

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Integration

Exercise A, Question 1

Question:

Evaluate the following definite integrals:

$$(a) \int_1^2 \left(\frac{2}{x^3} + 3x \right) dx$$

$$(b) \int_0^2 (2x^3 - 4x + 5) dx$$

$$(c) \int_4^9 \left(\sqrt{x} - \frac{6}{x^2} \right) dx$$

$$(d) \int_1^2 \left(6x - \frac{12}{x^4} + 3 \right) dx$$

$$(e) \int_1^8 \left(x^{-\frac{1}{3}} + 2x - 1 \right) dx$$

Solution:

$$\begin{aligned} (a) \int_1^2 \left(\frac{2}{x^3} + 3x \right) dx &= \int_1^2 (2x^{-3} + 3x) dx \\ &= \left[\frac{2x^{-2}}{-2} + \frac{3x^2}{2} \right]_1^2 \\ &= \left[-x^{-2} + \frac{3}{2}x^2 \right]_1^2 \\ &= \left(-\frac{1}{4} + \frac{3}{2} \times 4 \right) - \left(-1 + \frac{3}{2} \right) \\ &= \left(-\frac{1}{4} + 6 \right) - \frac{1}{2} \\ &= 5 \frac{1}{4} \end{aligned}$$

$$\begin{aligned} (b) \int_0^2 (2x^3 - 4x + 5) dx &= \left[\frac{2x^4}{4} - \frac{4x^2}{2} + 5x \right]_0^2 \\ &= \left[\frac{x^4}{2} - 2x^2 + 5x \right]_0^2 \end{aligned}$$

$$= \left(\frac{16}{2} - 2 \times 4 + 10 \right) - \left(0 \right)$$

$$= 8 - 8 + 10$$

$$= 10$$

$$(c) \int_4^9 \left(\sqrt{x} - \frac{6}{x^2} \right) dx$$

$$= \int_4^9 \left(x^{\frac{1}{2}} - 6x^{-2} \right) dx$$

$$= \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{6x^{-1}}{-1} \right]_4^9$$

$$= \left[\frac{2}{3}x^{\frac{3}{2}} + 6x^{-1} \right]_4^9$$

$$= \left(\frac{2}{3} \times 9^{\frac{3}{2}} + \frac{6}{9} \right) - \left(\frac{2}{3} \times 4^{\frac{3}{2}} + \frac{6}{4} \right)$$

$$= \left(\frac{2}{3} \times 3^3 + \frac{2}{3} \right) - \left(\frac{2}{3} \times 2^3 + \frac{3}{2} \right)$$

$$= 18 + \frac{2}{3} - \frac{16}{3} - \frac{3}{2}$$

$$= 16 \frac{1}{2} - \frac{14}{3}$$

$$= 11 \frac{5}{6}$$

$$(d) \int_1^2 \left(6x - \frac{12}{x^4} + 3 \right) dx$$

$$= \int_1^2 (6x - 12x^{-4} + 3) dx$$

$$= \left[\frac{6x^2}{2} - \frac{12x^{-3}}{-3} + 3x \right]_1^2$$

$$= [3x^2 + 4x^{-3} + 3x]_1^2$$

$$= \left(3 \times 4 + \frac{4}{8} + 6 \right) - \left(3 + 4 + 3 \right)$$

$$= 12 + \frac{1}{2} + 6 - 10$$

$$= 8 \frac{1}{2}$$

$$(e) \int_1^8 \left(x^{-\frac{1}{3}} + 2x - 1 \right) dx$$

$$\begin{aligned} &= \left[\begin{array}{l} \frac{x}{3} + \frac{2x^2}{2} - x \\ \frac{2}{3} \end{array} \right]_1^8 \\ &= \left[\begin{array}{l} \frac{3}{2}x^{\frac{2}{3}} + x^2 - x \\ 1 \end{array} \right]_1^8 \\ &= \left(\frac{3}{2} \times 2^2 + 64 - 8 \right) - \left(\frac{3}{2} + 1 - 1 \right) \\ &= 62 - \frac{3}{2} \\ &= 60 \frac{1}{2} \end{aligned}$$

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Edexcel Modular Mathematics for AS and A-Level

Integration

Exercise A, Question 2

Question:

Evaluate the following definite integrals:

$$(a) \int_1^3 \left(\frac{x^3 + 2x^2}{x} \right) dx$$

$$(b) \int_1^4 (\sqrt{x} - 3)^2 dx$$

$$(c) \int_3^6 \left(x - \frac{3}{x} \right)^2 dx$$

$$(d) \int_0^1 x^2 \left(\sqrt{x} + \frac{1}{x} \right) dx$$

$$(e) \int_1^4 \frac{2 + \sqrt{x}}{x^2} dx$$

Solution:

$$\begin{aligned} (a) \int_1^3 \left(\frac{x^3 + 2x^2}{x} \right) dx &= \int_1^3 (x^2 + 2x) dx \\ &= \left[\frac{x^3}{3} + x^2 \right]_1^3 \\ &= \left(\frac{27}{3} + 9 \right) - \left(\frac{1}{3} + 1 \right) \\ &= 18 - \frac{4}{3} \\ &= 16 \frac{2}{3} \end{aligned}$$

$$\begin{aligned} (b) \int_1^4 (\sqrt{x} - 3)^2 dx &= \int_1^4 (x - 6\sqrt{x} + 9) dx \\ &= \int_1^4 \left(x - 6x^{\frac{1}{2}} + 9 \right) dx \\ &= \left[\frac{x^2}{2} - \frac{6x^{\frac{3}{2}}}{\frac{3}{2}} + 9x \right]_1^4 \end{aligned}$$

$$\begin{aligned}
&= \left[\frac{x^2}{2} - 4x^{\frac{3}{2}} + 9x \right]_1^4 \\
&= \left(\frac{16}{2} - 4 \times 2^3 + 36 \right) - \left(\frac{1}{2} - 4 + 9 \right) \\
&= 8 - 32 + 36 - 5 \frac{1}{2} \\
&= 12 - 5 \frac{1}{2} \\
&= 6 \frac{1}{2}
\end{aligned}$$

$$\begin{aligned}
\text{(c)} \int_3^6 \left(x - \frac{3}{x} \right)^2 dx \\
&= \int_3^6 \left(x^2 - 6 + \frac{9}{x^2} \right) dx \\
&= \int_3^6 (x^2 - 6 + 9x^{-2}) dx \\
&= \left[\frac{x^3}{3} - 6x + \frac{9x^{-1}}{-1} \right]_3^6 \\
&= \left[\frac{x^3}{3} - 6x - 9x^{-1} \right]_3^6 \\
&= \left(\frac{216}{3} - 36 - \frac{9}{6} \right) - \left(\frac{27}{3} - 18 - \frac{9}{3} \right) \\
&= 72 - 36 - \frac{3}{2} - 9 + 18 + 3 \\
&= 48 - \frac{3}{2} \\
&= 46 \frac{1}{2}
\end{aligned}$$

$$\begin{aligned}
\text{(d)} \int_0^1 x^2 \left(\sqrt{x} + \frac{1}{x} \right) dx \\
&= \int_0^1 \left(x^{\frac{5}{2}} + x \right) dx \\
&= \left[\frac{x^{\frac{7}{2}}}{\frac{7}{2}} + \frac{x^2}{2} \right]_0^1 \\
&= \left[\frac{2}{7} x^{\frac{7}{2}} + \frac{x^2}{2} \right]_0^1 \\
&= \left(\frac{2}{7} + \frac{1}{2} \right) - \left(0 \right) \\
&= \frac{4}{14} + \frac{7}{14}
\end{aligned}$$

$$= \frac{11}{14}$$

$$\begin{aligned}
 \text{(e) } & \int_1^4 \left(\frac{2 + \sqrt{x}}{x^2} \right) dx \\
 &= \int_1^4 \left(\frac{2}{x^2} + \frac{1}{x^{\frac{3}{2}}} \right) dx \\
 &= \int_1^4 \left(2x^{-2} + x^{-\frac{3}{2}} \right) dx \\
 &= \left[\frac{2x^{-1}}{-1} + \frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} \right]_1^4 \\
 &= \left[-2x^{-1} - 2x^{-\frac{1}{2}} \right]_1^4 \\
 &= \left(-\frac{2}{4} - \frac{2}{2} \right) - \left(-2 - 2 \right) \\
 &= -1\frac{1}{2} + 4 \\
 &= 2\frac{1}{2}
 \end{aligned}$$

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Exercise B, Question 1

Question:

Find the area between the curve with equation $y = f(x)$, the x -axis and the lines $x = a$ and $x = b$ in each of the following cases:

(a) $f(x) = 3x^2 - 2x + 2$; $a = 0, b = 2$

(b) $f(x) = x^3 + 4x$; $a = 1, b = 2$

(c) $f(x) = \sqrt{x} + 2x$; $a = 1, b = 4$

(d) $f(x) = 7 + 2x - x^2$; $a = -1, b = 2$

(e) $f(x) = \frac{8}{x^3} + \sqrt{x}$; $a = 1, b = 4$

Solution:

$$\begin{aligned} \text{(a) } A &= \int_0^2 (3x^2 - 2x + 2) \, dx \\ &= \left[\frac{3x^3}{3} - \frac{2x^2}{2} + 2x \right]_0^2 \\ &= [x^3 - x^2 + 2x]_0^2 \\ &= (8 - 4 + 4) - (0) \\ &= 8 \end{aligned}$$

$$\begin{aligned} \text{(b) } A &= \int_1^2 (x^3 + 4x) \, dx \\ &= \left[\frac{x^4}{4} + \frac{4x^2}{2} \right]_1^2 \\ &= \left(\frac{16}{4} + 2 \times 4 \right) - \left(\frac{1}{4} + 2 \right) \\ &= 4 + 8 - 2 \frac{1}{4} \\ &= 9 \frac{3}{4} \end{aligned}$$

$$\begin{aligned} \text{(c) } A &= \int_1^4 (\sqrt{x} + 2x) \, dx \\ &= \int_1^4 \left(x^{\frac{1}{2}} + 2x \right) \, dx \\ &= \left[\frac{\frac{3}{2}}{\frac{3}{2}} + x^2 \right]_1^4 \\ &= \left[\frac{3}{2} + x^2 \right]_1^4 \end{aligned}$$

$$\begin{aligned}
&= \left[\frac{2}{3}x^{\frac{3}{2}} + x^2 \right]_1^4 \\
&= \left(\frac{2}{3} \times 2^3 + 16 \right) - \left(\frac{2}{3} + 1 \right) \\
&= \frac{16}{3} + 16 - \frac{2}{3} - 1 \\
&= 15 + \frac{14}{3} \\
&= 19 \frac{2}{3}
\end{aligned}$$

$$\begin{aligned}
\text{(d)} \quad A &= \int_{-1}^2 (7 + 2x - x^2) \, dx \\
&= \left[7x + x^2 - \frac{x^3}{3} \right]_{-1}^2 \\
&= \left(14 + 4 - \frac{8}{3} \right) - \left(-7 + 1 + \frac{1}{3} \right) \\
&= 18 - \frac{8}{3} + 6 - \frac{1}{3} \\
&= 24 - \frac{9}{3} \\
&= 21
\end{aligned}$$

$$\begin{aligned}
\text{(e)} \quad A &= \int_1^4 \left(\frac{8}{x^3} + \sqrt{x} \right) \, dx \\
&= \int_1^4 \left(8x^{-3} + x^{\frac{1}{2}} \right) \, dx \\
&= \left[\frac{8x^{-2}}{-2} + \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_1^4 \\
&= \left[-4x^{-2} + \frac{2}{3}x^{\frac{3}{2}} \right]_1^4 \\
&= \left(-\frac{4}{16} + \frac{2}{3} \times 2^3 \right) - \left(-4 + \frac{2}{3} \right) \\
&= -\frac{1}{4} + \frac{16}{3} + 4 - \frac{2}{3} \\
&= 3 \frac{3}{4} + 4 \frac{2}{3} \\
&= 8 \frac{5}{12}
\end{aligned}$$

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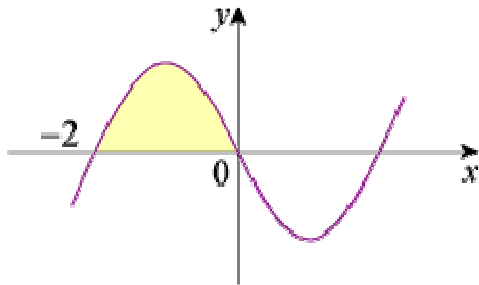
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Exercise B, Question 2

Question:

The sketch shows part of the curve with equation $y = x(x^2 - 4)$.
Find the area of the shaded region.



