# Math 1497 – Cole 2

Last class we consider u substitution

Ex

$$\int x^3 (x^4+2)^3 dx$$

let $u = x^4+2$ $du = 4x^3 dx$ $\frac{du}{4} = x^3 dx$

$$\int u^3 \frac{du}{4} = \frac{u^4}{16} + C = \frac{1}{16}(x^4+2) + C$$

we now consider a new technique for integrals that involve products

Ex $\int x \ln x \, dx$, $\int x e^x dx$ $\int e^x \sin x \, dx$

Consider

$$\frac{d}{dx} x e^x = 1 \cdot e^x + x e^x$$

so $d(xe^x) = 1 \cdot e^x + x e^x dx$

so $\int d(xe^x) = \int e^x\,dx - \int xe^x\,dx$

$$xe^x = e^x - \int xe^x\,dx$$

so $\int xe^x\,dx = e^x - xe^x + c$

Now $d(uv) = u\,dv + v\,du$

$$\int d(uv) = \int u\,dv + \int v\,du$$

$$\boxed{\int u\,dv = uv - \int v\,du}$$ Integratcin by parts formula

$\int u\,dv$ - given - need to pick

$u$ & $dv$

need to find $du$ $v$ then use formula

Ex 1 $\int x e^x \, dx$

$u = x$ $\quad\downarrow$

$\uparrow$ $dv = e^x dx$

So $u = x$ $\quad v = e^x$

$du = dx$ $\quad dv = e^x dx$

formula $x e^x - \int e^x dx$

$= x e^x - e^x + c$ as before

Ex 2 $\int x \cos x \, dx$

$u = x$ $\quad v = \sin x$

$du = 1 dx$ $\quad dv = \cos x \, dx$

$= x \sin x - \int \sin x \, dx = x \sin x + \cos x + C$

Ex 3 $\int x \ln x \, dx$

$u = x$

$dv = \ln x \, dx$ ← but how to integrate this?

instead try

$u = \ln x \quad v = x^2/2$

$du = \frac{dx}{x} \quad dv = x \, dx$

$= \frac{x^2}{2} \ln x - \int \frac{x^2}{2} \frac{dx}{x}$

$= \frac{x^2}{2} \ln x - \frac{1}{2} \int x \, dx$

$= \frac{x^2}{2} \ln x - \frac{x^2}{4} + c$

Ex 4 $\int \ln x \, dx$

$u = \ln x \qquad v = x$

$du = \frac{dx}{x} \qquad dv = dx$

$= x \ln x - \int x \frac{dx}{x} \quad \Leftarrow \quad \int 1 \, dx$

$= x \ln x - x + c$

Ex 5 $\int \tan^{-1} x \, dx$

$u = \tan^{-1} x \qquad v = x$

$du = \frac{1}{1+x^2} dx \qquad dv = dx$

$= x \tan^{-1} x + \int \frac{x}{1+x^2} dx$

$= x \tan^{-1} x + \frac{1}{2} \ln |1+x^2| + c$

Aside

$w = 1+x^2$

$dw = 2x\,dx$

$\int \frac{dw}{2w}$

$\frac{1}{2} \ln |w| + c$

ex 6 $\int x^2 e^x \, dx$

$u = x^2 \quad v = e^x$

$du = 2x\,dx \quad dv = e^x dx$

$= x^2 e^x - \int 2x e^x \, dx$

$= x^2 e^x - 2 \int \underline{x e^x dx}$

parts again

$= x^2 e^x - 2 \left[ x e^x - e^x \right] + C$

ex 7 $\int e^x \sin x \, dx$

$u = \sin x \quad v = e^x$

$du = \cos x\,dx \quad dv = e^x dx$

$= e^x \sin x - \int e^x \cos x \, dx$

parts on second $\int$

$u = \cos x \qquad v = e^x$

$du = -\sin x\,dx \quad dv = e^x dx$

$$\int e^x \sin x\,dx = e^x \sin x - \left[ e^x \cos x - \int -e^x \sin x\,dx \right]$$

$$= e^x \sin x - e^x \cos x - \int e^x \sin x\,dx$$

$$2\int e^x \sin x\,dx = e^x \sin x - e^x \cos x$$

$$\int e^x \sin x\,dx = \frac{1}{2} e^x \sin x - \frac{1}{2} e^x \cos x + C$$

Ex $\int \sec^3 x \, dx$

$$\int \sec x \sec^2 x \, dx$$

$u = \sec x \qquad v = \tan x$

$du = \sec x \tan x \, dx \qquad dv = \sec^2 x \, dx$

$$= \sec x \tan x - \int \sec x \tan^2 x \, dx$$

$$= \sec x \tan x - \int \sec x (\sec^2 x - 1) \, dx$$

$$= \sec x \tan x - \int \sec^3 x \, dx + \int \sec x \, dx$$

$$2 \int \sec^3 x \, dx = \sec x \tan x + \ln|\sec x + \tan x|$$

$$\int \sec^3 x \, dx = \frac{1}{2} \sec x \tan x + \frac{1}{2} \ln|\sec x + \tan x| + C$$