

The Science of Math Within an MTSS Framework



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Introduce yourself.

Describe your role as an educator.

Describe the mathematics you support.



Share fun things from today and tag @sarahpowellphd!





What is SCIENCE?

Knowledge about the world that is
based on facts learned through
experiments and observation



Science of Reading

A body of research that relies on knowledge accrued across disciplines – including general education, special education, educational psychology, communication sciences, and neuroscience.

The focus is on
how students learn
to read.

Science of Math

A body of research that relies on knowledge accrued across disciplines – including general education, special education, educational psychology, mathematical cognition, and neuroscience.

The focus is on
how students learn
to do math.

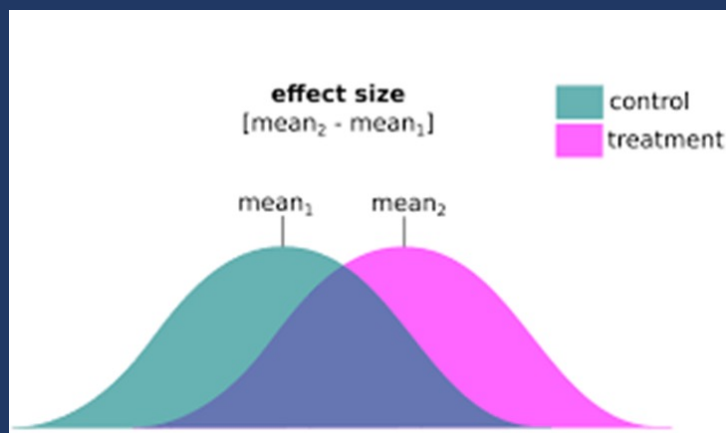
This is not a general education effort or a special education effort – this is to ensure all students can read and do math.





How do
you find the
science?

“Research evidence” comes from quasi-experimental and experimental studies demonstrating significant differences between treatment and control conditions.



- There are different types of metrics for effect sizes, but a practical and meaningful effect means that the effect size favoring treatment over control or comparison is **greater than 0.20**.
- An effect size of 1.0 would mean students in the treatment condition outperformed students in the control condition by an entire standard deviation.
- You and your school team (including caregivers) may hear various terms.
- Caveat: “No evidence” does not always mean it does not work, often just that it has not been tested empirically yet!

Solari, 2022





How do
you find the
science?

Research meta-analyses or syntheses

Learning Disabilities Research & Practice, 26(1), 36-47
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**Meta-Analysis of Mathematic Basic-Fact Fluency Interventions:
A Component Analysis**

Robin S. Coddling
University of Massachusetts, Boston
Matthew K. Burns
University of Minnesota
Gracia Lukito
University of Massachusetts, Boston

Mathematics fluency is a critical component of mathematics learning yet few attempts have been made to synthesize this research base. Seventeen single-case design studies with 55 participants were reviewed using meta-analytic procedures. A component analysis of practice elements was conducted and treatment intensity and feasibility were examined. Findings suggest that drill and practice with more than 3 steps a combination of treatments pertaining to pretest treatment setting are also

Review of Educational Research
October 2017, Vol. 87, No. 5, pp. 899-920
DOI: 10.3102/0034654317720163
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A Meta-Analysis of Schema Instruction on the Problem-Solving Performance of Elementary School Students

Corey Peltier and Kimberly J. Vannest
Texas A&M University

A variety of instructional practices have been recommended to increase the problem-solving (PS) performance of elementary school children. The purpose of this meta-analysis was to systematically review research on the use of schema instruction to increase the PS performance of elementary school-age students. A total of 21 studies, with 3,408 elementary school student participants, and 324 students identified with disabilities, met criteria. Moderator analysis includes assignment, implementer, instruction, problem type, and treatment duration. Results indicate an overall effect size (Hedge's g) for schema instruction of 1.57 (.043), CI_{95} [1.52, 1.61] for immediate PS and 1.09 (.046), CI_{95} [1.04, 1.14] for transfer PS. Moderator analysis, future research, and implications for practice are discussed.

Journal of Educational Psychology
2013, Vol. 105, No. 2, 380-400

© 2012 American Psychological Association
0022-0665/13/\$12.00 DOI: 10.1037/a0028108

A Meta-Analysis of the Efficacy of Teaching Mathematics With Concrete Manipulatives

Kira J. Carboneau, Scott C. Marley, and James P. Selig
University of New Mexico

The use of manipulatives to teach mathematics is often prescribed as an efficacious teaching strategy. To examine the empirical evidence regarding the use of manipulatives during mathematics instruction, we conducted a systematic search of the literature. This search identified 55 studies that compared instruction with manipulatives to a control condition where math instruction was provided with only abstract math symbols. The sample of studies included students from kindergarten to college level ($N = 7,237$). Statistically significant results were identified with small to moderate effect sizes, as measured by Cohen's d , in favor of the use of manipulatives when compared with instruction that only used abstract math symbols. However, the relationship between teaching mathematics with concrete manipulatives and student learning was moderated by both instructional and methodological characteristics of the studies. Additionally, separate analyses conducted for specific learning outcomes of retention ($k = 53$, $N =$

Article

The Concrete-Representational-Abstract Approach for Students With Learning Disabilities: An Evidence-Based Practice Synthesis

Emily C. Bouck¹, Rajiv Satsangi², and Jiyoung Park¹

Abstract
As researchers and practitioners have increasingly become interested in what practices are evidence based and for whom in education, different sets of quality indicators and evidence-based practice standards have emerged in the field of special education. Practices are commonly suggested as evidence based, even without a best evidence synthesis on the existing research, such as the case with the concrete-representational-abstract (CRA) instructional framework to support students with disabilities in mathematics. This study sought to support the classification of the CRA instructional framework as an evidence-based approach for students with learning disabilities by applying quality indicators and standards of evidence-based practice by Cook et al. (2014). Based on the application of the indicators and standards, the CRA instructional framework was determined to be an evidence-based practice for students with learning disabilities who struggle in mathematics relative to computational problems, such as addition, subtraction, and multiplication, largely with regrouping.

Learning Disabilities Research & Practice, 38(3), 144-155
© 2018 The Division for Learning Disabilities of the Council for Exceptional Children
DOI: 10.1111/ldrp.12172

Effects of Data-Based Individualization for Students with Intensive Learning Needs: A Meta-Analysis

Pyung-Gang Jung
Evans Women's University
Kristen L. McMaster and Amy K. Kunkel
University of Minnesota
Jaehyun Shin
Gyeongin National University of Education
Pamela M. Stecker
Clemson University

We examined the mean effect of teachers' use of data-based individualization (DBI) on the performance of students with intensive learning needs across academic areas and factors influencing the effects of DBI on student achievement. A total of 57 effect sizes from 14 studies were categorized into two comparisons: DBI Only (comparisons between DBI and a business-as-usual control) and DBI Plus (comparisons in which DBI implementers had access to additional information on student performance while they implemented DBI, compared to a control). The mean effect of DBI Only on student performance was $g = 0.37$; the mean effect of DBI Plus was $g = 0.38$. Differential effects of DBI were found depending on the nature of CBM tasks, frequency of CBM administration, and type and frequency of supports provided to teachers. Findings support the use of DBI to enhance student outcomes across academic areas.

Journal of Educational Psychology
2019, Vol. 111, No. 6, 980-1002
https://doi.org/10.1037/a0054914

The Effects of Early Numeracy Interventions for Students in Preschool and Early Elementary: A Meta-Analysis

Gena Nelson and Kristen L. McMaster
University of Minnesota

The purpose of this meta-analysis was to examine the effectiveness of early numeracy interventions for young students, including students with disabilities or those at risk for math difficulty (MD). This study evaluated preschool, kindergarten, and 1st-grade interventions on early numeracy content, instructional features, and methodological components that improved students' math achievement. A total of 34 studies met inclusion criteria for this meta-analysis, with 52 treatment groups. The average weighted effect size for numeracy interventions with two outliers removed was moderate ($g = 0.64$), and the 95% confidence interval did not include zero (0.52, 0.76). Results of the final meta-regression model predicted larger treatment effects for interventions that included counting with 1-to-1 correspondence and were 8 weeks or shorter in duration. The results of the meta-regression also showed that, on average, interventions were more effective for students with lower levels of risk for MD according to screening criteria than for students with higher levels of risk. The results of the meta-regression also showed that, on average, interventions were more effective for students with lower levels of risk for MD according to screening criteria than for students with higher levels of risk.

Article

A Synthesis of Elementary Mathematics Interventions: Comparisons of Students With Mathematics Difficulty With and Without Comorbid Reading Difficulty

Sarah R. Powell, PhD¹, Christian T. Doabler, PhD¹, Olayemi A. Akinola, PhD¹, William J. Therrien, PhD², Steven A. Maddox, MEd¹, and Katherine E. Hess, BA¹

Abstract
In this synthesis, we reviewed 65 studies involving elementary students (i.e., grades 1–5) identified with mathematics difficulty (MD) in which authors implemented a mathematics intervention. Of these studies, we identified 33 group designs, 9 quasi-experimental designs, and 23 single-case designs. We aimed to synthesize performance differences between students with MD with and without reading difficulty (RD). We identified three categories of studies for analysis: Students with MD + RD, MD-alone, or MD-nonspecified (i.e., no reading information provided). Overall, 80% of studies included students with MD-nonspecified, and the interventions for these students demonstrated strong effects. For the limited number of studies with students with MD + RD or MD-alone, intervention effects were strong for students with MD + RD and variable for students with MD-alone. In the MD-alone, we noted differential pattern of results for students with MD + RD and MD-alone. In the MD-alone, we noted differential pattern of results for students with MD + RD and MD-alone. In the MD-alone, we noted differential pattern of results for students with MD + RD and MD-alone. In the MD-alone, we noted differential pattern of results for students with MD + RD and MD-alone.

