Shedding light on the obscure concepts seen in class

- Lemke-Howson algorithm

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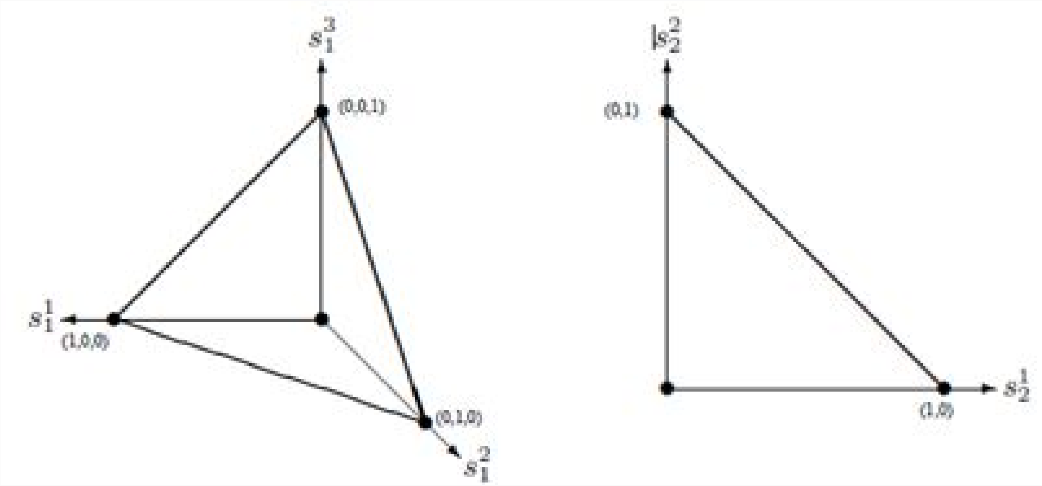
Motivation

- Extensive theory literature
- Lack of extensive examples
- Waste of time
- Thinking on following years

