

Commanding evidence for a third kind of relativistic effect

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About the Papers

Problems of current interest are considered and *quantitative* solutions to them achieved through introduction of a new relativistic theory. The theory is presented in the first discussion (I). The problems are then taken up in the sequence listed below. Roman numerals in parentheses specify the location of the respective discussion. Capsulizing the results:

1. Dark matter: It does not exist. The halo material is responsible. (I)
2. Solar neutrino deficit: Material in the Sun's core is a bit cooler and a little less dense than thought. (I)
3. Apsidal motion of eclipsing binary stars: With inclusion of the additional relativistic effects, measured rates are reconciled with theory. (I)
4. Hill's measurement of the Sun's oblateness and his conclusion: Hill was correct. These results agree with his to two significant figures. (I)
5. The Pioneer 10,11 gravitational anomalies: There is no anomalous acceleration. The third-relativistic effects have been ignored in calibration procedures, resulting in an error. (II)
6. The distance to M33 is greater than thought: Not so. The value of the binary system's semimajor axis has been incorrectly inferred, a consequence of ignoring the third-relativistic effects. (III)
7. "Missing" baryonic matter: Third-relativistic effects bring current measurements into excellent agreement with the Einstein-de Sitter model. The baryons are not missing. (IV)
8. Gravity is the result of a scale difference between separate electrostatic spacetimes of attraction and repulsion. Light is explained as an oscillatory energy transfer between the spacetimes, which leads to a direct evaluation of the Hubble constant and an accounting of the "dark energy" effect. (V)

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Summaries of the Discussions

Summaries of the papers in plain language and without the mathematics are available for the asking. I will email them to you as PDF attachments.

About the Author

I received my BS degree from the United States Military Academy, West Point, and my PhD in physics from the University of California, Berkeley. I was appointed to West Point by the Army from the enlisted ranks and served after graduation in the 82nd Airborne Division. I worked as a professional physicist for a number of years, principally at Goddard Space Flight Center and at the University of Denver, but decided that a career, any career, was not something I would ever be comfortable with. I have lived for many years now with my family at this same wilderness location near Meadow Creek. We have solar power and, as of July 2006, satellite-direct Internet access.

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About my Motives

The first two papers presented here were submitted to professional journals for publication. They were rejected for reasons that in each case were easily refuted. Editors either refused to reconsider the papers or stated that they did not want to publish papers they considered speculative. As for the third paper, I contacted one of the authors of the referenced work, told him of my results and suggested we collaborate. He was not interested.

On a happier note, I quote some beautiful lines from Robert Browning's "Abt Vogler" that succinctly describe why I persist.

But here is the finger of God, a flash of the will that can,
 Existent behind all laws, that made them and, lo, they are!
 And I know not if, save in this, such gift be allowed to man,
 That out of three sounds he frame, not a fourth sound, but a
 star.

Consider it well: each tone of our scale in itself is naught;
 It is everywhere in the world – loud, soft, and all is said:
 Give it to me to use! I mix it with two in my thought:
 And, there! Ye have heard and seen: consider and bow the
 head!

The Photo: M101, the "Pinwheel Galaxy." Courtesy of NASA, STScI.

Does ionized matter exhibit a third kind of relativistic effect?

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Abstract: Special and general relativistic effects are manifested through direct changes in space and time measures. A number of current astrophysical problems suggest that there may be a third relativistic effect, associated with ionization. It is proposed that there are two natural measures of spacetime, gravitational and electromagnetic. For its gravitational interactions, ionized matter scales the former with respect to the latter. A quantitative model of the scaling is developed from simple basic assumptions analogous to those underlying special relativity. Introducing this scaling effect by means of suitable approximations into the theoretical treatments of the problems of “dark matter” in the Galaxy, “missing” solar neutrinos, and the eclipsing binary systems DI Herculis and AS Camelopardalis leads to agreement between theory and observation in each case, and with no additional assumptions. The theory has no adjustable parameters. A class of experiments to test directly for the effect is proposed.

1. Introduction

Years ago when I wrote the article on the concept of *force* for a popular encyclopedia, I was pleased to learn that the philosophical background was to be contributed by Max Jammer, a man whose work I greatly admired. His closing thought there has stayed with me: “Certainly [force] is not an illusion . . . it enables us to discuss the general laws of motion irrespective of the particular physical situation with which these laws happen to be associated” [1].

Paradoxically, the underlying premise of this discussion is that the statement is false. I propose that ionized matter obeys altered laws of gravitation. The changes are introduced through a scaling of space and time at the atomic level by the ionized material. It is the failure to incorporate these differences of scale into theoretical treatments that gives rise to the well-known discrepancies between prediction and observation in the three classes of phenomena to be discussed: galactic dynamics, or the presence of what is presumed to be “dark matter”; the apparent deficit in solar neutrino production as observed at Earth; and the unexpectedly slow apsidal motion of the eclipsing binary star systems DI Herculis and AS Camelopardalis. An incorporation of the scaling effects as developed here leads to agreement between the corrected theory and observation in each case.

This is an initial investigation. It introduces quantitative evidence for an unanticipated electromagnetic effect. The problems selected share four simplifying characteristics: The gravitating bodies involved (1) are composed predominantly of hydrogen; (2) are nearly completely ionized throughout, to

include helium and the more-abundant heavier nuclei such as oxygen; (3) may be treated as perfect gases in thermal equilibrium (composed of stripped nuclei and electrons); and (4) are very nearly spherically symmetric. The present discussion is restricted to problems displaying these characteristics. If there is in fact a connection at the quantum level between the electromagnetic force and gravitation as some have proposed (see e.g. [2, 3]), it is reasonable to expect that one might observe its effects under these conditions.

2. Development of theory

Gravitational effects originate ultimately within each atom. Our starting premise is that the electromagnetic characteristics of each atom, under conditions to be specified shortly, alter spacetime for it, but *only* with respect to the gravitational force. That is, there are two natural representations of spacetime: one electromagnetic, here to be denoted by E , and the other gravitational, to be denoted by G_* . When matter is ionized, E and G_* differ in scaling of time and distance.

Since only nondegenerate, fully ionized matter is considered, quantization of states is not a direct concern. Each atomic unit, that is, each nucleus and its neighboring electrons, scales G_* in terms of its own space and time dimensions. (Where it is helpful to make the distinction explicit, star subscripts will be used to denote variables evaluated in G_* .) For matter in bulk, the gravitational scaling factors are functions of the temperature T , the density ρ , and the elemental composition. If the temperature and density in a problem are static, the

gravitational scaling factors are constant. Deductions from astronomical observations often involve employment of gravitational laws. When they do, it is implicitly assumed that E and G_* are the same. Thus astrophysical problems are a likely place for the effects to manifest and a good place to test the theory.

We begin by considering a fully ionized plasma at thermal equilibrium, consisting of hydrogen. Let a be a measure of the mean distance between electrons and protons, and v_e the RMS electron speed. We postulate that distances in E and G_* are scaled at the atomic level according to the relationship

$$\Delta r_* = \Delta r \gamma^{-1}. \quad (1)$$

As well as being dimensionless, γ is assumed to be a function of v_e and a and not singular at $v_e = 0$. We set $\gamma = \gamma(v_e, a)$, $d\gamma = (\partial\gamma/\partial v_e)dv_e + (\partial\gamma/\partial a)da$. At fixed temperature T , $dv_e = 0$ and $d\gamma = (\partial\gamma/\partial a)da$. A simple solution meeting the above criteria is obtained by setting $(\partial\gamma/\partial a) = (v_e/c)(1/a)$, c being the speed of light. Then $(d\gamma)_T = (v_e/c)(da/a)$. Setting $\beta = v_e/c$ and integrating, $\gamma - \gamma_0 = \beta \ln(a/a_0)$. At $a = a_0$, a_0 taken to be the radius of the first Bohr orbit, we set $\gamma = \gamma_0 = 1$. Generalizing to elements of atomic number Z is accomplished by replacing a_0 with a_0/Z . The Debye-Hückel radius R_D is adopted as the measure of a . Finally, v_e is set equal to $(3kT/m_e)^{1/2}$, where k is Boltzmann's constant and m_e is the mass of the electron.

In summary, the gravitational scaling factor (GSF), ${}_j\gamma$, for element Z_j in a homogeneous plasma made up of elements of atomic number Z_i and atomic weight A_i , each with fractional mass abundance X_i , is

$${}_j\gamma = 1 + \beta \ln(Z_j R_D / a_0), \quad (2)$$

where $\beta = (3kT/m_e c^2)^{1/2}$, $R_D = (kT m_p / 4\pi e^2 \rho \zeta)^{1/2}$, $\zeta = \sum_i (X_i / A_i)(Z_i^2 + Z_i)$,

$a_0 = h^2 / 4\pi^2 e^2 m_e$, m_p is the mass of the proton, h is Planck's constant and e the charge of the electron.

It is assumed that the speed of light is the same in E and G_* and that the systems are related through a Lorentz transformation in vector form [4]. One obtains the transformation equations:

$$\mathbf{r}_* = \gamma \mathbf{r} - (\gamma^2 - 1)^{1/2} ct \hat{\mathbf{e}} \quad ct_* = -(\gamma^2 - 1)^{1/2} \mathbf{r} \cdot \hat{\mathbf{e}} + \gamma ct \quad (3a,b)$$

$$\mathbf{r} = \gamma \mathbf{r}_* + (\gamma^2 - 1)^{1/2} ct_* \hat{\mathbf{e}}_* \quad ct = (\gamma^2 - 1)^{1/2} \mathbf{r}_* \cdot \hat{\mathbf{e}}_* + \gamma ct_* \quad (4a,b)$$

where $\hat{\mathbf{e}} = \Delta \mathbf{r} / \Delta r = \Delta \mathbf{r}_* / \Delta r_* = \hat{\mathbf{e}}_*$.

In the E system, one considers two events that are at the origin and separated by a time interval Δt . From Eq. (3b),

$$\Delta t_* = \gamma \Delta t. \quad (5)$$

The gravitational force exerted on or by a body depends linearly on its mass. To calculate the effective GSF, γ_1 , for a spherically symmetric body of mass m_1 for which ρ , T and the X_i are known everywhere, one first determines

the weighted mean GSF at each value of r , using equation (2), as it is determined by the local composition. That is, one obtains $[\sum_i X_i(r) \gamma(r)]$. One then has

$$\gamma_1 = (1/m_1) \int_V [\sum_i X_i(r) \gamma(r)] dm_1. \quad (6)$$

We now consider two spherically symmetric, gravitationally interacting bodies of masses m_1 and m_2 respectively. The distance between their centers is Δr in the E system. We set

$$\Delta r_* = \Delta r \gamma_*^{-1}, \quad (7)$$

where $\gamma_* = \gamma_1 \gamma_2$. The scaling effects of each body are independently imposed. As a consequence, Newton's Third Law of motion holds. As in Eq. (5) then,

$$\Delta t_* = \gamma_* \Delta t. \quad (8)$$

3. "Dark matter"

The rotational speeds of stars about galactic centers often imply galactic masses significantly greater than the collective mass of stars, gas and dust in the galaxy. The movements of galaxies within clusters often exhibit the effect to a marked degree, typically implying galactic masses about an order of magnitude greater than that observed [5-9].

Our own galactic halo is reasonably approximated by a uniform sphere of dilute, mostly ionized gas in which cosmic rays are evenly distributed, trapped by random, weak magnetic fields [10-12]. The gravitational effect of this material is greatly enhanced by the scaling effect. The magnitude of the enhancement can be easily estimated.

To start, mean values of halo variables must be adopted. These are: radius of the halo, $R \cong .5 \times 10^{23} \text{cm}$; composition of the gas, $X \cong .73$, $Y \cong .25$, $Z \cong .02$ (X, Y, Z the fractional mass abundances of H, He and heavier elements, respectively); mean mass density, $\rho \cong 1.67 \times 10^{-26} \text{g cm}^{-3}$; mean magnetic field strength, $B \cong 6 \times 10^{-6} \text{gauss}$. Hayakawa suggests $B \cong 3 \times 10^{-6} \text{gauss}$, with an uncertainty of a factor of 2, as a reasonable estimate of the average field strength as inferred from cosmic rays [10]. Radio-luminosity data, however, suggest $B \cong 6 \times 10^{-6} \text{gauss}$ [10,13]. The higher value has been adopted. Proceeding, we first get an estimate of the mean temperature of the gas.