Exceptional Children
2018, Vol. 84(2), 177-196
© The Author(s) 2017
DOI: 10.1177/0014402917737467
journals.sagepub.com/home/txc
SAGE

Mathematical Interventions for Secondary Students With Learning Disabilities and Mathematics Difficulties: A Meta-Analysis

Asha K. Jitendra¹, Amy E. Lein², Soo-hyun Im¹, Ahmed A. Alghamdi¹, Scott B. Hefte¹, and John Mouanoutoua¹

Abstract
This meta-analysis is the first to provide a quantitative synthesis of empirical evaluations of mathematical intervention programs implemented in secondary schools for students with learning disabilities and mathematics difficulties. Included studies used a treatment-control group design. A total of 19 experimental and quasi-experimental studies containing 20 independent samples met study inclusion criteria. Results of a random effects model analysis indicated that mathematical interventions influence mathematics outcomes ($g = 0.37$, 95% confidence interval [0.18, 0.56]) for students with learning disabilities and mathematics difficulties. In addition, instructional time moderated the relation between mathematics interventions and student learning. Limitations of the study, future directions for research, and implications for practice are discussed.



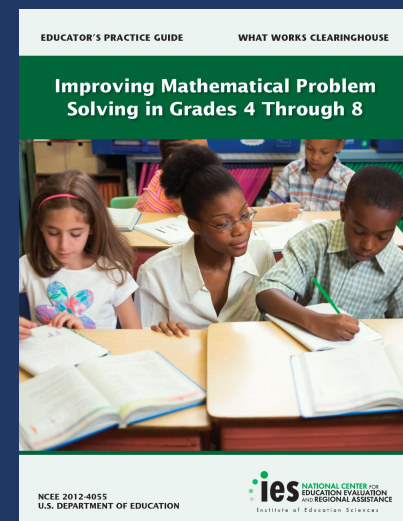
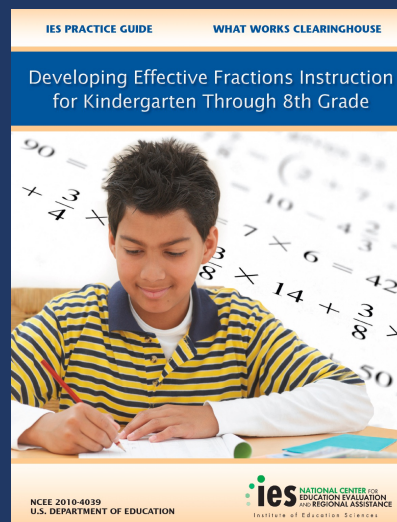
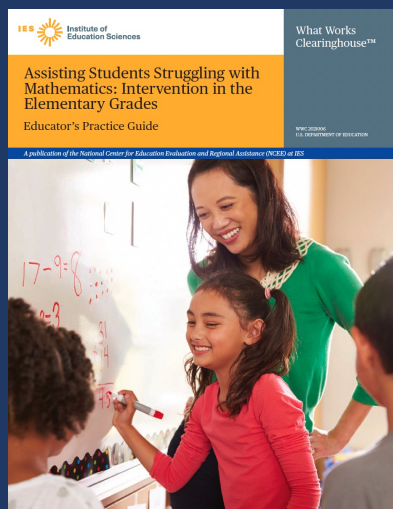


How do
you find the
science?

Compilations of research

Select topics to **Find What Works** based on the evidence

Literacy	Mathematics	Science	Behavior
Children and Youth with Disabilities	English Learners	Teacher Excellence	Charter Schools
Early Childhood (Pre-K)	Kindergarten to 12th Grade	Path to Graduation	Postsecondary





How do
you find the
science?



Share resources for evidence-based
practices that could be used to
describe the Science of Math.



What is **the Science of Math**?

The Science of Math is a movement focused on using **objective evidence** about how students learn math in order to make educational decisions and to inform policy and practice.

What guides the Science of Math?

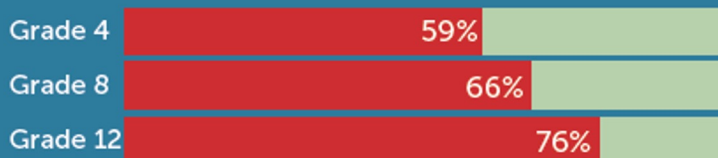
Advocates for the Science of Math rely on **well-researched instructional approaches** and **research about how students learn**.

This includes descriptive, qualitative, quantitative, and correlational research.

Why is **the Science of Math** needed?

Access to learning math is a basic educational right.

Currently, the majority of students in the U.S. do not meet minimum levels of math proficiency^a



At Grade 8, the percentage of students scoring below proficient differs by race/ethnicity^a

56%
White

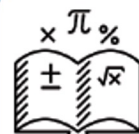
86%
Black

80%
Hispanic



At Grade 8, 82% of National School Lunch Program students score below proficient, compared to 52% of students not in the Lunch Program

At Grade 4, only 83% of students with disabilities score below proficient. At Grade 8, this is 91% of students with disabilities.



At Grade 4, 84% of English learners score below proficient. This is 95% in Grade 8 and 97% in Grade 12.

Our goal is to ensure that all students, regardless of background or status, have equitable access to high-quality math instruction.



A Look at Misconceptions in the Teaching and Learning of Math...



MISCONCEPTION

Some educators believe students should not be exposed to procedural instruction until they have demonstrated adequate conceptual understanding of a topic.

TRUTH

Conceptual knowledge supports procedural knowledge AND procedural knowledge supports conceptual knowledge. They should be taught together!

There is a two-way relationship between conceptual and procedural knowledge -

CONCEPTS



PROCEDURES

conceptual understanding and procedural fluency develop together^a

CONCEPTS

PROCEDURES

Concepts and procedures overlap - they are difficult to measure independently^b

WHEN TEACHING MATH



Teaching and practicing conceptual understanding can help with the selection and use of problem-solving procedures.^c



Teaching and practicing procedures helps to develop and deepen understanding of concepts. Procedures include more than algorithms.^d



Teaching conceptual and procedural knowledge together can help strengthen each over time.^e



MISCONCEPTION

Many educators believe interventions targeting a growth mindset will improve academic achievement.

Many educators are concerned with fostering a growth mindset (i.e., "I can work hard to improve my success in math each day") rather than a fixed mindset (i.e., "I'm just not good at math") in students to promote math achievement.

TRUTH

Intervention research on stand-alone growth mindset interventions yield minimal gains on GPA in mathematics courses^a and replication attempts have failed.^b The most effective way to improve academic achievement is to deliver skill-building intervention.^c

What is Growth Mindset Theory (GMT)?

Individuals who believe intelligence is malleable will obtain higher attainment than students who view intelligence as fixed.

In GMT, teachers support students to:

- (1) believe they can improve their performance;
- (2) identify the effort and persistence required;
- (3) seek input or feedback to improve performance;
- (4) try new strategies or approaches if old ones fail.

ADVICE FOR USING GROWTH MINDSET IN INTERVENTIONS

Use praise statements based on students' effort, understanding, and persistence on challenging math work.^d

Encourage students to master skills by providing choice of interventions, feedback, and goals on learning, and opportunities to monitor their own progress, reflect on learning goals, and record learning accomplishments.^e

Establish individual learning goals rather than promoting work exemplars from high achievers.^d



MISCONCEPTION

Inquiry-based instruction should be the primary tactic used to teach math. Explicit instruction only is beneficial for struggling learners. Explicit instruction is an instructional tactic where students are provided with correct answers and this only promotes rote learning.

TRUTH

Explicit instruction offers value through sequencing of tasks in increments of difficulty, fluency building that promotes effective practice, and scaffolded opportunities for students to combine learned skills with new knowledge. Explicit instruction facilitates creativity and is effective for all learners^a.

What are the common misconceptions about inquiry-based instruction?

Inquiry-based approaches increase math achievement

Inquiry-based approaches increase students' feelings about math

Students are more likely to remember information they have "discovered"

Students learn better when they are curious and interested in the problem

Discovery and application are the most useful tactics for teaching math

HOW DOES EXPLICIT INSTRUCTION PROMOTE CREATIVITY?



Explicit instruction provides sequences of instruction tied to students' needs to promote mastery of the fundamental skill and provide opportunities to expand new understandings.



The process of mastering fundamental skills and demonstrating new knowledge is identical to that followed by athletes, musicians, artists, and experts in all fields.



MISCONCEPTION

Many educators believe that struggling or grappling with challenging math tasks causes students to gain a deeper understanding than would be achieved if they learned the same skill without a struggle.

TRUTH

Productive struggle does not deepen understanding, grit, or creative problem solving. Productive struggle can lead to frustration and cause students to develop misconceptions.^a In addition, the 'false starts' involved in struggling with challenging tasks without adequate support or guidance lead to lost instructional time and inefficiency.^b

What is the problem with productive struggle?

No evidence suggests giving students partial information for making connections leads to learning.^c

The idea comes from constructivism, which runs counter to what we understand about math learning.^d

Students learning new skills require clear demonstrations and guided practice with immediate feedback.

New concepts are not learned by struggling. Making connections relies on a foundation of learned knowledge.

REFRAMING PRODUCTIVE STRUGGLE



Using productive struggle for generalization involves providing effective explicit instruction for learning and building proficiency with new math content first.



After verifying students have learned the content, teachers can provide practice opportunities for productive struggle in which students work to generalize their learning to a novel, challenging problem or task (i.e., moving the 'struggle' from the beginning to the end of the instructional sequence).



MISCONCEPTION

Many educators believe algorithms promote memorization, and this would contribute to a superficial understanding of steps, conventions, and rules. This belief leads to the idea that students should not be taught algorithms.

TRUTH

An algorithm is a step-by-step procedure for solving a problem. Using an algorithm requires conceptual understanding of what is happening in the problem and procedural knowledge to accurately solve. Algorithms can serve as a link between conceptual understanding and procedural knowledge.



Examples of algorithms

$$\begin{array}{r} 23 \\ \times 6 \\ \hline 120 \\ + 18 \\ \hline 138 \end{array}$$

$$\begin{array}{r} 192 \\ + 133 \\ \hline \end{array}$$

$$\begin{array}{l} 100 + 100 = 200 \\ 90 + 30 = 120 \\ 2 + 3 = 5 \end{array}$$

$$200 + 120 + 5 = 325$$

$$\begin{array}{r} 711 \\ \cancel{817} \\ - 653 \\ \hline 164 \end{array}$$

ALGORITHMS^a



Promote
flexibility



Lead to deeper
understanding



Help know when and
how to use strategies



MISCONCEPTION

Targeted interventions on increasing executive functioning will increase mathematics performance. Many people believe that improving executive functioning through direct training (e.g., working memory, cognitive training programs) will improve mathematics achievement.