Solution:

$$\begin{aligned}
 A &= \int_{-2}^0 x(x^2 - 4) \, dx \\
 &= \int_{-2}^0 (x^3 - 4x) \, dx \\
 &= \left[\frac{x^4}{4} - \frac{4x^2}{2} \right]_{-2}^0 \\
 &= \left[\frac{x^4}{4} - 2x^2 \right]_{-2}^0 \\
 &= \left(0 \right) - \left(\frac{16}{4} - 2 \times 4 \right) \\
 &= -4 + 8 \\
 &= 4
 \end{aligned}$$

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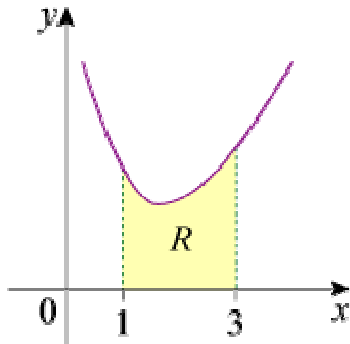
Integration

Exercise B, Question 3

Question:

The diagram shows a sketch of the curve with equation $y = 3x + \frac{6}{x^2} - 5$, $x > 0$.

The region R is bounded by the curve, the x -axis and the lines $x = 1$ and $x = 3$. Find the area of R .



Solution:

$$\begin{aligned}
 A &= \int_1^3 \left(3x + \frac{6}{x^2} - 5 \right) dx \\
 &= \int_1^3 (3x + 6x^{-2} - 5) dx \\
 &= \left[\frac{3x^2}{2} + \frac{6x^{-1}}{-1} - 5x \right]_1^3 \\
 &= \left[\frac{3}{2}x^2 - 6x^{-1} - 5x \right]_1^3 \\
 &= \left(\frac{3}{2} \times 9 - \frac{6}{3} - 15 \right) - \left(\frac{3}{2} - 6 - 5 \right) \\
 &= \frac{27}{2} - 17 - \frac{3}{2} + 11 \\
 &= \frac{24}{2} - 6 \\
 &= 6
 \end{aligned}$$

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Exercise B, Question 4

Question:

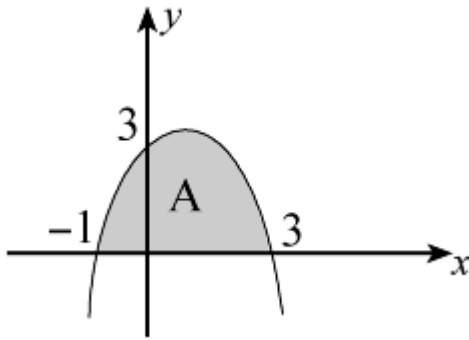
Find the area of the finite region between the curve with equation $y = (3 - x)(1 + x)$ and the x -axis.

Solution:

$y = (3 - x)(1 + x)$ is \cap shaped

$$y = 0 \Rightarrow x = 3, -1$$

$$x = 0 \Rightarrow y = 3$$



$$\begin{aligned} A &= \int_{-1}^3 (3 - x)(1 + x) \, dx \\ &= \int_{-1}^3 (3 + 2x - x^2) \, dx \\ &= \left[3x + x^2 - \frac{x^3}{3} \right]_{-1}^3 \\ &= \left(9 + 9 - \frac{27}{3} \right) - \left(-3 + 1 + \frac{1}{3} \right) \\ &= 9 + 1 \frac{2}{3} \\ &= 10 \frac{2}{3} \end{aligned}$$

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Exercise B, Question 5

Question:

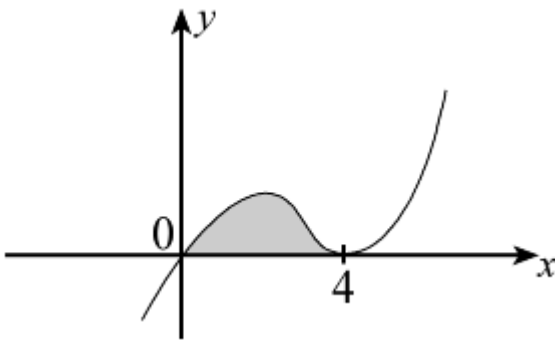
Find the area of the finite region between the curve with equation $y = x(x - 4)^2$ and the x -axis.

Solution:

$$y = x(x - 4)^2$$

$$y = 0 \Rightarrow x = 0, 4 \text{ (twice)}$$

Turning point at $(4, 0)$



$$\text{Area} = \int_0^4 x(x - 4)^2 dx$$

$$= \int_0^4 x(x^2 - 8x + 16) dx$$

$$= \int_0^4 (x^3 - 8x^2 + 16x) dx$$

$$= \left[\frac{x^4}{4} - \frac{8x^3}{3} + 8x^2 \right]_0^4$$

$$= \left(64 - \frac{8}{3} \times 64 + 128 \right) - \left(0 \right)$$

$$= \frac{64}{3} \text{ or } 21 \frac{1}{3}$$

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Exercise B, Question 6

Question:

Find the area of the finite region between the curve with equation $y = x^2(2 - x)$ and the x -axis.

Solution:

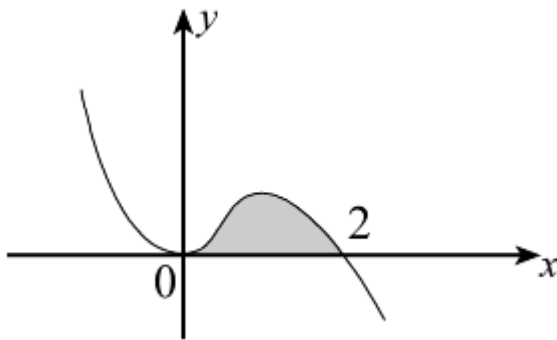
$$y = x^2(2 - x)$$

$$y = 0 \Rightarrow x = 0 \text{ (twice), } 2$$

Turning point at $(0, 0)$

$$x \rightarrow -\infty, y \rightarrow \infty$$

$$x \rightarrow \infty, y \rightarrow -\infty$$



$$\text{Area} = \int_0^2 x^2(2 - x) \, dx$$

$$= \int_0^2 (2x^2 - x^3) \, dx$$

$$= \left[\frac{2x^3}{3} - \frac{x^4}{4} \right]_0^2$$

$$= \left(\frac{16}{3} - \frac{16}{4} \right) - \left(0 \right)$$

$$= \frac{4}{3} \text{ or } 1 \frac{1}{3}$$

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Exercise C, Question 1

Question:

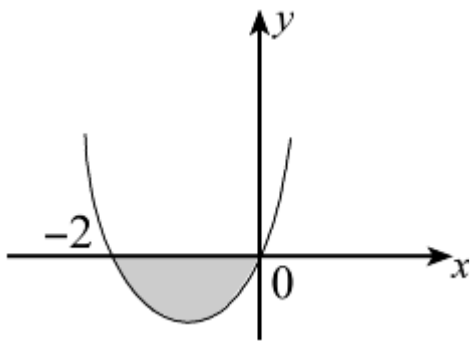
Sketch the following and find the area of the finite region or regions bounded by the curve and the x -axis:

$$y = x(x + 2)$$

Solution:

$y = x(x + 2)$ is \cup shaped

$$y = 0 \Rightarrow x = 0, -2$$



$$\begin{aligned} \text{Area} &= - \int_{-2}^0 x(x + 2) \, dx \\ &= - \int_{-2}^0 (x^2 + 2x) \, dx \\ &= - \left[\frac{x^3}{3} + x^2 \right]_{-2}^0 \\ &= - \left\{ \left(0 \right) - \left(-\frac{8}{3} + 4 \right) \right\} \\ &= - \left(-\frac{4}{3} \right) \\ &= \frac{4}{3} \text{ or } 1\frac{1}{3} \end{aligned}$$

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Exercise C, Question 2

Question:

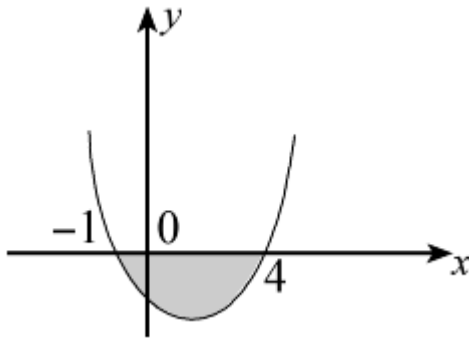
Sketch the following and find the area of the finite region or regions bounded by the curve and the x -axis:

$$y = (x + 1)(x - 4)$$

Solution:

$y = (x + 1)(x - 4)$ is \cup shaped

$$y = 0 \Rightarrow x = -1, 4$$



$$\begin{aligned} & \int_{-1}^4 (x + 1)(x - 4) \, dx \\ &= \int_{-1}^4 (x^2 - 3x - 4) \, dx \\ &= \left[\frac{x^3}{3} - \frac{3x^2}{2} - 4x \right]_{-1}^4 \\ &= \left(\frac{64}{3} - \frac{3}{2} \times 16 - 16 \right) - \left(-\frac{1}{3} - \frac{3}{2} + 4 \right) \\ &= \frac{64}{3} - 40 + \frac{11}{6} - 4 \\ &= -20 \frac{5}{6} \end{aligned}$$

$$\text{So area} = 20 \frac{5}{6}$$

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Exercise C, Question 3

Question:

Sketch the following and find the area of the finite region or regions bounded by the curve and the x -axis:

$$y = (x + 3)x(x - 3)$$

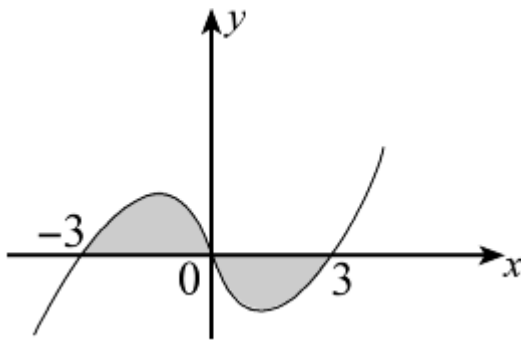
Solution:

$$y = (x + 3)x(x - 3)$$

$$y = 0 \Rightarrow x = -3, 0, 3$$

$$x \rightarrow \infty, y \rightarrow \infty$$

$$x \rightarrow -\infty, y \rightarrow -\infty$$



$$\int y dx = \int (x^3 - 9x) dx = \left[\frac{x^4}{4} - \frac{9}{2}x^2 \right]$$

$$\int_{-3}^0 y dx = \left(0 \right) - \left(\frac{81}{4} - \frac{9}{2} \times 9 \right) = + \frac{81}{4}$$

$$\int_0^3 y dx = \left(\frac{81}{4} - \frac{9}{2} \times 9 \right) - \left(0 \right) = - \frac{81}{4}$$

$$\text{So area} = \frac{81}{4} + \frac{81}{4} = \frac{81}{2} \text{ or } 40 \frac{1}{2}$$