From the density and composition of the gas, the mean particle density n is obtained, assuming H and He to be fully ionized and the heavier nuclei not appreciably ionized. It is, taking $^{16}_8\text{O}$ as representative of the heavier elements, $n = [2X + (3/4)Y + (1/16)Z](\rho/m_H) = 1.65 \times 10^{-2} \text{cm}^{-3}$.

Halo energy appears to be equipartitioned between gas, cosmic rays and magnetic fields [10]. The total pressure is the sum of the partial pressures of these components, $P = P_g + P_{\text{CR}} + P_B$. Let u represent the energy density contributed by each component. Then $P = (2/3)u + (1/3)u + u = 2u$. From the ideal gas law one has $P = 2u = 2 \times (B^2/8\pi) = nkT$, $T = 1.26 \times 10^6 \text{K}$.

With these values, using Eq. (2) and Eq. (6) one gets $_{\text{H}}\gamma_1 = 1.757$, $_{\text{He}}\gamma_1 = 1.774$, $_{\text{Z}}\gamma_1 = 1.000$, and $\gamma_1 = .73 \times 1.757 + .25 \times 1.774 + .02 \times 1.000 = 1.746$. For gravitational interactions of this halo material with an element of gas near the

edge of the halo or with the halo of another, similar galaxy in our own Local Group, $\gamma_* = \gamma_1\gamma_2 \cong \gamma_1^2 = 3.05$.

Periods and distances used to estimate galactic masses are determined spectroscopically and photometrically. They are related, however, through the virial theorem or, alternatively for two bodies, Kepler's Third Law. Setting $t_* = \gamma_* t$ and $r_* = r/\gamma_*$, and assuming for a cluster of galaxies a uniform value of γ_* (that is, assuming the same value of γ_1 for each galaxy), the virial theorem becomes $\langle T \rangle = - (1/2) \gamma_*^5 \langle U \rangle$, where T is the summed kinetic energy of the galaxies and U is their gravitational potential energy. As written in the G_* system for two galaxies of halo masses M_{h1} and M_{h2} respectively, with orbital motions characterized by period P_* and semimajor axis A_* , Kepler's Third Law is

$$P_*^2 = (4\pi^2 A_*^3)/G(M_{h1} + M_{h2}); \text{ in the E system, } P^2 = (4\pi^2 A^3)/\gamma_*^5 G(M_{h1} + M_{h2}).$$

The mass of our own galactic halo is, using the mean density ρ and radius R adopted above, $M_{h1} \cong .44 \times 10^{10} m_\odot$. Taking $M_{h1} \cong M_{h2}$, one has

$$\gamma_*^5 (M_{h1} + M_{h2}) \cong 2.3 \times 10^{12} m_\odot. \text{ The halo itself produces a gravitating effect an order}$$

of magnitude greater than that ordinarily attributed to stars, gas and dust. It thus becomes unnecessary to postulate "dark matter."

To gain a measure of how this result varies with values of halo variables chosen, it is useful to divide the values of ρ and B adopted above by 2, while keeping the composition unchanged (with the result that $T \rightarrow T/2$ also). Expanding

the halo radius then by a factor of 1.97 to 10^{23} cm leaves the net result unchanged. Recent measurements of intergalactic magnetic fields in a sampling of 16 normal Abell clusters are consistent with such extended halos [14].

4. Solar Neutrino Problem

A detailed, standard model of the Sun produced some years back has led to a perplexing result. The rate of energy production in the solar interior can be directly inferred from measurement of sunlight at Earth. Associated with the energy production is a predictable rate of neutrino emission, the magnitude of the predicted rate following from the model. The predicted neutrino flux, however, is not observed. The measured flux at Earth is only about one-third that expected [15-19]. A widely held opinion is that both the model and the neutrino flux measurements are unassailable; the discrepancy is likely due to unexpected phenomena.

Models of the solar interior are based on reasonable suppositions about its structure and composition, and upon the numerical solution of five equations plus boundary conditions [5,20,21]. Gravitational effects enter through the equation of hydrostatic equilibrium, $dP/dr = -\rho GM(r)/r^2$, where $M(r)$ is the solar mass within radius r . It is my contention that the unexpected phenomena are introduced at this point.

First, the value of the solar mass is acquired through the use of Kepler's laws and observations of the orbital motions of planetary objects. Again, relationships involving gravitational forces must first be written in G_* and then transformed to E . Otherwise, errors are introduced. Calculating a weighted-mean

GSF for the Sun using equations (2) and (6), and introducing it into Kepler's Third Law as corrected, leads one to a value for the solar mass that is slightly lower than that presently accepted.

Secondly, inside the Sun there is a variation in the GSF with distance from the Sun's center. Since the pressure near the center is the result of the gravitational attraction of the outer material by the inner material, a further change in the central pressure is introduced.

It is in this central region that neutrino production predominantly occurs. Though the two effects are small, they compound in the Sun's core to produce sufficiently lowered temperatures and densities to account for the reduced neutrino flux observed.

To obtain a revised estimate of the expected neutrino flux, the following procedure is adopted: (a) Table VII in Ref. [17] is used as a basis. In it there are 27 entry lines, each for a radial distance r/R_{\odot} from the Sun's center. Each entry line provides data to support the calculation of the $j\gamma$ according to Eq. (2), the weighted-mean GSF at that r value, or $[\sum_i X_i(r) j\gamma(r)]$, and finally, an overall GSF for the Sun using Eq. (6).¹ (b) The GSF obtained are then used to effect consecutive homology transformations, one for each of the two physical changes described above. Homology transformations carry the ρ and T values of the model over into those that would have resulted had the GSF been introduced at the outset. In this way, reduced values for temperature and density near the Sun's center are arrived at. (c) With these revised ρ and T values, reduction factors for each of the neutrino fluxes expected from the five nuclear reactions

contributing to the measured flux are calculated. From these reduction factors and the updated theoretical fluxes and capture rates tabulated in Ref. [19], a revised total neutrino capture rate is obtained.

The overall mean solar GSF obtained with these data and with the use of Eq. (2) and Eq. (6) is $\gamma_1 = 1.0035$. A complication arises in calculating it: heavier elements are not completely ionized throughout the Sun. To accommodate this fact, two calculations of γ_1 were made, the first assuming the heavier elements are completely ionized throughout, the second assuming they are not ionized at all where $T \leq 9 \times 10^6$ K. The results are $\gamma_1 = 1.0042$ and $\gamma_1 = 1.0028$, respectively. The average of the two values is $\gamma_1 = 1.0035$. For planetary objects in the solar system, $\gamma_2 = 1.0000$.² Then $\gamma_* = \gamma_1 \gamma_2 = 1.0035$. The solar mass is thus overestimated by a factor of $\gamma_*^5 = 1.0176$. To correct for this in the solutions of the equations plus boundary conditions that serve as foundations for models of the solar interior, a homology transformation can be used. If $M \rightarrow \gamma_*^{-5} M = .9827M$, then in the solutions, $\rho \rightarrow .9827\rho$ and $T \rightarrow .9827T$ everywhere [20].

The pressure near the center of the Sun is due to the gravitational attraction by the material there of material overlying it. The GSF in this particular circumstance, γ_c , is approximated by the product of the GSF value at the center of the Sun, $\gamma_0 = .9616$, and $\gamma_1 = 1.0035$; or $\gamma_c = .9650$.

At this point the Lane-Emden functions can be used to effect a

homology transformation specific to standard models [21]. For the temperature T_c and density ρ_c at or near the Sun's center, if $r \rightarrow r = r\gamma_c^{-1}$ in solutions, then $T_c \rightarrow \gamma_c T_c$ and $\rho_c \rightarrow \gamma_c^3 \rho_c$. Alternatively, one can argue directly that if $r \rightarrow r = r\gamma_c^{-1}$, then $\rho_c \rightarrow \gamma_c^3 \rho_c$. From $dP/dr = -\rho GM(r)/r^2$, one concludes that $P \rightarrow \gamma_c^4 P$. From the ideal gas law then, $T_c \rightarrow \gamma_c T_c$. The two effects compound giving the net result, $T_c \rightarrow \gamma_c^{-5} \gamma_c T_c = .9483 T_c$ and $\rho_c \rightarrow \gamma_c^{-5} \gamma_c^3 \rho_c = .8831 \rho_c$. The fractional changes in the neutrino production rates brought about by these reductions in density and temperature can now be estimated.

The reaction rates for nonresonant nuclear reactions in the Sun's core can be approximated by the relationship [5,21]

$$r_{12} \cong C_i \rho^2 T^n, \quad (9)$$

where C_i is a proportionality constant, $n = (\tau - 2)/3$, $T_6 \equiv T/10^6$ K, and

$\tau = 42.46 [Z_1^2 Z_2^2 \{(A_1 A_2)/(A_1 + A_2)\} / T_6]^{1/3}$. Each reaction is taken up separately.

Source, ${}^7\text{Be}$: ${}^3\text{He} + {}^4\text{He} \rightarrow {}^7\text{Be} + \gamma$, ${}^7\text{Be} + e^- \rightarrow {}^7\text{Li} + \nu_e$,

$$r_{12} \cong C_1 \rho^2 T^{16.64} \rightarrow (.8831)^2 (.9483)^{16.64} r_{12} = .3224 r_{12} \equiv f_{\text{Be}} r_{12}.$$

Source, ${}^8\text{B}$: ${}^7\text{Be} + p \rightarrow {}^8\text{B} + \gamma$, ${}^8\text{B} \rightarrow {}^8\text{Be} + e^+ + \nu_e$,

$$r_{12} \cong C_2 \rho^2 T^{13.16} \rightarrow (.8831)^2 (.9483)^{13.16} r_{12} = .3878 r_{12} \equiv f_{\text{B}} r_{12}.$$

Source, ${}^{13}\text{N}$: ${}^{12}\text{C} + p \rightarrow {}^{13}\text{N} + \gamma$, ${}^{13}\text{N} \rightarrow {}^{13}\text{C} + e^+ + \nu_e$,

$$r_{12} \cong C_3 \rho^2 T^{17.78} \rightarrow (.8831)^2 (.9483)^{17.78} r_{12} = .3035 r_{12} \equiv f_{\text{N}} r_{12}.$$

Source, ${}^{15}\text{O}$: ${}^{14}\text{N} + p \rightarrow {}^{15}\text{O} + \gamma$, ${}^{15}\text{O} \rightarrow {}^{15}\text{N} + e^+ + \nu_e$,

$$r_{12} \cong C_4 \rho^2 T^{19.86} \rightarrow (.8831)^2 (.9483)^{19.86} r_{12} = .2717 r_{12} \equiv f_{\text{O}} r_{12}.$$

Source, p-e-p: $p + e^- + p \rightarrow {}^2\text{H} + \nu_e$.

The p-e-p reaction can be related approximately to the p-p reaction by

$r_{\text{pep}} \cong r_{\text{pp}} C_5 (1+X) \rho T_6^{-1/2} (1+.02T_6)$, [17]. These reactions occur in a more extensive region of the solar interior than those considered above. For the approximation here, the above GSF are used, but at $T_6 = 12.41$, very nearly the midpoint for the p-p reaction. The preceding relationship at that temperature becomes

approximately

$$r_{\text{pep}} \cong C_6 (1+X) \rho T_6^{-1/2} (1+.02T_6) \rho^2 T^{4.19} \rightarrow$$

$$1.0163 (.8831)^3 (.9483)^{4.19} r_{\text{pep}} = .5603 r_{\text{pep}} \equiv f_{\text{pep}} r_{\text{pep}}.$$

In Table 1 of Ref. [19], expected neutrino capture rates for the ${}^{37}\text{Cl}$ detector employed in the observations discussed there are listed by neutrino-source reaction. The rates are in solar neutrino units: 1 SNU = 10^{-36} captures per target atom per second. In Table 1, these values are listed and then multiplied by the respective reduction factors, f_x , arrived at above for each contributing reaction. Totals can then be compared with the observed capture rate of 2.1 ± 0.3 SNU.