TRUTH

Most evidence suggests there is a small to negligible relationship between cognitive measures and student response to intervention.^a In the few studies examining the causal link between executive function interventions and academic outcomes, researchers only found improvements on measures of executive function but no improvements on academic achievement.^b

What Does the Evidence Support?

Students at-risk for math disabilities also may have difficulties with attention, motivation, self-regulation, and working memory.^{cd}

Interventions should be tailored and intensified according to students' needs using direct evaluation of students' math skills to make low-inference decisions about intervention tactics.

Effective use of evidence-based instructional approaches negates the potential influence of executive function difficulties.^e

IMPLICATIONS FOR PRACTICE



Interventions should:

- (1) include self-regulation and reinforcement strategies;
- (2) minimize cognitive load on working memory through explicit instruction and breaking down problems into smaller, more manageable parts;
- (3) minimize language load by using visual representations;
- (4) include fluency-building practice.^f



MISCONCEPTION

Executive function has a stronger relation to math achievement than other content areas. Scholars and educators suggest cognitive measures are helpful for designing individualized interventions.^a The belief that matching cognitive aptitudes, more recently executive functions, to plan for academic instruction and interventions persists.

TRUTH

Extensive research on executive function indicates there is no uniqueness in the relationship between executive functioning and math and no evidence that impacts on executive function lead to increases in academic achievement.^b

What is Executive Functioning?



The cognitive processes that learners use to self-regulate their thoughts and actions,^c which includes working memory, inhibitory control, attention, and planning.^d



IMPLICATIONS FOR PRACTICE



Address the learning needs of students who struggle by conducting a thorough assessment of their math skills. Identify strengths, areas for improvement, and the skills/concepts students have/have not mastered.



Assess students through curriculum-based assessments, review of classroom math work, interviews of teachers and caregivers, and systematic direct observations of students' engagement during math compared to other subjects.^e



MISCONCEPTION

Many educators believe math anxiety is caused by instructional activities and timed tests.

In schools, educators may interpret students disengaging in math activities or saying they dislike math as math anxiety. Educators may reduce the difficulty of a math lesson or remove timed tests as a way to reduce math anxiety.

TRUTH

No studies have determined that timed tests cause math anxiety - defined as feelings of apprehension, tension, or fear that may interfere with performance on math-related tasks.^a In fact, timed tactics improve math performance.^b

Why are timed tests helpful?

- ⌚ Timed tests provide critical information for student mastery of key skills and concepts.
- ⌚ Once a student reaches 100% accuracy, the metric cannot capture additional learning.
- ⌚ Rate-based metrics are reliable and better indicate a student's instructional level.^c

Why are timed activities helpful?

- ⌚ Timed tasks are fluency-building activities.
- ⌚ Timed activities are necessary to promote math mastery when students have established a high level of accuracy and conceptual understanding.
- ⌚ Fluency is a necessary dimension of math mastery associated with robust understanding and flexible problem solving.^d

HOW DO YOU ADDRESS MATH ANXIETY?

Promote skill development through effective instruction.^e

Use language focused on working hard and showing growth rather than attaining a benchmark criterion.

Include fluency-building tactics in core instruction every day.

Avoid tasks where students have to "figure it out," and save those tasks for students who have mastered the fundamentals.

Support practices with tasks that increase in difficulty as students master skills.



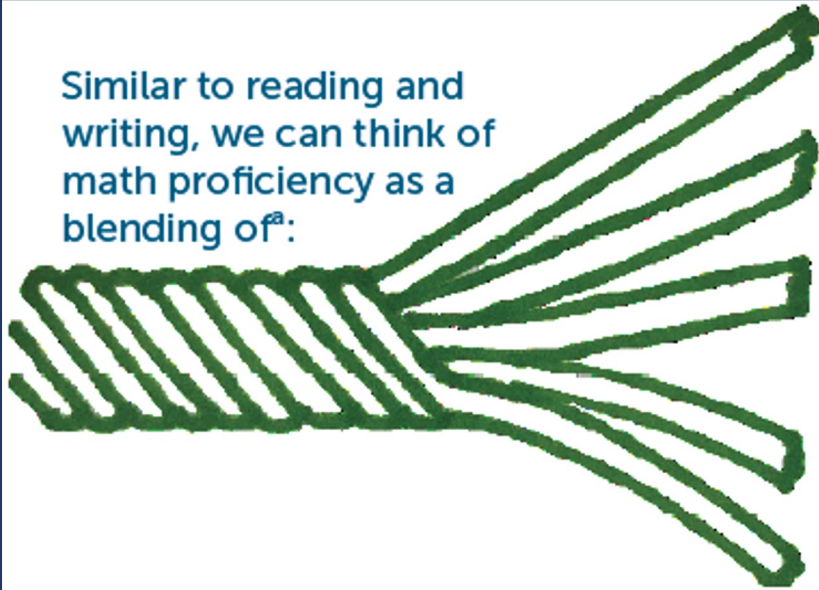


Share misconceptions and truths
you have encountered.

A Focus on Math Proficiency...



Similar to reading and writing, we can think of math proficiency as a blending of^a:



CONCEPTS

Understanding concepts, operations, and relations

PROCEDURES

Using procedures flexibly, accurately, and efficiently

STRATEGIES

Formulating, representing, and solving problems

REASONING

Reflecting, explaining, and justifying

DISPOSITION

Seeing math as sensible, useful, and worthwhile



To help students achieve math proficiency, teachers should^b:

Use a focused, coherent progression of math learning with emphasis on proficiency in key topics

Develop conceptual understanding, procedural fluency, and problem-solving skills at the same time

Use multiple approaches to meet the needs of students; explicit instruction should be used regularly

Focus on proficiency with whole numbers, fractions, geometry, and measurement; these are critical for algebra

Use formative assessment on a regular basis to assess student learning



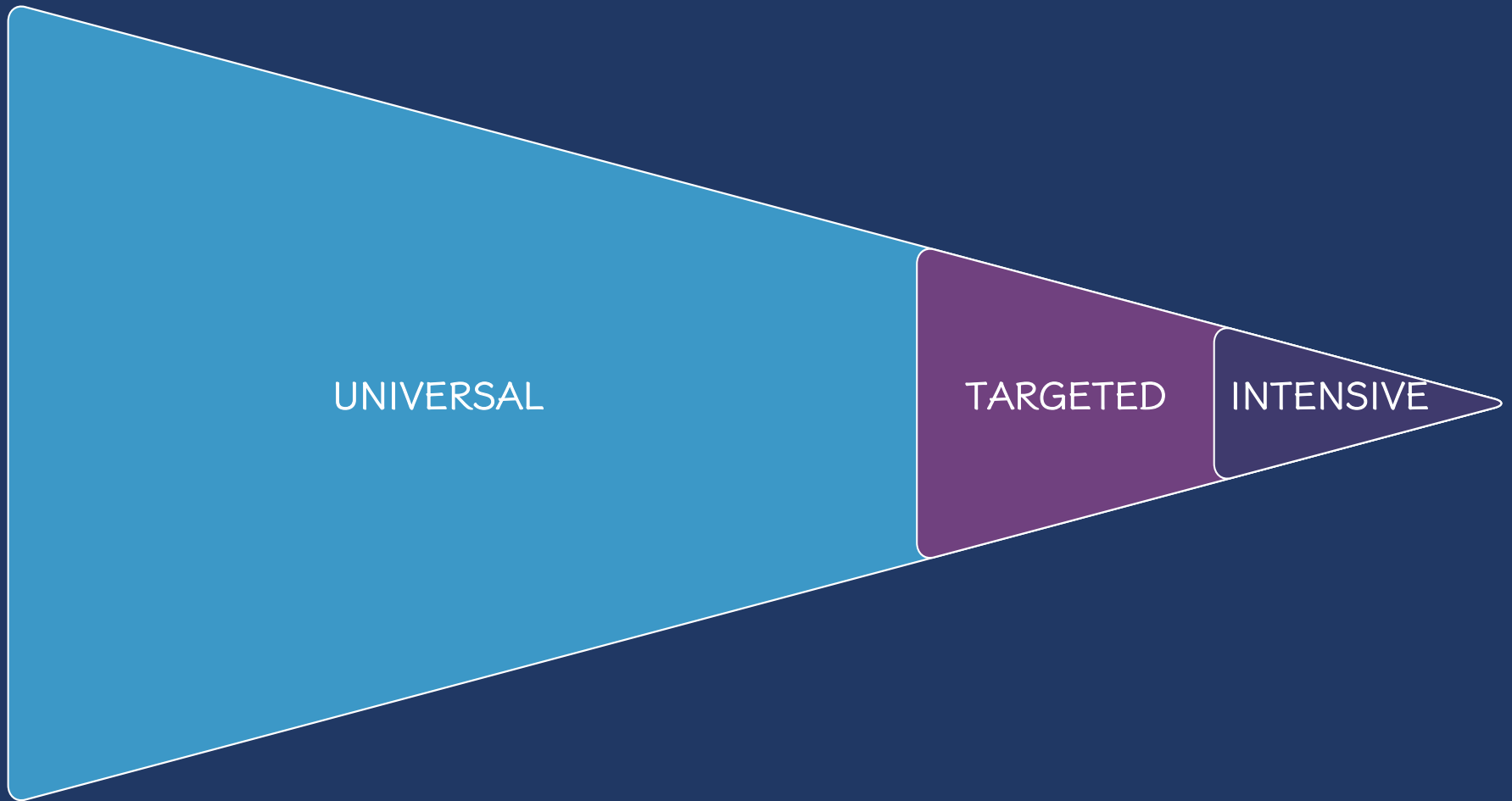
To help *students experiencing math difficulty* with math proficiency, teachers should^c:

1. Use explicit instruction
2. Teach clear and concise math language
3. Use concrete, pictorial, and virtual representations
4. Use number lines for learning concepts and procedures
5. Provide deliberate instruction on solving word problems
6. Use timed activities as one way to build math fluency



The Science of Math and MTSS...





