Edge/Odds – The Kelly Formula and Maximizing Returns

Professional money managers, particularly in our strategies (event-driven, deep value and the like) seek to distinguish themselves with the construction of portfolios spiced with unique calls where they perceive that they have an "edge." It is what should differentiate us from the market, or *beta,* and from each other in attracting capital. However, it is my view that very few managers spend any time attempting to define the appropriate sizing of positions to mitigate downside and maximize returns. We at Tiburon use our own proprietary variation on what is known as "The Kelly Formula." It is part of every decision to put a position in our portfolio. While the *Kelly Formula* has some well-known advocates in the investing world, such as Warren Buffett, Charlie Munger and Bill Gross, *Kelly* requires some meaningful modification in order to be an effective investment sizing tool. We will discuss here, the *Kelly Formula*, its flaws and modifications we make at Tiburon in order to use it effectively to optimize position sizing.

John Kelly and an Edge

John Kelly, a scientist who worked at Bell Labs in the 1950's, is best known for formulating what has become known as the "Kelly Formula" or "Kelly Criterion."[1] It is an algorithm for maximizing winnings in bets. Kelly's early work was based on sizing bets when the gambler had an "edge." The issue was in what circumstances would a gambler have an "edge" in games of chance? Dancing around the matter of morality, Kelly's examples were mostly rigged horse races and game shows. Privately he'd mentioned the logical application to investing as well. By analogy, some had suggested that the "edge" necessary to effectively use *Kelly* to size investments was inside information.[2] However, we would argue its justifiable use predicated on the real edge we and others obtain due to our unique investment process and accurate interpretation of information available in the public domain.

J.L. Kelly, Jr.

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[1] See "A New Interpretation of Information Rate" J.L. Kelly, March 21, 1956 (ATT)
http://www.racing.saratoga.ny.us/kelly.pdf

[2] Claude Shannon, Kelly's Bell Labs mentor and collaborator. See "Fortune's Formula", William Poundstone, Hill & Wang 2005.

The Kelly Formula Explained

For simple bets with two outcomes, one involving losing the entire amount bet, and the other involving winning the bet amount multiplied by the payoff odds, the *Kelly* bet is:

$$f^* = \frac{bp-q}{b} \qquad \frac{\text{Percentage of}}{\text{Bankroll}} = \frac{(\text{Odds Received x Probability of Winning}) - \text{Probability of Losing}}{\text{Odds Received}}$$

where:

- f^* is the fraction of the current bankroll to wager;
- b is the net odds received on the wager (that is, odds are usually quoted as "b to 1")
- p is the probability of winning;
- q is the probability of losing, which is $1 - p$.

For example, you have \$1,000 and are offered 2-1 on coin flips – you win \$2 if it comes up heads; you'll lose \$1 if it comes up tails. With a 50% chance of winning ($p = 0.50$, $q = 0.50$), you receive 2-to-1 odds on a winning bet ($b = 2$), then you should bet 25% of the bankroll at each opportunity ($f^* = 0.25$), in order to maximize the long-run growth rate of the bankroll.

If the edge is negative ($b < q/p$) the formula gives a negative result, indicating that the gambler should take the other side of the bet.

Why *Kelly* Needs Modification to Apply to Investments

Distinctions between Games and Investments

Kelly assumes sequential bets that are *independent*.[3] That may be a good model for some gambling games, but generally does not apply in investing. The roll of dice is not influenced by the price of oil, the outbreak of war, the failure of financial systems, but securities prices are. *Kelly requires lack of correlation between bets – this is a difficult task when applied to a portfolio*. A game of Poker starts with a hand dealt and ends with players displaying cards. The game then starts anew. Investing professionally usually means there's a portfolio. Even if a portfolio is concentrated, there are a variety of "bets." Considering the bets *one at a time*, let's say *Kelly* says to bet 10% of wealth on each, which means the investor's entire wealth is at risk. That risks ruin, especially if the payoffs of the bets are correlated. If, as an investor, you put on 10 positions in this way, there would need to be zero correlation in order for *Kelly* to be effective (where correlation is defined as a dependent statistical relationship). Further, the portfolio has

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[3] In probability theory, independent means that the occurrence of one event makes others no more or less probable.

these 10 bets on *simultaneously*. The sequential nature of *Kelly* is more applicable to gambling games than to investing.