To best interpret the significance of Table 1, however, it is more meaningful to anticipate the contingency in the model of extension of the energy production region of the Sun to compensate for the lowered core densities and temperatures. Setting the p-e-p reduction factor equal to unity while keeping the other reduction factors unchanged reasonably approximates the consequence of this. The estimated resultant capture rate is then 2.3 SNU.

5. Eclipsing binary stars

The eclipsing binary star systems DI Herculis and AS Camelopardalis offer exceptional opportunities to test gravitational theories. In each of these systems, appreciable apsidal-advance rates are expected to arise from both general relativistic and classical effects, these latter a result of tidal and rotational distortions [22,23].

Sizable differences between observation and theory for the apsidal-advance rates of these two binary systems are reported in Refs. [22-25]. Maloney, Guinan and Mukherjee conclude that the differences call attention to possible problems with our understanding of classical or general relativistic gravitation as it occurs in close binary systems [25].

The theory introduced here implies discrepancies in both the classical and general relativistic contributions if calculations are done without consideration of the scale differences between the E and G_* systems. Again, periods and distances are determined from spectroscopic and photometric measurements in the E system. They are then related through gravitational laws that are correct as conventionally written only in the G_* system. Inclusion of the scale differences between systems through introduction of the GSF, however, leads to reconciliation between theory and observation for both DI Her and AS Cam.

Apsidal motion is treated in a very general way by Robertson and Noonan [4]. The procedure adopted below is patterned after theirs.

Central force equations of motion can be written

$$d^2r_*/dt_*^2 - r_*(d\theta/dt_*)^2 = -F(r_*) \equiv -F_* \quad (10a)$$

$$(dr_*/dt_*)^2 + r_*^2(d\theta/dt_*)^2 = 2J(r_*) \equiv 2J_* \quad (10b)$$

$$r_*^2(d\theta/dt_*) = H(r_*) \equiv H_* \quad (10c)$$

where F , J and H are *arbitrary* differentiable functions. Equations of motion under a gravitational force as conventionally written above are correct only in the G_* system. The radius vector in the orbital plane, r_* , and t_* , the time in G_* , and F_* , J_* , and H_* are subscripted to make this explicit. The angle θ is the angle in the orbital plane as conventionally defined; it is invariant.

Setting $u_* = 1/r_*$, one gets from Equations (10),

$$d^2u_*/d\theta^2 + u_* = N_*, \quad (11)$$

where $N_* = (1/H_*^2)(dJ_*/du_*) - 2(J_*/H_*^3)(dH_*/du_*)$. One then sets $\eta_* = u_* - u_{0*}$,

where $u_{0*} = N_*(u_{0*})$ is the solution to Eq. (11) when $d^2u_*/d\theta^2 = 0$. That is,

$u_{0*} = 1/r_{0*}$, where r_{0*} is the radius of a circular orbit. Deviations from a circular orbit satisfy the equation

$$d^2\eta_*/d\theta^2 + [1 - (dN_*/du_*)_{u_{0*}}]\eta_* \cong 0. \quad (12)$$

The fraction of an orbit advanced each orbit is then

$$\sigma_* = (1/2)(dN_*/du_*)_{u_{0*}}, \quad (13)$$

and the rate of apsidal advance is

$$\langle d\omega/dt \rangle_* = 2\pi\sigma_*/P_*, \quad (14)$$

P_* the binary period in G_* .

One then transforms σ_* and P_* to obtain $\langle d\omega/dt \rangle$, the rate of apsidal advance in E.

It is useful at this point to quote again from Max Jammer's discussion: "Long ago, George Berkeley (*On Motion*, 1721) said that the notion of force is a fiction just like that of epicycles in astronomy. He declared that concepts such as force, attraction, and gravitation are convenient for purposes of computation but do not increase real understanding. David Hume, Pierre de Maupertuis, and the early proponents of modern positivism (Gustav Kirchoff, Heinrich Hertz, Ernst Mach) similarly contended that the concept of force is only a methodological device devoid of any real content. At best, according to Kirchoff, it is an abbreviation for the product of mass and acceleration" [1].

Setting $r_* \rightarrow r\gamma_*^{-1}$, $t_* \rightarrow \gamma_* t$ in Equations (10), and adopting the above viewpoint, one gets the scale changes

$$F_* = F(r_*) \rightarrow \gamma_*^3 F(r) \quad (15a)$$

$$J_* = J(r_*) \rightarrow \gamma_*^4 J(r) \quad (15b)$$

$$H_* = H(r_*) \rightarrow \gamma_*^3 H(r) \quad (15c)$$

From Equations (15), $N_* \rightarrow \gamma_*^{-3} N(u)$, $dN_*/du_* \rightarrow \gamma_*^{-4} dN/du$. Since $P_* = \gamma_* P$,

$$\langle d\omega/dt \rangle = (\pi/P) \gamma_*^{-5} (dN/du)_{u_{0*}}. \quad (16)$$

To find the transformed value of u_{0*} , Eq. (10c) can be used to eliminate t_* in Eq.

(10b) which, on setting $u_* = 1/r_*$, becomes

$$(du_*/d\theta)^2 + u_*^2 = 2J_*/H_*^2. \quad (17)$$

Setting $(du_*/d\theta) = 0$ in Eq. (17) and making use of Eq. (15), one has that

$$u_{0*}^2 = 2J_{0*}/H_{0*}^2 \rightarrow \gamma_*^{-2}(2J_0/H_0^2) = \gamma_*^{-2}u_0^2, \text{ so that } u_{0*} \rightarrow \gamma_*^{-1}u_0, \text{ the subscript "0"}$$

denoting functions evaluated at u_{0*} and u_0 respectively, and $u_0 = 1/r_0$, where r_0 is

the radius of a circular orbit in E. Then

$$\langle d\omega/dt \rangle = (\pi/P) \gamma_*^{-5} (dN/du)_u, \quad u = \gamma_*^{-1}u_0. \quad (18)$$

The general relativistic and classical treatments of apsidal motion coincide to this point but now separate. The general relativistic case is taken up first.

Levi-Civita first treated the relativistic two-body problem as a perturbation on a Newtonian representation [26]. He arrived at a potential function that leads one directly to an expression for $N(u)$,

$$N(u) = \beta_0 + \beta_1 u + \beta_2 u^2, \quad (19)$$

where $\beta_1 = 6[1 - (1/2)(\mu^2/m_1 m_2)][G^2 m_1^2 m_2^2 / L^2 c^2]$ and

$\beta_2 = (3/2)[(\mu^2 m_1 m_2 G^2 A(1-e^2)) / L^2 c^2]$; m_1 and m_2 are the masses of the gravitating

bodies, μ is their reduced mass, and L is the system's angular momentum. From

Eq. (19) one has for σ evaluated at $u = \gamma_*^{-1}u_0$,

$$\sigma = (1/2)(dN/du)_u = \beta_1/2 + \beta_2\gamma_*^{-1}u_0. \quad (20)$$

A simplifying approximation can be justified by setting $m_1 = m_2$ and taking $\gamma_* \sim 4/3$ in Eq. (20). One finds that ignoring γ_* in Eq. (20) introduces an error of less than 3% into the calculated apsidal advance rates. From Eq. (18) then, the corrected general relativistic rate of apsidal advance, $\langle d\omega/dt \rangle_{GR,GSF}$, is given to sufficient accuracy by

$$\langle d\omega/dt \rangle_{GR,GSF} \cong \gamma_*^{-5} \langle d\omega/dt \rangle_{GR}. \quad (21)$$

The classical apsidal-advance rate relationship made use of in Refs. [22-25] has been derived by Cowling [27]. From Ref. [27] one obtains an expression for $N(u)$,

$$N(u) = [G(m_1 + m_2)/h^2](1 + \delta_R u^2 + \delta_T u^5), \quad (22)$$

where $h = r^2 d\theta/dt$; $\delta_R = (k_1 R_1^5/Gm_1)(d\theta_1/dt)^2 + (k_2 R_2^5/Gm_2)(d\theta_2/dt)^2$;

$\delta_T = 6k_1 R_1^5(m_2/m_1) + 6k_2 R_2^5(m_1/m_2)$; $d\theta/dt$ is the orbital angular velocity; $d\theta_1/dt$ and $d\theta_2/dt$ are the rotational angular velocities of the stars; R_1 and R_2 are their radii; and k_1, k_2 are weakly varying functions of their masses and compositions [28].

Choosing an orbit that approaches arbitrarily close to circular, setting $(d\theta/dt)^2 = G(m_1 + m_2)u_{00}^3$ and $h = (d\theta/dt)/u_{00}^2$, one gets from Eq. (22),

$$(1/2)(dN/du)_{u_0} \cong u_{00}[\delta_R u_0 + (5/2)\delta_T u_0^4]. \quad (23)$$

The δ_R term represents apsidal motion resulting from rotational distortion, the δ_T term that from tidal distortion. To compare their typical magnitudes, we set

$d\theta_1/dt = d\theta_2/dt = d\theta/dt$ and $m_1 = m_2$. One notes that $u_{00} \cong u_0$. Then

$$\delta_R u_0 = 2k_1 R_1^5 u_{00}^3 u_0 + 2k_2 R_2^5 u_{00}^3 u_0, \quad (24a)$$

$$(5/2)\delta_T u_0^4 = 15k_1 R_1^5 u_0^4 + 15k_2 R_2^5 u_0^4. \quad (24b)$$

As a simplifying approximation, the correction to the larger term, that due to tidal distortion, is taken as representative. From Eq. (18) and Eq. (23) one has then for the corrected classical rate of apsidal advance, $\langle d\omega/dt \rangle_{Cl,GSF}$,

$$\langle d\omega/dt \rangle_{Cl,GSF} \cong \gamma_*^{-9} \langle d\omega/dt \rangle_{Cl}. \quad (25)$$

To estimate the GSF for DI Her and AS Cam, approximate values of the masses of the stars are needed so that temperatures and densities in their interiors can be estimated from an acceptable model. Since the masses may be inaccurately determined to start with, this conceivably presents a difficulty. In a comprehensive assessment of the published orbital parameters of DI Her, Popper has concluded that for DI Her these are well established and consistent with those of similar main sequence B stars [29]. Maloney *et al.* conclude that the orbital parameters of AS Cam are also self-consistent and those expected of similar main sequence B stars [25].

For our purposes, the mass values used in predicting the published apsidal advance rates are acceptable since we use them merely as an entry point into tabulated (by stellar mass) T_c and ρ_c values for model stellar interiors. As a further simplifying approximation, the stars in each binary system are taken as a pair to be alike with respect to T_c and ρ_c . To approximate a weighted mean GSF for each star, representative mean temperatures and densities are adopted. These are $T \cong T_c/2$ and $\rho \cong \rho_c/8$ (corresponding to a polytropic index of 3). These

choices appear to be the most reasonable when compared to the results obtained earlier in detailed calculations for the Sun. For the composition, the values adopted for all four stars are $X=.60$, $Y=.37$, $Z=.03$, [21], the heavier nuclei represented by oxygen in calculating the GSF. The GSF are then calculated using these values in equation (2) and from $\gamma_1 = \gamma_2 = [\sum_i X_i(r) i\gamma(r)]$.

The mass values for DI Her are $m_1 = 5.15m_\odot$, $m_2 = 4.53m_\odot$ [29]. From Ref. [21], $T_c \cong 26 \times 10^6$ K and $\rho_c \cong 21$ g cm⁻³. Then $T \cong 13 \times 10^6$ K and $\rho \cong 2.6$ g cm⁻³. For AS Cam, the mass values are $m_1 = 3.3m_\odot$, $m_2 = 2.5m_\odot$ [23]. From Ref. [21], $T_c \cong 23 \times 10^6$ K and $\rho_c \cong 42$ g cm⁻³. Then $T \cong 11.5 \times 10^6$ K and $\rho \cong 5.2$ g cm⁻³.