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Exercise C, Question 4

Question:

Sketch the following and find the area of the finite region or regions bounded by the curves and the x -axis:

$$y = x^2 (x - 2)$$

Solution:

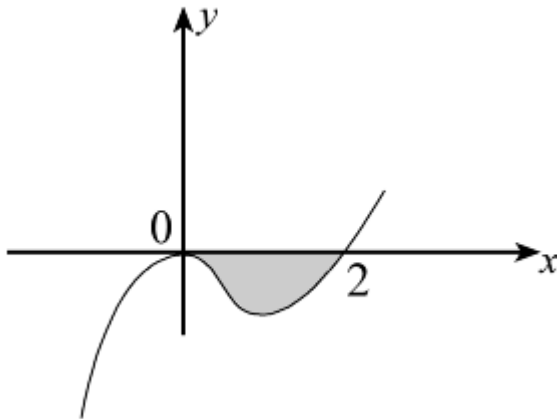
$$y = x^2 (x - 2)$$

$$y = 0 \Rightarrow x = 0 \text{ (twice), } 2$$

Turning point at $(0, 0)$

$$x \rightarrow \infty, y \rightarrow \infty$$

$$x \rightarrow -\infty, y \rightarrow -\infty$$



$$\text{Area} = - \int_0^2 x^2 (x - 2) \, dx$$

$$= - \int_0^2 (x^3 - 2x^2) \, dx$$

$$= - \left[\frac{x^4}{4} - \frac{2}{3}x^3 \right]_0^2$$

$$= - \left\{ \left(\frac{16}{4} - \frac{2}{3} \times 8 \right) - \left(0 \right) \right\}$$

$$= - \left(4 - \frac{16}{3} \right)$$

$$= \frac{4}{3} \text{ or } 1 \frac{1}{3}$$

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Exercise C, Question 5

Question:

Sketch the following and find the area of the finite region or regions bounded by the curve and the x -axis:

$$y = x(x - 2)(x - 5)$$

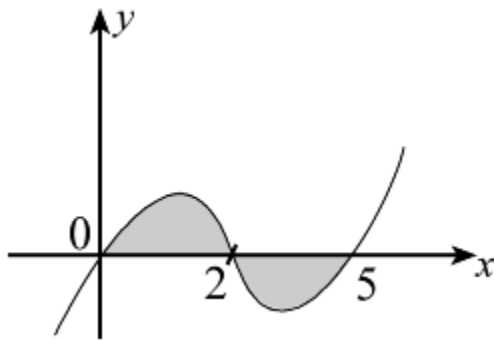
Solution:

$$y = x(x - 2)(x - 5)$$

$$y = 0 \Rightarrow x = 0, 2, 5$$

$$x \rightarrow \infty, y \rightarrow \infty$$

$$x \rightarrow -\infty, y \rightarrow -\infty$$



$$\int y dx = \int x(x^2 - 7x + 10) dx = \int (x^3 - 7x^2 + 10x) dx$$

$$\int y dx = \left[\frac{x^4}{4} - \frac{7}{3}x^3 + 5x^2 \right]$$

$$\int_0^2 y dx = \left(\frac{16}{4} - \frac{7}{3} \times 8 + 20 \right) - \left(0 \right) = 24 - \frac{56}{3} = 5 \frac{1}{3}$$

$$\int_2^5 y dx = \left(\frac{625}{4} - \frac{7}{3} \times 125 + 125 \right) - \left(5 \frac{1}{3} \right) = -15 \frac{3}{4}$$

$$\text{So area} = 5 \frac{1}{3} + 15 \frac{3}{4} = 21 \frac{1}{12}$$

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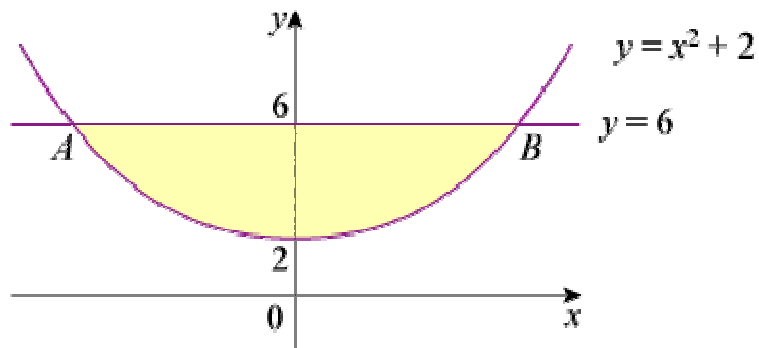
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Exercise D, Question 1

Question:

The diagram shows part of the curve with equation $y = x^2 + 2$ and the line with equation $y = 6$. The line cuts the curve at the points A and B .



- Find the coordinates of the points A and B .
- Find the area of the finite region bounded by AB and the curve.

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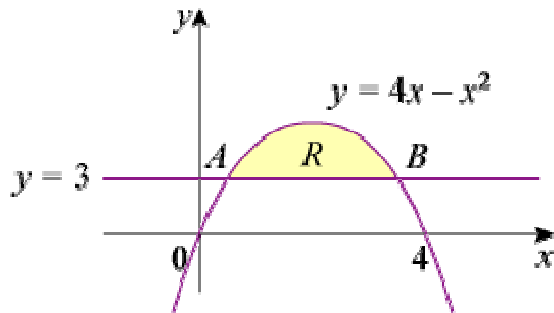
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Exercise D, Question 2

Question:

The diagram shows the finite region, R , bounded by the curve with equation $y = 4x - x^2$ and the line $y = 3$. The line cuts the curve at the points A and B .



- (a) Find the coordinates of the points A and B .
- (b) Find the area of R .

Solution:

(a) A, B are given by

$$3 = 4x - x^2$$

$$x^2 - 4x + 3 = 0$$

$$(x - 3)(x - 1) = 0$$

$$x = 1, 3$$

So A is $(1, 3)$ and B is $(3, 3)$

$$(b) \text{ Area} = \int_1^3 [(4x - x^2) - 3] dx$$

$$= \int_1^3 (4x - x^2 - 3) dx$$

$$= \left[2x^2 - \frac{x^3}{3} - 3x \right]_1^3$$

$$= \left(18 - 9 - 9 \right) - \left(2 - \frac{1}{3} - 3 \right)$$

$$= 1 \frac{1}{3}$$

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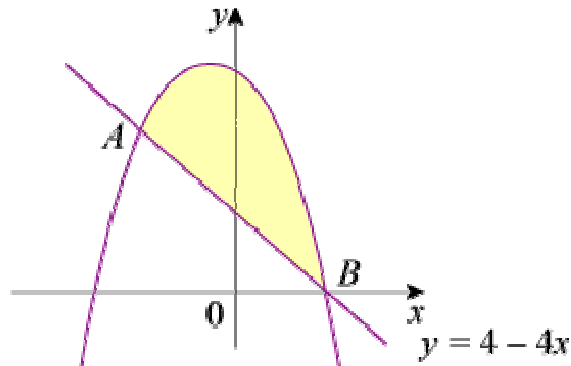
Edexcel Modular Mathematics for AS and A-Level

Integration

Exercise D, Question 3

Question:

The diagram shows a sketch of part of the curve with equation $y = 9 - 3x - 5x^2 - x^3$ and the line with equation $y = 4 - 4x$. The line cuts the curve at the points $A(-1, 8)$ and $B(1, 0)$.



Find the area of the shaded region between AB and the curve.

Solution:

$$\begin{aligned}
 \text{Area} &= \int_{-1}^1 (\text{curve} - \text{line}) \, dx \\
 &= \int_{-1}^1 [9 - 3x - 5x^2 - x^3 - (4 - 4x)] \, dx \\
 &= \int_{-1}^1 (5 + x - 5x^2 - x^3) \, dx \\
 &= \left[5x + \frac{x^2}{2} - \frac{5}{3}x^3 - \frac{x^4}{4} \right]_{-1}^1 \\
 &= \left(5 + \frac{1}{2} - \frac{5}{3} - \frac{1}{4} \right) - \left(-5 + \frac{1}{2} + \frac{5}{3} - \frac{1}{4} \right) \\
 &= 10 - \frac{10}{3} \\
 &= \frac{20}{3} \text{ or } 6\frac{2}{3}
 \end{aligned}$$

Solutionbank C2

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Integration

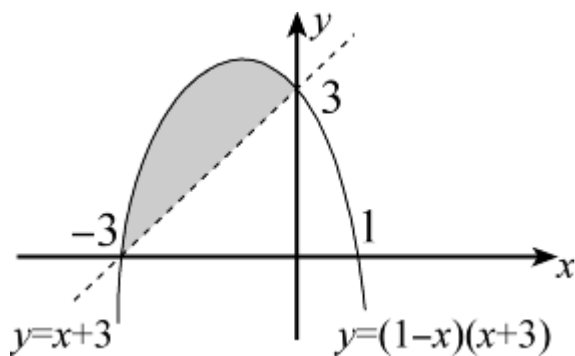
Exercise D, Question 4

Question:

Find the area of the finite region bounded by the curve with equation $y = (1 - x)(x + 3)$ and the line $y = x + 3$.

Solution:

$y = (1 - x)(x + 3)$ is \cap shaped and crosses the x -axis at $(1, 0)$ and $(-3, 0)$
 $y = x + 3$ is a straight line passing through $(-3, 0)$ and $(0, 3)$



Intersections when

$$x + 3 = (1 - x)(x + 3)$$

$$0 = (x + 3)(1 - x - 1)$$

$$0 = -x(x + 3)$$

$$x = -3 \text{ or } 0$$

$$\text{Area} = \int_{-3}^0 [(1 - x)(x + 3) - (x + 3)] dx$$

$$= \int_{-3}^0 (-x^2 - 3x) dx$$

$$= \left[-\frac{x^3}{3} - \frac{3}{2}x^2 \right]_{-3}^0$$

$$= \left(0 \right) - \left(\frac{27}{3} - \frac{27}{2} \right)$$

$$= \frac{27}{6} \text{ or } \frac{9}{2} \text{ or } 4.5$$

Solutionbank C2

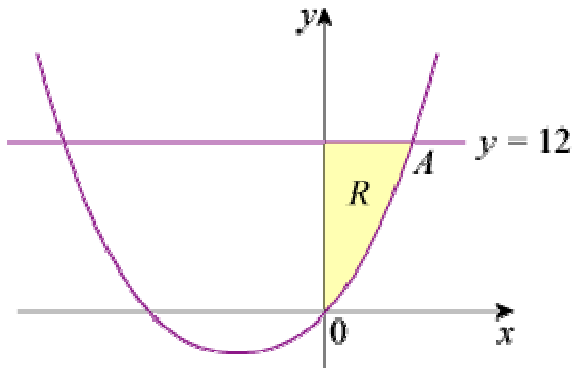
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Integration

Exercise D, Question 5

Question:

The diagram shows the finite region, R , bounded by the curve with equation $y = x(4 + x)$, the line with equation $y = 12$ and the y -axis.



(a) Find the coordinate of the point A where the line meets the curve.