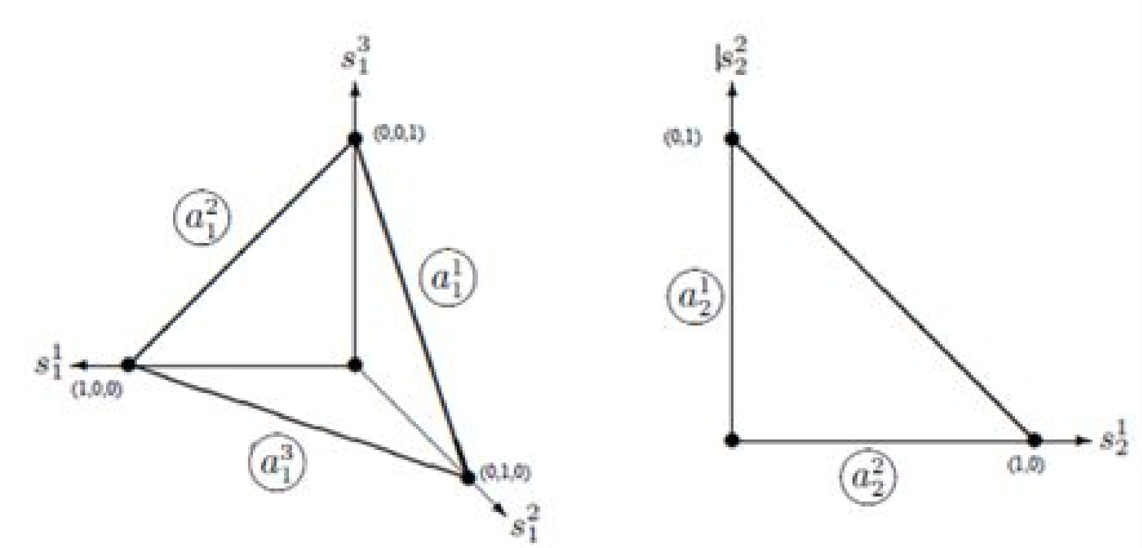
Lemke-Howson Algorithm

- LCP formulation for NE

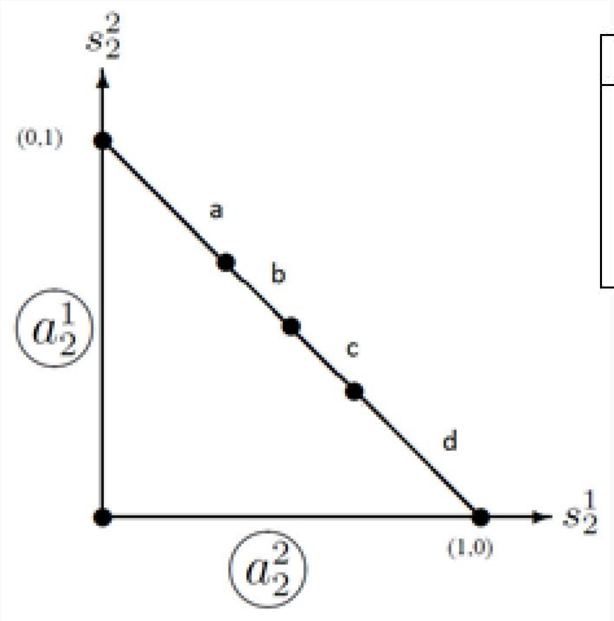
	Player 2		
Player 1		s_2^1	s_2^2
	s_1^1	0,1	6,0
	s_1^2	2,0	5,2
	s_1^3	3,4	3,3



Lemke-Howson Algorithm



Lemke-Howson Algorithm



		Player 2	
Player 1		s_2^1	s_2^2
	s_1^1	0,1	6,0
	s_1^2	2,0	5,2
	s_1^3	3,4	3,3

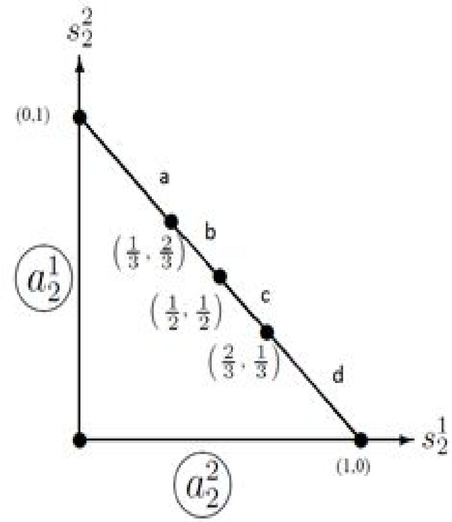
- $u(a_1^1) = p \cdot 0 + (1-p) \cdot 6 = 6 - 6p$
- $u(a_1^2) = p \cdot 2 + (1-p) \cdot 5 = 5 - 3p$
- $u(a_1^3) = p \cdot 3 + (1-p) \cdot 3 = 3$

- $u(a_1^1) = u(a_1^2) \rightarrow 6 - 6p = 5 - 3p \rightarrow p = 1/3$
- $u(a_1^1) = u(a_1^3) \rightarrow 6 - 6p = 3 \rightarrow 6p = 3 \rightarrow p = 1/2$
- $u(a_1^2) = u(a_1^3) \rightarrow 5 - 3p = 3 \rightarrow 3p = 2 \rightarrow p = 2/3$

Lemke-Howson Algorithm

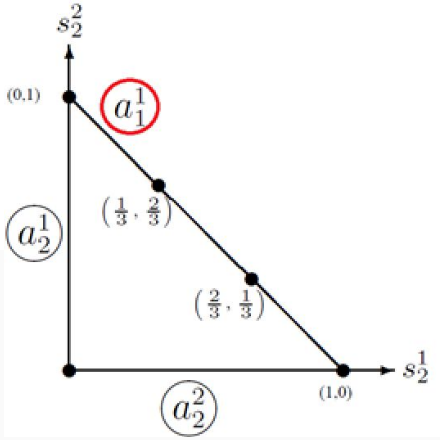
- $u(a_1^1) = u(a_1^2) \to 6 - 6p = 5 - 3p \to p = 1/3$
- $u(a_1^1) = u(a_1^3) \to 6 - 6p = 3 \to 6p = 3 \to p = 1/2$
- $u(a_1^2) = u(a_1^3) \to 5 - 3p = 3 \to 3p = 2 \to p = 2/3$

- $u(a_1^1) = p \cdot 0 + (1-p) \cdot 6 = 6 - 6p$
- $u(a_1^2) = p \cdot 2 + (1-p) \cdot 5 = 5 - 3p$
- $u(a_1^3) = p \cdot 3 + (1-p) \cdot 3 = 3$



For region a, which is between (0, 1) and $\left(\frac{1}{3}, \frac{2}{3}\right)$, we choose the point $\left(\frac{1}{4}, \frac{3}{4}\right)$ which is in between, then we replace p with it in the formulas:

- $6 - 6p = 6 - \frac{1}{4} \cdot 6 = 4.5$
- $5 - 3p = 5 - 3 \cdot \frac{1}{4} = 4.25$
- 3

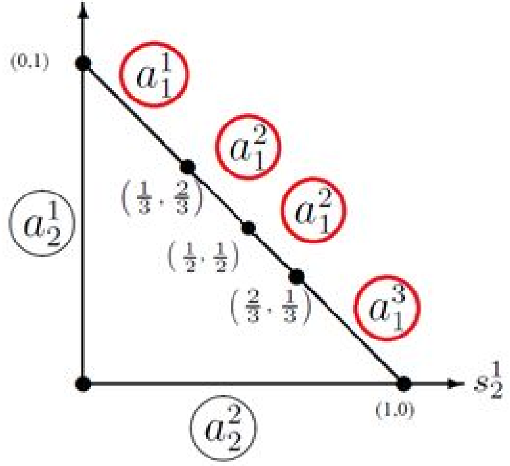


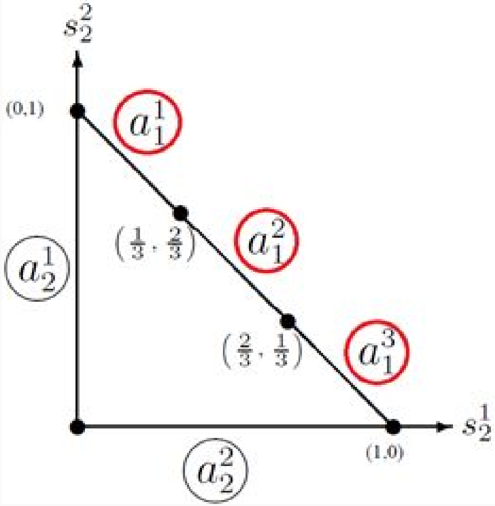
Lemke-Howson Algorithm

- $6 - 6p = 6 - \frac{5}{12} \cdot 6 = 3.5$
- $5 - 3p = 5 - 3 \cdot \frac{5}{12} = 3.75$
- 3

- $6 - 6p = 6 - \frac{7}{12} \cdot 6 = 2.5$
- $5 - 3p = 5 - 3 \cdot \frac{7}{12} = 3.25$
- 3

- $6 - 6p = 6 - \frac{3}{4} \cdot 6 = 1.5$
- $5 - 3p = 5 - 3 \cdot \frac{3}{4} = 2.75$
- 3

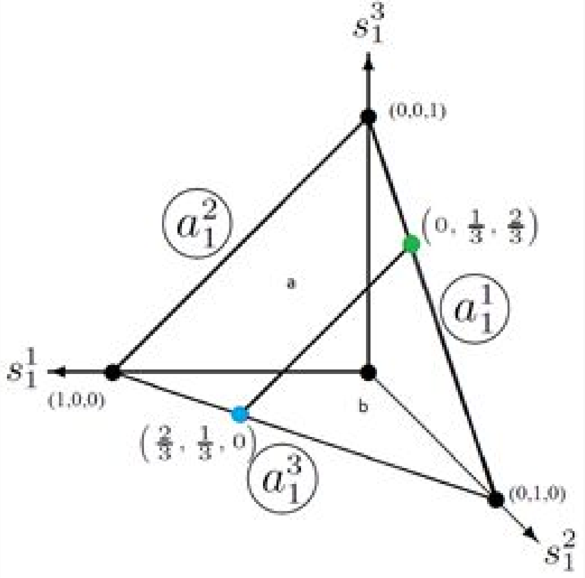




Lemke-Howson Algorithm

		Player 2	
		s_2^1	s_2^2
Player 1	s_1^1	0,1	6,0
	s_1^2	2,0	5,2
	s_1^3	3,4	3,3

- $u(a_2^1) = s_1^1 \cdot 1 + s_1^2 \cdot 0 + s_1^3 \cdot 4$
- $u(a_2^2) = s_1^1 \cdot 0 + s_1^2 \cdot 2 + s_1^3 \cdot 3$



Lemke-Howson Algorithm

- We zero s_1^1:

  $0 \cdot 1 + s_1^3 \cdot 4 = s_1^2 \cdot 2 + s_1^3 \cdot 3 \rightarrow s_1^2 = s_1^3 \cdot 2.$