The results are as follows: DI Her: $\gamma_* = \gamma_1\gamma_2 = 1.23$. From Ref. [22], the predicted apsidal advance rates are $\langle d\omega/dt \rangle_{GR} = 2.34$ °/100yr, $\langle d\omega/dt \rangle_{CI} = 1.93$ °/100yr. From Eq. (21) and Eq. (25) one has for the corrected total apsidal advance rate $\langle d\omega/dt \rangle_{GSF}$,

$$\langle d\omega/dt \rangle_{GSF} \cong \gamma_*^{-5} \langle d\omega/dt \rangle_{GR} + \gamma_*^{-9} \langle d\omega/dt \rangle_{CI} = 1.13 \text{ °/100yr}, \quad (26)$$

in agreement with the observed rate as revised in Ref. [24],

$$\langle d\omega/dt \rangle_{obs} = 1.00 \text{ °/100yr} \pm .30 \text{ °/100yr}.$$

AS Cam: $\gamma_* = \gamma_1\gamma_2 = 1.14$. From Ref. [23], the predicted apsidal advance rates are $\langle d\omega/dt \rangle_{GR} = 8.5$ °/100yr, $\langle d\omega/dt \rangle_{CI} = 35.8$ °/100yr. From Eq. (21) and Eq. (25) one has for the corrected total apsidal advance rate $\langle d\omega/dt \rangle_{GSF}$,

$$\langle d\omega/dt \rangle_{GSF} \cong \gamma_*^{-5} \langle d\omega/dt \rangle_{GR} + \gamma_*^{-9} \langle d\omega/dt \rangle_{CI} = 15.4 \text{ °/100yr}, \quad (27)$$

in agreement with the observed rate, $\langle d\omega/dt \rangle_{\text{obs}} = 15.0 \text{ }^\circ/100\text{yr} \pm 5.3 \text{ }^\circ/100\text{yr}$, as reported in Ref. [23].

6. Conclusion.

Though other possible explanations for each of the three classes of anomalies considered have been proposed, each relies on adjustable parameters, compounds assumptions, and is narrowed to the single problem at hand. By contrast, the proposed theory has no adjustable parameters and solves problems of all three classes; in each case it accomplishes this with no additional assumptions. One eliminates the troublesome and embarrassing need to postulate a persistently elusive, invisible form of matter that comprises nearly all of the physical universe; and our present-day formulations of known *fundamental* physical laws are not challenged. In light of this rather striking comparative economy and the theory's simplicity, it is reasonable that it be given further attention by others, both in furthering its basic development and in applying it to similar and related problems.

In this latter regard, one notes in particular that over 1000 eclipsing binary stars are known [21]. It is clear that upper main-sequence binary stars, with their high interior temperatures and comparatively low densities, should exhibit these gravitational scaling effects to a pronounced degree, while those of the lower main sequence should be little affected.

A solar system demonstration of the effect appears to have been secured some years ago by H. Hill, who considered the Sun's oblateness and its effect on the apsidal motion of Mercury [30]. His result, a correction factor to the predicted

general relativistic apsidal advance rate, is in agreement with the result that immediately follows from the discussion of apsidal motion presented here. The value for the correction factor found here, taken directly from the solar-neutrino discussion: .9827; Hill's values: $.987 \pm .006$, or $.991 \pm .006$, depending on the set of planetary radar observations chosen. Although Hill's work apparently remains controversial, its potential importance is unquestionable. As Hill points out, his approach involves redundancy – unlike the binary pulsar results for example which, though they indeed demonstrate consistency of approach, cannot be relied upon to reveal unanticipated effects.

Jupiter also may reveal the scaling effect. This follows from the expectation that there may be appreciable ionized material deep in its interior. Specifically, if Jupiter is used in ranging calibrations for deep-space probes, a small error should be introduced in that accepted (ephemeris) values of its range should, as a consequence of the ionization, be slightly in error.

Finally, and this is a cardinal point, a connection at the quantum level between the electromagnetic force and gravitation is indicated, and in a way that should suggest a number of potentially fruitful experimental procedures to examine its fundamental properties in greater detail. In particular, different materials ionized to varying degrees and subjected to either the Sun's or Earth's gravitational field should accelerate at rates different from those of non-ionized material. These rates can be correlated with the material's controlled properties.

Acknowledgments

I am grateful to Wayne Logus for useful discussion, and to him, David Douglas, Linda Harris and John Brunstein for their generous help in securing reference material.

Notes

1. It can be argued that one requires $\langle \gamma^n \rangle$ and not $\langle \gamma \rangle^n$ in what follows. In any event, writing $\gamma_k = 1 + \varepsilon_k$ for the m_k mass element, one has that the two are equal to first order in the ε_k .
2. Jupiter may present an exception.

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Table I. The reduction factors discussed in the text are listed in the third column. The last column lists the products of the capture rates from Ref. [19] and their corresponding reduction factors.

Neutrino Source	Capture Rate, Ref. [19] (SNU)	Reduction Factor, f_x	Revised Rate (SNU)
p-e-p24	.5603	.1345
^7Be95	.3224	.3063
^8B	4.3	.3878	1.6675
^{13}N08	.3035	.0243
^{15}O	<u>.24</u>	.2717	<u>.0652</u>
Totals	5.8		2.2

Space-probe gravitational anomalies as artifacts of calibration on Jupiter

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All salient features of the Pioneer 10,11 gravitational anomalies are readily explained as consequences of calibrations on Jupiter if the mean Sun-Jupiter distance is presumed to be in error by 1.64 parts per million. Since an error of this magnitude cannot be attributed to observational limitations, one has persuasive evidence that there is a currently unrecognized gravitational effect operative here, likely unique to Jupiter among solar system planets. A fundamental mechanism that provides for that effect is cited.

PACS numbers: 95.10.Eg, 95.55.Pe, 96.30.Kf

From a straightforward, relatively simple calculation, one can predict apparent anomalies in the motion of deep-space probes that duplicate those reported for Pioneers 10 and 11 [1,2]. It is proposed that an error arising from calibrations on Jupiter is ultimately responsible for these anomalies. That error in turn stems from a constant, small fractional error in the accepted values for the Jupiter-Sun distance. Though the cause of this error is immaterial to the calculated results, current estimates of the error-bounds on ephemeris values suggest that it cannot be due to the limitations of observation. Rather, it is proposed that the error is a striking manifestation of an unanticipated gravitational effect that is likely unique to Jupiter among solar-system planets, as suggested by Brunstein [3]. The Pioneer 10, 11 results are based on measurements of phase differences between transmitted and received signals.

These phase differences are unavoidably altered in passage through the solar environment, which is itself changing constantly. The alterations must be compensated for. The confusion introduced by them is particularly troublesome out to, and a little beyond, the orbit of Jupiter. Jupiter, then, being a reliable, strongly reflective microwave target, nicely functions as a range “benchmark,” and one makes use of it to subtract out the phase-shifting disturbance introduced by the solar environment.

Jupiter, however, contains appreciable ionized material deep in its interior. Since one is unaware of the “anomalous” gravitational effect proposed in Ref. [3], the difference between gravitational and electromagnetic spacetimes, a consequence of the ionization, is not taken into consideration. One mixes the two differing scales of space and time in a single equation, for example, Kepler’s Third Law, a gravitational-spacetime law, and an error is introduced. Because of the faith one has in the ephemeris values for the Sun-Jupiter distance, the error passes for a solar-environment effect and is absorbed into the database for the Earthbound observing station. We will, in concluding, estimate the amount of ionized material necessary to produce the anomalous effects observed.

Let A be the mean Sun-Jupiter distance as determined through the use of Newtonian mechanics, the timing of Jupiter’s orbital period furnishing the observational input. That value of A is presumed to be too large. We take $A+\delta A$ to be a more-accurate value (more accurate in the sense that, used in place of the corresponding ephemeris values in calculations, it will remove the gravitational anomalies attributed to the spacecraft). The increment δA is then a

negative quantity, $(\delta A/A) = -1.64 \times 10^{-6}$. The important features of our discussion are quickly brought out by assuming circular orbits for Earth and Jupiter and a perpendicular orientation to the ecliptic of the Earth's rotational axis – a “back-of-the-envelope” formulation, justified finally by the disencumbered insight it lends. Implicit in this approach, there is the inference that the Pioneer anomalies we compare results with are the product of an analysis that has adequately treated the complexities and effects we ignore – that analysis being flawed only in the value of A . *The intent here most certainly is not to mimic the analytic procedures followed anywhere, but rather to demonstrate what final results one might expect from a straightforward, logical procedure.* The only free parameter that enters the discussion is the magnitude of $(\delta A/A)$; the sense will be justified presently.

In addition to the symbols readily identified In Fig. 1, one needs the following:

V_{Jr} = Jupiter's velocity relative to an equatorial, Earthbound transmitting/receiving station, as calculated based on the accepted value of A .

V_{Jrm} = Jupiter's velocity relative to an equatorial, Earthbound transmitting/receiving station as measured and then set in a calibration procedure or its equivalent.

V_J = the calculated value of Jupiter's velocity in a fixed-Sun system.

δV_J = the increment in V_J due solely to the increment δA in A .

V_E = the velocity of the Earth's center in the fixed-Sun system.

V_r = the rotational velocity of the transmitting/receiving station with respect to the Earth's axis. Then,

$$V_{Jr} = V_J \sin(\alpha + \beta) - V_E \sin \theta - V_r \sin \phi \quad (1)$$

$$V_{Jrm} = (V_J + \delta V_J) \sin(\alpha + \beta) - V_E \sin \theta - V_r \sin \phi \quad (2)$$

The Doppler residual in observing Jupiter is

$$V_{Jrm} - V_{Jr} = \delta V_J \sin(\alpha + \beta). \quad (3)$$

Set $f \equiv V_{Jrm} / V_{Jr}$, (4)

$$f = 1 + (\delta V_J / V_J) \sin(\alpha + \beta), \quad (5)$$

$$f = 1 + (\delta V_J / V_J) [(R/s) \sin \theta + (r/z) \sin \phi]. \quad (6)$$

Substituting from (1), noting that $V_r \ll V_E$, $(r/z) \ll (R/s)$, substituting

$(\delta V_J / V_J) = -(\delta A / 2A)$, and approximating (R/s) by

$$(R/s) \cong (R/A) [1 - (1/2)(R/A)^2 + (R/A) \cos \theta], \quad (7)$$

one arrives at

$$f \cong 1 + .546 (V_J / V_E) (\delta A / A) (R/s) \quad (8)$$

and $f - 1 \cong -7.404 \times 10^{-8} (1 + .196 \cos \theta)$. (9)

At this point one introduces the effect of the calibration operation by generalizing (4) as

$$f = V_{Em} / V_E. \quad (10)$$

The measured and calibration-adjusted, “corrected,” velocity of the Earth’s center is taken, locked away in the complexity of the analysis, to be V_{Em} . In this way an error δV_E in V_E is introduced as a hidden consequence of one’s confident use of accepted values of the Sun-Jupiter distance:

$$\delta V_E = V_{Em} - V_E = (f - 1) V_E, \quad (11)$$

$$\delta V_E \cong -.2205 (1 + .196 \cos \theta) \text{ cm/s}. \quad (12)$$

The consequent error δa_E in the acceleration of the Earth's center toward the Sun is $\delta a_E = -2\omega_a \delta V_E$, ω_a the orbital angular velocity of the Earth. Then

$$\delta a_E \cong 8.78 \times 10^{-8} (1 + .196 \cos \theta) \text{ cm/s}^2. \quad (13)$$

The Earth thus appears to have a receding (away from the Sun) acceleration of this incremental amount. When the relative motion is attributed instead to the space probe, the probe naturally appears to have an anomalous acceleration inward, reversing the sense,

$$\delta a_p \cong -8.78 \times 10^{-8} (1 + .196 \cos \theta) \text{ cm/s}^2. \quad (14)$$

There is an apparent annual variation of amplitude $1.7 \times 10^{-8} \text{ cm/s}^2$. Our procedure to this point is summarized in Table 1.

The diurnal variation inherent in the $\sin \phi$ terms ignored after Eq. (6) derives from the error δV_E . It is negligible. That due to the error δV_r , arising from a calibration adjustment to V_r , however, is comparatively large. The authors of Ref. [2] indeed obtained a diurnal acceleration anomaly for Pioneer 10 of amplitude $100.1 \times 10^{-8} \text{ cm/s}^2$. Our procedure can be readily extended to reproduce that result. One writes

$$\delta a = \delta a_E + \delta a_r \cos \eta \quad (15)$$

for the total apparent acceleration anomaly of the Earthbound station, radially in the fixed-Sun system. Here δa_r is the station's apparent anomalous radial acceleration with respect to the Earth's center, due to the error introduced by the effective calibration of the Earth's rotational speed. For the space probe then,

$$\delta a_p = -\delta a_E - \delta a_r \cos \eta, \quad \text{or} \quad \delta a_p = -\delta a_E [1 + (\delta a_r / \delta a_E) \cos \eta]. \quad (16)$$

One has that

$$a_r = -V_r^2/r, \quad \delta a_r = -2\omega_d \delta V_r, \quad a_E = -V_E^2/R, \quad \delta a_E = -2\omega_a \delta V_E, \quad (17a,b,c,d)$$

ω_d being the Earth's rotational angular velocity.