Instructional Platform

INSTRUCTIONAL DELIVERY

Explicit
instruction

Precise
language

Multiple
representations

INSTRUCTIONAL STRATEGIES

Fluency building

Problem solving
instruction



Instructional Platform

INSTRUCTIONAL DELIVERY

Explicit
instruction

INSTRUCTIONAL STRATEGIES



MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

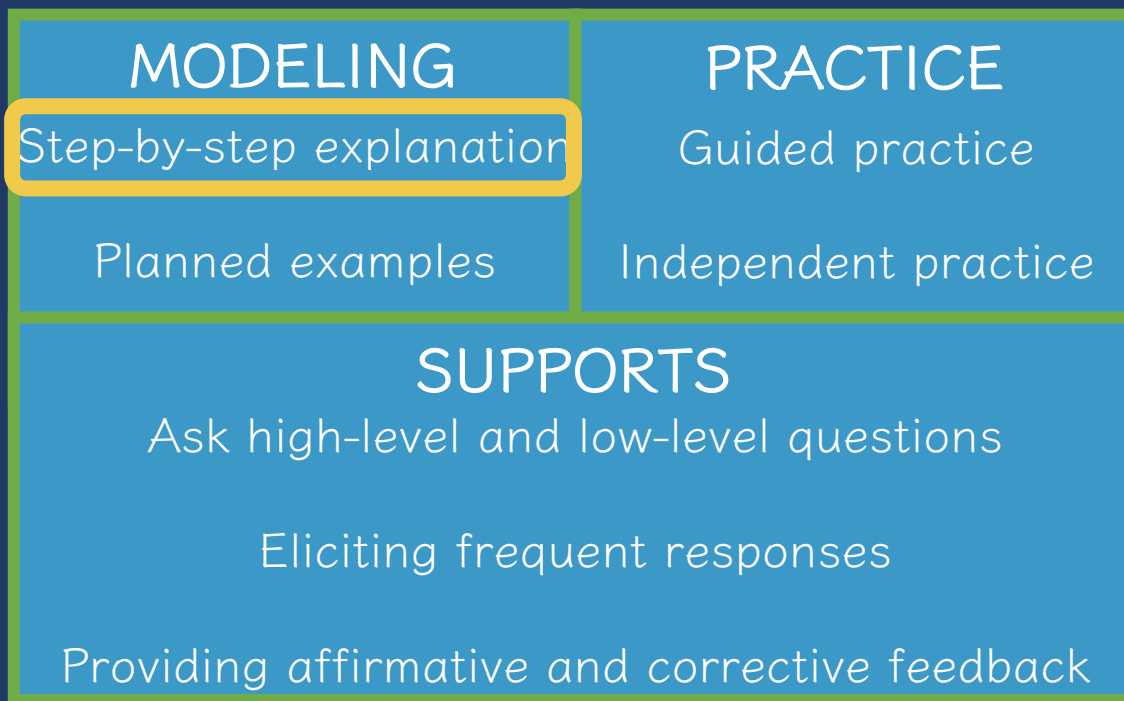
Eliciting frequent responses

Providing affirmative and corrective feedback

Modeling is a dialogue between the teacher and students.

In **Modeling**, a teacher introduces or reviews mathematical content.





Modeling includes a step-by-step explanation of how to do a mathematical problem.

A teacher may do 1 modeled problem or several.



MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

Modeling needs to include planned examples. These examples should be sequenced so easier skills lead to more difficult skills.

Planned examples in **Modeling** may also include worked examples – both correct and incorrect worked examples.



MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback



Think of the Science of Math.
What math content would be
important to model?



MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

Practice continues as a dialogue between the teacher and students.

During **Practice**, students have multiple opportunities to practice problems with varying levels of teacher support.



MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

Guided practice is practice in which the teacher and students practice problems together.



MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

Independent practice is practice in which the students practice independently with teacher support.



MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback



What practice opportunities would be important for your students?

MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

These **Supports** should be used in both **Modeling** and **Practice**.



MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

During **Modeling** and **Practice**, it is essential to engage students and check for understanding.

Ask a combination of high-level and low-level questions.



MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

During **Modeling** and **Practice**, students should frequently respond. The frequent responses keeps student attention and keeps student learning active.



MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

- Oral
- Written
- With manipulatives
- With drawings
- With gestures



MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

During **Modeling** and **Practice**, students should receive immediate feedback on their responses.

Students should receive affirmative and (when necessary) corrective feedback.



MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback



Which math supports would be important for your students?



