The Performance of PCC-OFDM in Multipath Fading Channels

Kusha R. Panta, Himal A. Suraweera and Jean Armstrong

***Abstract*—Polynomial cancellation coding (PCC) is a very effective technique that reduces the sensitivity of orthogonal frequency division multiplexing (OFDM) to frequency errors at the cost of reduced spectral efficiency. This paper investigates the effect of PCC on the fading characteristics of multipath channels and on the error performance OFDM in a two-path fading channels. Channel coding is employed for forward error correction (FEC). It is shown that PCC improves the error performance of OFDM. Moreover, the overall effect of PCC and the channel coding is taken into consideration while analyzing the spectral efficiency of OFDM.**

***Index Terms*—Orthogonal frequency division multiplexing, fading channels, polynomial cancellation coding, error performance.**

I. Introduction

Orthogonal frequency division multiplexing (OFDM) is a modulation technique used in many new digital data transmission systems such as digital video broadcasting (DVB), digital audio broadcasting (DAB) and wireless local area networks (WLANs). However OFDM is very sensitive to frequency errors [1]. Its performance also suffers from multipath distortions in multipath propagation. Polynomial cancellation coding (PCC) is a very effective technique that reduces the sensitivity of OFDM to frequency errors [2],[3]. In PCC-OFDM, data to be transmitted are mapped onto weighted groups of subcarriers instead of individual subcarriers. In its simplest form each data in PCC-OFDM is mapped onto pairs of weighted subcarriers. We will be only considering PCC-OFDM of the simplest form in this paper and the weighted subcarrier pairs will be called the subchannels.

In addition to providing an effective way of reducing the sensitivity of OFDM to frequency errors, PCC also provides other advantages. It has been shown that multipath propagation causes much less intersymbol interference (ISI) and intercarrier interference (ICI) in PCC-OFDM [2]. As a result a cyclic prefix is not required. In this paper, we will investigate the performance of PCC-OFDM in multipath fading channels. The error performance of PCC-OFDM is compared with that of the OFDM for a two-path fading channel. Channel coding for forward error correction (FEC) is considered and the combined effect of channel coding and PCC on the spectral efficiency of PCC-OFDM is presented.

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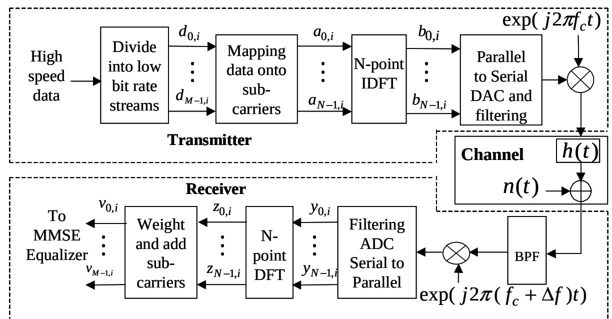

Fig. 1. PCC-OFDM symbol in a two-path channel

II. PCC-OFDM Communication System

Fig. 1 shows a simple block diagram of a PCC-OFDM communication system. High speed serial complex data are converted into M lower speed parallel data substreams using a serial to parallel (S/P) converter. The S/P converter takes M data symbols from the serial input stream and outputs them onto M substreams at each symbol period, T. Data $d_{0,i}, d_{1,i}, \ldots d_{M-1,i}$ represent the ith data block to be transmitted and are mapped onto values $\mathbf{A}_i = [a_{0,i}, a_{1,i}, \ldots a_{N-1,i}]$ that modulate N subcarriers of inverse discrete Fourier transform (IDFT) in ith symbol period. In OFDM, $M = N$ and data are mapped onto individual subcarriers on a one-to-one basis. In this case, each $d_{k,i}$ is mapped onto a weighted pairs of subcarriers so that $a_{0,i} = d_{0,i}$, $a_{1,i} = -d_{0,i}$, $a_{2,i} = d_{1,i}$, $a_{3,i} = -d_{1,i}$, $\ldots$, $a_{N-1,i} = -d_{N/2-1,i}$ The corresponding weighting coefficients are $+1$, -1 and $M = N/2$.