Consistent with the mistaken impression that δV_E is attributable to the space probe and is therefore independent of ω_a , one writes

$$a_E = -\omega_a V_E, \quad \delta a_E = -\omega_a \delta V_E. \quad (17e,f)$$

Then using Eq. (17f) instead of (17d), one writes

$$(\delta a_r / \delta a_E) = 2(\omega_d / \omega_a)(\delta V_r / \delta V_E). \quad (18)$$

From the calibration procedure one has again as in Eq. (11), $\delta V_E = (f-1)V_E$,

$\delta V_r = (f-1)V_r$. Eq. (16) becomes

$$\delta a_p \cong -\delta a_E [1 + 2(\omega_d / \omega_a)(V_r / V_E) \cos \eta], \quad (19)$$

and with the use of Eq. (13),

$$\delta a_p \cong -8.78 \times 10^{-8} (1 + 0.196 \cos \theta)(1 + 11.4 \cos \eta) \text{ cm/s}^2. \quad (20)$$

There is now also an apparent diurnal variation of amplitude $100 \times 10^{-8} \text{ cm/s}^2$.

The results are summarized in Table 2, alongside analogous features for Pioneers 10 and 11 as summarized in Ref. [2].

Coraddu *et al.* [4] offer an analysis with what may be a related conclusion. They assume some fully ionized hydrogen and helium deep in Jupiter's interior, 80% and 20% respectively by relative mass abundance, and of density 5 g/cm^3 and temperature $kT=2\text{ev}$. They find that moderate deviations from a Maxwell-Boltzmann energy distribution can be expected in this material and may increase deuterium reaction rates sufficiently to contribute to Jupiter's excess heat radiation.

Their conclusion aside, starting with these same conditions, one can employ the theory introduced in Ref. [3] to estimate the fraction of Jupiter's mass that is ionized and as a consequence would result in $(\delta A/A) = -1.64 \times 10^{-6}$, the incremental adjustment proposed here. It is $(m_{\text{ionized}}/m_{\text{total}}) = 1.2 \times 10^{-4}$, which does not seem unreasonable. The negative sense of δA follows from the particular values of temperature and density assumed. Perhaps the space-probe gravitational anomalies and Jupiter's excess heat radiation are related.

I thank Wayne Logus and Linda Harris for their help in obtaining reference material.

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Table 1. Summary of the logic path initiated by an unrecognized error in the mean Jupiter-Sun distance. The net result is an apparent anomalous acceleration attributed to the observed space probe.

1. The accepted value A of the mean Jupiter-Sun distance is too large. Taking $A+\delta A$ to be the correct value, δA is a *negative* quantity. {Justified in concluding comments}
 2. An unanticipated, unrecognized increment δV_J in Jupiter's orbital velocity is the consequence. It is *positive*.
 3. A calibration adjustment using Jupiter as a standard then absorbs this velocity increment. In doing so, it masks its origin and introduces an error δV_E to the Earth's orbital velocity. δV_E takes on a *negative* sense. {Eqs. (10), (11), (12)}
 4. Its real nature being unrecognized, δV_E results in an apparent anomalous acceleration of the Earth's center in a *positive* sense, away from the Sun. {Eq. (13)}
 5. The acceleration is attributed to the space probe, reversing the sense. The probe appears to have a *negative* incremental acceleration, toward the Sun. {Eq. (14)}
-

Table 2. Features of a space probe's apparent anomalous acceleration as discussed and calculated in the text are listed in Col. 2. Results for analogous features for Pioneers 10,11 are listed in Col. 3. Page numbers in parentheses refer to Ref. [2].

Feature of anomalous acceleration, δa	As calculated	Pioneer 10,11
1. magnitude of constant component	$8.78 \times 10^{-8} \text{ cm/s}^2$	$8.74 \times 10^{-8} \text{ cm/s}^2$
2. sense	toward Sun	toward Sun
3. longer periodicity	annual	annual, more or less. (pp. 25,26)
4. form of longer periodicity	sinusoidal	essentially sinusoidal
5. amplitude of longer periodicity	$1.7 \times 10^{-8} \text{ cm/s}^2$	about $1.6 \times 10^{-8} \text{ cm/s}^2$ (p. 40)
6. phase of longer periodicity	peak at conjunction	peak at conjunction
7. shorter periodicity	1 day	1 day
8. form of shorter periodicity	sinusoidal	more or less sinusoidal (p. 41)
9. amplitude of shorter periodicity	$100. \times 10^{-8} \text{ cm/s}^2$	$100.1 \times 10^{-8} \text{ cm/s}^2$ (p. 41)

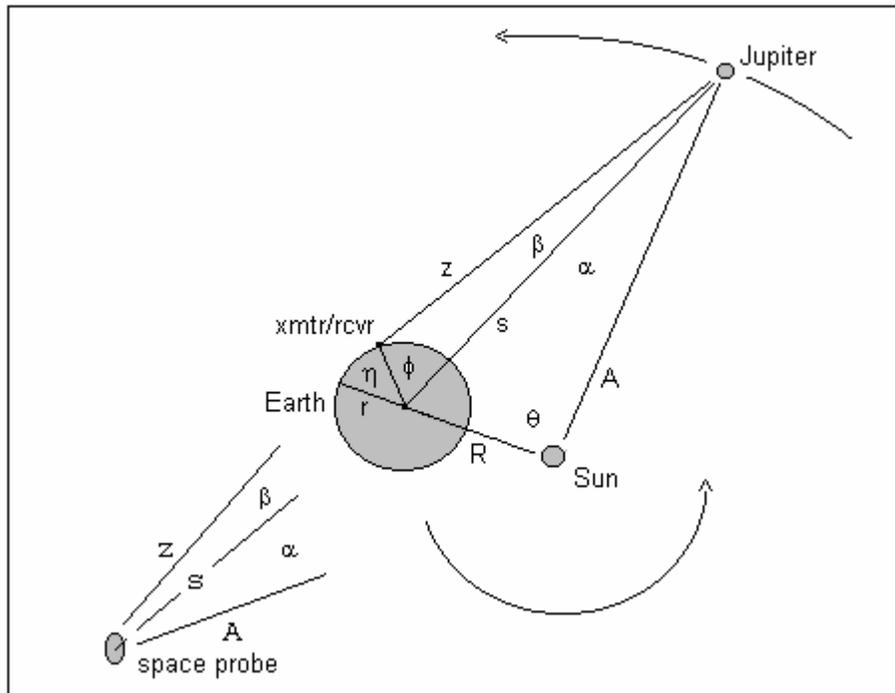


Fig.1

Angles and distances made use of in the formulation

The first “direct” distance measurement to M33: Actually, it was inferred and it’s spurious

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A recent determination of the distance to M33 is examined and its conclusion questioned. Observations of the eclipsing binary star system that serve as its basis have been related through standard Newtonian orbital relationships. The stars are relatively massive and hot: the third-relativistic effects that follow from ionization are considerable. Modifying the spacetime scales to reflect this, one finds that the inferred distance to M33 is too great by a factor of 1.15, which brings the measurement into good agreement with the accepted value.

Introduction

A recent determination [1] of the distance to M33 has garnered much attention because of the implication that the Hubble constant is smaller, and the universe presumably larger and older, than thought. Though this implication follows from the result, it does not properly characterize the work reported in Ref. [1]. Rather, what has been accomplished is an evaluation of the distance to an eclipsing binary star system in M33. Since this type of distance determination should be inherently accurate, one wonders at the disparate result.

Using this type of measurement as a calibration standard, however, involves the implicit assumption that one understands the underlying physics involved. The discussion in Ref. [1] illustrates that this was not so. One can, in fact, easily trace the error from its introduction to the final, flawed result.

Summary of relevant E and G* spacetime relationships

The analysis developed here should be transparent. Therefore, to facilitate one's quick understanding, the content of Ref. [2] that is immediately relevant is summarized as follows.

There are two spacetimes, electromagnetic (E) and gravitational (G*). They are related through a Lorentz transformation, the results of which are the changes in scale:

$$\Delta r_* = \Delta r \gamma_*^{-1}, \quad \Delta t_* = \gamma_* \Delta t .$$

Gravitational laws are correct as ordinarily written only in gravitational spacetime, G*. For example, Kepler's Third Law is properly written in G* as

$$P_*^2 = (4\pi^2 a_*^3) / G(m_1 + m_2) . \quad (1a)$$

Astronomical observations, that is, *direct* observations, are naturally electromagnetic in nature and are thus in the E system. The preceding relationship is then correctly written in the E system as

$$\gamma_*^2 P^2 = (4\pi^2 a^3 \gamma_*^{-3}) / G(m_1 + m_2) , \quad (1b)$$

where P and a are now the directly observed, or E-system, values. The gravitational scaling factor, γ_* , (the GSF) is evaluated as discussed in Ref. [2].

For non-ionized matter, γ_* is identically equal to one and Eq. (1a) and Eq. (1b) do not differ.

Distance to M33: the underlying physics

The following assumptions are made about the measurements and the procedures followed, as they are presented in Ref. [1].

1. The masses of the stars are inferred principally from their spectral characteristics and are thus presumably correct.
2. The radial-velocity and light-curve direct measurements are correct. That is, the *measured* values are E system values and are accurately determined.
3. The semimajor axis a is an inferred value. It is obtained from what is effectively a variant form of Kepler's Third Law. That is, the basic physics underlying the determination of a is properly represented by Kepler's Third Law, but neither Eq. (1a) nor Eq. (1b) has been used. Rather, a mixed system has inadvertently been introduced.
4. The energy flux measurement is naturally in the E system and is correct.

The period P has been directly measured (that is, in E), but the semimajor axis, a , has been inferred from *orbital* characteristics. It is thus inferred from G_* relationships, and one is therefore using a mixed system. Using Kepler's Third Law to express both the physics and this error, one writes for the *mixed* system,

$$P^2 = (4\pi^2 a^3 \gamma_*^{-2}) / G(m_1 + m_2) . \quad (1c)$$

From Eq. (1c) one infers incorrectly that the semimajor axis has the value a_μ ,

$$a_\mu = a \gamma_*^{-2/3} ,$$

in the E system. (See the discussion in the Appendix.) Thus it is a_μ that

carries the error inherent in failure to consider the difference in scales between electromagnetic and gravitational spacetimes.

Based on the procedure described in Ref. [1], it is useful at this point to bring in the radii of the stars, R_1 and R_2 . We introduce a conceptual device – an operational, or measurement, procedure – that in an apparent way introduces the relevant physics in what is termed a “light-curve” analysis and at the same time brings in the radii. Justification for this approach follows from the disencumbered insight that follows. *The intent here most certainly is not to mimic the analytic procedures followed anywhere, but rather to demonstrate what final results one might expect from a straightforward, logical procedure.*

Consider the transit time τ for the eclipsing effect. To encompass the physics while avoiding irrelevant complexity, we let the orbital eccentricity approach zero, assuming a circular orbit, and use Kepler’s Second Law. One gets for τ as one would ordinarily write it, ignoring for the moment the difference in spacetime scales,

$$\tau \cong (P/\pi) [(R_1+R_2)/2a + O^3], \quad (2a)$$

where the third- and higher-order terms may be neglected, since the error so introduced is less than one percent, smaller by a factor of four than the estimated fractional uncertainty in a as reported in Ref. [1].

Now let

$$R_1 \rightarrow R_{1*}, \quad R_2 \rightarrow R_{2*}, \quad P \rightarrow P_*, \quad a \rightarrow a_*.$$

One then has

$$\tau \cong \gamma_*(P/\pi)[(R_1+R_2)/2a] . \quad (2b)$$

This is the correct expression as written in the E system.