Instructional Platform

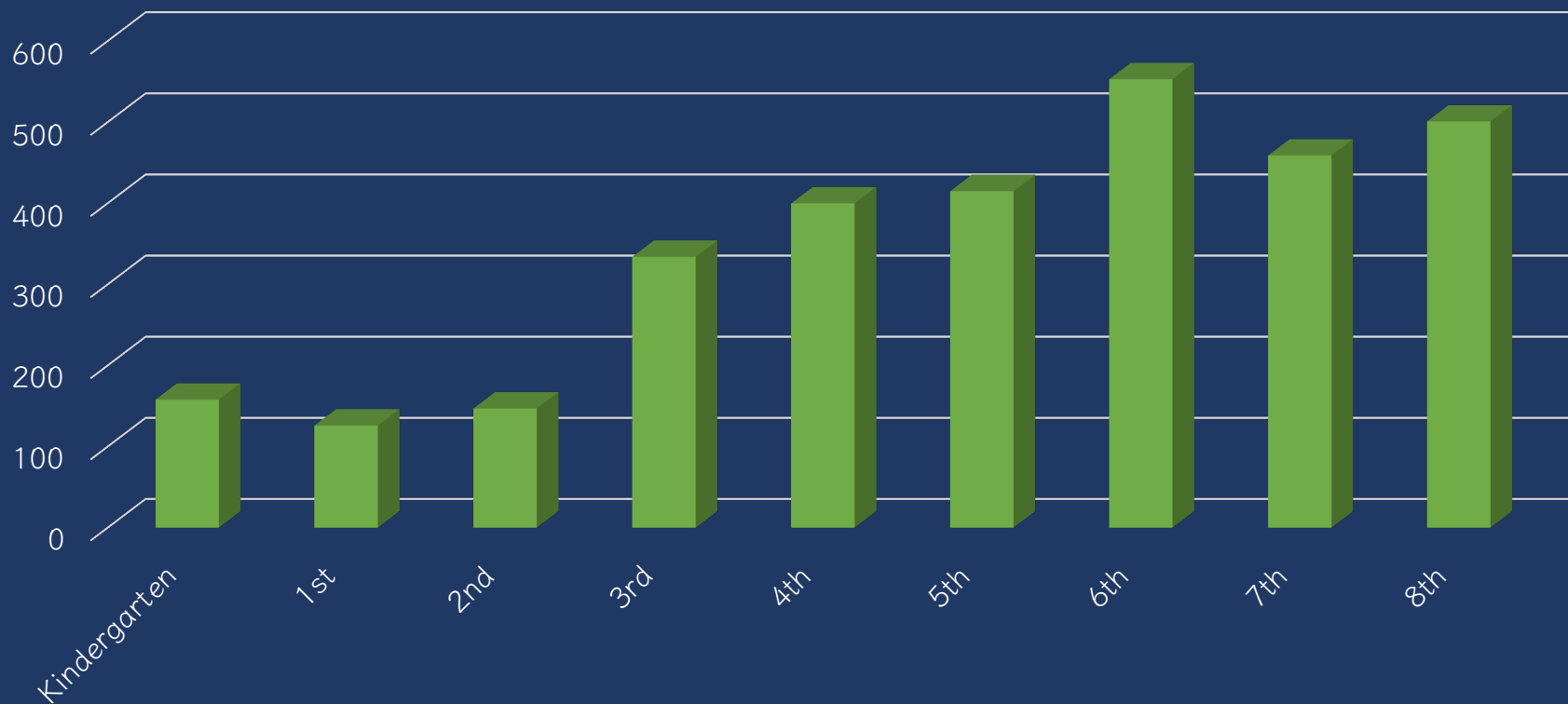
INSTRUCTIONAL DELIVERY

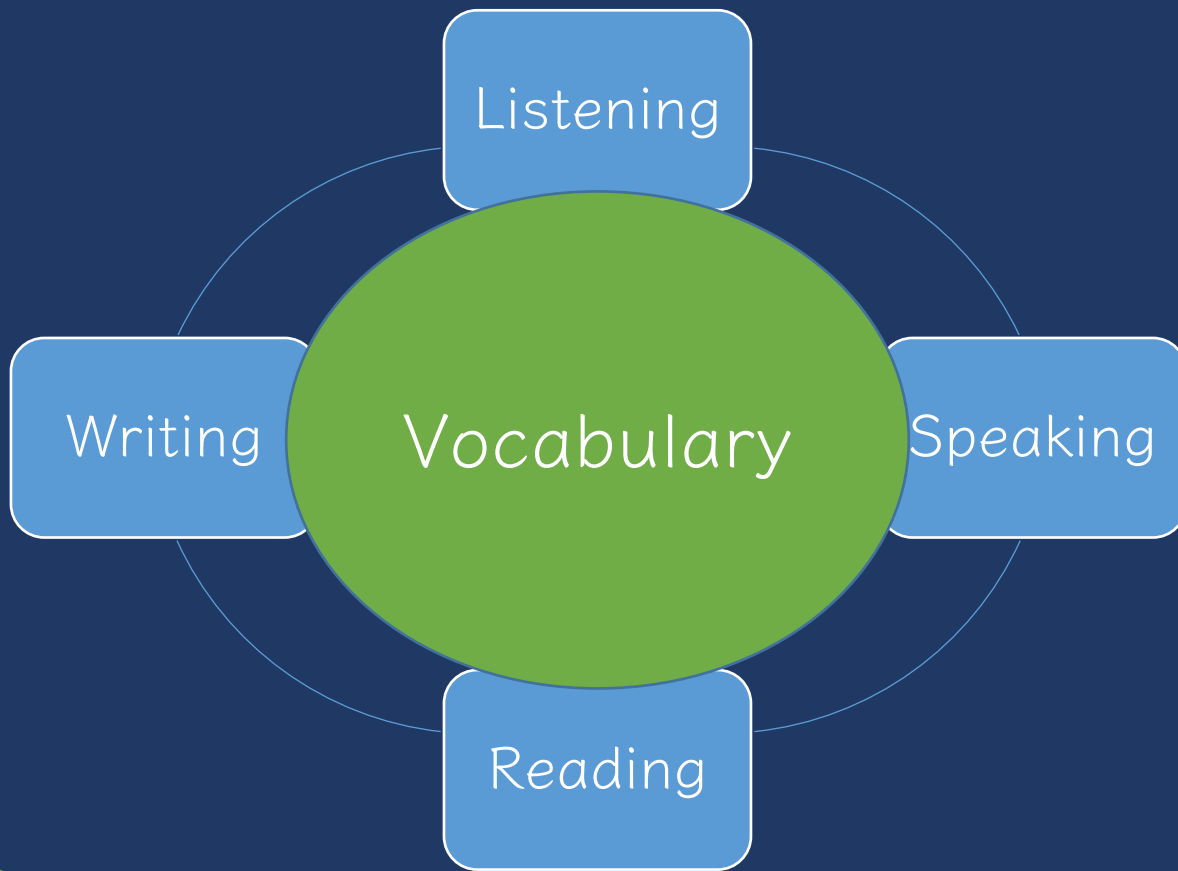
Explicit
instruction

Precise
language

INSTRUCTIONAL STRATEGIES







Which are areas of concern with regard to the language of mathematics?

1. Some math terms are shared with English but have different meanings

base

right

degree

Rubenstein & Thompson (2002)



1. Some math terms are shared with English but have different meanings

2. Some math words are shared with English with similar meanings
(but a more precise math meaning)

difference

even

Rubenstein & Thompson (2002)



1. Some math terms are shared with English but have different meanings
2. Some math words are shared with English with similar meanings
(but a more precise math meaning)
3. Some math terms are only used in math

trapezoid

numerator

parallelogram

Rubenstein & Thompson (2002)



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4. Some math terms have more than one meaning

round

square

second

base

Rubenstein & Thompson (2002)



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5. Some math terms are similar to other content-area terms with different meanings

divide vs.
Continental
Divide

variable vs.
variably
cloudy

Rubenstein & Thompson (2002)



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eight vs. ate

sum vs. some

rows vs. rose

base vs. bass

Rubenstein & Thompson (2002)



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factor vs.
multiple

hundreds vs.
hundredths

numerators
vs.
denominator

Rubenstein & Thompson (2002)



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mesa vs.
tabla

Rubenstein & Thompson (2002)



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9. English spelling and usage may have irregularities

four vs. forty

Rubenstein & Thompson (2002)



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10. Some math concepts are verbalized in more than one way

skip count
vs. multiples

one-fourth
vs. one
quarter



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rhombus vs.
diamond

vertex vs.
corner



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Which of
these
cause the
most
difficulty
for your
students?



Use formal math language

Use terms precisely



Instructional Platform

INSTRUCTIONAL DELIVERY

Explicit
instruction

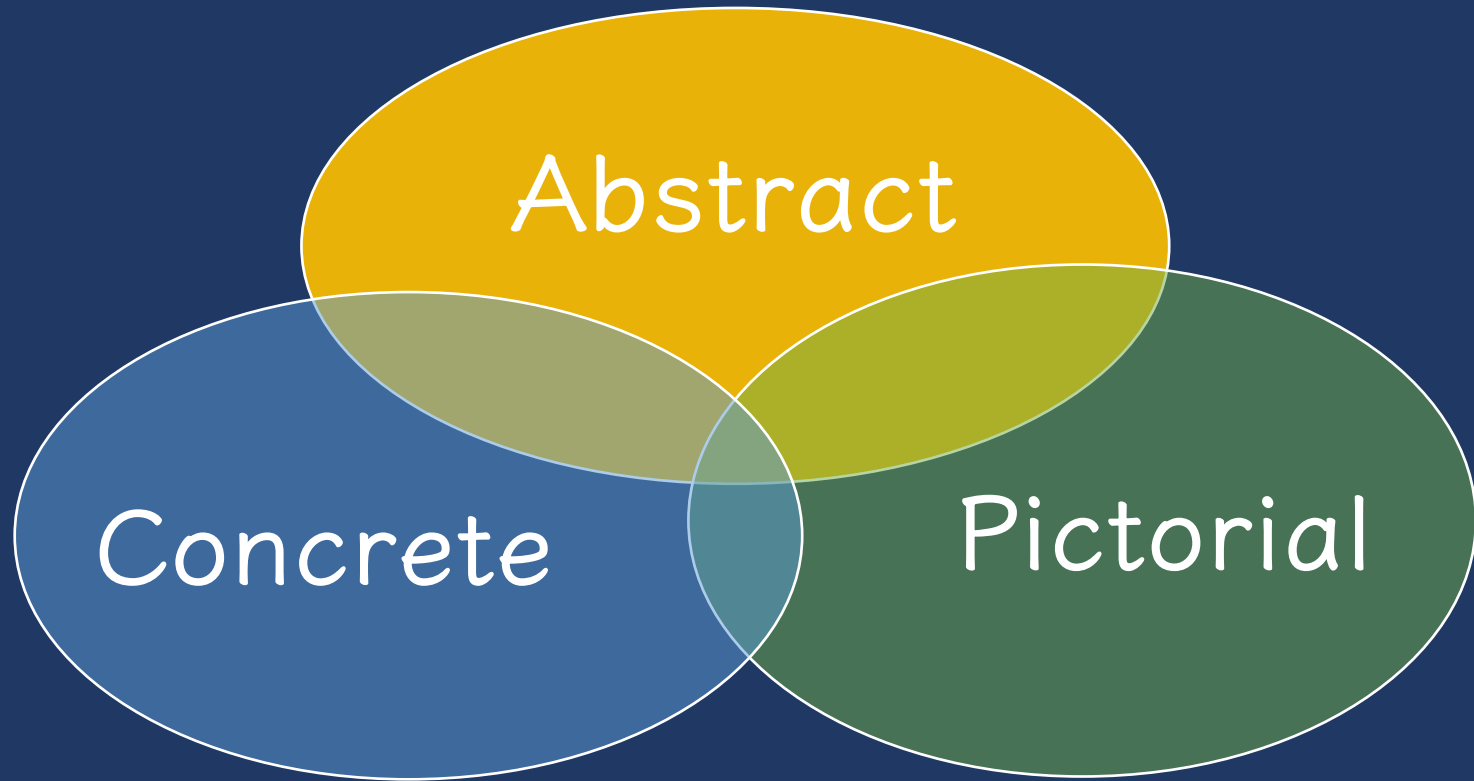
Precise
language

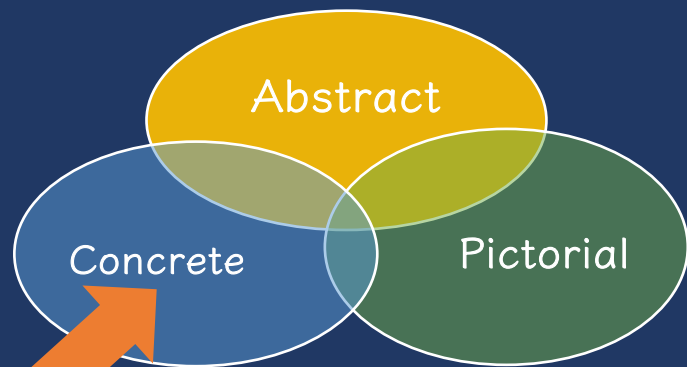
Multiple
representations

INSTRUCTIONAL STRATEGIES

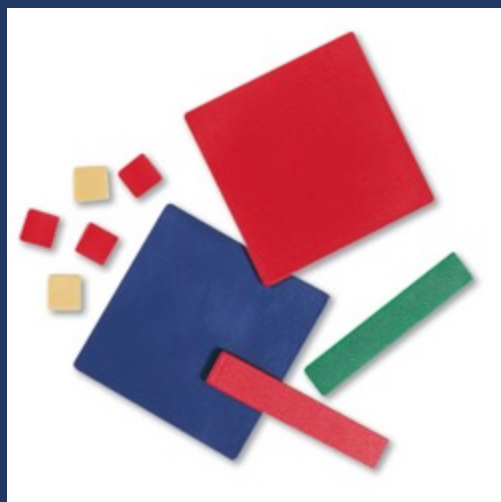
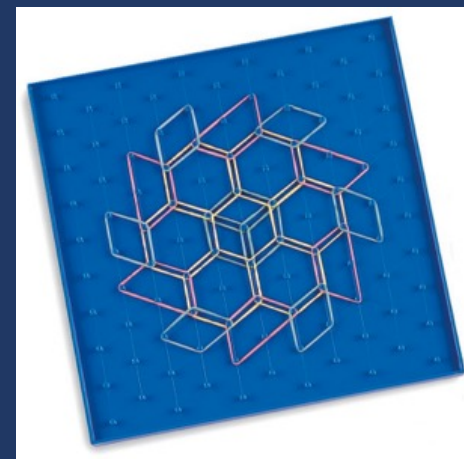


