

UK Thermoelectric Network Meeting, February 2018, Edinburgh

# Quantum transport simulations for nanostructured thermoelectric materials

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Laura Oliveira<sup>1</sup>, Mischa Thesberg<sup>2</sup>, Hossein Karamitaheri<sup>3</sup>

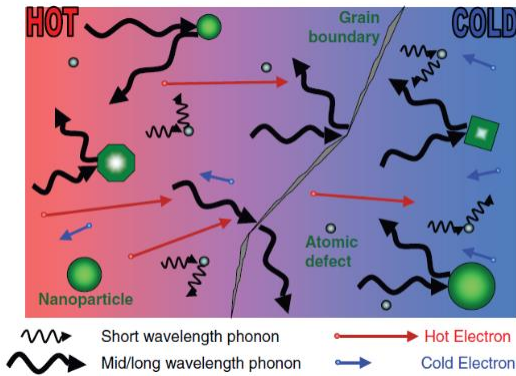
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<sup>2</sup>Institute for Microelectronics, TU Vienna, Austria

<sup>3</sup>Department of Electrical Engineering, University of Kashan, Iran

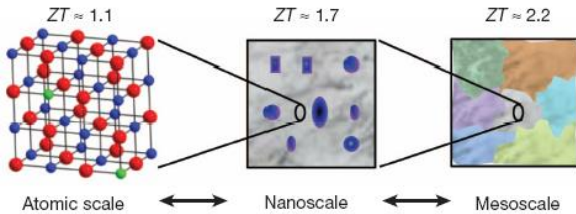


# Nanostructuring lowers $\kappa$ (significantly)

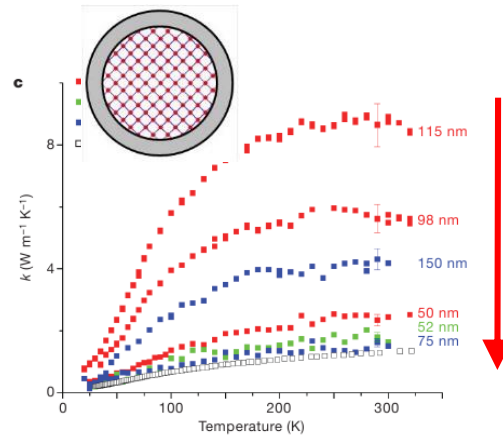


Vineis, *Adv. Mat.*, 2010

- $\blacktriangleright$  Nanostructuring
- $\blacktriangleright$  phonon engineering

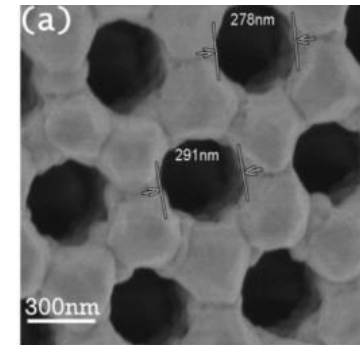


Biswas et al, *Nature*, 2012.  
(p-type PbTe)



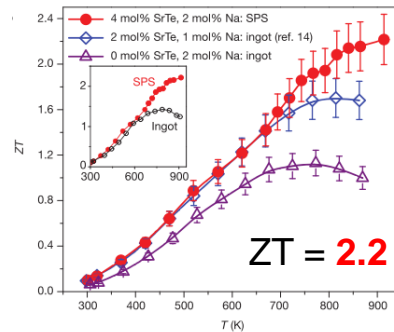
Hochbaum, *Nature*, 2008

$$\kappa = 1\text{-}2 \text{ W K}^{-1}\text{m}^{-1}$$



Perez et al., *Scientific Reports* 6, (2016): 32778.

$\kappa = 0.55 \text{ W K}^{-1}\text{m}^{-1}$   
 ~30 nm SiGe nanopore film

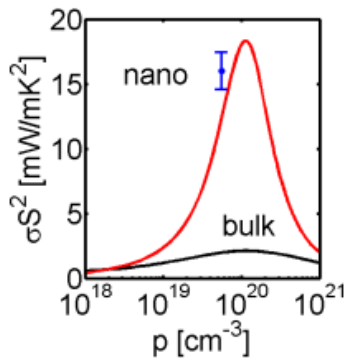
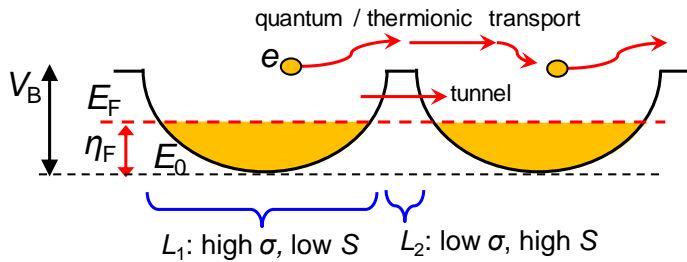


$$ZT = \frac{\sigma S^2 T}{k_e + k_l}$$

(Annotations:  $\sigma S^2 T$  is circled in blue with a downward arrow;  $k_l$  is circled in red with two downward arrows.)