One does not, however, transform quantities as above. Rather, in effect one uses the transformations,

$$R_1 \rightarrow R_1, \quad R_2 \rightarrow R_2, \quad P \rightarrow P, \quad a \rightarrow a_\mu ,$$

and spuriously writes

$$\tau \cong \gamma_*^{2/3}(P/\pi)[(R_1+R_2)/2a] . \quad (2c)$$

The τ and P values in both Eq. (2b) and Eq. (2c) are those one would *directly measure* – that is, in the E system. They are correctly *related*, however, in Eq. (2b). Since one is attempting to relate them through the incorrect relationship, Eq. (2c), one must infer, “adjust,” the values of the radii. Somewhere in the train of software programs used in the data handling procedure, one sets

$$R_1 \rightarrow \gamma_*^{1/3} R_1, \quad R_2 \rightarrow \gamma_*^{1/3} R_2 .$$

These spurious values of the radii are then carried forward into the evaluation of the distance d to the binary-star system, and, as reflected in Eq. (4) of Ref. [1],

$$d \rightarrow \gamma_*^{1/3} d ,$$

so that the directly measured flux values are made finally consistent with the chain of previous inferences.

To evaluate γ_* for two gravitationally interacting stars, each of about 30 m_\odot , core-density and core-temperature values are taken as $\rho_c = 3.55 \text{ g/cm}^3$ and $T_c = 38.9 \times 10^6 \text{ K}$ [3]. Adopting the same procedure and the mass-fraction values

used in Ref. [2] in evaluating the GSF for the binary pairs considered there, one sets $\rho = \rho_c/8$, $T = T_c/2$ as representative mean values, and $X=.60$, $Y=.37$, $Z=.03$.

One gets

$$\gamma_* = 1.535, \quad \gamma_*^{1/3} = 1.153.$$

The distance d is wrongly inferred to be greater by about 15 percent than it actually is, which brings the result into good agreement with the previously accepted value.

I thank Marion Brunstein for help in obtaining reference material.

Appendix

The logic path leading to Eq. (1c) can be characterized as follows:
One tacitly assumes there are no differences in scale between the E and G_* systems. Specifically, one takes P and P_* to be equal, a and a_* to be equal. One then writes Kepler's Third Law in the conventional manner,

$$P^2 = (4\pi^2 a^3)/G(m_1 + m_2).$$

This is merely Eq. (1a) without the star subscripts. Though minus the notation, the expression continues to hold true *only* in G_* : it correctly relates the values of the semimajor axis and the period *only* in G_* .

The period P has of course been directly measured – that is, in the E

system. As expressed in G_* , $P \rightarrow P_* = \gamma_* P$. Kepler's Third Law is then "properly" written in the mixed system as

$$\gamma_*^2 P^2 = (4\pi^2 a^3) / G(m_1 + m_2) . \quad (1c)$$

If the expression is still to relate quantities correctly, the semimajor axis a must now be implicitly understood to have the value a_* , its value in the G_* system.

Since P has been accurately measured and m_1, m_2 correctly inferred from spectral characteristics, one necessarily infers from Eq. (1c) that the value of the semimajor axis is

$$a_\mu = a \gamma_*^{-2/3} .$$

The correct E-system value of a is gotten directly from Eq. (1b),

$$a = \gamma_*^{5/3} [G(m_1 + m_2) P^2 / 4\pi^2]^{1/3} .$$

Substituting a_* for a in Eq. (1c), $a_* = a \gamma_*^{-1}$, one gets this same correct result.

One notes that τ , the transit time for the eclipsing effect given in Eq. (2b), is a function of R_1/a and R_2/a . Both the orbital semimajor axis and the radii of the stars enter into the calculation of τ through an orbital, *gravitational*, relationship, and both are newly introduced in that calculation. They both transform between E and G_* in the same linear fashion. Ratios formed of them are then independent of the system in which one is working. The conclusion that

$$R_1 \rightarrow \gamma_*^{1/3} R_1 , \quad R_2 \rightarrow \gamma_*^{1/3} R_2 ,$$

is then not affected by failure to recognize the difference in scale between systems, provided the radii and the semimajor axis are dealt with consistently.

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Relativistic effects of ionization account for the “missing” baryons

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Current views of Big Bang nucleosynthesis fit well with the Einstein-de Sitter model and recent observational data, provided one includes the relativistic effects of ionization.

Introduction

The late Richard Feynman is reported to have summed up a meeting of cosmologists that he once attended thusly: “Like a bunch of worms, crawling all over one another, trying to get out of the can.” Those who have done some fishing will understand [1]. There must be many thousands of papers currently available that treat the evolution and dynamics of . . . well, the universe, the entire universe, to be specific. This discussion is another attempt to turn the can over. I wonder if anyone will notice.

In what follows, the notions of “dark energy,” “vacuum-energy density,” etc. are not considered. They are taken to be stop-gap measures. They do not appear to be susceptible to any convincing manner of testing, and, more to the point, they cannot be subjected to direct testing. I suggest their fate will likely follow that of “dark matter.” On page 24 of this series, I have proposed a class of

simple experiments to test *directly* for the dark-matter effect elucidated there and in the now numerous examples that follow that original discussion. That these direct tests, or their equivalent, have not by now been performed and reported on in an open way must be termed scandalous, an embarrassment to the physics and astronomy communities.

We now proceed to examine the value of the baryon density of the universe as a whole, as it has been inferred from a coupling of contemporary cosmological views with recent measurements. We then examine how this value must be reinterpreted in light of the relativistic effects of ionization. But before doing that, a short aside is needed to introduce a viewpoint with respect to the Hubble parameter, a matter of foremost significance in this discussion.

One can arrive in a straightforward manner, and very quickly, at Newtonian gravity starting with an immediately obvious hypothesis that follows naturally from the dark-matter explanation given earlier in this series. The hypothesis is this: The spacetime scales of the electron-proton Coulomb interaction and that of the proton-proton, or electron-electron, Coulomb interaction are not the same. That is, the spacetime of the attractive force is not the same as that of the force of repulsion. They differ by about 1 part in 10^{36} , with the speed of light the same in both systems. Light can then be regarded as an oscillatory, a back-and-forth, transfer of energy between systems. Pursuing this line of reasoning further, one comes easily to the conclusion that the Hubble parameter is in fact a fundamental constant,

$$H_0 \equiv Gc m_a^2 / 2a_0 e^2,$$

where m_a is the mean mass universally of an atomic, or charge-neutral, unit. Taking $m_a = 1.6704 \times 10^{-24}$ g, one directly evaluates the Hubble parameter: $H_0 = 2.2859 \times 10^{-18} \text{ s}^{-1}$, or $H_0 = 70.53 \text{ km s}^{-1} \text{ Mpc}^{-1}$. One notes that this value is in excellent agreement with the most recent observational determinations of H_0 . Whether H_0 or any of the constants that comprise it are functions of time does not concern us here. *In this discussion, H_0 will be taken to be this fundamental constant.* A greater detailing of the procedure that leads to this specific understanding will be taken up at a later time. At this point, it will suffice to demonstrate what this *single, restricted* conclusion leads to. (One notes in passing that the notion of “dark energy” appears to follow as a compounded misunderstanding of the Hubble parameter’s meaning. Further, I would suggest to those whose interest in these matters does not stray far from general relativity that Einstein’s field equations apparently owe their obvious success to an implicit fusing of the two spacetimes.)

The dilemma

A particularly favored solution that follows from Einstein’s field equations as applied to the universe as a whole is the Einstein-de Sitter model. One presumes that the universe is expanding and that the rate is controlled by gravitation. From this particular model, one concludes that there is a critical density of matter and energy in the universe, ρ_c . From the theory one obtains an expression for this critical density,

$$\rho_c = (3/8\pi G)H_0^2, \quad (1)$$

where G is the gravitational constant and H_0 the Hubble parameter.

To further quantify the discussion, one conventionally introduces a density parameter Ω_b by setting

$$\Omega_b \equiv \rho_b / \rho_c, \quad (2)$$

where ρ_b is the matter, or baryon, density obtained from direct measurement. For the universe to be flat (that is, Euclidean, but in the four dimensions of spacetime), one must have that $\Omega_b = 1$ to an *extremely* high degree of accuracy – both presently and at all previous times [2]. Otherwise, following the Big Bang the universe should either have quickly collapsed back to a singularity ($\Omega_b > 1$), or expanded so rapidly that there should presently be no evidence of any structure – no stars, no galaxies ($\Omega_b < 1$).

Estimating the density ρ_b is a difficult undertaking. In recent years, the detection of deuterium in high-redshift clouds of primordial gas, along with accurate estimates of its abundance relative to the accompanying ordinary hydrogen, has made the task possible. Results have been combined with standard theories of the nucleosynthesis believed to have ensued immediately after the Big Bang to yield what appear to be reliable estimates of ρ_b and Ω_b [3,4]. The discussion that follows is based on the result presented in Refs. [3,4],

$$\Omega_b h^2 = .020 \pm (.002). \quad (3)$$

A dimensionless parameter h has been introduced in a conventional manner by the authors to reflect the uncertainty in the value of H_0 that one adopts; $h \equiv H_0 / 100 \text{ km s}^{-1} \text{ Mpc}^{-1}$. We set $h = .705$. Their result seems to imply that the measured value of Ω_b is not reconcilable with the Einstein-de Sitter model.

The solution

In the reasoning that has led to the dilemma, there are two implicit factors that must be given careful consideration.

(a) In general relativity, the Hubble parameter is often interpreted to represent a recessional velocity v , presumably a residual of the Big Bang and defined by $H_0 = v/R$, where R is the distance to the observed object. We here replace that interpretation with the specific notion that H_0 is a fundamental constant with the value defined earlier. As such, it does not scale with change of systems. One is reminded that the ready notion that distant stars, galaxies, are receding from us does not in fact accurately reflect the meaning of general relativity. It is *space* that is expanding. The view commonly expressed in terms of classical dynamics is assuredly more comprehensible, but it is a misconception.

(b) Nevertheless, Equation (1) does express a gravitational relationship, while all direct measurements are naturally made in electromagnetic spacetime. Because predominantly throughout the universe the baryonic matter is ionized, this consideration is of absolutely fundamental importance.

In view of (a) and (b), one concludes that Equation (1), *as written*, is incorrect. In that form, it must be understood to hold true only in gravitational spacetime, G_* . One has thus in Equation (1) made use of a mixed system. To correct for this, the critical density ρ_c must be transformed to the E system:

$$r \rightarrow r_* = \gamma_*^{-1} r, \quad \rho_c \rightarrow \rho_{c*} = \gamma_*^3 \rho_c . \quad (4)$$

(One is referred to earlier discussion in this series, especially pp. 5–8, for details of the definition and calculation of γ_* , the gravitational scaling factor.) Equation (1), correctly written in the electromagnetic system, E, becomes

$$\rho_c = \gamma_*^{-3}(3/8\pi G)H_0^2. \quad (5)$$

With ρ_c defined by Equation (1), Equation (2) accordingly should be written in E as

$$\Omega_b \equiv \gamma_*^3 \rho_b / \rho_c, \quad (6)$$

with the result from Refs. [3,4] becoming, again in E,

$$\Omega_b h^2 = \gamma_*^3 [.020 \pm (.002)]. \quad (7)$$

It is worth emphasizing here that ρ_b has been inferred directly from *electromagnetic measurements* – that is, without intercession of any gravitational relationships in the process.

Evaluation of Ω_b

With input from Equation (1), a value for ρ_b can be retrieved from the observational result expressed in Equation (3). It is, setting $h=.705$, $\rho_b = .1868 \times 10^{-30} \text{ g cm}^{-3}$. With this value of the density fixed, one can then estimate the gravitational scaling factor γ_* as a function of temperature T. It is assumed that warm/hot gas, in this case, the intergalactic material, is dominant in the “dark-matter” effect, and that this is indeed the predominant material one is concerned with in Equation (1). Relative mass-abundances of the intergalactic material are assumed to be X (hydrogen) $\cong .76$, Y (helium) $\cong .24$.

Again, one is referred to pp. 5–8 for a detailed discussion of the gravitational scaling factors and their calculation. The notation $\gamma_* \equiv \gamma_1\gamma_2$, used until now, will at this point be altered to $\gamma_{ab} \equiv \gamma_a\gamma_b$ to reflect more clearly its meaning as the product of factors arising from the gravitational interaction of the two systems, *a* and *b*. This is done in anticipation of its practicality in subsequent work.