The time domain output samples of the IDFT are then converted parallel to serial, digital to analog before being filtered. The filtered baseband signal is then modulated onto a high frequency carrier and transmitted. The channel distorts the signal and adds Gaussian noise $n(t)$. The channel impulse response is given by

$$h(t) = \sum_i \alpha_i \delta(t - \tau_i) \tag{1}$$

where α_i and τ_i represent the signal attenuation and the channel delay on the signal received on path i.

Compared to the OFDM receiver, there is an extra “weighting and adding” block at the output of the DFT block at the PCC-OFDM receiver. This can be shown to be equivalent to matched filtering of the signal. The function of all other blocks is the same for OFDM. In PCC-OFDM, however, data $\mathbf{Z}_i = [z_{0,i}, z_{1,i}, \ldots z_{N-1,i}]$ at the output of the DFT are then weighted and added to generate the estimates of $d_{0,i}, d_{1,i} \ldots d_{M-1,i}$ The outputs at the "weighting and adding" block are given by $v_{0,i}, v_{1,i} \ldots v_{M-1,i}$ and

$$v_{i,j} = \frac{(z_{i,2j} - z_{i,2j+1})}{2} \quad \text{for } j = 0, 1, \ldots, N/2 - 1 \quad (2)$$

The outputs of the "weighting and adding" block are then fed into a single tap equalizer. In this paper a minimum mean square error (MMSE) equalizer is used.

III. Performance of PCC-OFDM in Multipath Fading

In a multipath channel, several echoes of the transmitted signal are received. Each echo is subject to a unique delay and Doppler shift. One effect of multipath propagation is to introduce ICI and ISI in the received signals. Another effect is to cause frequency selective fading. Previous results have shown that PCC also increases the performance of OFDM in the presence of multipath distortions [2],[4]. The improved performance of PCC-OFDM in multipath fading channels is due in part to the energy distribution within the transmitted symbol. In PCC-OFDM symbols most of the symbol energy is concentrated in the middle of the symbol due to the windowing effect of PCC in the time domain [3]. Since the symbol energy at either end of the symbol is significantly less than that in OFDM, there will be reduced power in the ISI and the ICI that would be caused by the delay spread. The weighting and adding in the receiver result in a further windowing effect in the receiver which further reduces the effect of ISI. In addition, the fading characteristics experienced by the PCC-OFDM will be different than those experienced by normal OFDM.

IV. Characteristics of Frequency Selective Fading in PCC-OFDM

The actual characteristics of frequency selective fading and its effect on the performance of PCC-OFDM depend on many factors. One factor is the increased subchannel spacing. For the same N, the subchannels in PCC-OFDM will be twice as wide as those in OFDM. However a number of properties of PCC-OFDM may allow N to be reduced in practical implementations. In OFDM, a cyclic prefix is required. To limit the loss in spectral efficiency this causes, it is desirable to have N as large as possible. As no cyclic prefix is required in PCC-OFDM this constraint is removed. Another reason for choosing large N in OFDM is to improve the spectral roll-off. As the spectral roll-off of PCC-OFDM is much more rapid, this constraint is also removed. Thus, the subchannel bandwidth in PCC-OFDM will be at least twice as large as in OFDM. This means for a given fade in the channel, fewer subchannels will be affected compared to OFDM. The depth of the fade experienced by individual subchannels will also be less because the channel will be averaged over the larger subchannel width. The symbols shape of PCC-OFDM also reduces the depth of the nulls which multipath fading causes. Figs. 2-3 show a block diagram of a PCC-OFDM symbol and an OFDM symbol in a two-path channel together with its replica in the delayed path. The shape of the symbol was chosen to highlight the energy distribution of the symbol in both cases. In PCC-OFDM, even an equal amount of signal energy in both signal paths, will not produce a complete notch in the spectrum, as the energy distribution within the symbol is not symmetrical like OFDM. However, a direct path signal and a delayed signal with equal signal power may produce a complete notch in OFDM. The overall effect of the larger subchannel spacing and the symbol energy distribution is to make the effect of frequency selective fading in PCC-OFDM less severe than in OFDM.