# How about the power factor ?

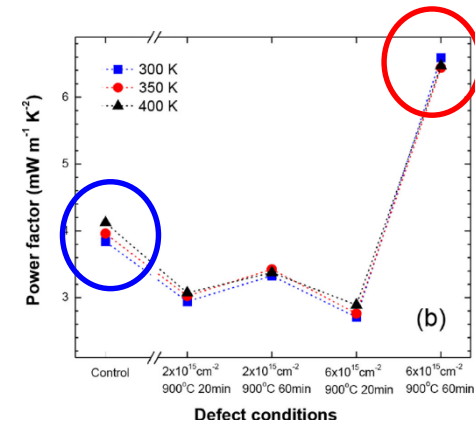
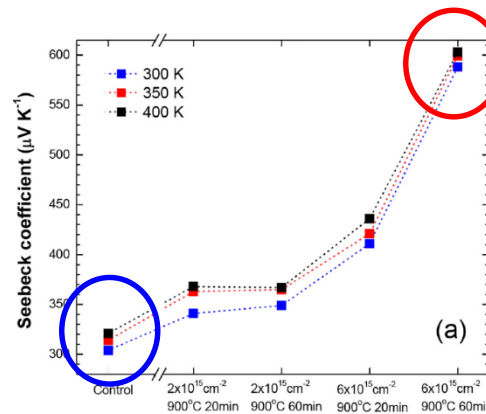
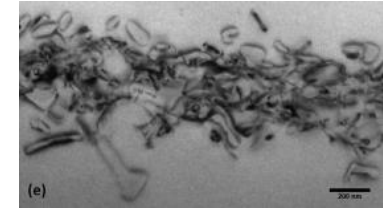
with  
Dario Narducci  
(Milano-Bicocca)



$\sigma \uparrow$     $S \uparrow$   
Large  $\sigma$  and  $S$

Very high PF:  
poly-c materials: **15 mW/K²m⁻¹**  
(~5x compared to bulk p-type Si)

with  
Nick Bennett  
(Heriot Watt, UK)



$S \uparrow$

$PF \uparrow$

High PF:  
Dislocations in Si: **6.5 mW/K²m⁻¹**  
(~70% higher than bulk n-type Si)

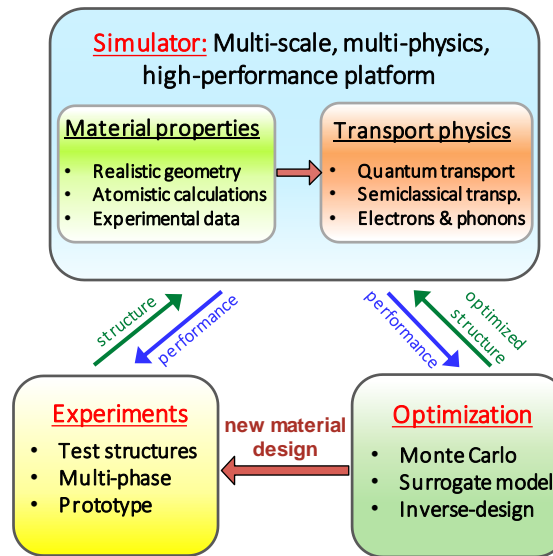
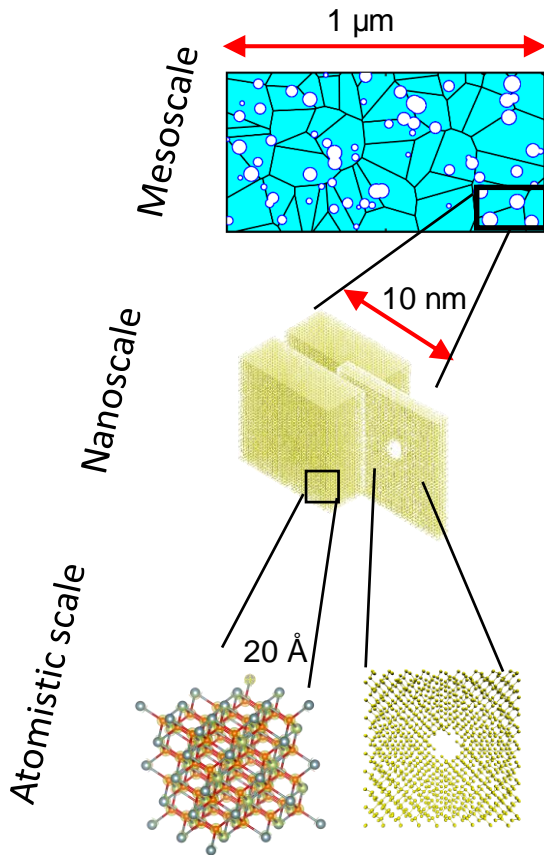
# Goals of this talk