(b) Find the area of R .

Solution:

(a) A is given by

$$x(4 + x) = 12$$

$$x^2 + 4x - 12 = 0$$

$$(x + 6)(x - 2) = 0$$

$$x = 2 \text{ or } -6$$

So A is $(2, 12)$

(b) R is given by taking $\int_0^2 x(4 + x) dx$ away from a rectangle of area $12 \times 2 = 24$.

So area of R

$$= 24 - \int_0^2 (x^2 + 4x) dx$$

$$= 24 - \left[\frac{x^3}{3} + 2x^2 \right]_0^2$$

$$= 24 - \left\{ \left(\frac{8}{3} + 8 \right) - \left(0 \right) \right\}$$

$$= 24 - \frac{32}{3}$$

$$= \frac{40}{3} \text{ or } 13 \frac{1}{3}$$

Solutionbank C2

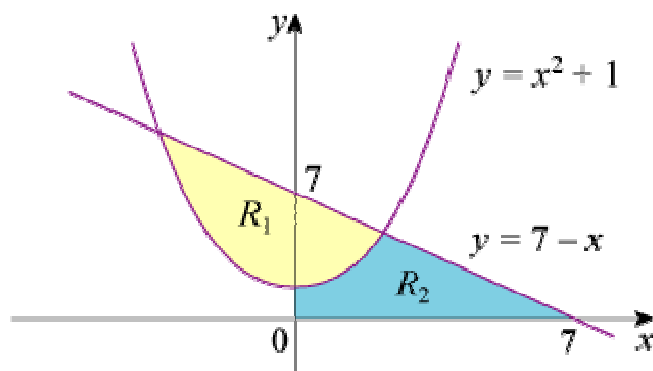
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Integration

Exercise D, Question 6

Question:

The diagram shows a sketch of part of the curve with equation $y = x^2 + 1$ and the line with equation $y = 7 - x$. The finite region R_1 is bounded by the line and the curve. The finite region R_2 is below the curve and the line and is bounded by the positive x - and y -axes as shown in the diagram.



(a) Find the area of R_1 .

(b) Find the area of R_2 .

Solution:

(a) Intersections when

$$7 - x = x^2 + 1$$

$$0 = x^2 + x - 6$$

$$0 = (x + 3)(x - 2)$$

$$x = 2 \text{ or } -3$$

(a) Area of R_1 is given by $\int_{-3}^2 [7 - x - (x^2 + 1)] dx$

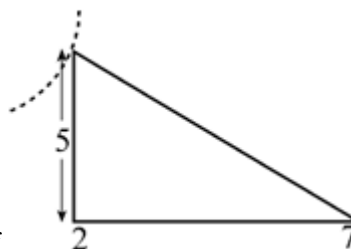
$$= \int_{-3}^2 (6 - x - x^2) dx$$

$$= \left[6x - \frac{x^2}{2} - \frac{x^3}{3} \right]_{-3}^2$$

$$= \left(12 - \frac{4}{2} - \frac{8}{3} \right) - \left(-18 - \frac{9}{2} + \frac{27}{3} \right)$$

$$= 20 \frac{5}{6}$$

(b) Area of R_2 is given by $\int_0^2 (x^2 + 1) dx$ + area of



$$\begin{aligned} &= \left[\frac{x^3}{3} + x \right]_0^2 + \frac{1}{2} \times 5 \times 5 \\ &= \left(\frac{8}{3} + 2 \right) - \left(0 \right) + \frac{25}{2} \\ &= 17 \frac{1}{6} \end{aligned}$$

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Exercise D, Question 7

Question:

The curve C has equation $y = x^{\frac{2}{3}} - \frac{2}{x^{\frac{1}{3}}} + 1$.

- (a) Verify that C crosses the x -axis at the point $(1, 0)$.
- (b) Show that the point $A(8, 4)$ also lies on C .
- (c) The point B is $(4, 0)$. Find the equation of the line through AB .
The finite region R is bounded by C , AB and the positive x -axis.
- (d) Find the area of R .

Solution:

(a) $x = 1, y = 1 - \frac{2}{1} + 1 = 0$

So $(1, 0)$ lies on C

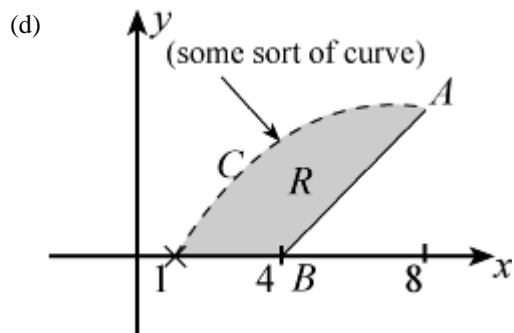
(b) $x = 8, y = 8^{\frac{2}{3}} - \frac{2}{8^{\frac{1}{3}}} + 1 = 2^2 - \frac{2}{2} + 1 = 4$

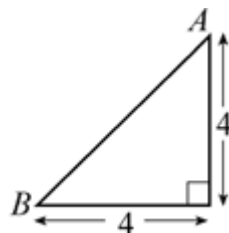
So $(8, 4)$ lies on C

(c) A is $(8, 4)$ and B is $(4, 0)$

Gradient of line through AB is $\frac{4-0}{8-4} = 1$.

So equation is $y - 0 = x - 4$, i.e. $y = x - 4$





The area of R is given by $\int_1^8 (\text{curve}) \, dx - \text{area of}$

$$\begin{aligned}
 &= \int_1^8 \left(x^{\frac{2}{3}} - \frac{2}{x^{\frac{1}{3}}} + 1 \right) dx - \frac{1}{2} \times 4 \times 4 \\
 &= \left[\frac{3}{5} x^{\frac{5}{3}} - \frac{2x^{\frac{2}{3}}}{\frac{2}{3}} + x \right]_1^8 - 8 \\
 &= \left(\frac{3}{5} \times 32 - 3 \times 4 + 8 \right) - \left(\frac{3}{5} - 3 + 1 \right) - 8 \\
 &= \frac{93}{5} - 4 + 2 - 8 \\
 &= 8 \frac{3}{5}
 \end{aligned}$$

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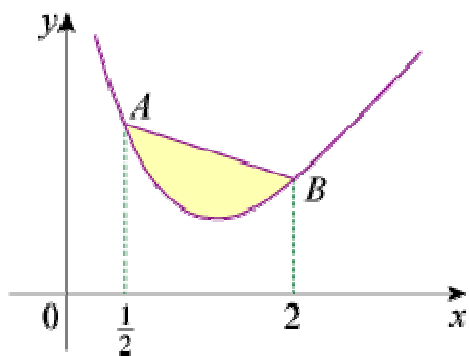
Integration

Exercise D, Question 8

Question:

The diagram shows part of a sketch of the curve with equation $y = \frac{2}{x^2} + x$.

The points A and B have x -coordinates $\frac{1}{2}$ and 2 respectively.



Find the area of the finite region between AB and the curve.

Solution:

$$\text{Area} = \int_{\frac{1}{2}}^2 \left[\text{line } AB - \left(\frac{2}{x^2} + x \right) \right] dx$$

$$A \text{ is } \left(\frac{1}{2}, 8\frac{1}{2} \right) \text{ and } B \text{ is } \left(2, 2\frac{1}{2} \right)$$

$$\text{Gradient} = -\frac{6}{1\frac{1}{2}} = -4$$

$$\text{So equation is } y - 2\frac{1}{2} = -4 \left(x - 2 \right), \text{ i.e. } y = 10\frac{1}{2} - 4x$$

$$\begin{aligned} \text{Area} &= \int_{\frac{1}{2}}^2 \left(10\frac{1}{2} - 5x - 2x^{-2} \right) dx \\ &= \left[\frac{21}{2}x - \frac{5}{2}x^2 - \frac{2x^{-1}}{-1} \right]_{\frac{1}{2}}^2 \\ &= \left[\frac{21}{2}x - \frac{5}{2}x^2 + \frac{2}{x} \right]_{\frac{1}{2}}^2 \\ &= \left(21 - 10 + 1 \right) - \left(\frac{21}{4} - \frac{5}{8} + 4 \right) \end{aligned}$$

$$= 12 - 8 \frac{5}{8}$$

$$= 3 \frac{3}{8} \text{ or } 3.375 \text{ or } 3.38 \text{ (3 s.f.)}$$

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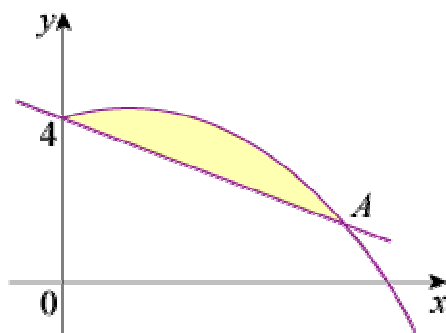
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Integration

Exercise D, Question 9

Question:

The diagram shows part of the curve with equation $y = 3\sqrt{x} - \sqrt{x^3} + 4$ and the line with equation $y = 4 - \frac{1}{2}x$.



- (a) Verify that the line and the curve cross at the point $A(4, 2)$.
- (b) Find the area of the finite region bounded by the curve and the line.

Solution:

(a) $x = 4$ in line gives $y = 4 - \frac{1}{2} \times 4 = 2$

$x = 4$ in curve gives $y = 3 \times \sqrt{4} - \sqrt{64} + 4 = 6 - 8 + 4 = 2$
So $(4, 2)$ lies on line and curve.

(b) Area = $\int_0^4 \left[3x^{\frac{1}{2}} - x^{\frac{3}{2}} + 4 - \left(4 - \frac{1}{2}x \right) \right] dx$

$$= \int_0^4 \left(3x^{\frac{1}{2}} - x^{\frac{3}{2}} + \frac{1}{2}x \right) dx$$

$$= \left[\frac{3x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + \frac{x^2}{4} \right]_0^4$$

$$= \left[2x^{\frac{3}{2}} - \frac{2}{5}x^{\frac{5}{2}} + \frac{x^2}{4} \right]_0^4$$

$$= \left(2 \times 8 - \frac{2}{5} \times 32 + 4 \right) - \left(0 \right)$$

$$= 20 - \frac{64}{5}$$

$$= \frac{36}{5} \text{ or } 7.2$$

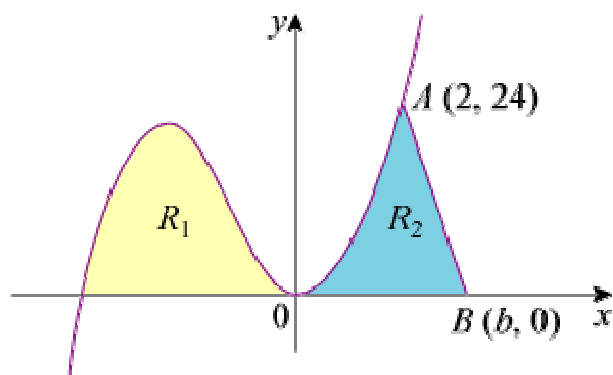
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Integration

Exercise D, Question 10

Question:



The sketch shows part of the curve with equation $y = x^2(x + 4)$. The finite region R_1 is bounded by the curve and the negative x -axis. The finite region R_2 is bounded by the curve, the positive x -axis and AB , where $A(2, 24)$ and $B(b, 0)$.

The area of $R_1 =$ the area of R_2 .

(a) Find the area of R_1 .

(b) Find the value of b .