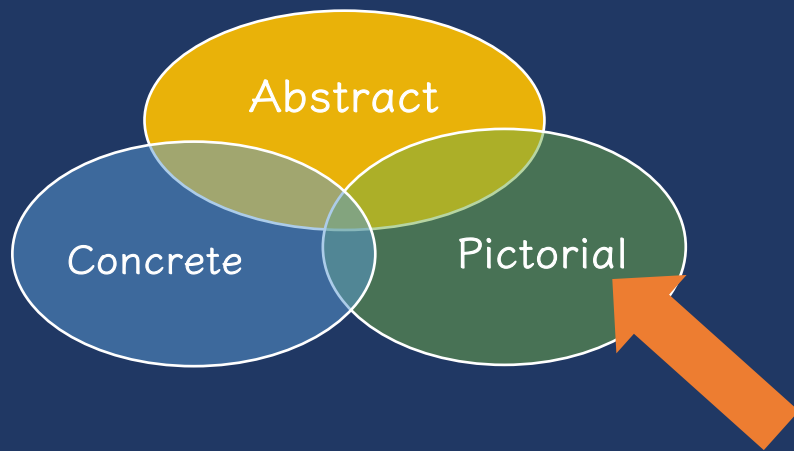
Multiple Representations



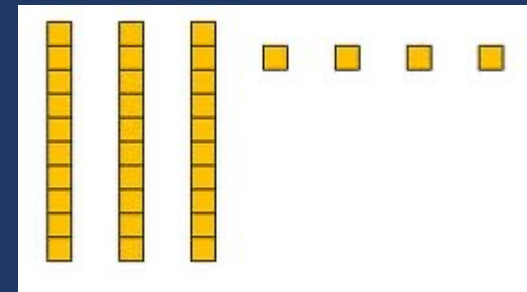
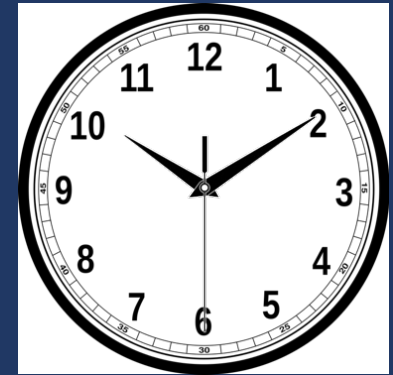
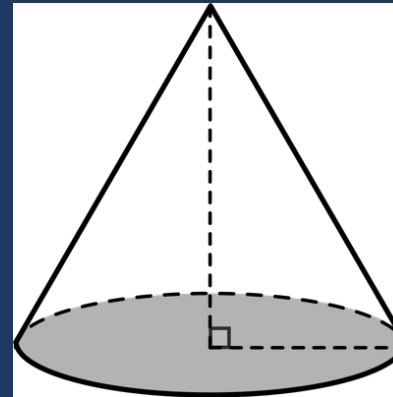


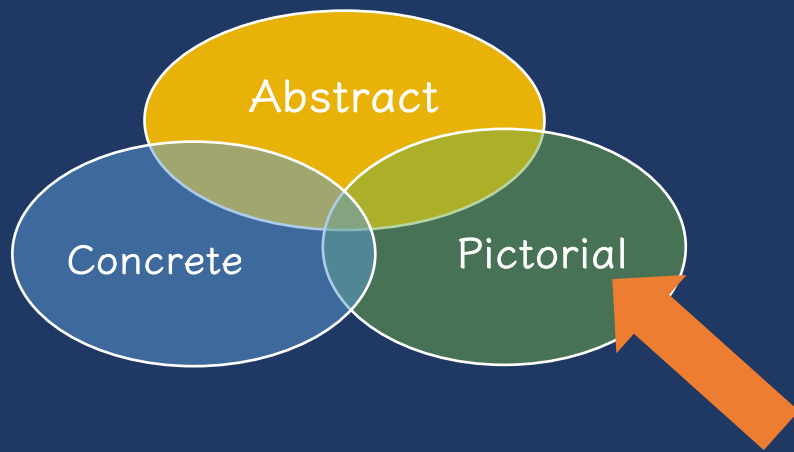
Three-dimensional objects



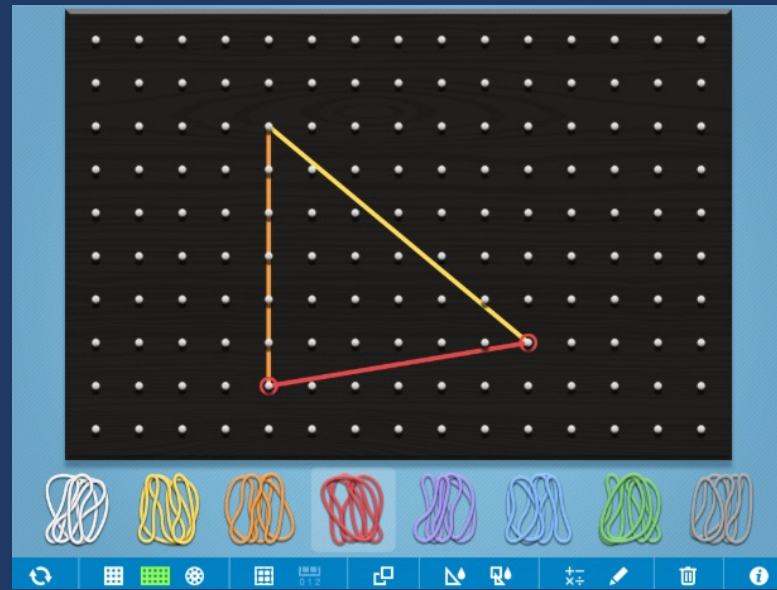


Two-dimensional images

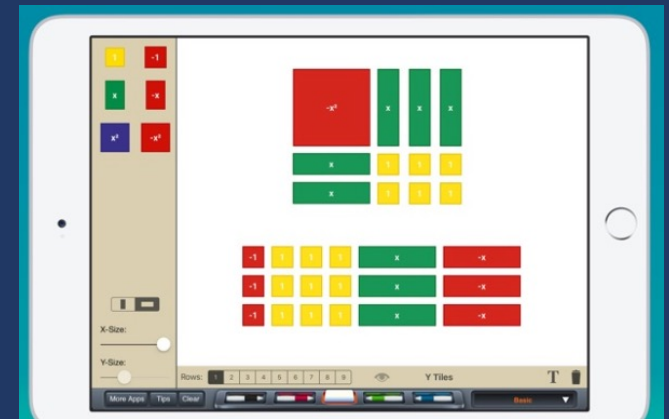
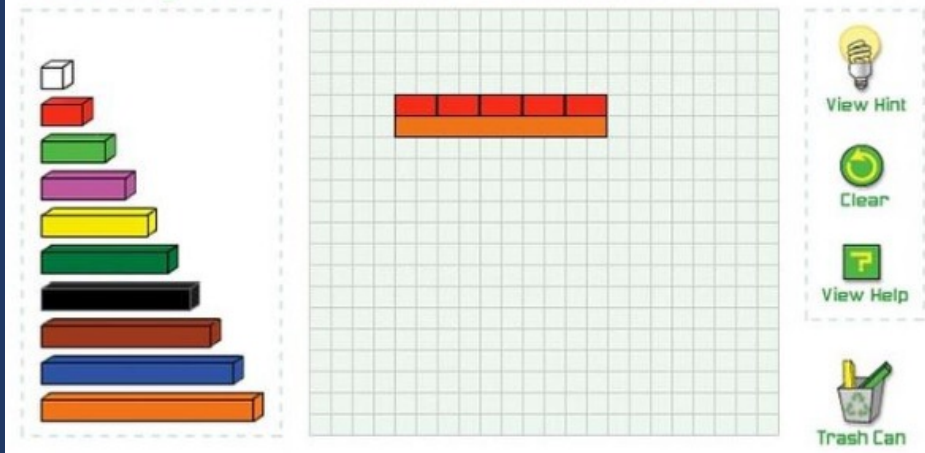


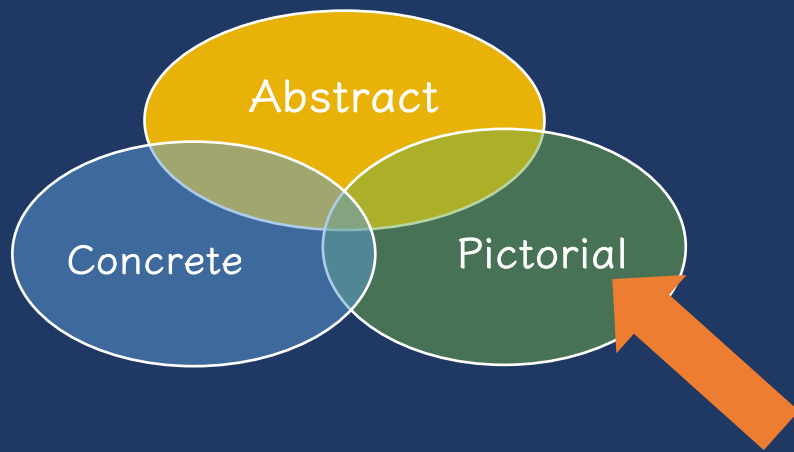


Two-dimensional images

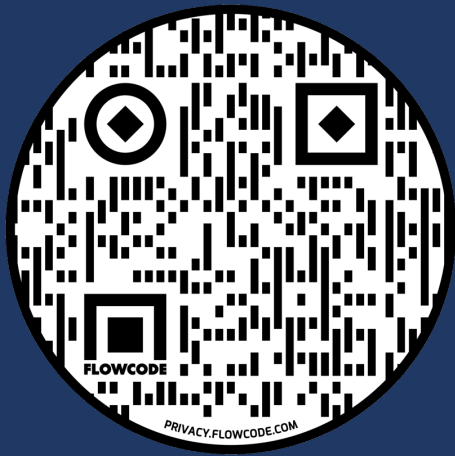


Modeling Fractions with Cuisenaire Rods





Two-dimensional images



Virtual Manipulatives

Number & Operations

Fractions & Decimals

Geometry

Data & Probability

Place Value

Integers & Algebra

Time & Money

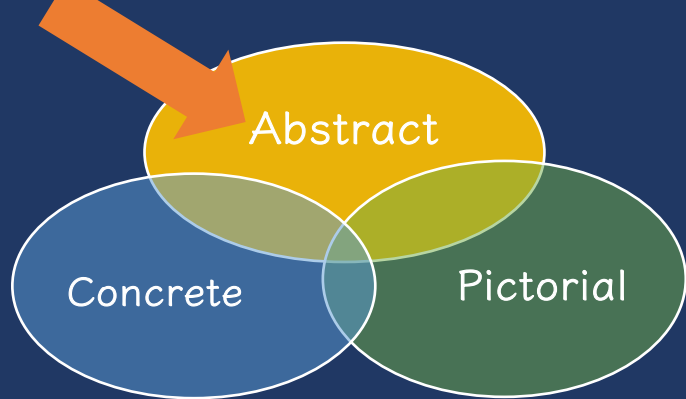
Extras

Fall 2020
EDC 370E

Sarah R. Powell, Ph.D.
srpowell@austin.utexas.edu
www.sarahpowellphd.com
@sarahpowellphd