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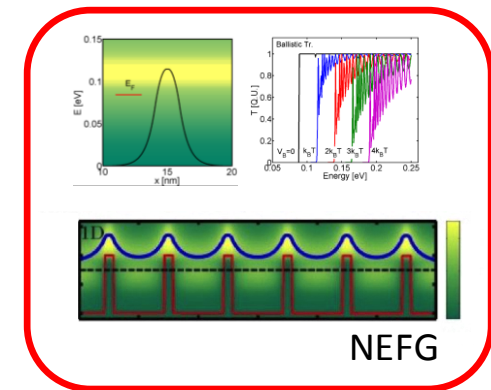
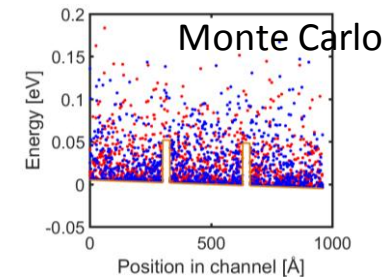
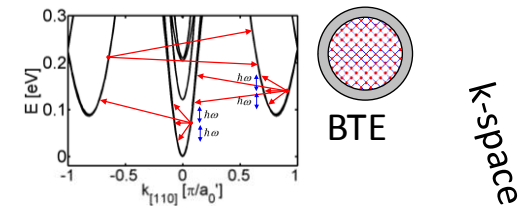
- *The Non-Equilibrium Green's Function (NEGF):  
Studies on superlattices and nanocomposites*
- *Expansion to multi-physics, multi-scale capabilities*

# Overview of nanostructure simulation activities

## MULTI-SCALE GEOMETRIES (atomistic to continuum)



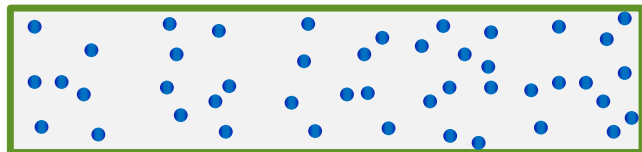
## MULTI-PHYSICS TRANSPORT (semiclassical to quantum)



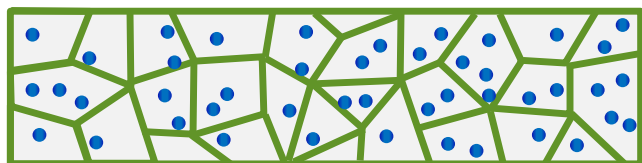
# Features for PF improvement



Polycrystalline (PC)

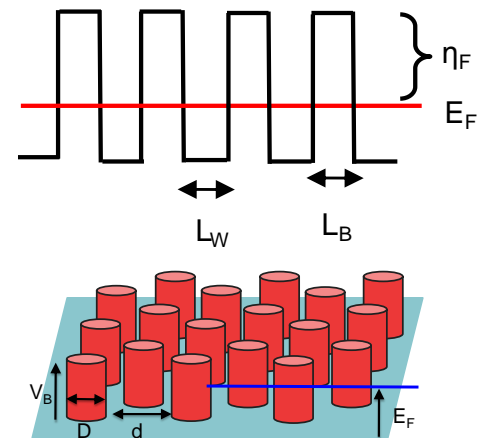


Nano-inclusions (NI)



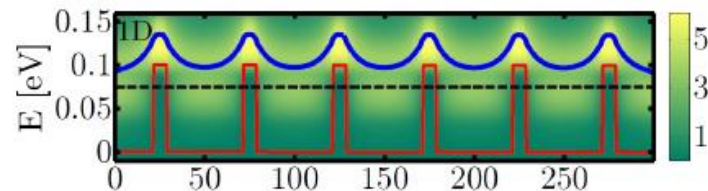
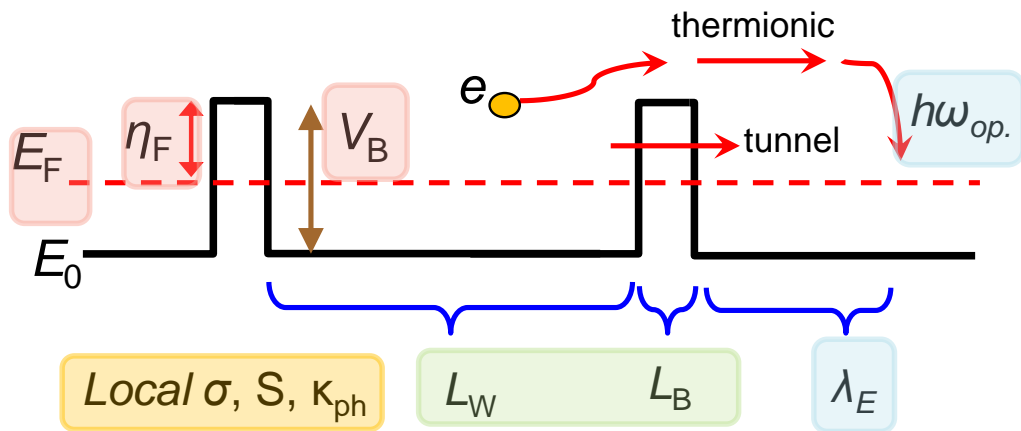
PC+NI