Solution:

$$(a) y = x^2(x + 4)$$

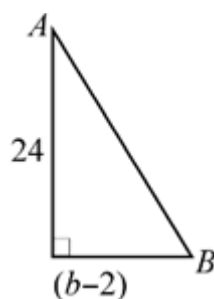
$$y = 0 \Rightarrow x = 0 \text{ (twice), } -4$$

$$\text{Area of } R_1 \text{ is } \int_{-4}^0 (x^3 + 4x^2) dx$$

$$= \left[\frac{x^4}{4} + \frac{4}{3}x^3 \right]_{-4}^0$$

$$= \left(0 \right) - \left(\frac{4^4}{4} - \frac{4^4}{3} \right)$$

$$= \frac{4^4}{12} = \frac{4^3}{3} = \frac{64}{3} \text{ or } 21 \frac{1}{3}$$



(b) Area of R_2 is $\int_0^2 (x^3 + 4x^2) dx +$ area of

$$\begin{aligned} &= \left[\frac{x^4}{4} + \frac{4}{3}x^3 \right]_0^2 + 12 \left(b - 2 \right) \\ &= \left(\frac{16}{4} + \frac{32}{3} \right) - \left(0 \right) + 12 \left(b - 2 \right) \\ &= 14 \frac{2}{3} + 12b - 24 \\ &= -9 \frac{1}{3} + 12b \end{aligned}$$

$$\text{Area of } R_2 = \text{area of } R_1 \Rightarrow -9 \frac{1}{3} + 12b = 21 \frac{1}{3}$$

$$\text{So } 12b = 30 \frac{2}{3} \Rightarrow b = 2 \frac{5}{9} \text{ or } 2.56 \text{ (3 s.f.)}$$

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Integration

Exercise E, Question 1

Question:

Copy and complete the table below and use the trapezium rule to estimate $\int_1^3 \frac{1}{x^2+1} dx$:

x	1	1.5	2	2.5	3
$y = \frac{1}{x^2+1}$	0.5	0.308		0.138	

Solution:

$$x = 2, y = 0.2; x = 3, y = 0.1$$

$$h = 0.5$$

$$\text{So } A \approx \frac{1}{2} \times 0.5 \left[0.5 + 2 \left(0.308 + 0.2 + 0.138 \right) + 0.1 \right]$$

$$= \frac{1}{4} \left[1.892 \right]$$

$$= 0.473$$

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Exercise E, Question 2

Question:

Use the table below to estimate $\int_1^{2.5} \sqrt{2x-1} \, dx$ with the trapezium rule:

x	1	1.25	1.5	1.75	2	2.25	2.5
$y = \sqrt{2x-1}$	1	1.225	1.414	1.581	1.732	1.871	2

Solution:

$$\begin{aligned}
 A &\approx \frac{1}{2} \times 0.25 \left[1 + 2 \left(1.225 + 1.414 + 1.581 + 1.732 + 1.871 \right) + 2 \right] \\
 &= \frac{1}{8} \left[18.646 \right] \\
 &= 2.33075 \\
 &= 2.33 \text{ (3 s.f.)}
 \end{aligned}$$

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Exercise E, Question 3

Question:

Copy and complete the table below and use it, together with the trapezium rule, to estimate $\int_0^2 \sqrt{x^3 + 1} \, dx$:

x	0	0.5	1	1.5	2
$y = \sqrt{x^3 + 1}$	1	1.061	1.414		

Solution:

$$x = 1.5, y = \sqrt{1.5^3 + 1} = 2.09165 \dots \text{ or } 2.092 \text{ (4 s.f.)}$$

$$x = 2, y = \sqrt{2^3 + 1} = 3$$

$$\int_0^2 \sqrt{x^3 + 1} \, dx$$

$$\approx \frac{1}{2} \times 0.5 \left[1 + 2 \left(1.061 + 1.414 + 2.092 \right) + 3 \right]$$

$$= \frac{1}{4} \left[13.134 \right]$$

$$= 3.2835$$

$$= 3.28 \text{ (3 s.f.)}$$

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Exercise E, Question 4

Question:

(a) Use the trapezium rule with 8 strips to estimate $\int_0^2 2^x dx$.

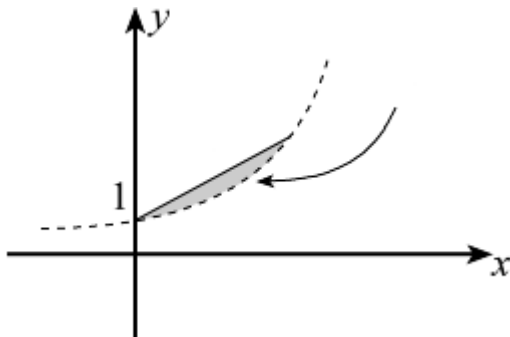
(b) With reference to a sketch of $y = 2^x$ explain whether your answer in part (a) is an underestimate or an overestimate of $\int_0^2 2^x dx$.

Solution:

$$h = 0.25$$

x	0	0.25	0.5	0.75	1.0	1.25	1.5	1.75	2.0
y	1	1.189	1.414	1.682	2	2.378	2.828	3.364	4

$$\begin{aligned} \int_0^2 2^x dx &\approx \frac{1}{2} \times 0.25 \left[1 + 2 \left(1.189 + 1.414 + 1.682 + 2 + 2.378 + 2.828 + 3.364 \right) + 4 \right] \\ &= \frac{1}{8} \left[34.71 \right] \\ &= 4.33875 \\ &= 4.34 \text{ (3 s.f.)} \end{aligned}$$



(b)

Curve bends beneath straight line of trapezium so trapezium rule will **overestimate**.

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Integration

Exercise E, Question 5

Question:

Use the trapezium rule with 6 strips to estimate $\int_0^3 \frac{1}{\sqrt{x^2+1}} dx$.

Solution:

$$h = 0.5$$

x	0	0.5	1	1.5	2	2.5	3
y	1	0.894	0.707	0.555	0.447	0.371	0.316

$$\begin{aligned}
 A &\approx \frac{1}{2} \times 0.5 \left[1 + 2 \left(0.894 + 0.707 + 0.555 + 0.447 + 0.371 \right) + 0.316 \right] \\
 &= \frac{1}{4} \left[7.264 \right] \\
 &= 1.816 \text{ or } 1.82 \text{ (3 s.f.)}
 \end{aligned}$$

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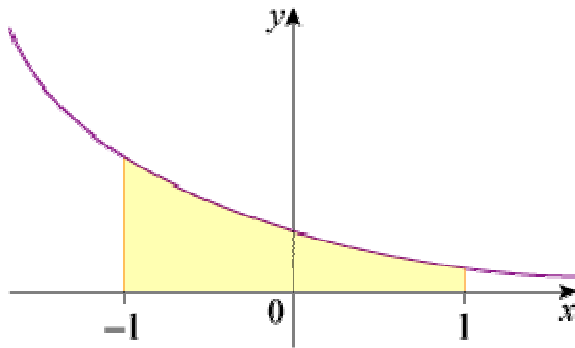
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Exercise E, Question 6

Question:

The diagram shows a sketch of part of the curve with equation $y = \frac{1}{x+2}$, $x > -2$.



(a) Copy and complete the table below and use the trapezium rule to estimate the area bounded by the curve, the x -axis and the lines $x = -1$ and $x = 1$.

x	-1	-0.6	-0.2	0.2	0.6	1
$y = \frac{1}{x+2}$	1	0.714			0.385	0.333

(b) State, with a reason, whether your answer in part (a) is an overestimate or an underestimate.

Solution:

(a) $h = 0.4$

$$x = -0.2, y = \frac{1}{1.8} = 0.555 \dots = 0.556 \text{ (3 d.p.)}$$

$$x = 0.2, y = \frac{1}{2.2} = 0.4545 \dots = 0.455 \text{ (3 d.p.)}$$

$$\text{area} \approx \frac{1}{2} \times 0.4 \left[1 + 2 \left(0.714 + 0.556 + 0.455 + 0.385 \right) + 0.333 \right]$$

$$= 0.2 [5.553]$$

$$= 1.1106$$

$$= 1.11 \text{ (3 s.f.)}$$

(b) Curve bends down below the straight lines of the trapezia so trapezium rule will give an **overestimate**.

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Integration

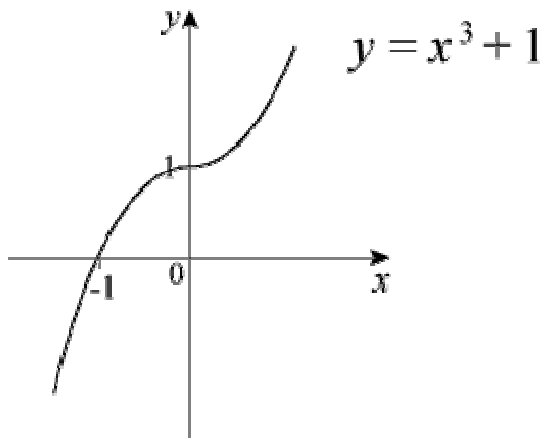
Exercise E, Question 7

Question:

- (a) Sketch the curve with equation $y = x^3 + 1$, for $-2 < x < 2$.
- (b) Use the trapezium rule with 4 strips to estimate the value of $\int_{-1}^1 (x^3 + 1) dx$.
- (c) Use integration to find the exact value of $\int_{-1}^1 (x^3 + 1) dx$.
- (d) Comment on your answers to parts (b) and (c).

Solution:

- (a) $y = x^3 + 1$ is a vertical translation (+ 1) of $y = x^3$



- (b) $h = 0.5$

$$x \quad -1 \quad -0.5 \quad 0 \quad 0.5 \quad 1$$

$$y \quad 0 \quad 0.875 \quad 1 \quad 1.125 \quad 2$$

$$\int_{-1}^1 (x^3 + 1) dx \approx \frac{1}{2} \times 0.5 \left[0 + 2 \left(0.875 + 1 + 1.125 \right) + 2 \right] = \frac{1}{4} \left[8 \right] = 2$$

$$(c) \int_{-1}^1 (x^3 + 1) dx = \left[\frac{x^4}{4} + x \right]_{-1}^1 = \left(\frac{1}{4} + 1 \right) - \left(\frac{1}{4} - 1 \right) = 2$$

- (d) Same. Curve has rotational symmetry of order 2 about (0, 1) and trapezia cut curve above and below symmetrically.

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Integration

Exercise E, Question 8

Question:

Use the trapezium rule with 4 strips to estimate $\int_0^2 \sqrt{3^x - 1} \, dx$.

Solution:

$$h = 0.5$$

$$x \quad 0 \quad 0.5 \quad 1 \quad 1.5 \quad 2$$

$$y \quad 0 \quad 0.856 \quad 1.414 \quad 2.048 \quad 2.828$$

$$\int_0^2 \sqrt{3^x - 1} \, dx \approx \frac{1}{2} \times 0.5 \left[0 + 2 \left(0.856 + 1.414 + 2.048 \right) + 2.828 \right]$$

$$= \frac{1}{4} \left[11.464 \right]$$

$$= 2.866$$

$$= 2.87 \text{ (3 s.f.)}$$

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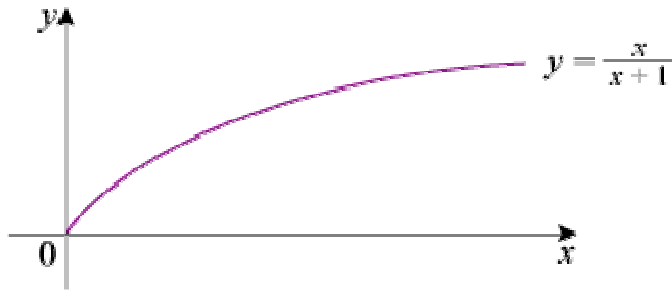
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Integration

Exercise E, Question 9

Question:

The sketch shows part of the curve with equation $y = \frac{x}{x+1}, x \geq 0$.