Fractions & Decimals			





Numerals and symbols and words

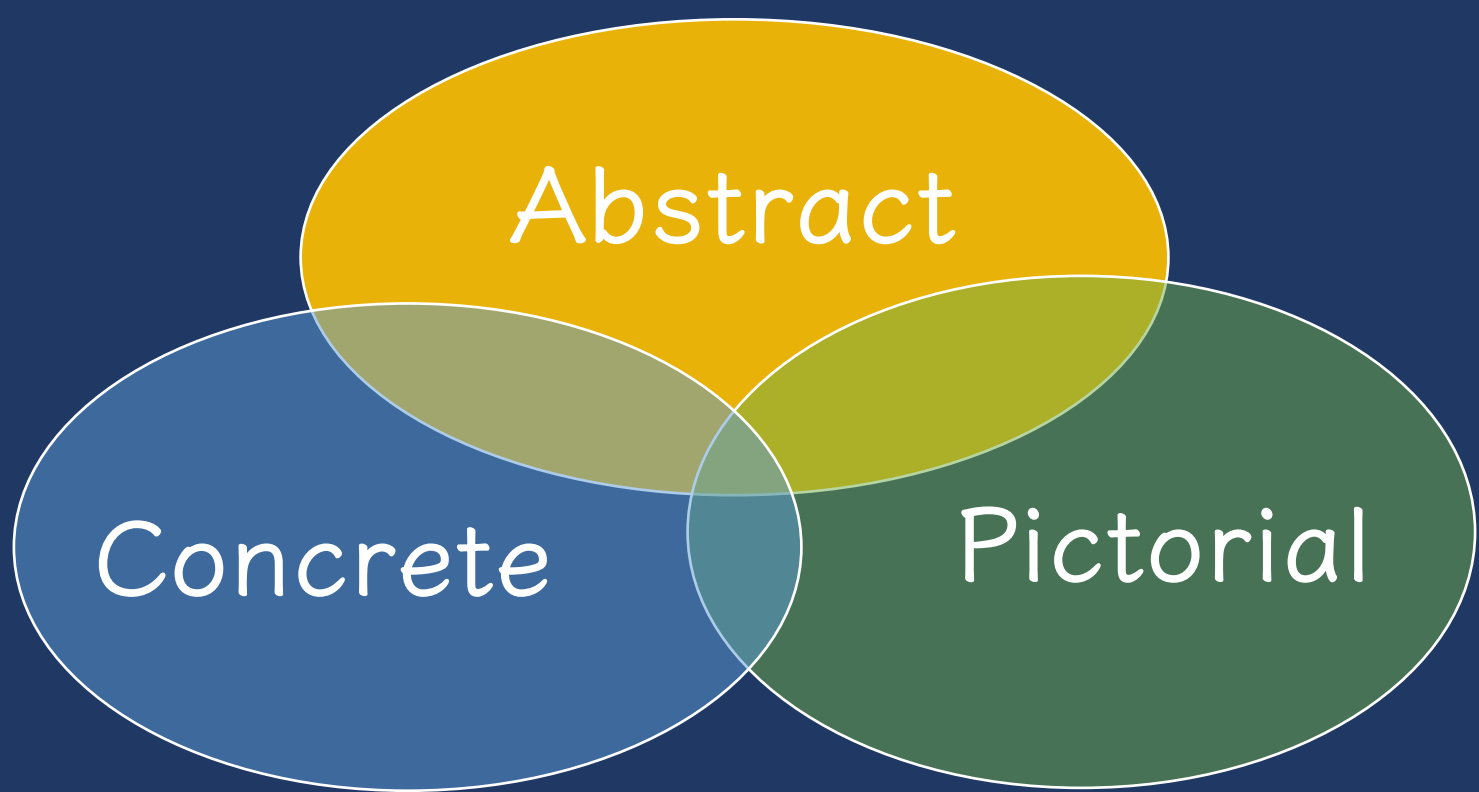
$$2 + 8 = 10$$

$$34 = 3 \text{ tens and } 4 \text{ ones}$$

$$x - 6 = 8$$

$$\begin{array}{r} 4,179 \\ + \quad 569 \\ \hline \end{array}$$





Think of the Science of Math.
How could multiple
representations help students
understand math?

Instructional Platform

INSTRUCTIONAL DELIVERY

Explicit
instruction

Precise
language

Multiple
representations

INSTRUCTIONAL STRATEGIES

Fluency building



Building Fluency

Addition	Subtraction
Multiplication	Division

- Fluency is doing mathematics easily and accurately.
- Fluency in mathematics makes mathematics easier.
- Fluency provides less stress on working memory.
- Fluency helps students build confidence with mathematics.



Addition	Subtraction
Multiplication	Division

- Build fluency with math facts.
 - Addition: single-digit addends
 - Subtraction: single-digit subtrahend
 - Multiplication: single-digit factors
 - Division: single-digit divisor

$$\begin{array}{r}
 5 \\
 + 8 \\
 \hline
 \end{array}
 \begin{array}{r}
 9 \\
 - 4 \\
 \hline
 \end{array}
 \begin{array}{r}
 6 \\
 \times 7 \\
 \hline
 \end{array}
 \begin{array}{r}
 56 \\
 \div 8 \\
 \hline
 \end{array}$$



Addition	Subtraction
Multiplication	Division

- Build fluency with whole-number computation

$$\begin{array}{r} 15 \\ + 28 \\ \hline \end{array}$$

$$\begin{array}{r} 1009 \\ - 724 \\ \hline \end{array}$$

$$\begin{array}{r} 23 \\ \times 9 \\ \hline \end{array}$$

$$\begin{array}{r} 7250 \\ \div 15 \\ \hline \end{array}$$



Addition	Subtraction
Multiplication	Division

- Build fluency with rational-number computation

$$\begin{array}{r} 1.4 \\ + 3.9 \\ \hline \end{array}$$

$$\begin{array}{r} 7.892 \\ \div 0.14 \\ \hline \end{array}$$

$$\frac{2}{3} \times \frac{3}{4}$$

$$\frac{9}{4} - \frac{3}{8}$$



Addition	Subtraction
Multiplication	Division

- Build fluency with integer computation

$$-135 \div 2 =$$

$$\begin{array}{r} 6 \\ \times -12 \\ \hline \end{array}$$

$$-14 - (-7) =$$

$$\begin{array}{r} 1.4 \\ + -3.9 \\ \hline \end{array}$$



Addition	Subtraction
Multiplication	Division



Think of the Science of Math.
What are evidence-based fluency
practices you could use with
students?

Instructional Platform

INSTRUCTIONAL DELIVERY

Explicit
instruction

Precise
language

Multiple
representations

INSTRUCTIONAL STRATEGIES

Fluency building

Problem solving
instruction



Teaching Problem Solving

Have an attack strategy
Teach word-problem schemas



Have an attack strategy

RIDE

Read the problem.

Identify the relevant information.

Determine the operation and unit for the answer.

Enter the correct numbers and calculate, then check the answer.

RIDGES

Read the problem.

I know statement.

Draw a picture.

Goal statement.

Equation development.

Solve the equation.



Have an attack strategy

STAR

Stop and read the problem carefully.

Think about your plan and the strategy you will use.

Act. Follow your plan and solve the problem.

Reverview your answer.

RICE

Read and record the problem.

Illustrate your thinking.

Compute.

Explain your thinking.



Have an attack strategy

SUPER

Slowly read the story problem twice.
Underline the question and circle the numbers you need.
Picture it. Draw the scenario to show what is happening.
Explain the problem with a number sentence.
Rewrite the answer in a sentence.

SHINES

Slowly and carefully read the problem.
Highlight or underline key information.
Identify the question by drawing a circle around it.
Now solve the problem. Show your work.
Examine your work for precision, accuracy, and clarity.
Share your answer by writing a sentence.



Have an attack strategy

SOLVE

Study the problem.

Organize the facts.

Line up the plan.

Verify the plan with computation.

Examine the answer.

R-CUBES

Read the problem.

Circle key numbers.

Underline the question.

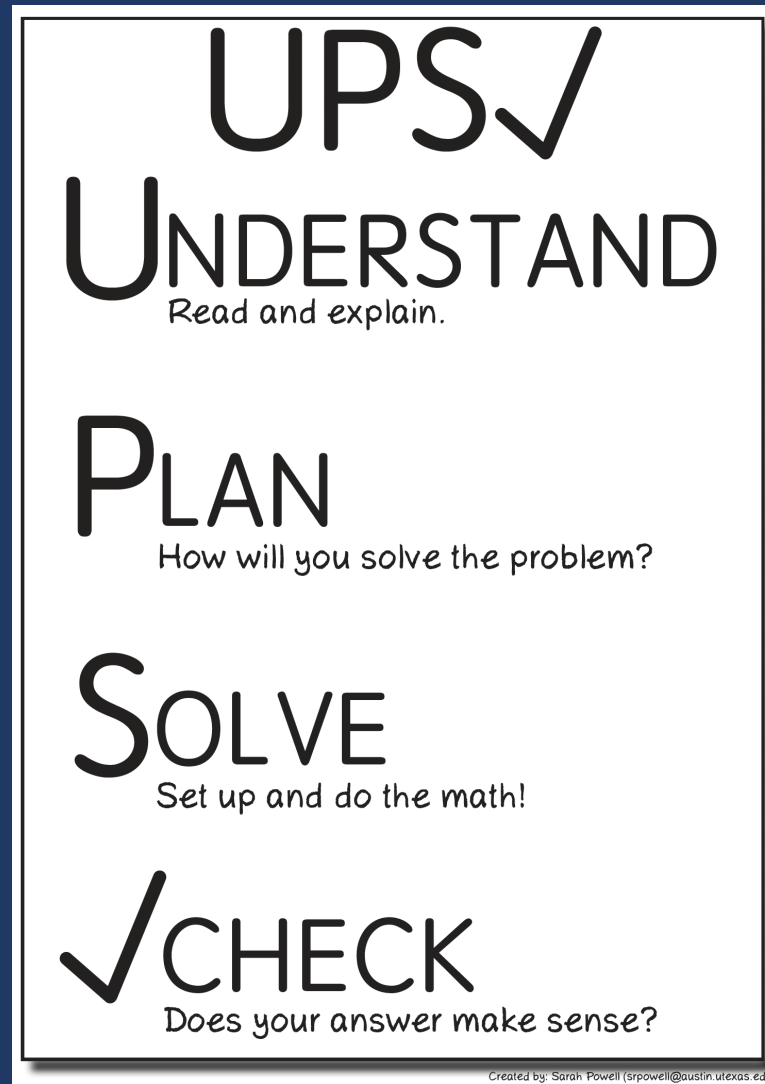
Box action words.