Gambling game results are statistical whereas investments have an idiosyncratic nature. Winning and losing in games of chance leave no room for second guessing or changes in assumptions based on qualitative matters. Security prices are impacted not only by market recognition of intrinsic value and exogenous macro events, but by the behaviors of rational interested parties and irrational uninformed ones.[4] Determining "edge" when considering investments is most often qualitative and based on the analyst or portfolio manager's personal perspective.

In the Long Run We All Die

The *Kelly Formula* works out "in the long run" (that is, it is an asymptotic property). *Kelly* considers long-term wealth solely. This is one of the reasons that value investors discuss using the *Kelly Formula* to size investments. However, for many us, the near term matters as well. Sizing trades using *Kelly* leads to highly volatile short-term outcomes regardless of what might happen in the long run.

Do Not Bet Thy Whole Wad

One of the most unrealistic assumptions of the *Kelly Formula*'s application to investments is that wealth is both the goal and the limit to what one can bet. Most people cannot bet their entire fortune. As a manager with *absolute return* criteria, we still afford our investors with quarterly liquidity. How do we reconcile sizing positions fully with the prospect of loss (no matter what edge we might have) and/or volatility in market value when the sources of capital are varied and with short term liquidity rights? So clearly *Kelly*, if applied to investments, has relevance regarding personal investment choices, less so with professional investment managers, particularly when the sources of capital also value liquidity. One last comment here: the answer isn't locking up investor capital for long periods. Look at 2008. Many deep value investors lost copious amounts of money as they sized up positions based on a sense of edge (over odds or not). My point is why not seek to make money over the long term *while not losing money in the short term*?

A natural assumption is that taking more risk increases the probability of both very good and very bad outcomes. One of the most important ideas in *Kelly* is that betting more than the *Kelly* amount *decreases* the probability of very good results, while still increasing the probability of very bad results. Since in reality we seldom know the precise probabilities and payoffs, and since over betting is worse than under betting, it makes sense to err on the side of caution and bet less than the *Kelly* amount.

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[4] See discussion of Tiburon's Five Pronged Methodology – "Rational Actors Assessment."
http://www.distressed-debt-investing.com/2009/09/exclusive-interview-with-hedge-fund.html
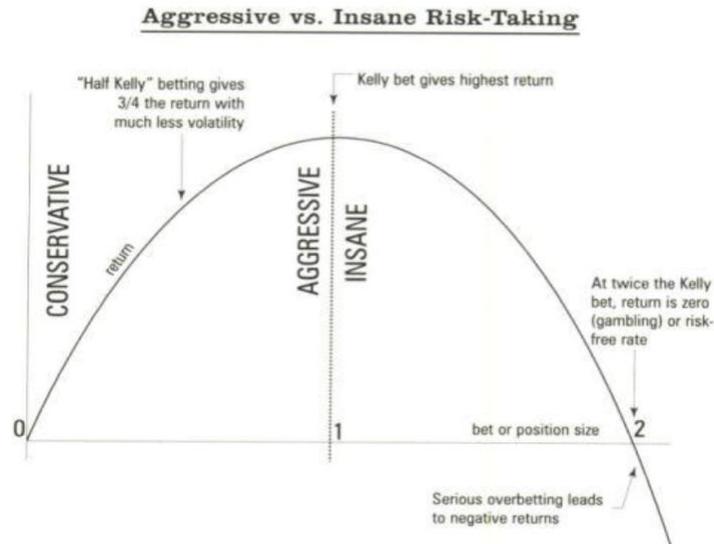
Source: "Fortune's Formula," William Poundstone

The Tiburon/Kelly Formula Variance

The *Kelly Formula* is an essential tool we use to size positions that enter our portfolio. However, as noted above, there are a number of flaws in the strict adherence to *Kelly*. Just to innumerate them once more succinctly:

- *Kelly* applies to sequential bets with, therefore, no correlation. Professionally run pools of capital are done so in a portfolio with underlying "bets" influenced by macro factors, markets and each other. There is some inherent correlation.
- Gambling games are won and lost in ways that can be statistically derived. Movement and terminal value of investments have idiosyncratic properties.
- Determining the "edge" in gambling is quantitative and precise. Determining the "edge" in investments is most often qualitative and based on personal perspective, and therefore hard to define precisely.
- Professionally run investment pools rarely have the ability to place highly concentrated bets with no care for short term volatility, while seeking long term absolute returns. In the real world, there are competing objectives.