(a) Use the trapezium rule with 6 strips to estimate $\int_0^3 \frac{x}{x+1} dx$.

(b) With reference to the sketch state, with a reason, whether the answer in part (a) is an overestimate or an underestimate.

Solution:

(a) $h = 0.5$

x	0	0.5	1	1.5	2	2.5	3
y	0	0.333	0.5	0.6	0.667	0.714	0.75

$$\begin{aligned} \int_0^3 \frac{x}{x+1} dx &\approx \frac{1}{2} \times 0.5 \left[0 + 2 \left(0.333 + 0.5 + 0.6 + 0.667 + 0.714 \right) + 0.75 \right] \\ &= \frac{1}{4} \left[6.378 \right] \\ &= 1.5945 \\ &= 1.59 \text{ (3 s.f.)} \end{aligned}$$

(b) Curve bends outwards above straight lines of trapezia so trapezium rule is an **underestimate**.

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Exercise E, Question 10

Question:

(a) Use the trapezium rule with n strips to estimate $\int_0^2 \sqrt{x} \, dx$ in the cases (i) $n = 4$ (ii) $n = 6$.

(b) Compare your answers from part (a) with the exact value of the integral and calculate the percentage error in each case.

Solution:

(a) (i) $h = 0.5$

$$\begin{array}{l} x \quad 0 \quad 0.5 \quad 1 \quad 1.5 \quad 2 \\ y \quad 0 \quad 0.707 \quad 1 \quad 1.225 \quad 1.414 \end{array}$$

$$\int_0^2 \sqrt{x} \, dx \approx \frac{1}{2} \times 0.5 \left[0 + 2 \left(0.707 + 1 + 1.225 \right) + 1.414 \right] = \frac{1}{4} \left[7.278 \right] = 1.8195$$

(ii) $h = \frac{1}{3}$

$$\begin{array}{l} x \quad 0 \quad \frac{1}{3} \quad \frac{2}{3} \quad 1 \quad \frac{4}{3} \quad \frac{5}{3} \quad 2 \\ y \quad 0 \quad 0.577 \quad 0.816 \quad 1 \quad 1.155 \quad 1.291 \quad 1.414 \end{array}$$

$$\int_0^2 \sqrt{x} \, dx \approx \frac{1}{2} \times \frac{1}{3} \left[0 + 2 \left(0.577 + 0.816 + 1 + 1.155 + 1.291 \right) + 1.414 \right] = \frac{1}{6} \left[11.092 \right] = 1.8486$$

$$(b) \int_0^2 \sqrt{x} \, dx = \left[\frac{2}{3} x^{\frac{3}{2}} \right]_0^2 = \left(\frac{2}{3} \times 2 \sqrt{2} \right) - \left(0 \right) = \frac{4}{3} \sqrt{2} = 1.8856 \dots$$

$$(i) \% \text{ error} = \frac{100 \left(\frac{4}{3} \sqrt{2} - 1.8195 \right)}{\frac{4}{3} \sqrt{2}} = 3.51 \%$$

$$(ii) \% \text{ error} = \frac{100 \left(\frac{4}{3} \sqrt{2} - 1.8486 \right)}{\frac{4}{3} \sqrt{2}} = 1.96 \%$$

Solutionbank C2

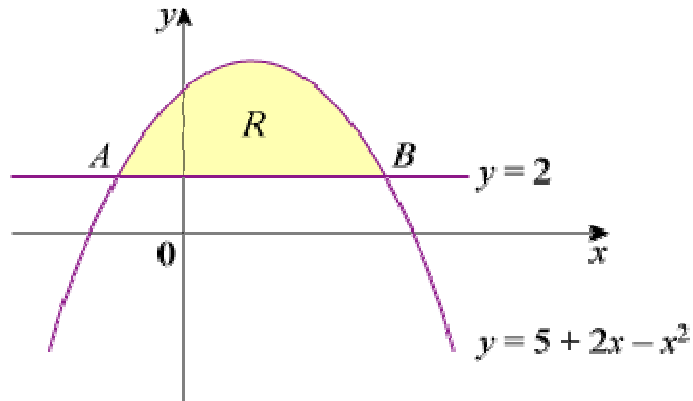
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Integration

Exercise F, Question 1

Question:

The diagram shows the curve with equation $y = 5 + 2x - x^2$ and the line with equation $y = 2$. The curve and the line intersect at the points A and B .



- (a) Find the x -coordinates of A and B .
- (b) The shaded region R is bounded by the curve and the line. Find the area of R .

[E]

Solution:

$$\begin{aligned} \text{(a)} \quad 2 &= 5 + 2x - x^2 \\ &\Rightarrow x^2 - 2x - 3 = 0 \\ &\Rightarrow (x - 3)(x + 1) = 0 \\ &\Rightarrow x = -1 \text{ (A)}, 3 \text{ (B)} \end{aligned}$$

$$\begin{aligned} \text{(b) Area of } R &= \int_{-1}^3 (5 + 2x - x^2 - 2) \, dx \\ &= \int_{-1}^3 (3 + 2x - x^2) \, dx \\ &= \left[3x + x^2 - \frac{1}{3}x^3 \right]_{-1}^3 \\ &= \left(9 + 9 - \frac{27}{3} \right) - \left(-3 + 1 + \frac{1}{3} \right) \\ &= 9 + 2 - \frac{1}{3} \\ &= 10 \frac{2}{3} \end{aligned}$$

Solutionbank C2

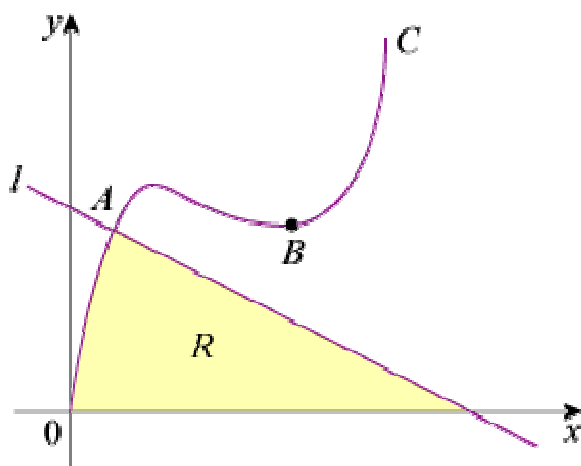
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Integration

Exercise F, Question 2

Question:

The diagram shows part of the curve C with equation $y = x^3 - 9x^2 + px$, where p is a constant. The line l has equation $y + 2x = q$, where q is a constant. The point A is the intersection of C and l , and C has a minimum at the point B . The x -coordinates of A and B are 1 and 4 respectively.



(a) Show that $p = 24$ and calculate the value of q .

(b) The shaded region R is bounded by C , l and the x -axis. Using calculus, showing all the steps in your working and using the values of p and q found in part (a), find the area of R .

[E]

Solution:

(a) When $x = 1$: $q - 2x = x^3 - 9x^2 + px$

$$\Rightarrow q - 2 = 1 - 9 + p$$

$$\Rightarrow q + 6 = p \quad \textcircled{1}$$

When $x = 4$: $\frac{dy}{dx} = 3x^2 - 18x + p = 0$

$$\Rightarrow 48 - 72 + p = 0$$

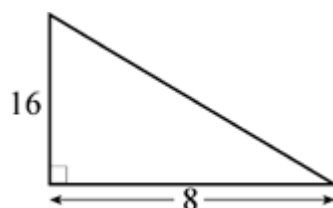
$$\Rightarrow p = 24$$

Substitute into $\textcircled{1}$: $q = p - 6 = 18$

(b) Line is $y = 18 - 2x$

So A is $(1, 16)$ and the line cuts the x -axis at $(9, 0)$

Area of R is given by



$$\int_0^1 (x^3 - 9x^2 + 24x) dx + \text{area of}$$

$$\begin{aligned} &= \left[\frac{x^4}{4} - \frac{9}{3}x^3 + \frac{24}{2}x^2 \right]_0^1 + \frac{1}{2} \times 8 \times 16 \\ &= \left[\frac{x^4}{4} - 3x^3 + 12x^2 \right]_0^1 + 64 \\ &= \left(\frac{1}{4} - 3 + 12 \right) - \left(0 \right) + 64 \\ &= 73 \frac{1}{4} \end{aligned}$$

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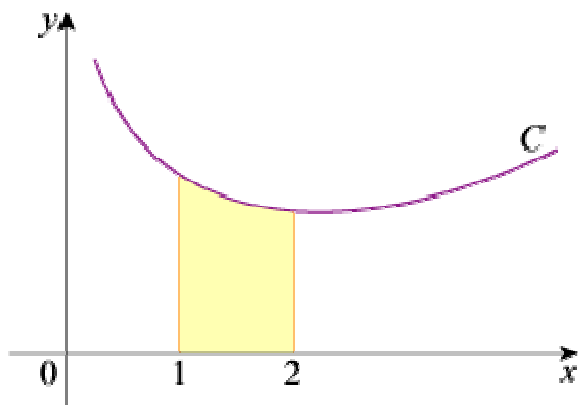
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Integration

Exercise F, Question 3

Question:

The diagram shows part of the curve C with equation $y = f(x)$, where $f(x) = 16x^{-\frac{1}{2}} + x^{\frac{3}{2}}$, $x > 0$.



(a) Use calculus to find the x -coordinate of the minimum point of C , giving your answer in the form $k\sqrt{3}$, where k is an exact fraction.

The shaded region shown in the diagram is bounded by C , the x -axis and the lines with equations $x = 1$ and $x = 2$.

(b) Using integration and showing all your working, find the area of the shaded region, giving your answer in the form $a + b\sqrt{2}$, where a and b are exact fractions.

[E]

Solution:

$$(a) f' \left(x \right) = -8x^{-\frac{3}{2}} + \frac{3}{2}x^{\frac{1}{2}}$$

$$f' \left(x \right) = 0 \Rightarrow \frac{8}{x^{\frac{3}{2}}} = \frac{3}{2}x^{\frac{1}{2}} \text{ or } x^2 = \frac{16}{3}$$

$$(x \text{ must be positive}) \text{ So } x = \frac{4}{\sqrt{3}} \text{ or } \frac{4}{3}\sqrt{3}$$

$$(b) \text{ Area} = \int_1^2 \left(16x^{-\frac{1}{2}} + x^{\frac{3}{2}} \right) dx$$

$$= \left[\frac{16x^{\frac{1}{2}}}{\frac{1}{2}} + \frac{x^{\frac{5}{2}}}{\frac{5}{2}} \right]_1^2$$

$$= \left[32x^{\frac{1}{2}} + \frac{2}{5}x^{\frac{5}{2}} \right]_1^2$$

$$\begin{aligned} &= \left(32\sqrt{2} + \frac{2}{5} \times 2^2\sqrt{2} \right) - \left(32 + \frac{2}{5} \right) \\ &= \frac{168}{5}\sqrt{2} - \frac{162}{5} \end{aligned}$$

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Integration

Exercise F, Question 4

Question:

(a) Find $\int \left(x^{\frac{1}{2}} - 4 \right) \left(x^{-\frac{1}{2}} - 1 \right) dx$.

(b) Use your answer to part (a) to evaluate

$$\int_1^4 \left(x^{\frac{1}{2}} - 4 \right) \left(x^{-\frac{1}{2}} - 1 \right) dx.$$

giving your answer as an exact fraction.