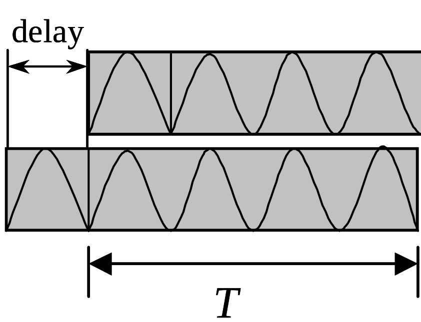

Fig. 2. OFDM symbol in a two-path channel

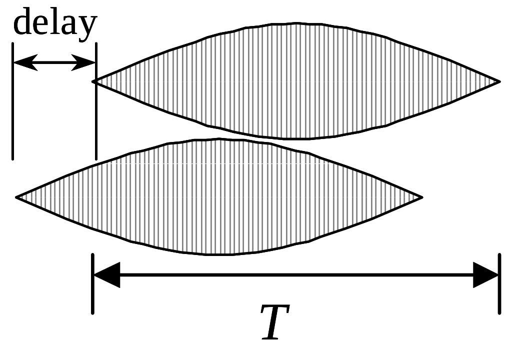

Fig. 3. PCC-OFDM symbol in a two-path channel

In PCC-OFDM, frequency selective fading still remains a problem, though not as severe as in OFDM. The fading will corrupt data transmission on *bad* subchannels. The use of channel coding for FEC is needed to improve the error performance PCC-OFDM system.

V. Error Performance of PCC-OFDM

This subsection provides bit error rate (BER) performance of PCC-OFDM with square M-QAM with Gray bit mapping. Let

$$H[k] = H_Q[k] + jH_I[k] = r_j e^{j\psi} \quad (3)$$

where $H[k]$ is the discrete frequency domain coefficients of the channel impulse response. The receiver "averages" the N DFT outputs to extract the sent $N/2$ v_j data per block. The fact that PCC-OFDM does not use a CP will introduce ISI for the current evaluated block and by incorporating its effects on the noise term we can write,

$$\begin{aligned} v_j &= \frac{\{r_{2j}e^{j\psi}|a_{2j}|e^{j\theta} - r_{2j+1}e^{j\psi}|a_{2j+1}|e^{j\theta+\pi}\}}{2} \\ &+ \frac{\{n_{2j} - n_{2j+1}\}}{2} \end{aligned} \quad (4)$$

n_{2j} is the complex frequency domain sample value of AWGN affecting the $2j$th subcarrier with its real and imaginary component variance equal to σ_n^2. For 16-QAM $|a_{2j}| \in 1, \sqrt{10}$ and $\sqrt{18}$. (4) can be simplified further by assuming that the adjacent channel phase values are likely to be equal. Hence,

$$v_j = \frac{|a_{2j}|e^{j\theta}e^{j\psi}\,(r_{2j} + r_{2j+1})}{2} + \frac{(n_{2j} - n_{2j+1})}{2} \quad (5)$$

Clearly it can be seen from (5) that the channel coefficients are averaged for PCC-OFDM and this guarantees better system performance even in channels with *deep nulls*. Defining a new random variable λ as $(r_{2j} + r_{2j+1})$ we note that λ is also Rayleigh distributed however with a mean square value Ω_r higher than the same for either r_j or r_{j+1}. With this in mind we simplify (5) as,

$$\begin{aligned} v_j &= \lambda_j\,|a_{2j}|\cos(\theta+\phi) + \lambda_j\,|a_{2j+1}|\sin(\theta+\phi) \quad (6)\\ &+ N_Q + jN_I \\ &= \hat{V_Q} + j\hat{V_I} \end{aligned}$$