Conclusion

Corrected values of Ω_b obtained from Equation (7), along with their associated temperatures, are listed in the table below.

The density and temperature of the intergalactic material in the neighborhood of the Milky Way are concluded to be, respectively, $\rho_b = .1868 \times 10^{-30} \text{ g cm}^{-3}$ and $T = .8 \times 10^6 \text{ K}$. The gravitational scaling factors associated with these values and the relative mass abundances above are $\gamma_i = 1.717$ and $\gamma_{ii} = 2.948$.

T	γ_{ii}^3	Ω_b
$.9 \times 10^6 \text{ K}$	29.89	1.20
$.8 \times 10^6 \text{ K}$	25.61	1.03
$.7 \times 10^6 \text{ K}$	21.64	.87

Observations indicate that a large fraction, perhaps most, of the baryons now are in the form of ionized gas at temperatures in the range $10^5 - 10^7 \text{ K}$. This

is oftentimes referred to as the warm/hot intergalactic material, or WHIM [5,6,].

The temperatures that lead to $\Omega_b \cong 1$ are seen to be centered on this range.

It is the dark matter, so-called, that is apparently responsible for the exceedingly fine balance inherent in the severe constraint on the density parameter Ω_b discussed earlier. The details of primordial nucleosynthesis and the Big Bang aside, what one necessarily concludes is this: A flat-spacetime universe is a consequence of the relativistic-ionization effect, and it is realized predominantly through the gravitation of the warm/hot intergalactic material.

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Gravity, the cosmological redshift and the “dark energy” misunderstanding

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Gravity is explained as a relativistic effect, the result of a difference in scale between the attractive and repulsive spacetimes established by the interactions of electrons and protons. Light is viewed as an oscillatory energy transfer between the two systems. This leads to a direct evaluation of the Hubble constant in terms of other fundamental constants and to a quantitative accounting of the dark-energy effect. No adjustable parameters appear anywhere in the development.

*“If it was so, it might be; and if it were so, it would be: but as it isn't, it ain't. That's logic.”
(Tweedledee, in Lewis Carroll's *Through the Looking Glass* ... or was it Tweedledum?)*

I. Introduction, a fundamental shift in viewpoint

The stable charged particles, electron and proton, are postulated to establish the spacetimes of electromagnetic and gravitational interactions. The geometric representation of spacetime is interpretable as a utilitarian abstraction, natural and innate, but ultimately misleading since spacetime is physical [1].

The measurable properties of the spacetimes under consideration manifest in the interactions of electrons and protons. The two primary interactions are attraction and repulsion, and these define separate spacetimes. Each primary interaction is postulated to establish, at each event pair, a distinct, separate spacetime for that interaction. Hydrogen atoms in the ground state offer an entry point, a Rosetta-Stone-like key, in this regard. Their interactions are themselves an expression of the relationship between the two spacetimes, viewed at a very fundamental level.

II. Gravity as a dual-spacetime effect

Starting with a hypothesis that is immediately suggested by the dark-matter explanation that began this series, one easily arrives at Newtonian gravity. The hypothesis is this: The spacetime scales of the electron-proton Coulomb interaction and that of the proton-proton, or electron-electron, Coulomb interaction are not the same. They differ by about 1 part in 10^{36} , with the speed of light the same in both systems.

We first consider two hydrogen atoms. There are two scales, two measures, of spacetime: that of the force of attraction, designated by an asterisk subscript (e.g., r_* for distance: the “starred” system), and that of the force of repulsion, designated by lack of subscript (e.g., r for distance: the “unstarred” system). The potential energy shared by two hydrogen atoms separated by a distance $r \gg a_0$, a_0 the most probable distance of the electron from the proton in ground-state hydrogen atoms (the radius of the first Bohr orbit), is

$$V = e^2/r + e^2/r - e^2/r_* - e^2/r_* \quad (1)$$

As in the initial discussion and using here the same notation (one is referred to pages 5-8), the relationship between r and r_* for *ionized hydrogen* is

$$r_* = r/\gamma_*, \text{ where } \gamma_* \equiv \gamma_1\gamma_2 = \gamma_1^2, \quad (2)$$

and for two gravitationally interacting hydrogen masses,

$$\gamma_1 = \gamma_2 = 1 + (v/c)\ln(a/a_0). \quad (3)$$

In the initial discussion, the distance a represents the mean separation between electrons and protons in the plasma, taken there to be fully ionized.

At this point we generalize this to include non-ionized hydrogen:

$$\gamma_1 = 1 + (v/c)\ln[(a/a_0)(1+\varepsilon_0/2)], \quad \varepsilon_0 \ll 1. \quad (4)$$

For hydrogen in the ground state, $v/c = e^2/\hbar c \equiv \alpha$, $a = a_0$, and

$$\gamma_* = \gamma_1^2 \cong 1 + \alpha\varepsilon_0, \quad \gamma_* - 1 \cong \alpha\varepsilon_0. \quad (5)$$

Then from Equations (1) and (5),

$$V = -2e^2(\gamma_* - 1)/r = -2e^2\alpha\varepsilon_0/r. \quad (6)$$

One now interprets this electrostatic potential-energy difference as the gravitational energy shared by the two hydrogen atoms. A complication arises at this point: the gravitational constant G is, in practice, measured using heavy elements. To accommodate this fact in a practical manner and in a way that lends itself most readily to astrophysical application, two steps are taken: (a) the neutron is treated as the *charge-equivalent* of an electron superimposed on a proton, and (b) elements in universal relative abundances of $X=.73$, $Y=.25$, $Z=.02$ are assumed, rather than, say, terrestrial abundances. One arrives at a universally averaged mass of a charge-neutral unit (proton+electron or neutron), $m_a = 1.6704 \times 10^{-24}$ g. This procedure is somewhat arbitrary, and it also assumes that the accepted value of G has been reliably established. There is in fact reason to question the value of G [2]. One is thus lead to restrict confidence in results to three, possibly four, significant figures.

Setting the gravitational potential energy $V_g = -Gm_1m_2/r = V$, one has

$$\alpha\varepsilon_0 = Gm_1m_2/2e^2 = Gm_a^2/2e^2. \quad (7)$$

Then $\alpha\varepsilon_0 = .40351 \times 10^{-36}$, a value we will have need of shortly. Newtonian gravity

is in this way explained as an electrostatic effect, a consequence of the scale differences between the two spacetimes.

The earlier characterization of the spacetimes as electromagnetic and gravitational now, of course, loses significance. The spacetime referred to as “electromagnetic” prior to the present discussion is now understood to be a *merger* of the physically distinct spacetimes of attraction and of repulsion. Gravitation prior to this point has been treated, conventionally, as a force distinct from the electrostatic force. This historically founded practice has been effective until recently but has failed to detect anomalies, consequences that may be attributed to two factors: (1) the exceedingly fine difference between the scales of the spacetimes of attraction and of repulsion when matter is not ionized, and (2) a former lacking in technological capability, resulting in imprecision in measurements on matter in the plasma state, on stars and interstellar and intergalactic material. (For plasma, the scales of the two spacetimes can differ appreciably, as we have earlier shown.) Using the system associated with *direct*, electromagnetic astronomical measurements as a standard reference frame as we have done to this point has been a pragmatic and unwitting continuation of the historical approach. We shall return to this issue in concluding comments.

III. Light as a dual-spacetime phenomenon, the Hubble constant

The transformation equations relating the two systems (See p. 7) are,

$$\mathbf{r}_* = \gamma_* \mathbf{r} - (\gamma_*^2 - 1)^{1/2} ct \hat{\mathbf{e}} \quad ct_* = -(\gamma_*^2 - 1)^{1/2} \mathbf{r} \cdot \hat{\mathbf{e}} + \gamma_* ct \quad (8a,b)$$

$$\mathbf{r} = \gamma_* \mathbf{r}_* + (\gamma_*^2 - 1)^{1/2} ct_* \hat{\mathbf{e}}_* \quad ct = (\gamma_*^2 - 1)^{1/2} \mathbf{r}_* \cdot \hat{\mathbf{e}}_* + \gamma_* ct_* \quad (9a,b)$$

where $\hat{\mathbf{e}} = \Delta \mathbf{r} / \Delta r = \Delta \mathbf{r}_* / \Delta r_* = \hat{\mathbf{e}}_*$.

An electromagnetic signal emanating from an atom is postulated to have two wavefronts, W_* and W , arising from coupled events, one in each system. W_* arises from the event in the starred system, W from that in the unstarred system. The events are simultaneous in the unstarred system but separated by a distance $r = a_o$. *The distance a_o is assumed to be fundamental to the defining of spacetime, not merely characteristic of the hydrogen atom.* Again: the hydrogen atom, the basic unit of stable, charge-neutral matter, is postulated to manifest the properties of the spacetimes directly. The two events are summarized as follows:

Starred-system event:

At $r_* = 0$, $t_* = 0$. W_* advances as $r_* = ct_*$. (From the transformation relationships, the coordinates of this event in the other system are $r = 0$, $t = 0$.)

Unstarred-system event:

At $r = a_o$, $t = 0$. W advances as $r = a_o + ct$. (From the transformation relationships, the coordinates of this event in the starred system: $r_* = \gamma_* a_o$,

$$ct_* = -(\gamma_*^2 - 1)^{1/2} a_o, \text{ or } t_* = -1.5857 \times 10^{-37} \text{ s.})$$

The advancing wavefronts are separated by a distance a , a function of distance from the source and elapsed time since the emission event:

$$a \equiv r - r_* = r - ct_* \quad (10)$$

In the unstarred system at fixed r , measured elapsed times are related through Equation (8b) as $t = t_*/\gamma_*$. Then

$$a = r - \gamma_* ct = r - \gamma_* ct - \gamma_* a_o + \gamma_* a_o = r - \gamma_*(a_o + ct) + \gamma_* a_o; \quad \text{with } r = a_o + ct,$$

$$a = r(1 - \gamma_*) + \gamma_* a_o = -\alpha \varepsilon_o r + \gamma_* a_o = \gamma_* a_o (1 - \alpha \varepsilon_o r / \gamma_* a_o); \quad (11)$$

with $R \equiv a_o / \alpha \varepsilon_o$, $r_* = r / \gamma_*$,

$$a = r - r_* = \gamma_* a_o (1 - r_*/R). \quad (12)$$

As a consequence of the difference in spacetime scales, a potential-energy difference exists between the two advancing wavefronts. The difference as a function of r is

$$\phi(r) = (e^2/r) - (e^2/r_*) = -a_o(e^2/r_*^2)(1 - r_*/R), \quad \text{or} \quad (13)$$

$$\phi(r) \cong -a_o(e^2/r^2)(1 - r/R), \quad (14)$$

to order $1/10^{36}$.

One recalls that the long-accepted, simple relationship between frequency and energy continues to be a complete enigma. It is “the point of our departure, not the result of our thinking” [3]. The $1/r^2$ factor in Equation (14) is anticipated for energy carried by radiation. Assuming that (a) light is an oscillatory exchange of energy *between the systems*, (b) that this potential-energy difference provides in a direct linear manner the energy associated with the oscillations and carried

along with the wavefronts, and (c) that the conventional relationship between energy and frequency holds, one writes

$$v = v_o(1 - r/R), \quad \lambda = \lambda_o (1 - r/R)^{-1}. \quad (15)$$

The $(1 - r/R)$ term, a scaling factor, follows as a consequence of the difference in scale between the two spacetimes and is interpretable as the observed cosmological energy degradation, or redshift. Setting $R \equiv c/H_o$, one has that $H_o = Gcm_a^2/2a_o e^2 = 2.2859 \times 10^{-18} \text{ s}^{-1}$, or $H_o = 70.53 \text{ km s}^{-1} \text{ Mpc}^{-1}$. From Equation (15) one has directly the redshift parameter z as related to r ,

$$1+z = (1 - r/R)^{-1}. \quad (16)$$

The potential energy difference $\phi(r)$ as a function of z is then

$$\phi = - (a_o e^2) / [R^2 z^2 (1+z)^{-1}]. \quad (17)$$

In passing, it is interesting to note the analogies with general relativistic cosmologies. With respect to sequentially spaced systems fixed along the path of an advancing dual wavefront, one has from Equation (12),

$$(da/dt)/a_o = -c/R = -H_o, \quad (18)$$

in analogy with the general relativistic relationship. The negative sign follows here as a consequence of the decrease in the spatial separation a with time (or distance), the immediate cause of the redshift. Also, one has analogously that

$$a_o/a = 1+z. \quad (19)$$

A noteworthy difference is that the fundamental scaling parameter a_o is a

physical, *measurable* quantity here, while the corresponding unit in general relativistic cosmologies is necessarily a geometric abstraction.