Feature sizes:  
a few – 10s – 100s of nanometers



Barriers:  $S \sim \eta_F$   $\uparrow$   $\sigma \sim \exp(-\eta_F)$   $\downarrow$

Wells:  $S \sim E_F$   $\downarrow$   $\sigma \sim \exp(-E_F)$   $\uparrow$

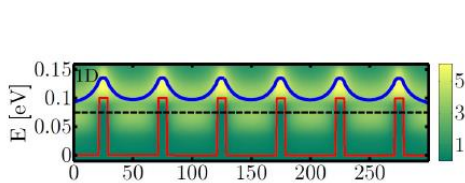
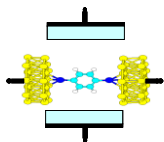
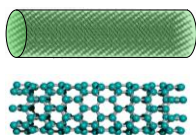
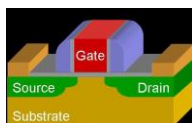
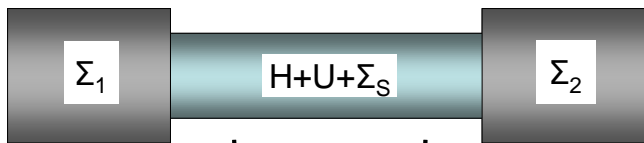


# Goals of this talk

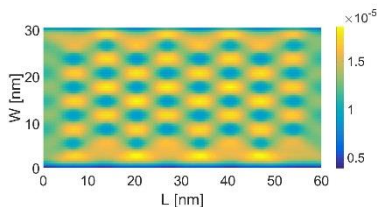
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- *The Non-Equilibrium Green's Function (NEGF):  
Studies on superlattices and nanocomposites*
- *Expansion to multi-physics, multi-scale capabilities*

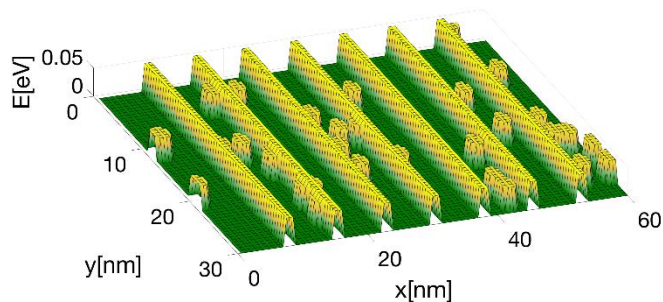
# Non-Equilibrium Green's Function (NEGF)