The conditional BER is $P_b(E|\lambda,\psi)$ and the overall conditional BER for 16-QAM can be written as,

$$P_b(E) = \frac{1}{16}\sum P_b(E|\lambda,\psi) \quad (7)$$

Simillar to the approach described in [6] each term of the above summation is in the form of,

$$\begin{aligned} \text{Pr}[\pm N_{I|Q} &> \alpha.\lambda\text{cos}(\theta+\psi)+\beta] \quad (8)\\ &= Q\left(\frac{\alpha.\lambda\text{cos}(\theta+\psi)+\beta}{\sigma_n^2}\right) \end{aligned}$$

or

$$\begin{aligned} \text{Pr}[\pm N_{I|Q} &> \alpha.\lambda\text{sin}(\theta+\psi)+\beta] \quad (9)\\ &= Q\left(\frac{\alpha.\lambda\text{sin}(\theta+\psi)+\beta}{\sigma_n^2}\right) \end{aligned}$$

Where $Q(x) = 1/\sqrt{2\pi}\int_x^\infty e^{-t^2/2}\,dt$. The coefficients of α and β depend upon the signal constellation used for modulation. More explicitly *each* symbol of the constellation have different values for α and β. P_b(E) is averaged over the probability density function $p(\lambda,\psi)$ to finally obtain BER as,

$$\text{P}_b(E) = \int_0^\infty\int_{-\pi}^{\pi} P_b(E|\lambda,\psi).p(\lambda,\psi)\,d\psi d\lambda \quad (10)$$

Expression BER given by (10) is a function of the amplitudes and the phase of the fading channel coefficients. Hence, the exact calculation of BER using (10) depends on the actual fading profile of the channel being available.

VI. Spectral Efficiency of PCC-OFDM

Mapping of data onto pairs of subcarriers in PCC-OFDM will reduce the spectral efficiency by half in comparison to that of OFDM. However some of the properties of PCC-OFDM allow some compensating gains in spectral efficiency [4]. The cyclic prefix can be eliminated. The faster spectral roll-off means that smaller frequency guard bands are required. The reduced sensitivity to frequency offset, phase noise and multipath transmission increase the effective SNR in a practical system. Other significant second order effects also increase the effective SNR. For example the reduction in ICI allows more accurate channel and frequency estimation. In this section it will be shown that the loss in the spectral efficiency is not that significant when the use of channel coding is considered for the FEC in PCC-OFDM. Moreover, PCC-OFDM still performs reasonably well in channels with large delay spreads whereas the OFDM will fail when the delay spreads are higher than its cyclic prefix.

VII. Simulation Results

In this section simulation results are presented for error performance of OFDM and PCC-OFDM in fading channels. Fig. 4-5 show the plot of symbol error rates (SER) for coded and uncoded PCC-OFDM and OFDM in a two-path fading channel. The channel is,

$$h(t) = 1 + \sqrt{\frac{3}{4}}\delta\,(t-\tau)$$

The channel coding is employed by applying Reed-Solomon codes *RS* (15,11) that outputs a 15 four bit code block for every 11 four bit message block. Simulations are performed for 16-QAM symbols using $N = 128$ and $M = 64$. The number of subcarriers can be of any value. A cyclic prefix of $30(T/128)$ is considered in OFDM. This particular length of the CP was chosen to enable OFDM to combat ISI/ICI that would otherwise be introduced by delay spreads of length up to almost a quarter of the symbol duration in the two-path channel. Transmitted signal energy per bit E_b is normalized to account for the energy used by the cyclic prefix in OFDM and by the mapping of data symbols onto subcarrier pairs in PCC-OFDM. The transmitted energy is normalized for the energy used by the extra bits generated for FEC as well as for the energy of echo path signal in the two-path channel.

The Fig. 4 presents simulation results for the error performance of PCC-OFDM and OFDM when the CP equals the delay spread of the channel. It shows that the SER of uncoded OFDM is almost flat against increasing E_b/N_0. However, the effect of the delay spread is not so severe in PCC-OFDM. When both systems are coded with *RS* (15,11), the coding gain of PCC-OFDM is about 2.0dB for the SER of 10^{-3}. However the gain will be higher when the delay spread is either longer or the shorter than the length of the CP.