IV. The cosmological redshift and “dark energy”

The notion of dark energy has come about from a comparison of the observed brightness versus redshift relationship displayed by supernovae of Type 1a (SNe 1a) – redshift being a measure of distance. This particular class of supernovae is thought to be sufficiently uniform in origin and development to be employed as standard candles. That is, SNe1a are thought to display *luminosity functions*, time-developments of their intrinsic brightness, that are relatively uniform from one to another.

As a function of redshift parameter z , they appear dimmer with increasing z than anticipated, at least for redshifts less than $z \sim 1.5$, much beyond which there is a scarcity of data. To account for this observed redshift-luminosity relationship using the theoretical structure of general relativistic cosmologies, one must unavoidably invoke the concept of “dark energy.” The explanation to be offered here is rather that a better understanding of the nature of light and of the meaning of the Hubble constant accounts for the effect without need of any exotic form of undetected matter or energy. One concludes the general relativistic cosmologies are flawed. The nature of the flaw will be discussed in closing comments.

The $1/r^2$ dependence of ϕ suggests an energy flux function F tentatively of the form

$$F = \eta(t)(c/a_0^3)(-\dot{\phi}) \quad (20)$$

for the measured radiation rate from the SNe 1a, assuming a separation of the space and time variables and that the dimensionless function $\eta(t)$ characterizing the time development of the supernovae is uniform from one to another. In terms of z , one has

$$F = \eta(ce^2/a_0^2)/[R^2z^2(1+z)^{-1}]. \quad (21)$$

An effective radius r_ϕ for energy flux measurements is then

$$r_\phi = Rz(1+z)^{-1/2}. \quad (22)$$

The luminosity function L is conventionally used to describe the intrinsic brightness of a light source in units of energy/time. L might be related to F by

$$F = L/4\pi r_\phi^2 \quad (23)$$

except for the fact that the energy and spacetime scales at the SNe, where L is to be evaluated, and locally at F , where measurements are made, are different. To surmount this difficulty, we do the following.

Consider the number N of photons emitted in a monochromatic light pulse of frequency ν_0 and duration Δt_0 as evaluated at the source. One must have

$$N = L\Delta t_0/h\nu_0 = 4\pi r_\phi^2 F\Delta t/h\nu, \quad (24)$$

since the number of photons in the pulse is independent of scale. One then has directly from the operational definition of time measurement that

$$\nu/\nu_0 = \Delta t_0/\Delta t = 1/(1+z), \quad (25)$$

and from Equation (24),

$$F = L/[4\pi r_\phi^2(1+z)^2]. \quad (26)$$

The luminosity distance d_L is by convention defined by

$$F = L / 4\pi d_L^2. \quad (27)$$

With Equation (22) one then has

$$d_L = Rz(1+z)^{1/2}. \quad (28)$$

The luminosity distance d_L is used in standardizing the relationship between the observed energy flux of an object and its redshift distance. This is done through introduction of the distance modulus μ , defined as $\mu = m - M$, where m is the apparent magnitude of the object and M its absolute magnitude, a measure of its luminosity. We are here considering “bolometric” magnitudes, those representing power integrated over all wavelengths. With d_L measured in megaparsecs, the relationship as conventionally written is $\mu = 5 \log_{10} d_L + 25$. In terms of natural logarithms,

$$\mu = (5/\ln 10)(\ln d_L) + 25. \quad (29)$$

With the substitution $R \equiv c/H_0 = 4.2503 \times 10^3$ megaparsecs, one has

$$\mu = 43.142 + 2.1715[\ln f(z)], \quad \text{where } f(z) \equiv z(1+z)^{1/2}. \quad (30)$$

The luminosity distance that follows from the particular general relativistic cosmology relying on the dark-energy concept and under consideration here is [4,5],

$$d_{L\Lambda}(z) = (c/H_0)(1+z) \int_0^z [(1+z)^2(1+\Omega_M z) - z(2+z)\Omega_\Lambda]^{-1/2} dz. \quad (31)$$

Setting

$$d_{L\Lambda} \equiv (c/H_0) f_\Lambda(z), \quad (32)$$

one has for the distance modulus μ_Λ ,

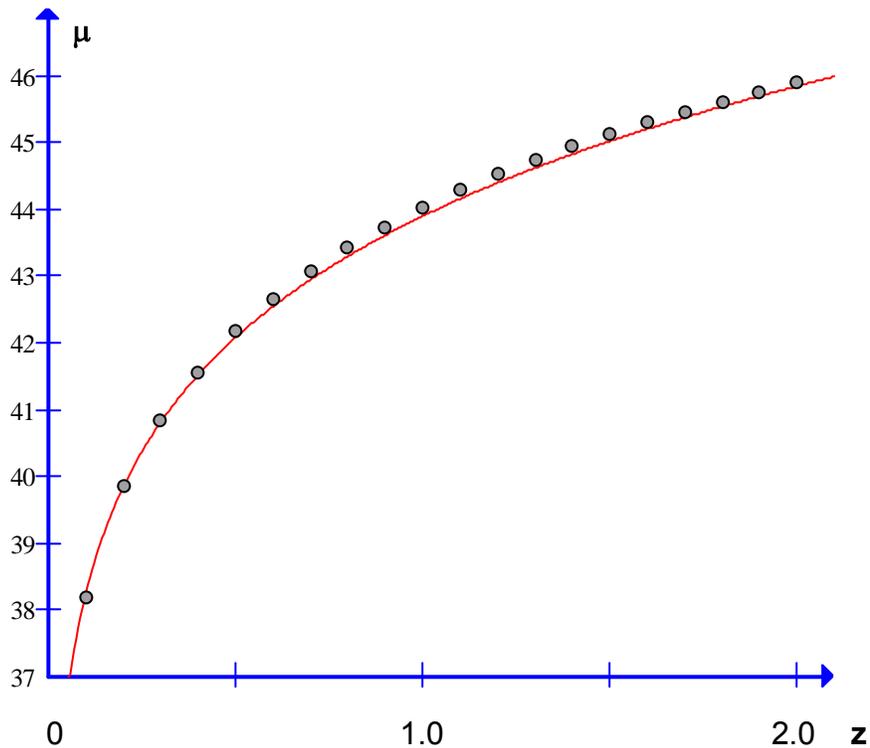
$$\mu_{\Lambda}(z) = 2.1715[\ln (c/H_0)] + 25 + 2.1715[\ln f_{\Lambda}(z)]. \quad (33)$$

At redshift z , differences between distance modulus values obtained from equation (30) and corresponding values obtained from Equation (33) are seen to depend functionally on differences between $f(z)$ and $f_{\Lambda}(z)$ and, more particularly, on the values chosen for the parameters H_0 , Ω_M and Ω_{Λ} . In the general relativistic cosmology, $\Omega_M + \Omega_{\Lambda} = 1$. Matter plus “dark matter” is assigned the fractional value Ω_M , and Ω_{Λ} represents the remainder, the “dark energy.” Thus while μ_{Λ} has two independent adjustable parameters, μ as evaluated in Equation (30) has none, H_0 having been earlier defined and evaluated.

The observational evidence for dark energy is most clearly demonstrated in the relationship between distance modulus μ and redshift parameter z [5,6,7,8]. One is referred to Fig. 4 of Ref. [5] and particularly to Fig. 6 of Ref. [6]. Here we want to compare the fixed result of Equation (30) with an example following from Equation (33). To that end, we set $\Omega_M = .27$ and $\Omega_{\Lambda} = .73$, and further set $\mu_{\Lambda}(2.0) = 45.9$ to specify the additionally required parameter explicitly. (See Fig. 6 of Ref. [6].) Results are presented in Figure 1 and Table 1.

Differences between distance modulus values predicted by Equation (30) and corresponding values of the fitted dark-energy relationship are of the order of the observational errors, though values predicted by Equation (30) are generally on the lower side. That is, the latter predict slightly brighter SNe 1a than does the fitted general relativistic relationship. If one accepts that the general relativistic

Figure 1



Distance modulus μ as a function of redshift parameter z . Equation (30) is represented by the solid line. Centers of the shaded circles represent results of equation (33) with $\Omega_M = .27$, $\Omega_\Lambda = .73$, and with $\mu_\Lambda(2.0) = 45.9$ to specify the additionally required parameter (see Fig. 6 of Ref. [6]). Integrals in Equation (33) have been performed numerically at intervals $\Delta z = .1$; observational errors introduce an uncertainty $\Delta\mu_\Lambda \approx .1$ to .15, about the radius of the circles.

relationship, as fitted, represents the observational data satisfactorily, this difference is not surprising. With the exception of gravitational lensing, the effects of intervening material – of dust, specifically – can only diminish apparent brightness (increase apparent magnitudes).

Table 1

Distance modulus values as functions of redshift parameter z . μ_Λ values are calculated using Equation (33) with $\Omega_M = .27$, $\Omega_\Lambda = .73$, and with $\mu_\Lambda(2.0) = 45.9$ to specify the additionally required parameter (see Fig. 6 of Ref. [6]). μ values are calculated using Equation (30).

z	μ_Λ	μ	$\Delta\mu = \mu_\Lambda - \mu$
.1	38.204	38.245	-.041
.2	39.850	39.845	.005
.3	40.854	40.812	.042
.4	41.588	41.518	.070
.5	42.169	42.077	.092
.6	42.650	42.543	.107
.7	43.062	42.944	.118
.8	43.421	43.296	.125
.9	43.739	43.610	.129
1.0	44.024	43.895	.129
1.1	44.283	44.155	.128
1.2	44.519	44.394	.125
1.3	44.736	44.616	.120
1.4	44.937	44.823	.114
1.5	45.124	45.017	.107
1.6	45.299	45.200	.099
1.7	45.462	45.373	.089
1.8	45.616	45.536	.080
1.9	45.762	45.692	.070
2.0	45.900	45.840	.060
2.5	46.495	46.492	.003
3.0	46.977	47.033	-.056

V. Conclusion

In earlier discussions, the unstarred system represented electromagnetic spacetime and the starred system the spacetime of gravity. With both electromagnetic radiation and gravity arising from interaction between the two

systems – now identified as the spacetimes of electrostatic repulsion and that of attraction – the original distinction loses meaning. Continuing to identify the unstarred system as the standard reference frame is then a convention, the root meaning of the appellation “relativity” arising again here.

The spacetime of contemporary physics, and in particular as it is treated in general relativity, is interpretable as a merger of the two spacetimes established in the electromagnetic interactions of non-ionized matter. The properties of electromagnetic radiation are directly measurable, one need not be reminded. As a consequence, an amalgamation of the two spacetimes as represented by electromagnetic radiation has naturally become the standard spacetime of reference.

The basic premise of general relativity is that the divergence of the stress-energy tensor vanishes. The explanation of the cosmological redshift presented here suggests that general relativity draws its success from its implicit merging of the two spacetimes. The energy transfer responsible for the cosmological redshift is contained within the amalgamation and masked by the formalism.

“I could have done it in a much more complicated way,” said the Red Queen, immensely proud.
Lewis Carroll [9]

Acknowledgment

I am grateful to Ivan Johansen and to John Strosahl and people associated with Digital River GmbH for making their effective mathematical software freely available through the Internet.

Notes and References

[1] This is of course an old idea, emphasized notably by Descartes. Einstein wrote a considerable number of words to this effect that may be summarized with: “On the basis of the general theory of relativity ... space, as opposed to ‘what fills space,’ ... has no separate existence” – an exceedingly important insight that seems to have escaped the serious attention it deserves. See Appendix V, *Relativity*, Crown Publishers, New York, 1961.

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[9] As quoted by C. Fröberg in *Introduction to Numerical Analysis* (2nd edition), Addison-Wesley, Reading, Massachusetts, 1967, p. 147. This particular passage is not in my own edition of *Through the Looking Glass*; I presume it is in others. (The version available to Prof. Fröberg was likely a Swedish translation.)