superlattices

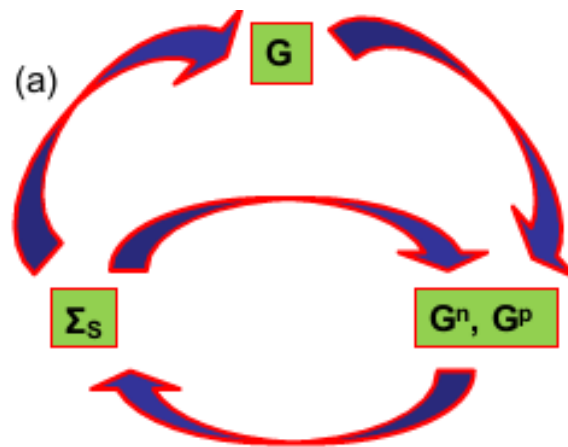


nano-inclusions

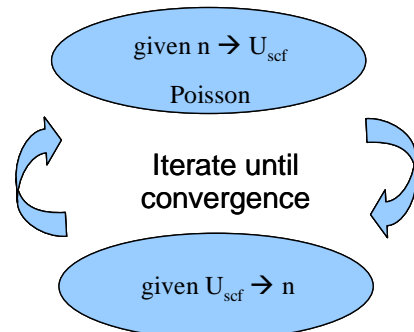


SLs + nano-inclusions

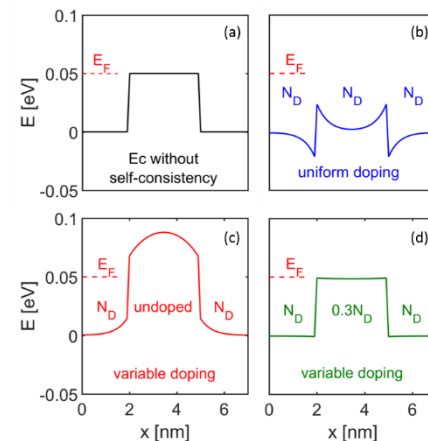
– Self-consistency



## ELECTROSTATICS

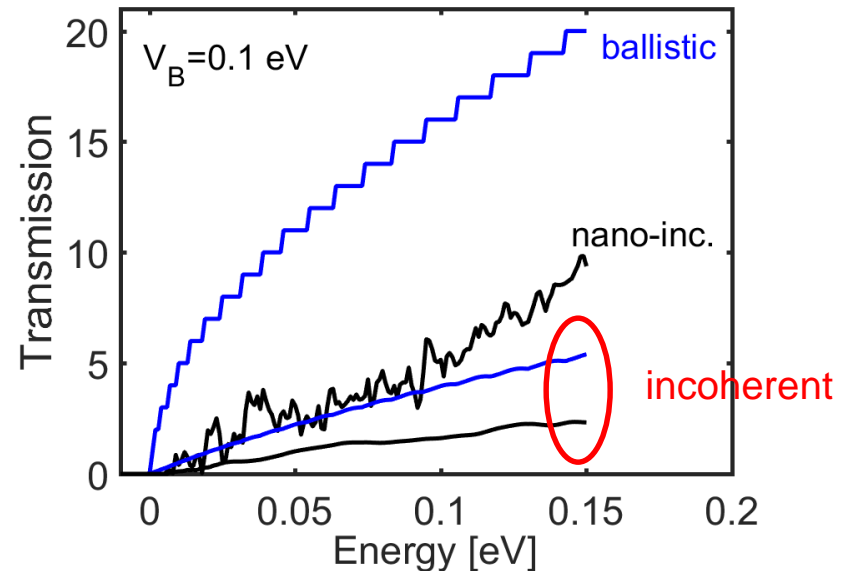
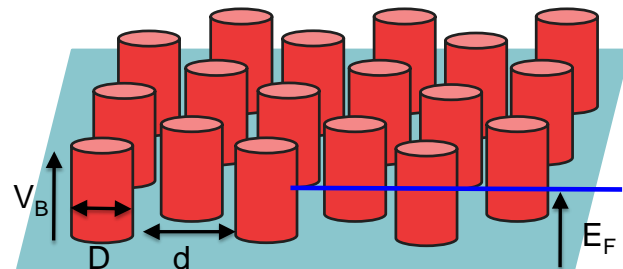


## TRANSPORT (NEGF)





# NEGF basic results: coherent vs incoherent



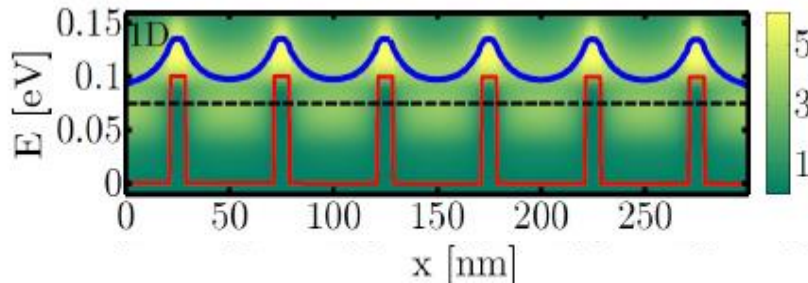
## *Coherent transport:*

- Usually NOT appropriate – can lead to localization (at room T)
- PF is limited by the  $G$  of the barrier region – worst case path

## *Incoherent transport:*

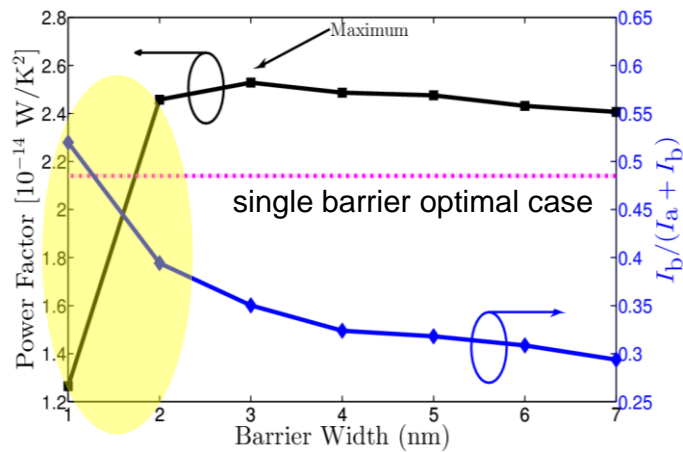
- Smoothened resonances
- The different regions can be decoupled

# Example 1: Superlattice - all features captured

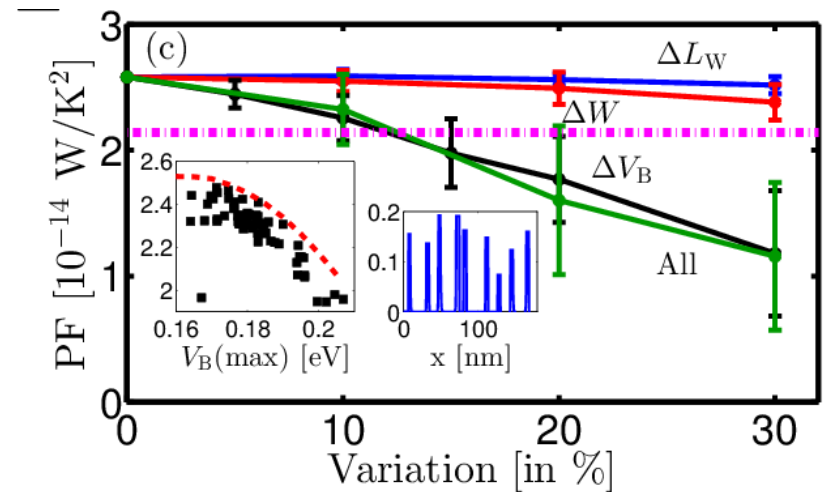


$$S = -\frac{k_B}{q} \left\langle \frac{E - E_F}{k_B T} \right\rangle = -\frac{\langle E - E_F \rangle}{qT}$$