Fig. 5 presents simulation results for the error performance of PCC-OFDM and OFDM when the delay spread is $35(T/128)$ $(>$ CP) and $20(T/128)$ $(<$ CP). When the delay spread is longer than the CP, the SERs of OFDM decreases very slowly against increasing E_b/N_0. The ISI and ICI introduced by the length of the delay spread that is not covered by the CP part is high. However PCC-OFDM is much less sensitive to delay spreads. The coding gain of PCC is also higher when the delay spread is shorter than the CP. Fig. 5 also shows that at the SER of 10^{-3} the coding gain of PCC is 3.0dB. This is because the error performance of OFDM mainly depends on the CP length. Its performance in channels with delay smaller than the CP will be very similar. However, in channels with delay longer than the CP, its performance collapses. Therefore, OFDM can not take advantage of the shorter channel delay and contributes to excessive loss in spectral efficiency than otherwise be required for shorter delay spreads in channels. In the case PCC-OFDM, the error performance depends on the delay spreads and the effect of the delay spread is not as severe as in OFDM.

Figs. 4-5 show that PCC provides a significant coding gain. Previously it has been perceived that its spectral efficiency is too low as each data symbol is transmitted on pairs of subcarriers. However if we consider the removal of the CP in OFDM due to PCC, the coding gain in multipath fading and faster roll-off of out-of-band spectral power, the resultant spectral efficiency of PCC-OFDM will be comparable to that of the OFDM. Moreover, PCC also makes OFDM more to tolerance to frequency errors.

VIII. Conclusions

In this paper, the effect of PCC on the fading characteristics of multipath channels has been discussed. It is highlighted that PCC will have some subtle effects on the fading characteristics of multipath channels and as a result, the fading experienced by PCC-OFDM might not be as severe as by OFDM. Moreover the simulation results for the error performance of PCC-OFDM has been presented in a frequency selective fading channels and it has been shown that PCC-OFDM outperforms OFDM in terms of error performance. Simulation results show that the error performance of PCC-OFDM is less sensitive to large delay spreads. Reed-Solomon codes have been applied for FEC and more it has been shown that the spectral efficiency of PCC-OFDM is comparable with that of OFDM when the effect of PCC is combined with that of the channel coding and the cyclic prefix.

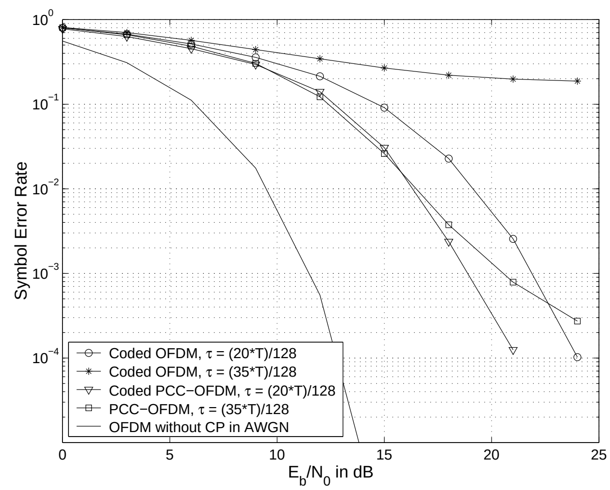

Fig. 5. PCC-OFDM symbol in a two-path channel when CP $\neq$ spread

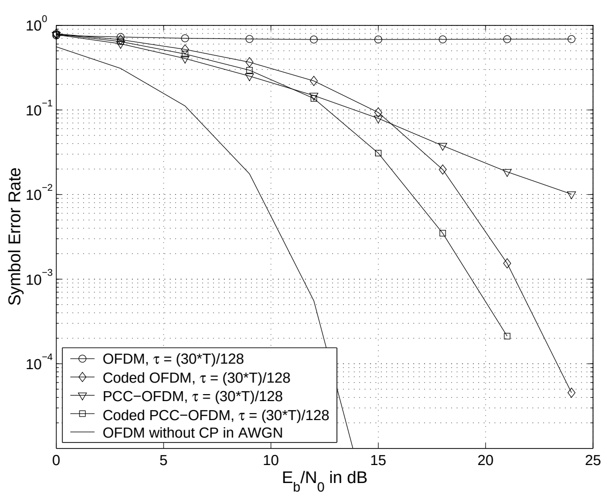

Fig. 4. PCC-OFDM symbol in a two-path channel when CP = delay spread.

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