = sw(\varpi_A)$ when $n = 2$, and $sw(\varpi_{\pi^*}) < sw(\varpi_A)$ when $n > 2$.

Comparison(continued)

Table: Egalitarian Social Welfare of ϖ_{π^*} and ϖ_A under g_B

	$n=2$			$n=3$			$n=4$		
m	π^*	sw_{π^*}	sw_A	π^*	sw_{π^*}	sw_A	π^*	sw_{π^*}	sw_A
4	$\underline{2}21$	6.000	6.146	$\underline{3}3$	3.750	4.432	$\underline{4}$	2.500	3.471
5	$1\underline{2}22$	9.000	9.313	$\underline{3}32$	5.000	6.794	$\underline{4}4$	4.500	5.338
6	$\underline{22}21$	13.125	13.198	$\underline{3}321$	9.000	9.613	$\underline{4}43$	5.833	7.594
7	$1\underline{2}2\underline{2}2$	17.333	17.698	$\underline{3}2133$	12.250	12.955			
8	$\underline{2}2\underline{2}1\underline{2}$	22.725	22.910	11332232	15.000	16.794			
9	$1\underline{22}22\underline{2}$	28.429	28.744						
10	$\underline{2}21\underline{22}21$	35.200	35.285						

- Conjecture 2: Under egalitarian criterion and convex scoring functions, $sw(\varpi_{\pi^*}) < sw(\varpi_A)$ for any number of n and m.

Strategy Issues under All-reporting Policy

- All-reporting policy is not *strategyproof*.
- Assumptions:
  - Manipulator is pessimistic(risk averse);
  - All agents but the only one manipulator act truthfully;
  - Manipulator knows all other agents' preferences.
- A manipulation problem M (for agent 1) consists of $\langle \succ_2, \ldots, \succ_n \rangle$, and a target set of objects $\mathcal{S} \subseteq \mathcal{O}$.

Strategy Issues(continued)

- A *manipulation problem* can be addressed in polynomial time.
- Problem of finding an optimal strategy under lexicographic scoring function can be solved in polynomial time.
- We conjecture that under the Borda scoring function, the problem of finding an optimal strategy is NP-hard.

Conclusion

- We proposed a parallel elicitation-free protocol for allocating indivisible goods.
- All-reporting policy outperforms the optimal sequential policies in our test cases.
- We addressed some strategy issues when manipulator is pessimistic.

Future Work

- Proof of the conjectures about the social welfare induced by All-reporting policy.
- Manipulation when the manipulator is risk neutral or risk seeking.
- Other parallel policies that can outperform All-reporting policy in some social welfare criteria.
- Elicitation-free protocol for allocating sharable goods.