  As $s_1^2 = 1 - s_1^3$

  $\rightarrow s_1^2 = 1/3$

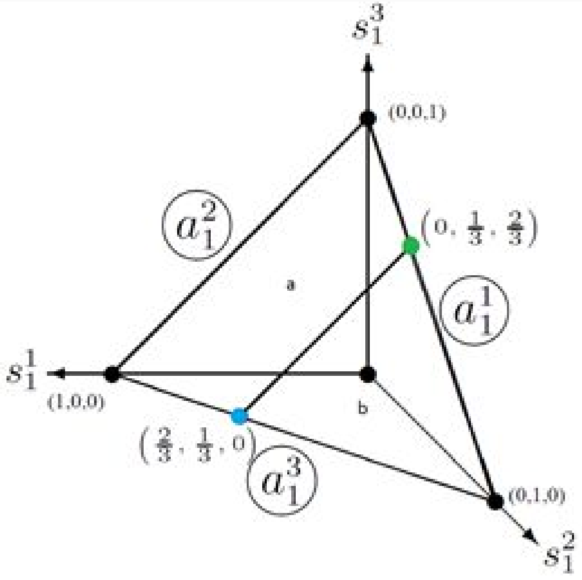
- We zero s_1^3:
  $s_1^1 \cdot 1 = s_1^2 \cdot 2$. As $s_1^1 = 1 - s_1^2 \rightarrow s_1^2 = 1/3$

- We zero s_1^2:

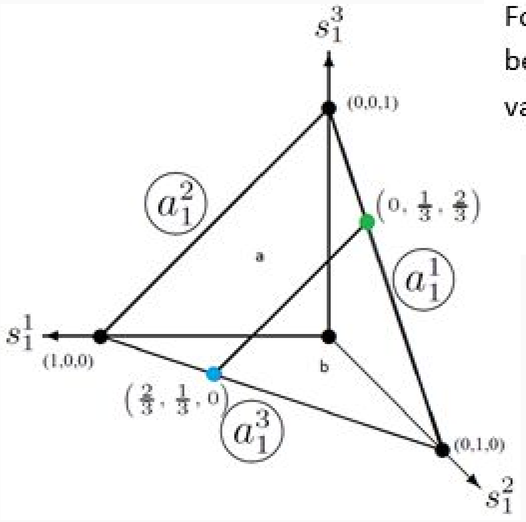
  $s_1^1 \cdot 1 + s_1^3 \cdot 4 = s_1^3 \cdot 3 \rightarrow s_1^1 = -s_1^3$

  $\rightarrow$ Contradiction $\rightarrow$ No valid point

- $u(a_2^1) = s_1^1 \cdot 1 + s_1^2 \cdot 0 + s_1^3 \cdot 4$
- $u(a_2^2) = s_1^1 \cdot 0 + s_1^2 \cdot 2 + s_1^3 \cdot 3$

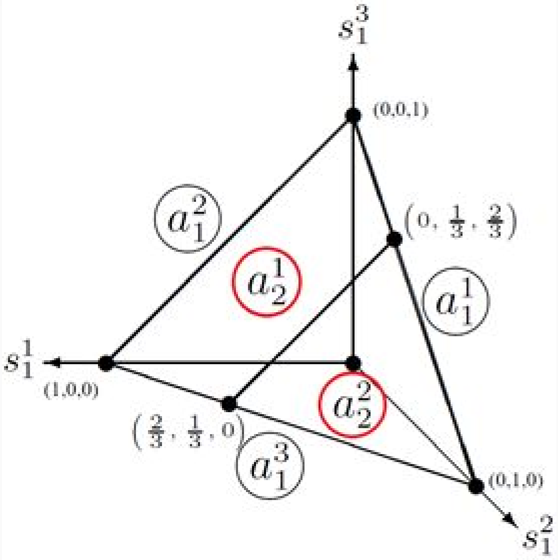


Lemke-Howson Algorithm



For region b, we choose the point (0, 1, 0) which is in between, then we replace p in the formulas with this value:

- 0
- $1 \cdot 2 = 2$



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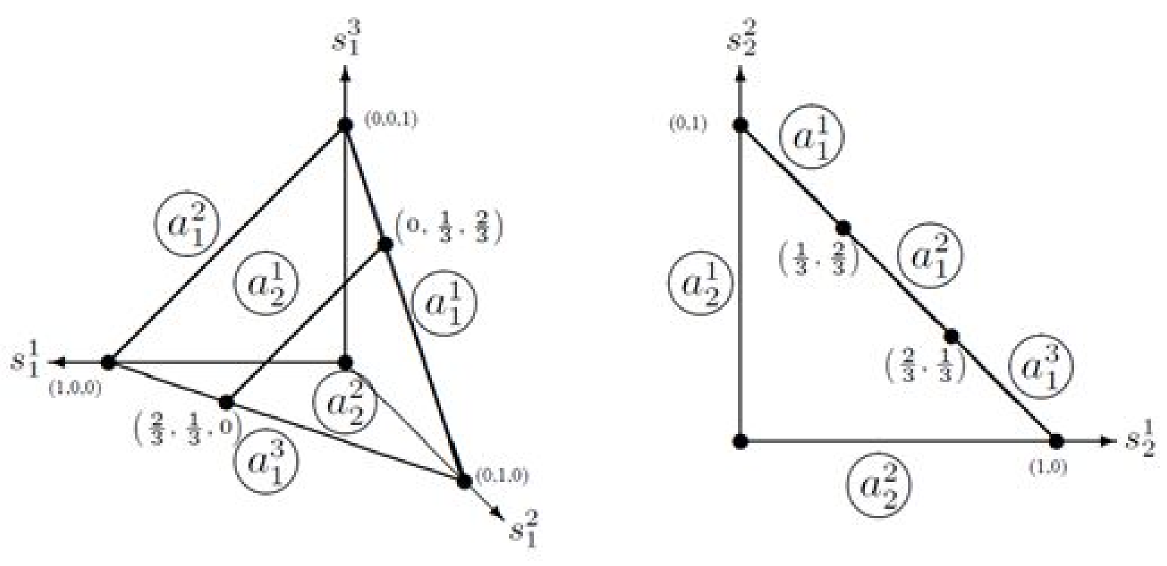

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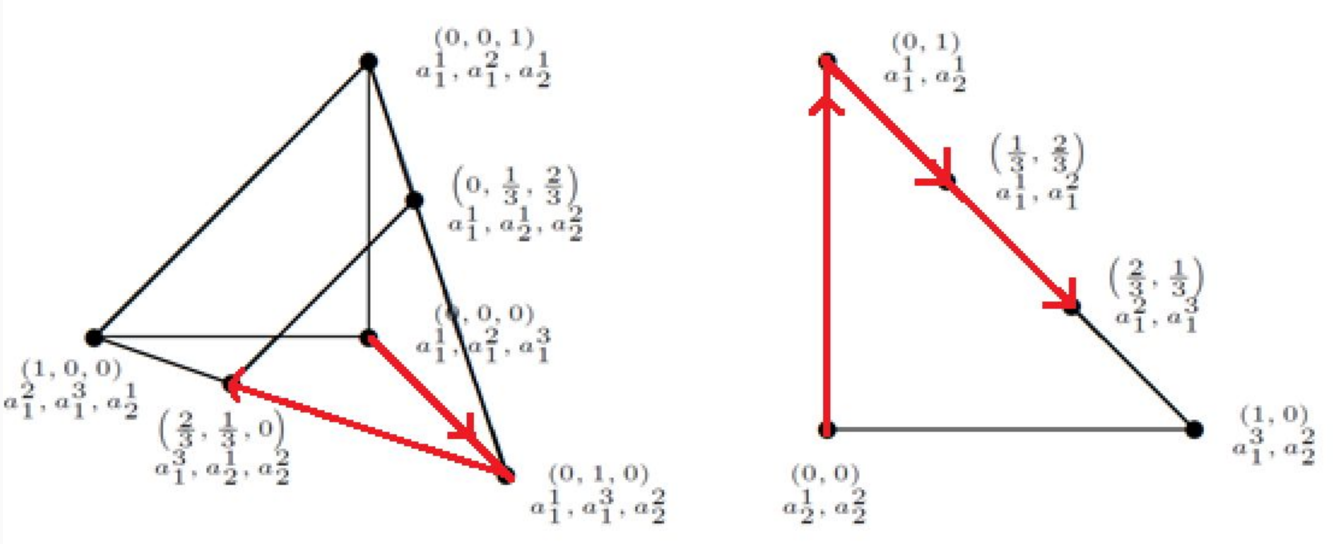

- $((0, 0, 1), (1, 0))$
- $((\frac{2}{3}, \frac{1}{3}, 0), (\frac{1}{3}, \frac{2}{3}))$
- $((\frac{2}{3}, \frac{1}{3}, 0), (\frac{2}{3}, \frac{1}{3}))$