Evaluate steps.

Solve and check.



Have an attack strategy



Teach word-problem schemas

Total

Difference

Change

Equal Groups

Comparison

Ratios/Proportions



Teaching Problem Solving

Have an attack strategy

Teach word-problem schemas



Think of the Science of Math.
How would you help students
approach word-problem solving?



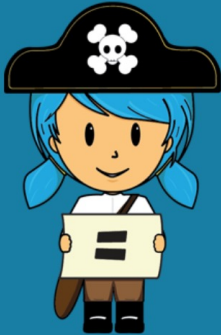


Pirate Math Equation Quest

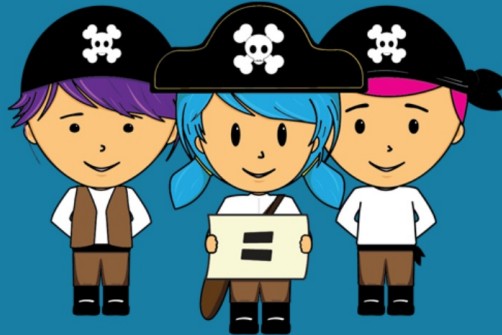
[About](#)[Research](#)[Individual](#)[Small Group](#)[STAAR](#)[Videos](#)

Welcome to Pirate Math Equation Quest!

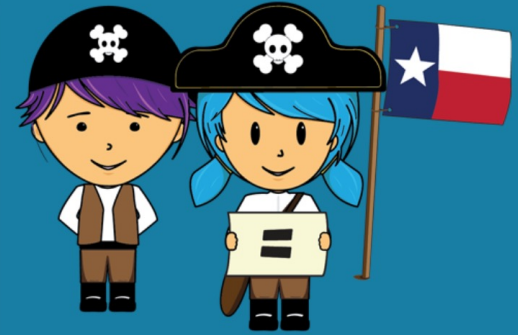
Individual Word-Problem Intervention



Small-Group Word-Problem Intervention



Small-Group Word-Problem Intervention for STAAR



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Instructional Platform

INSTRUCTIONAL DELIVERY

Explicit
instruction

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Multiple
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INSTRUCTIONAL STRATEGIES

Fluency building

Problem solving
instruction



National Center on
INTENSIVE INTERVENTION
at American Institutes for Research

Search

Intensive
Intervention ▾

Tools
Charts ▾

Implementation
Support ▾

Intervention
Materials ▾

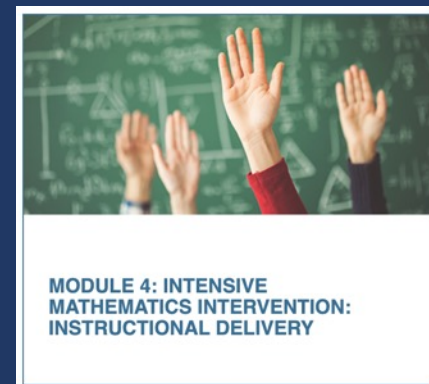
Information
For... ▾

Intensive Intervention in Mathematics Course Content

NCII, through a collaboration with the University of Connecticut, developed a set of course content focused on developing educators' skills in designing and delivering intensive mathematics instruction. This content is designed to support faculty and professional development providers with instructing pre-service and in-service educators who are developing and/or refining their implementation of intensive mathematics intervention.

Intensive instruction was recently identified as a [high-leverage practice in special education](#), and DBI is a research based approach to delivering intensive instruction across content areas (NCII, 2013). This course provides learners with an opportunity to extend their understanding of intensive instruction through in-depth exposure to DBI in mathematics, complete with exemplars from actual classroom teachers.

NCII, through a collaboration with the University of Connecticut and the [National Center on Leadership in Intensive Intervention](#) and with support from the [CEEDAR Center](#), developed course content focused on enhancing educators' skills in intensive mathematics intervention. The course includes eight modules that can support faculty and professional development providers with instructing pre-service and in-service educators who are learning to implement intensive mathematics intervention through data-based individualization (DBI). The content in this course complements concepts covered in the [Features of Explicit Instruction Course](#) and so we suggest that users complete both courses.

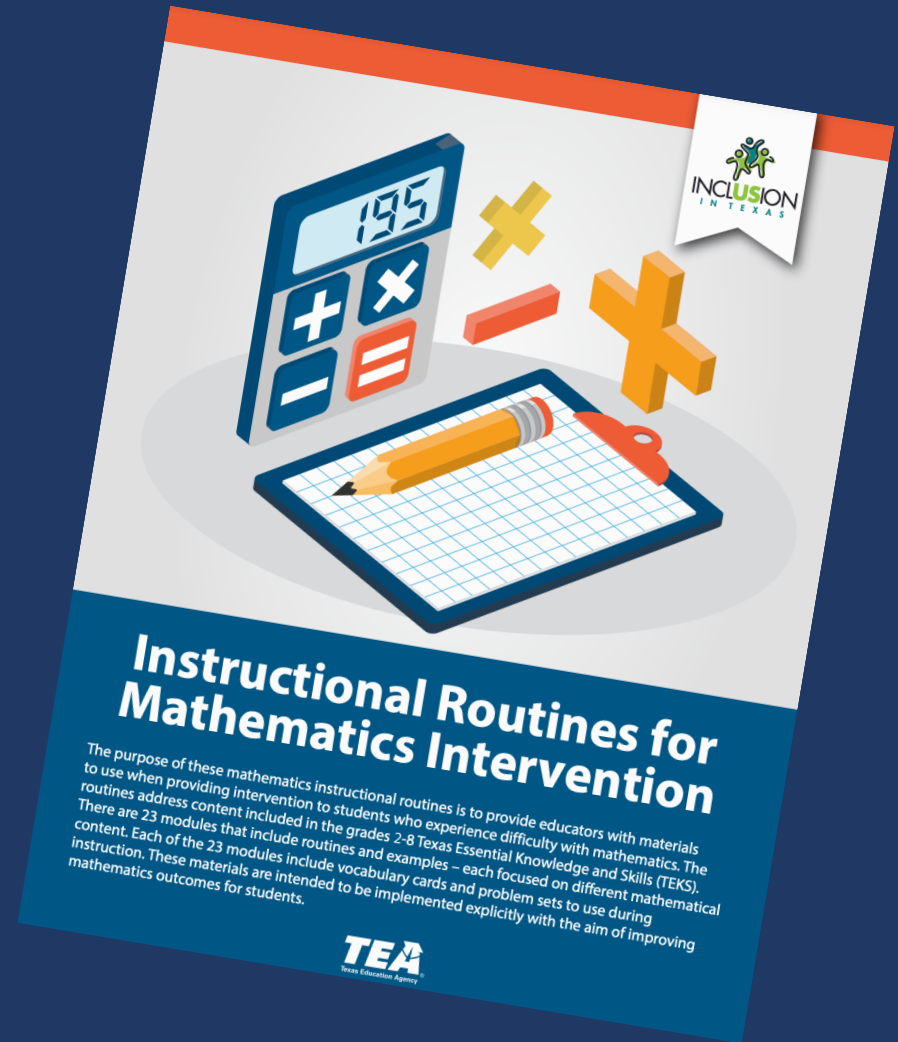
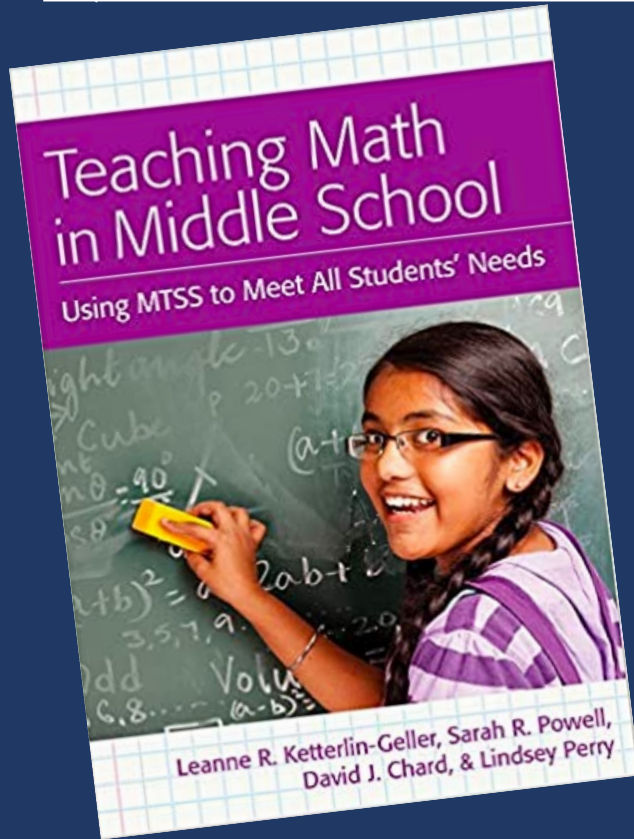


**MODULE 4: INTENSIVE
MATHEMATICS INTERVENTION:
INSTRUCTIONAL DELIVERY**



**MODULE 5: INTENSIVE
MATHEMATICS INTERVENTION:
INSTRUCTIONAL STRATEGIES**

<https://www.amazon.com/Teaching-Math-Middle-School-Students/dp/1598572741>



https://www.inclusionintexas.org/apps/pages/index.jsp?uREC_ID=2155039&type=d&pREC_ID=2169859





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@4ScienceofMath



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