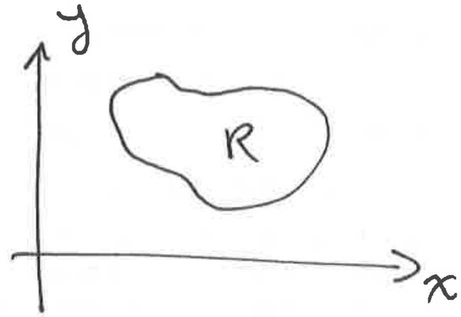
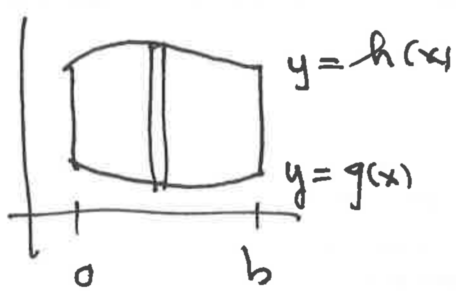
Math 2471 - Calc III

Double Integrals

$$\iint_R f(x,y)\,dA$$

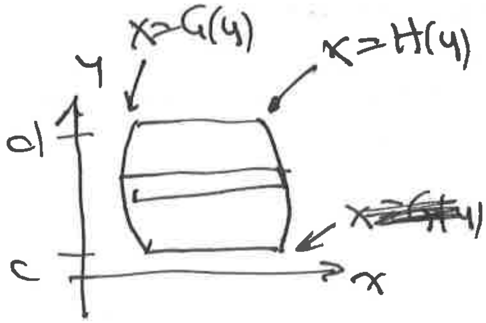


Types



$$\int_a^b \int_{y=g(x)}^{h(x)} f(x,y)\,dy\,dx$$

OR



$$\int_c^d \int_{x=G(y)}^{H(y)} f(x,y)\,dx\,dy$$

Ex1 $$\int_0^1 \int_0^x (xy^2+1)\,dy\,dx$$

$$\int_0^1 \frac{xy^3}{3}+y \Big|_0^x \,dx = \int_0^1 \frac{x^4}{3}+x\,dx$$

$$= \frac{x^5}{3\cdot 5}+\frac{x^2}{2}\Big|_0^1 = \frac{1}{15}+\frac{1}{2} = \frac{17}{30}$$

Ex2 $$\int_0^1 \int_y^1 (xy^2+1)\,dx\,dy$$

$$\int_0^1 \frac{x^2y^2}{2}+x \Big|_y^1 \,dy = \int_0^1 \left(\frac{y^2}{2}+1\right)-\left(\frac{y^4}{2}+y\right)dy$$

$$= \frac{y^3}{6}+y-\frac{y^5}{10}-\frac{y^2}{2}\Big|_0^1$$

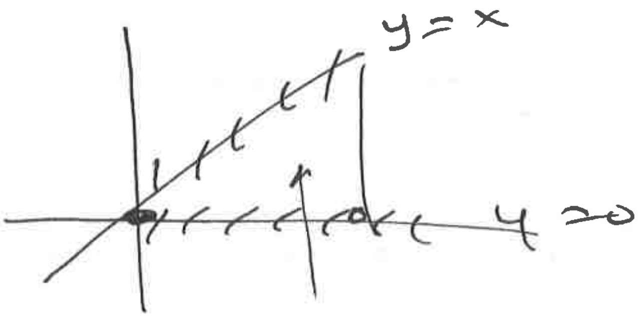
$$= \frac{1}{6}+1-\frac{1}{10}-\frac{1}{2} = \frac{5+30-3-15}{30} = \frac{17}{30}$$

Same ans.