[E]

Solution:

$$(a) \left(x^{\frac{1}{2}} - 4 \right) \left(x^{-\frac{1}{2}} - 1 \right) = 1 - 4x^{-\frac{1}{2}} - x^{\frac{1}{2}} + 4 = 5 - 4x^{-\frac{1}{2}} - x^{\frac{1}{2}}$$

$$\int \left(x^{\frac{1}{2}} - 4 \right) \left(x^{-\frac{1}{2}} - 1 \right) dx = 5x - \frac{4x^{\frac{1}{2}}}{\frac{1}{2}} - \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c = 5x - 8x^{\frac{1}{2}} - \frac{2}{3}x^{\frac{3}{2}} + c$$

$$(b) \int_1^4 \left(x^{\frac{1}{2}} - 4 \right) \left(x^{-\frac{1}{2}} - 1 \right) dx$$

$$= \left[5x - 8x^{\frac{1}{2}} - \frac{2}{3}x^{\frac{3}{2}} \right]_1^4$$

$$= \left(20 - 8 \times 2 - \frac{2}{3} \times 2^3 \right) - \left(5 - 8 - \frac{2}{3} \right)$$

$$= 4 - \frac{16}{3} + 3 + \frac{2}{3}$$

$$= 7 - \frac{14}{3}$$

$$= \frac{7}{3} \text{ or } 2\frac{1}{3}$$

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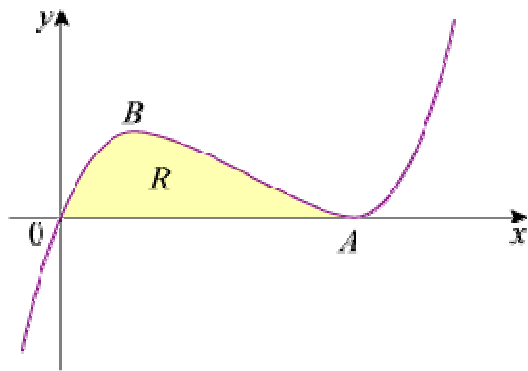
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Integration

Exercise F, Question 5

Question:

The diagram shows part of the curve with equation $y = x^3 - 6x^2 + 9x$. The curve touches the x -axis at A and has a maximum turning point at B .



- (a) Show that the equation of the curve may be written as $y = x(x-3)^2$, and hence write down the coordinates of A .
- (b) Find the coordinates of B .
- (c) The shaded region R is bounded by the curve and the x -axis. Find the area of R .

[E]

Solution:

(a) $(x-3)^2 = x^2 - 6x + 9$
 So $x(x-3)^2 = x^3 - 6x^2 + 9x$
 $y = 0 \Rightarrow x = 0$ [i.e. $(0, 0)$] or 3 (twice)
 So A is $(3, 0)$

(b) $\frac{dy}{dx} = 0 \Rightarrow 0 = 3x^2 - 12x + 9$
 $\Rightarrow 0 = 3(x^2 - 4x + 3)$
 $\Rightarrow 0 = 3(x-3)(x-1)$
 $\Rightarrow x = 1$ or 3
 $x = 3$ at A , the minimum, so B is $(1, 4)$

(c) Area of $R = \int_0^3 (x^3 - 6x^2 + 9x) dx$
 $= \left[\frac{1}{4}x^4 - 2x^3 + \frac{9}{2}x^2 \right]_0^3$
 $= \left(\frac{81}{4} - 2 \times 27 + \frac{9}{2} \times 9 \right) - \left(0 \right)$
 $= 6 \frac{3}{4}$

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Integration

Exercise F, Question 6

Question:

Given that $y^{\frac{1}{2}} = x^{\frac{1}{3}} + 3$:

(a) Show that $y = x^{\frac{2}{3}} + Ax^{\frac{1}{3}} + B$, where A and B are constants to be found.

(b) Hence find $\int y \, dx$.

(c) Using your answer from part (b) determine the exact value of $\int_1^8 y \, dx$.

[E]

Solution:

$$(a) y = \left(x^{\frac{1}{3}} + 3 \right)^2 = x^{\frac{2}{3}} + 6x^{\frac{1}{3}} + 9 \quad (A = 6, B = 9)$$

$$(b) \int y \, dx = \left[\begin{array}{l} x^{\frac{5}{3}} + \frac{6x^{\frac{4}{3}}}{\frac{4}{3}} + 9x + c \\ \frac{5}{3} \qquad \frac{4}{3} \end{array} \right]$$

$$= \frac{3}{5}x^{\frac{5}{3}} + \frac{9}{2}x^{\frac{4}{3}} + 9x + c$$

$$(c) \int_1^8 y \, dx = \left[\frac{3}{5}x^{\frac{5}{3}} + \frac{9}{2}x^{\frac{4}{3}} + 9x \right]_1^8$$

$$= \left(\frac{3}{5} \times 32 + \frac{9}{2} \times 16 + 72 \right) - \left(\frac{3}{5} + \frac{9}{2} + 9 \right)$$

$$= \frac{93}{5} + 135 - \frac{9}{2}$$

$$= 149 \frac{1}{10} \text{ or } 149.1$$

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Integration

Exercise F, Question 7

Question:

Considering the function $y = 3x^{\frac{1}{2}} - 4x^{-\frac{1}{2}}$, $x > 0$:

(a) Find $\frac{dy}{dx}$.

(b) Find $\int y \, dx$.

(c) Hence show that $\int_1^3 y \, dx = A + B\sqrt{3}$, where A and B are integers to be found.

[E]

Solution:

(a) $y = 3x^{\frac{1}{2}} - 4x^{-\frac{1}{2}}$

$$\frac{dy}{dx} = \frac{3}{2}x^{-\frac{1}{2}} + \frac{1}{2} \times 4x^{-\frac{3}{2}}$$

$$\frac{dy}{dx} = \frac{3}{2}x^{-\frac{1}{2}} + 2x^{-\frac{3}{2}}$$

(b) $\int y \, dx = \int \left(3x^{\frac{1}{2}} - 4x^{-\frac{1}{2}} \right) dx$

$$= \frac{3x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{4x^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$= 2x^{\frac{3}{2}} - 8x^{\frac{1}{2}} + c$$

(c) $\int_1^3 y \, dx = \left[2x^{\frac{3}{2}} - 8x^{\frac{1}{2}} \right]_1^3$

$$= (2 \times 3\sqrt{3} - 8\sqrt{3}) - (2 - 8)$$

$$= -2\sqrt{3} + 6$$

$$= 6 - 2\sqrt{3}$$

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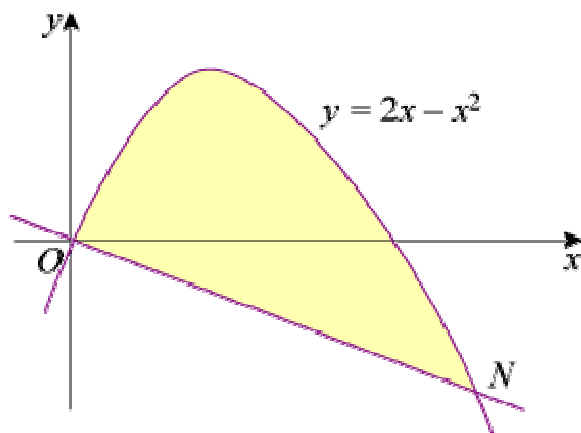
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Integration

Exercise F, Question 8

Question:

The diagram shows a sketch of the curve with equation $y = 2x - x^2$ and the line ON which is the normal to the curve at the origin O .



(a) Find an equation of ON .

(b) Show that the x -coordinate of the point N is $2\frac{1}{2}$ and determine its y -coordinate.

(c) The shaded region shown is bounded by the curve and the line ON . Without using a calculator, determine the area of the shaded region.

Solution:

$$(a) y = 2x - x^2$$

$$\frac{dy}{dx} = 2 - 2x$$

Gradient of tangent at $(0, 0)$ is 2.

$$\text{Gradient of } ON = -\frac{1}{2}$$

So equation of ON is $y = -\frac{1}{2}x$ or $2y + x = 0$

(b) N is point of intersection of ON and the curve, so

$$-\frac{1}{2}x = 2x - x^2$$

$$2x^2 - 5x = 0$$

$$x(2x - 5) = 0$$

$$x = 0, \frac{5}{2}$$

$$\text{So } N \text{ is } \left(2\frac{1}{2}, -1\frac{1}{4} \right)$$

$$\begin{aligned}
 \text{(c) Area} &= \int_0^{2\frac{1}{2}} (\text{curve} - \text{line}) \, dx \\
 &= \int_0^{2\frac{1}{2}} \left[2x - x^2 - \left(-\frac{1}{2}x \right) \right] dx \\
 &= \int_0^{2\frac{1}{2}} \left(\frac{5}{2}x - x^2 \right) dx \\
 &= \left[\frac{5}{4}x^2 - \frac{x^3}{3} \right]_0^{2\frac{1}{2}} \\
 &= \left(\frac{31.25}{4} - \frac{15.625}{3} \right) - \left(0 \right) \\
 &= \frac{125}{48}
 \end{aligned}$$

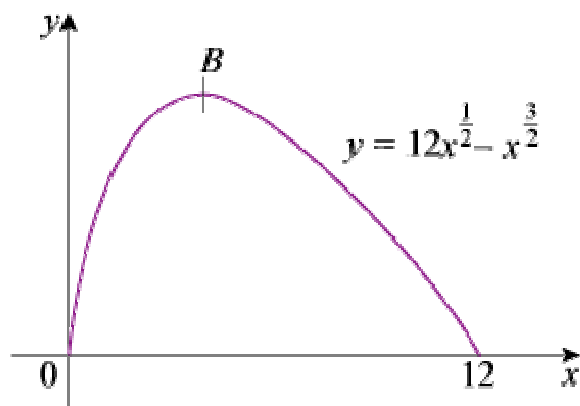
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Integration

Exercise F, Question 9

Question:



The diagram shows a sketch of the curve with equation

$$y = 12x^{\frac{1}{2}} - x^{\frac{3}{2}} \text{ for } 0 \leq x \leq 12.$$

(a) Show that $\frac{dy}{dx} = \frac{3}{2}x^{-\frac{1}{2}}(4 - x)$.

(b) At the point B on the curve the tangent to the curve is parallel to the x -axis. Find the coordinates of the point B .

(c) Find, to 3 significant figures, the area of the finite region bounded by the curve and the x -axis.

[E]

Solution:

(a) $y = 12x^{\frac{1}{2}} - x^{\frac{3}{2}}$

$$\frac{dy}{dx} = 6x^{-\frac{1}{2}} - \frac{3}{2}x^{\frac{1}{2}} = \frac{3}{2}x^{-\frac{1}{2}}(4 - x)$$

(b) $\frac{dy}{dx} = 0 \Rightarrow x = 4, y = 12 \times 2 - 2^3 = 16$

So B is $(4, 16)$

(c) Area = $\int_0^{12} \left(12x^{\frac{1}{2}} - x^{\frac{3}{2}} \right) dx$

$$= \left[\frac{12x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^{\frac{5}{2}}}{\frac{5}{2}} \right]_0^{12}$$

$$\begin{aligned} &= \left[8x^{\frac{3}{2}} - \frac{2}{5}x^{\frac{5}{2}} \right]_0^{12} \\ &= \left(8 \times \sqrt{12^3} - \frac{2}{5}\sqrt{12^5} \right) - \left(0 \right) \\ &= 133.0215 \dots \\ &= 133 \text{ (3 s.f.)} \end{aligned}$$

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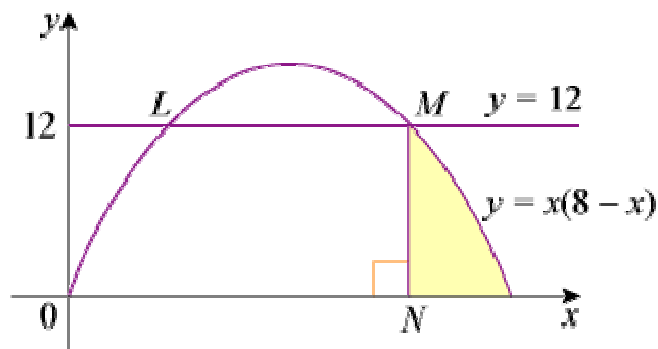
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Integration

Exercise F, Question 10

Question:

The diagram shows the curve C with equation $y = x(8 - x)$ and the line with equation $y = 12$ which meet at the points L and M .