current flow variations and  $\lambda_E$

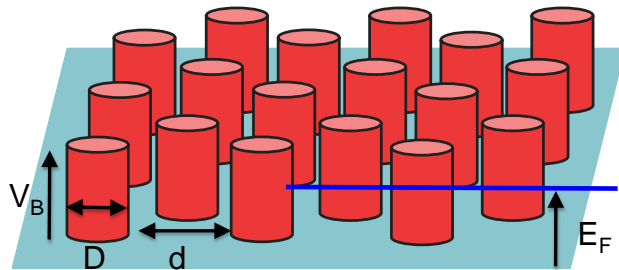


Tunneling is detrimental to PF



Variation in  $V_B$  reduces PF  
(could explain why filtering is not yet successful?)

# Example 2: nano-inclusions



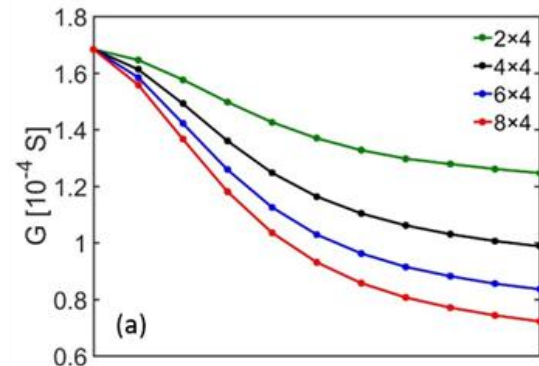
At small  $V_B$ :

PF is independent of density

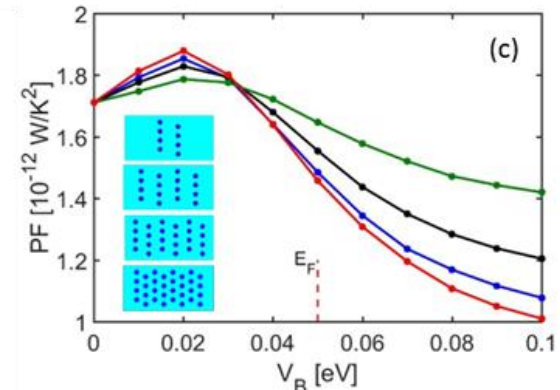
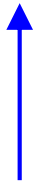
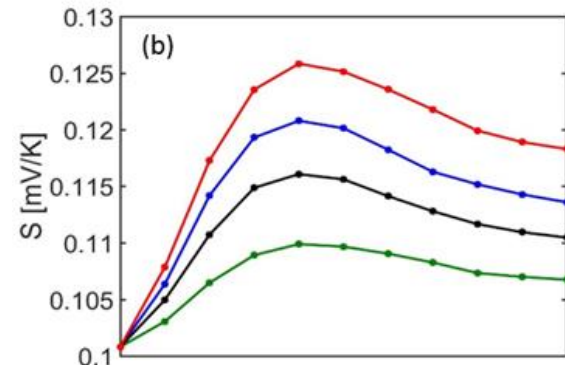
Geometry and randomness as well ?

At larger  $V_B$ :