let us consider the regions for each $\int$

$$\int_0^1 \int_0^x (xy^2+1)\,dy\,dx$$

curve → curve in y dir so $y=0 \to y=x$



then pt → pt

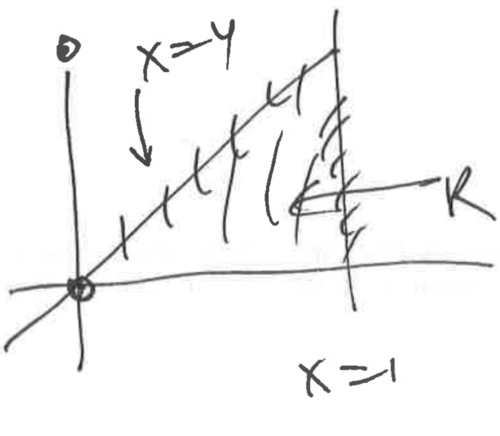
$x=0 \to x=1$

here is R

$$\int_0^1 \int_y^1 (xy^2+1)\,dx\,dy$$

curve → curve x direct

$x=y \to x=1$



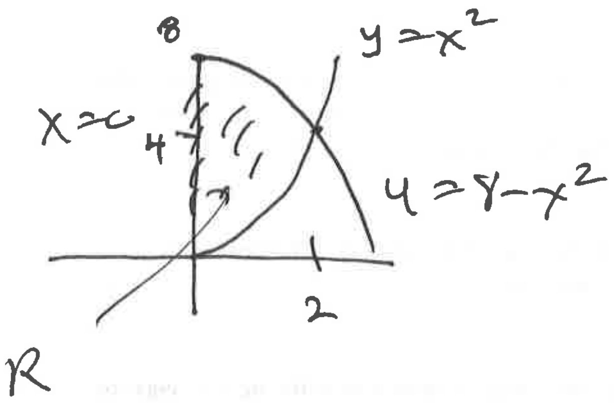
pt → pt in y direction

$y=0 \to y=1$

Same region R

Ex 3 Briggs pg 981 # 28

$$\iint_R (x+y)\,dA$$ R is region bound by $x=0$, $y=x^2$, $y=8-x^2$ in Q I



intersection pt

$8-x^2=x^2$

$2x^2=8 \quad x^2=4 \quad x=\pm 2$

Type 1

$$\int_0^2 \int_{x^2}^{8-x^2} (x+y)\,dy\,dx$$

Type 2

$$\int_0^4 \int_0^{\sqrt{y}} (x+y)\,dx\,dy + \int_4^8 \int_0^{\sqrt{8-y}} (x+y)\,dx\,dy$$

$$\int_0^2 xy+\frac{y^2}{2}\Big|_{x^2}^{8-x^2} dx = \int_0^2 x(8-x^2)+\frac{(8-x^2)^2}{2}-\left[x^3+\frac{x^4}{2}\right]dx$$

$$= \int_0^2 8x-2x^3+32-8x^2\,dx = 4x^2-\frac{x^4}{2}+32x-\frac{8x^3}{3}\Big|_0^2 = \frac{152}{3}$$

Consider

$$\int_0^4 \int_{\sqrt{x}}^2 \frac{x}{y^5+1}\, dy\, dx$$

Briggs pg 982 #66

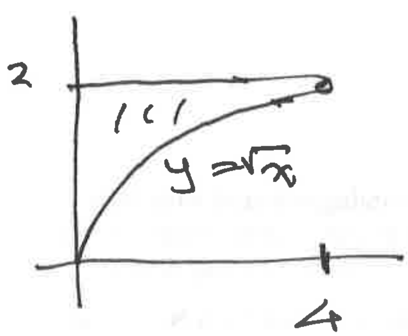
problem - we can't integrate wrt y!

So we re-write this as a type 2 integral

we will change the order of integration

Region $y=\sqrt{x} \to y=2$ curve → curve u dir

pt → pt $x=0 \to 4$



$y=\sqrt{x} \Rightarrow x=y^2$

$$\int_0^2 \int_{x=0}^{y^2} \frac{x\, dx\, dy}{y^5+1} = \int_0^2 \frac{\frac{x^2}{2}}{y^5+1}\Big|_0^{y^2} dy$$

$$= \frac{1}{2}\int_0^2 \frac{y^4}{y^5+1} dy = \frac{1}{10} \ln|y^5+1|\Big|_0^2$$

$$= \frac{1}{10} \ln(2^5+1)$$

$$\int_0^1 \int_{3y}^{3} e^{x^2}\,dx\,dy$$

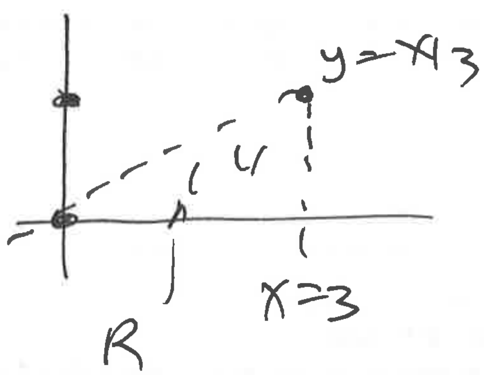
patrickJMT

We can't integrate this wrt x so we switch the order of integration

curve → curve in x direction $3y = x \Rightarrow x = ?$

$y = x/3$ to

the pt → pt is $y = 0 \to 1$



the lines intersect at $y = 1$

$$\int_0^3 \int_{y=0}^{x/3} e^{x^2}\,dy\,dx = \int_0^3 e^{x^2} y \Big|_0^{x/3} dx$$

$$= \frac{1}{3}\int_0^3 e^{x^2} x\,dx = \frac{e^{x^2}}{6}\Big|_0^3 = \frac{e^9 - 1}{6}$$

Ex $\displaystyle\int_0^1 \int_0^{\sqrt{1-x^2}} \frac{1}{x^2+y^2}\,dy\,dx$

$y=\sqrt{1-x^2}$

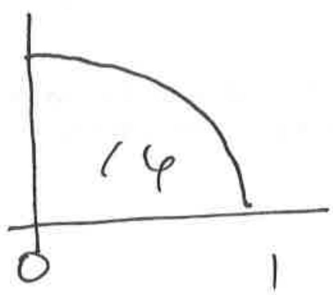
$y^2=1-x^2$ or $x^2+y^2=1$

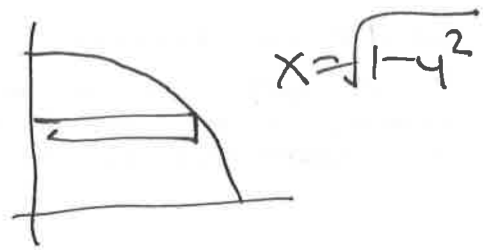
circle

Wow - hard integral!

Let's switch the order

$y=0 \to y=\sqrt{1-x^2}$ pt. $x=0 \to x=1$





Type 2 $\displaystyle\int_{y=0}^{1} \int_{x=0}^{\sqrt{1-y^2}} \frac{1}{x^2+y^2}\,dx\,dy$

Still hard - maybe a better way!

polar coordinates - tomorrow!