While Marty Whitman, my old mentor, has said, "Diversification is a surrogate, and usually a damn poor surrogate, for knowledge, control, and price consciousness," we deal, in part, with the varying goals of performance, liquidity for our investors and longevity as a manager by diversifying across approximately 40-50 positions. This amount of line items is small enough to have an "edge" in each investment, know them intimately and yet still moderate volatility.

We do correlation analysis on the portfolio to identify correlation between positions and between the portfolio broadly, and markets, interest rates, commodity prices, etc. We

wring out correlation among the investments in portfolio via hedges that mitigate risk exogenous to the thesis of each investment.

Getting a handle on "edge," we create "base," "best," and "worst" case scenarios, probability weight them, of course, after processing via our *Five Prong Methodology*.

Tiburon Variance A - Sizing

Our variation of the *Kelly Formula* solves two issues we have with *Kelly* when applying his methodology to portfolio allocation: 1. Tiburon risk rules and investment prudence limits us to 10% at market in any given position, and; 2. We won't invest in a situation where we don't see at least 40% potential upside. If we weight 40-50 portfolio positions by the traditional *Kelly Formula*, it would suggest that we use approximately 6x leverage. Therefore, we common size[5] the *Kelly* derived sizing recommendation. This enables us to assemble and appropriately size our portfolio without leverage.

Moderating the *Kelly* derived position size has some interesting mathematical properties. It cuts our risk of temporary loss (i.e., volatility) by a large amount while reducing our return expectation only a little. A 50% of *Kelly* bet, for example, gives a large margin of safety in the risk estimate. If you are off by a factor of two on your risk of loss estimate, a full *Kelly* bet will reduce your return expectation to zero. But a half *Kelly* bet will leave you with 2/3 of the return expectation.

With the full *Kelly* bet, your probability of temporary loss is a linear function of the amount of loss. For example, you stand a 90% chance of losing 10%, an 80% chance of losing 20%, a 50% chance of losing 50%, etc. Not many investors are comfortable with the prospect of a 50% probability of losing 50% of their money. With a reduced *Kelly* bet, your probability of temporary loss is a quadratic function of the amount of loss. For example, at half the *Kelly* bet you stand an 81% chance of losing 10%, a 64% chance of losing 20%, a 25% chance of losing 50%, etc.

Your expected gain with the half *Kelly* bet is reduced by 25%. For example, if your expected gain is 40% with the full *Kelly*, it is 30% with the half *Kelly*, and if your expected gain is 10% with the full *Kelly*, it is 7.5% with the half *Kelly*.

Tiburon Variance B – Beta Correlation

Every new trade is evaluated for its correlation to the rest of the portfolio. For the *Kelly Formula* to be effective in sizing positions, positions need to have little to no correlation to each other. Tiburon evaluates sizing trades as a function of the correlation between the new investment and the existing portfolio. The beta of the portfolio to the market and macro events is another matter, reviewed and potentially hedged as part of portfolio considerations and does not weigh on the sizing of the prospective new investment.

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[5] Common sizing is the expression of items as percentages rather than as dollar amounts.

Therefore, the **Tiburon/Kelly Formula Variance** is:

For any investment reviewed, subject to the trade's correlation to the Portfolio being less than 1[6],

$$Ts = \frac{\frac{bp-q}{b}}{\sum \frac{bp-q}{b}} \leq 10\%$$

where:

- Ts is Tiburon position sizing;
- Ts cannot exceed 10% of Tiburon portfolio at market;
- b is the net odds received on the wager (that is, odds are usually quoted as "b to 1")
- p is the probability of winning;
- q is the probability of losing, which is $1 - p;$
- $\sum$ is the summation of Kelly allocations across our portfolio.

For any investment professional building a portfolio, they first need to identify positions that meet their criteria and are part and parcel of their investment strategies. Given this, sizing the position is a function of the manager's *edge* in the trade and the *odds* of the favorable outcome. At Tiburon, every trade idea passes through our *Five Pronged Methodology*. As a function of this work, we probability weight the outcomes. As every trade idea requires a *Revaluation Catalyst*, and sizing is in part, a function of our conviction level about the catalyst (or the odds of its occurrence and impact on securities price), sizing naturally become edge/odds. We use the **Tiburon/Kelly Formula Variance** to, as best possible, accurately size positions to maximize profitability while minimizing downside.

Peter M. Lupoff   **March, 2010   Tiburon Capital Management, LLC**

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[6] Tiburon may reduce β via hedges to meet these criteria.