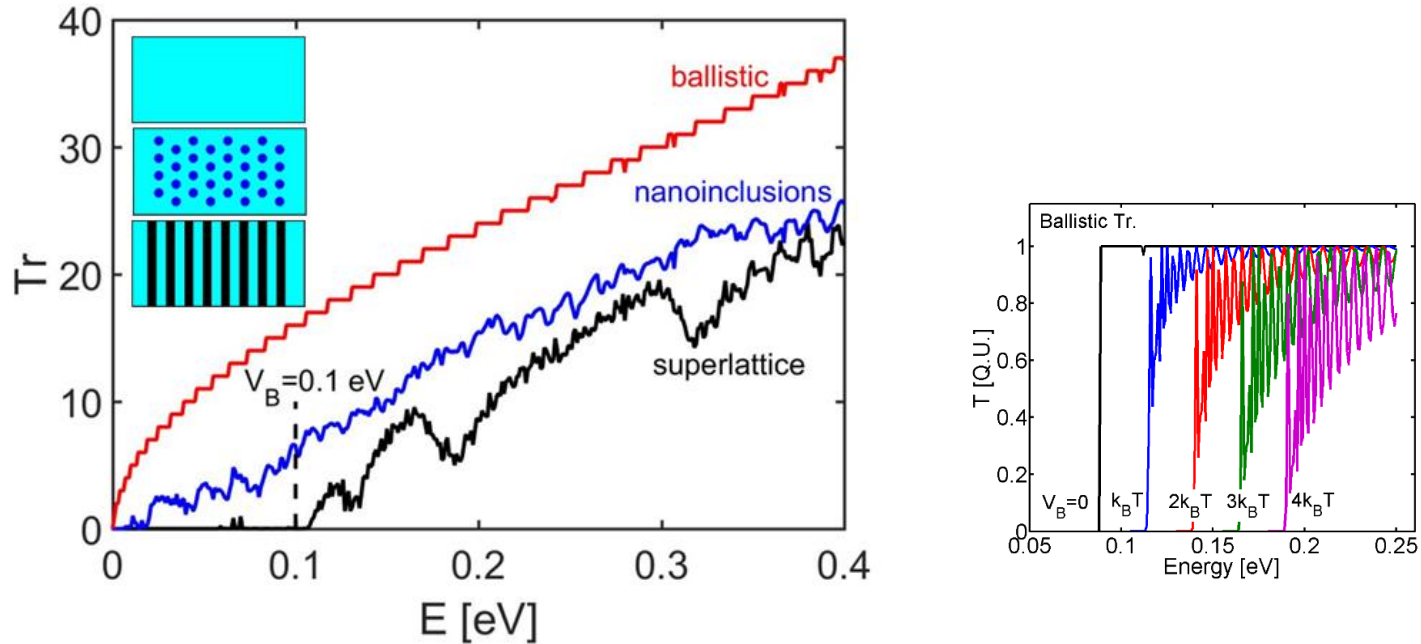
Density degrades the PF



porosity  
incr.

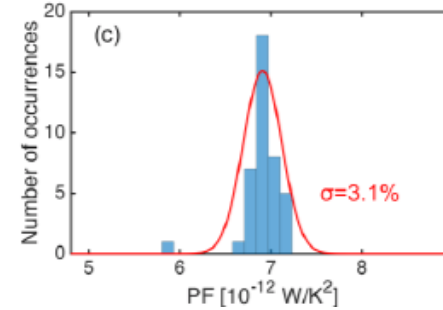
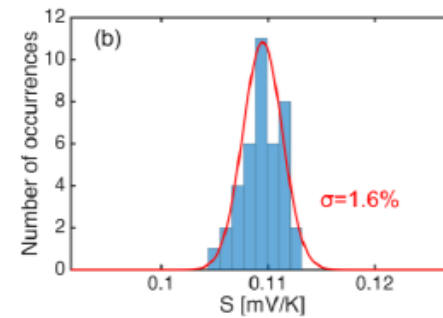
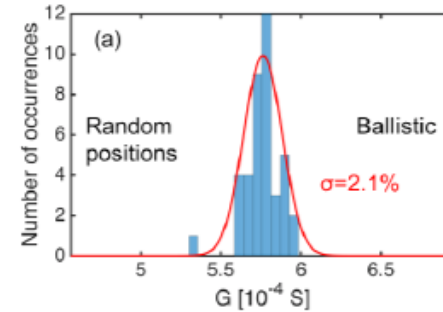
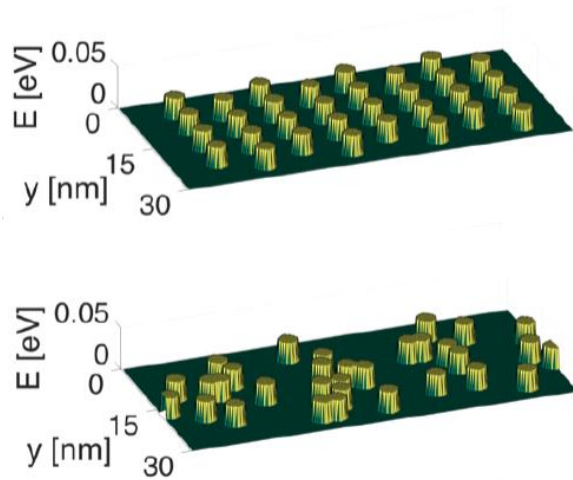


# Filtering using Nanoinclusions vs Superlattices



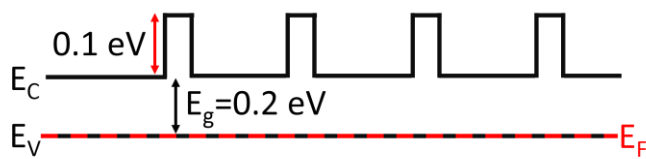
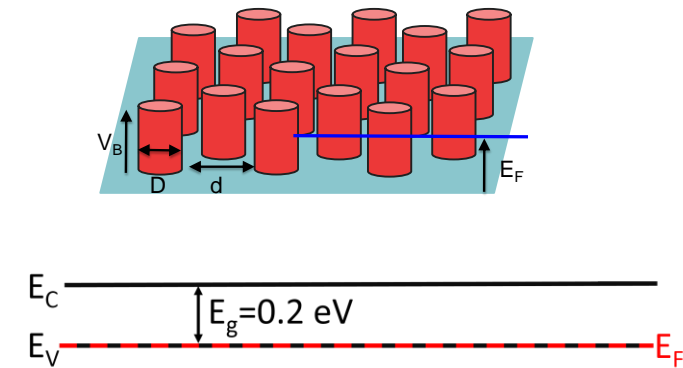
- Superlattices offer filtering capabilities
- *Nanoinclusions offer weak energy filtering capabilities (charge goes around them)*
- The transmission never recovers to the pristine value
- **A simple step function transmission is not adequate**