(a) Determine the coordinates of the point M .

(b) Given that N is the foot of the perpendicular from M on to the x -axis, calculate the area of the shaded region which is bounded by NM , the curve C and the x -axis.

[E]

Solution:

$$\begin{aligned}
 \text{(a) } x(8 - x) &= 12 \\
 \Rightarrow 8x - x^2 &= 12 \\
 \Rightarrow 0 &= x^2 - 8x + 12 \\
 \Rightarrow 0 &= (x - 6)(x - 2) \\
 \Rightarrow x &= 2 \text{ or } 6
 \end{aligned}$$

M is on the right of L , so M is $(6, 12)$

$$\begin{aligned}
 \text{(b) Area} &= \int_6^8 (8x - x^2) \, dx \\
 &= \left[4x^2 - \frac{x^3}{3} \right]_6^8 \\
 &= \left(4 \times 64 - \frac{512}{3} \right) - \left(4 \times 36 - \frac{216}{3} \right) \\
 &= 256 - 170 \frac{2}{3} - 144 + 72 \\
 &= 13 \frac{1}{3}
 \end{aligned}$$

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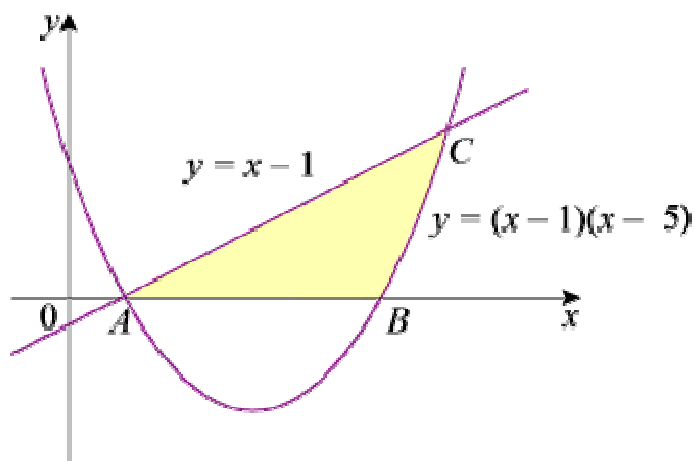
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Integration

Exercise F, Question 11

Question:

The diagram shows the line $y = x - 1$ meeting the curve with equation $y = (x - 1)(x - 5)$ at A and C . The curve meets the x -axis at A and B .



- (a) Write down the coordinates of A and B and find the coordinates of C .
- (b) Find the area of the shaded region bounded by the line, the curve and the x -axis.

Solution:

(a) A is $(1, 0)$, B is $(5, 0)$

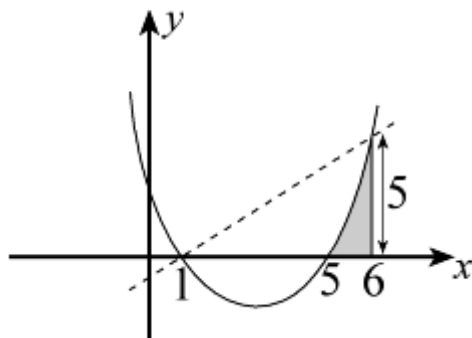
$$x - 1 = (x - 1)(x - 5)$$

$$\Rightarrow 0 = (x - 1)(x - 5 - 1)$$

$$\Rightarrow 0 = (x - 1)(x - 6)$$

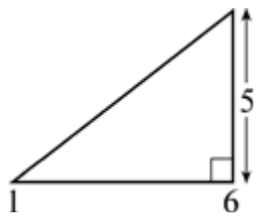
$$\Rightarrow x = 1, 6$$

So C is $(6, 5)$



(b)

$$\text{Shaded region is } \int_1^6 (x - 1) (x - 5) dx = \int_1^6 (x^2 - 6x + 5) dx$$



Required area = area of $\int_1^6 (x^2 - 6x + 5) dx$

$$= \frac{1}{2} \times 5 \times 5 - \left[\frac{x^3}{3} - 3x^2 + 5x \right]_1^6$$

$$= 12 \frac{1}{2} - \left[\left(\frac{216}{3} - 3 \times 36 + 30 \right) - \left(\frac{125}{3} - 75 + 25 \right) \right]$$

$$= 12 \frac{1}{2} + 6 - 50 + 41 \frac{2}{3}$$

$$= 10 \frac{1}{6}$$

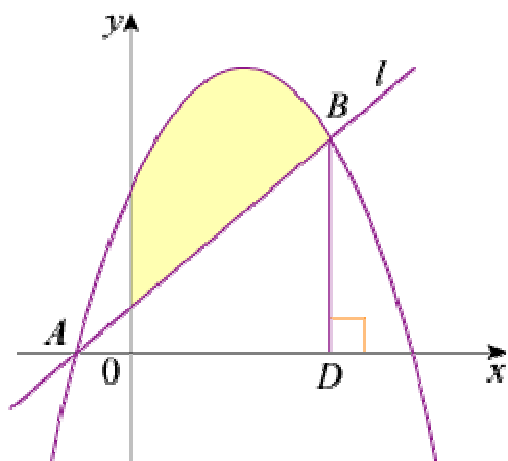
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Exercise F, Question 12

Question:



A and B are two points which lie on the curve C , with equation $y = -x^2 + 5x + 6$. The diagram shows C and the line l passing through A and B .

(a) Calculate the gradient of C at the point where $x = 2$.

The line l passes through the point with coordinates $(2, 3)$ and is parallel to the tangent to C at the point where $x = 2$.

(b) Find an equation of l .

(c) Find the coordinates of A and B .

The point D is the foot of the perpendicular from B on to the x -axis.

(d) Find the area of the region bounded by C , the x -axis, the y -axis and BD .

(e) Hence find the area of the shaded region.

[E]

Solution:

$$(a) \frac{dy}{dx} = -2x + 5$$

When $x = 2$ gradient of C is $-4 + 5 = 1$

(b) Equation of l is $y - 3 = 1(x - 2)$ i.e. $y = x + 1$

(c) A is $(-1, 0)$

B is given by

$$x + 1 = -x^2 + 5x + 6$$

$$x^2 - 4x - 5 = 0$$

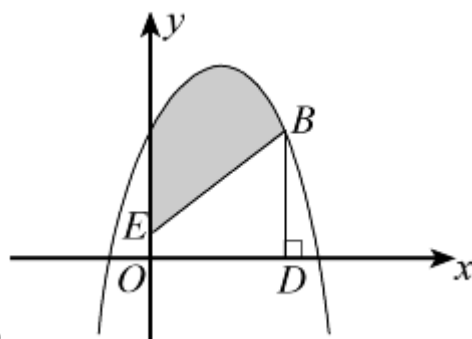
$$(x - 5)(x + 1) = 0$$

$$x = -1 \text{ or } 5$$

So B is $(5, 6)$

$$(d) \text{Area} = \int_0^5 (-x^2 + 5x + 6) dx$$

$$\begin{aligned}
 &= \left[-\frac{x^3}{3} + \frac{5x^2}{2} + 6x \right]_0^5 \\
 &= \left(-\frac{125}{3} + \frac{125}{2} + 30 \right) - \left(0 \right) \\
 &= \frac{125}{6} + 30 \\
 &= 50 \frac{5}{6}
 \end{aligned}$$



(e)

Required area is (d) – trapezium *OEBD*

$$\text{Area of trapezium} = \frac{1}{2} \times 5 \times \left(1 + 6 \right) = \frac{35}{2} = 17 \frac{1}{2}$$

$$\text{Shaded region} = 50 \frac{5}{6} - 17 \frac{1}{2} = 33 \frac{1}{3}$$

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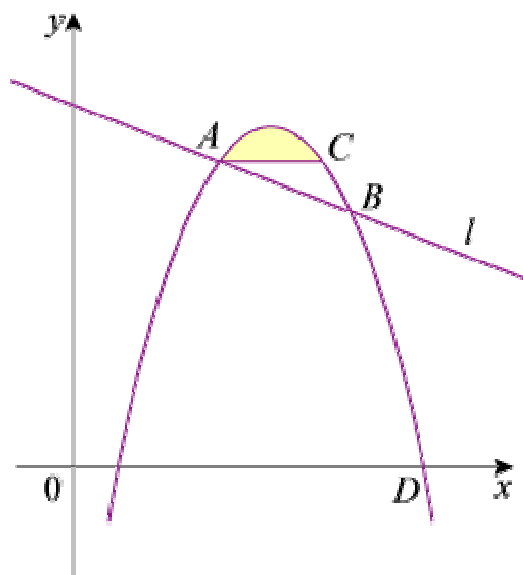
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Integration

Exercise F, Question 13

Question:



The diagram shows part of the curve with equation $y = p + 10x - x^2$, where p is a constant, and part of the line l with equation $y = qx + 25$, where q is a constant. The line l cuts the curve at the points A and B . The x -coordinates of A and B are 4 and 8 respectively. The line through A parallel to the x -axis intersects the curve again at the point C .

- (a) Show that $p = -7$ and calculate the value of q .
- (b) Calculate the coordinates of C .
- (c) The shaded region in the diagram is bounded by the curve and the line AC . Using algebraic integration and showing all your working, calculate the area of the shaded region.

[E]

Solution:

(a) Using A which lies on line and curve: $4q + 25 = p + 40 - 16$

$$\text{i.e. } 4q = p - 1 \quad \textcircled{1}$$

Using B which lies on line and curve: $8q + 25 = p + 80 - 64$

$$\text{i.e. } 8q = p - 9 \quad \textcircled{2}$$

$$\text{Solving } \textcircled{2} - \textcircled{1} \Rightarrow 4q = -8 \Rightarrow q = -2$$

$$\text{Substitute into } \textcircled{1} \Rightarrow p = 1 + 4q = -7$$

(b) At A , $y = 4q + 25 = 17$

So C is given by

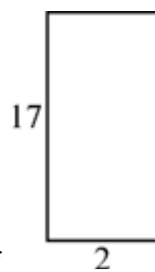
$$17 = -7 + 10x - x^2$$

$$x^2 - 10x + 24 = 0$$

$$(x - 6)(x - 4) = 0$$

$$x = 4, 6$$

So C is $(6, 17)$



(c) Area = $\int_4^6 (-7 + 10x - x^2) dx$ - area of

$$= \left[-7x + 5x^2 - \frac{1}{3}x^3 \right]_4^6 - 34$$

$$= \left(-42 + 180 - 72 \right) - \left(-28 + 80 - \frac{64}{3} \right) - 34$$

$$= \frac{4}{3} \text{ or } 1 \frac{1}{3}$$

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