

# Math 1496 - Calc 1

## Curve Sketching

1.  $x$  &  $y$  intercept
2. Symmetry
3. Domain, Range
4. Continuity
5. VA & HA (infinite limits)
6. differentiability
7. Extrema
8. increasing / decreasing
9. concavity
10. P.I

ex:  $y = x(x-4)^3$

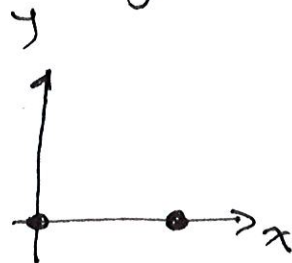
1)  $x$  &  $y$  intercept

$x$  intercept ( $y=0$ )

$$x=0, 4$$

$y$  intercept ( $x=0$ )

$$y=0$$



2. No symmetry
3. Domain - all  $x$
4. Range - all  $y$  (maybe)
5. no HA, VA

note  $\lim_{x \rightarrow \infty} x(x-4)^3 \rightarrow \infty$

$$\lim_{x \rightarrow -\infty} x(x-4)^3 \rightarrow -\infty$$

so maybe a abs. min

6. its diff everywhere - no holes or cusps

$$\begin{aligned} 7. \quad y &= 1(x-4)^3 + x[3(x-4)^2] = (x-4)^2[x-4+3x] \\ &= 4(x-4)^2(x-1) \end{aligned}$$

$$y' = 0 \text{ when } x = 1, 4$$

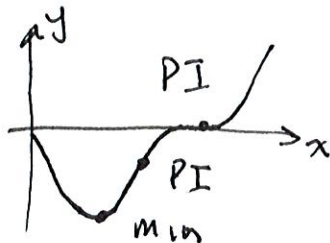
$$y'' = 8(x-4)(x-1) + 4(x-4)^2$$

$$= 4(x-4)[2(x-1) + x-4]$$

$$= 4(x-4)(3x-6) = 12(x-4)(x-2)$$

$$y'' = 0 \text{ when } x = 2, 4 \text{ (possible PI)}$$

$x$		1		2		4		
$x-1$	-	0	+	+	+	+	+	increasing (1,4) (4,∞)
$(x-4)^2$	+	+	+	+	+	0	+	decreasing (-∞, 1)
$(x-1)(x-4)^2$	-	0	+	+	+	0	+	min (1, -27)
slope	↓	-	/	/	/	-	/	max NA
$x-2$	-	-	-	0	+	+	+	concave ↑
$x-4$	-	-	-	-	-	0	+	(-∞, 2) (4, ∞)
$(x-2)(x-4)$	+	+	+	0	-	0	+	concave ↓ (2, 4)
$y''$		∪		Pi	∩	Pi	∪	PI (2, -16) (4, 0)



$$\text{Ex 2} \quad y = \frac{x^2}{x^2-1}$$

1. only intercept is at  $(0,0)$

2. Symm  $x \rightarrow -x$  same function so symmetry in y axis

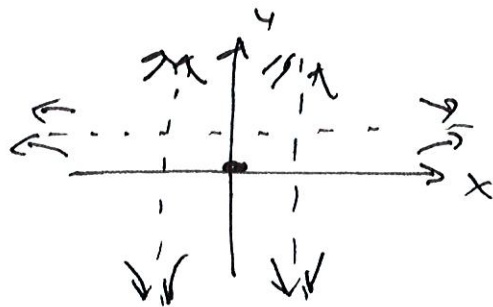
3) Domain  $D = \{x \mid x \neq \pm 1\}$

R well have to see

4) Continuity Everywhere except  $x = \pm 1$

5. VA  $x = \pm 1$

$$\text{HA} \quad \lim_{x \rightarrow \pm \infty} \frac{x^2}{x^2-1} = 1$$



6. Diff. Prob rot at  $x = \pm 1$

$$7. \quad y' = \frac{2x(x^2-1) - 2x(x^2)}{(x^2-1)^2} = \frac{2(x^3-2x-2x^3)}{(x^2-1)^2} = \frac{-2x}{(x^2-1)^2}$$

so  $y' = 0$  when  $x = 0$   $y'$  DNE when  $x = \pm 1$  (VA)

$$8. \quad y'' = \cancel{2 \left[ \frac{x^2-1}{(x^2-1)^3} \right]} - 2x \left[ \frac{1(x^2-1)^2 - x \{ 2(x^2-1) \cdot 2x \}}{(x^2-1)^4} \right]$$

$$= -2 \left[ \frac{x^2-1-4x^2}{(x^2-1)^3} \right] = \frac{2(3x^2+1)}{(x^2-1)^3} \quad y'' \neq 0 \text{ but changes sign}$$

$x$		-1		0		1	
$-2x$	+	+	+	0	-	-	-
$x-1$	-	-	-	-	-	0	+
$x+1$	-	0	+	+	+	+	+
$(x^2-1)^2$	+	0	+	+	+	0	+
$\frac{-2x}{(x^2-1)^2}$	+	$\infty$	+	0	-	$\infty$	-
slope	/		/	-	\		\
$3x^2+1$	+	+	+	+	+	+	+
$x-1$	-	-	-	-	-	0	+
$x+1$	-	0	+	+	+	+	+
$x^2-1$	+	0	-	-	-	0	+
$(x^2-1)^3$	+	0	-	-	-	0	+
$\frac{2(3x^2+1)}{(x^2-1)^3}$	+	und.	-	-	-	und.	+
h/v	U						U

in closing  
 $(-\infty, -1)$   $(-1, 0)$   
dec  
 $(0, 1)$   $(1, \infty)$   
max  $(0, 0)$   
min - none  
concave  $\uparrow$   
 $(-\infty, -1)$   $(1, \infty)$   
concave  $\downarrow$   
 $(-1, 1)$   
no PI

not a PI  
b/c  $x = \pm 1$   
not on graph