# Example 3: variability in complex geometries



Effect of geometry variations

# Example 4: Suppressing bipolar effects

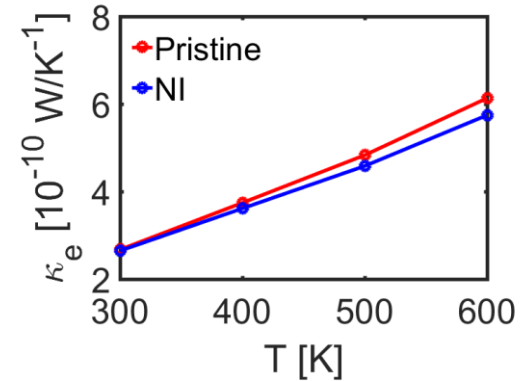
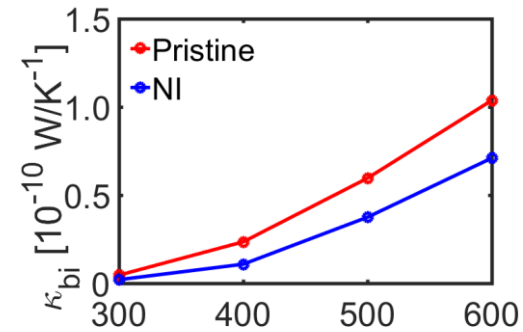
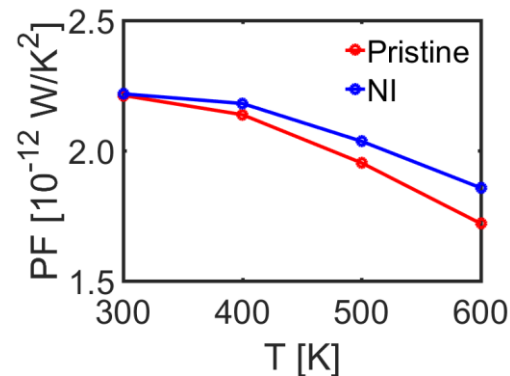
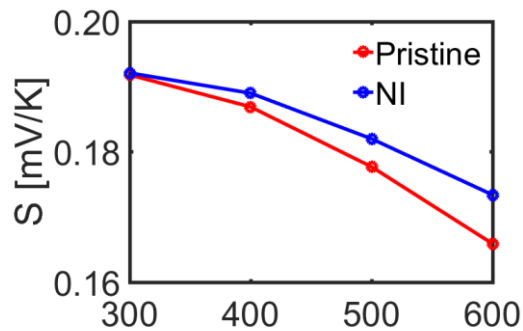
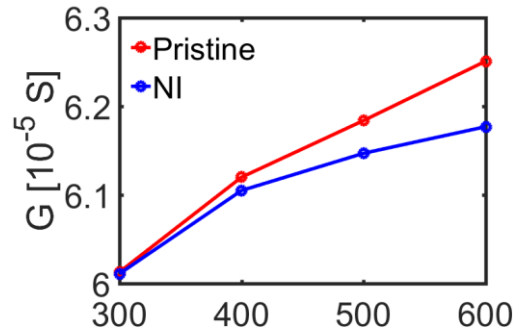


$$ZT = \frac{\sigma S^2 T}{\kappa_e + \kappa_l}$$

$$S = \frac{(G_e S_e + G_h S_h)}{(G_e + G_h)}$$

$$\kappa_{bi} = \frac{G_e G_h}{G_e + G_h} (S_e - S_h)^2 T$$

$$\kappa_{elec} = \kappa_{elec,e} + \kappa_{elec,h} + \kappa_{bi}$$



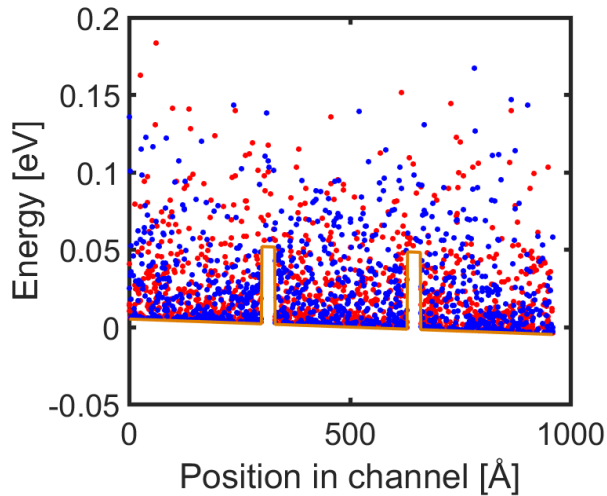
*How should the barriers need to be designed for bipolar suppression?*

# Goals of this talk

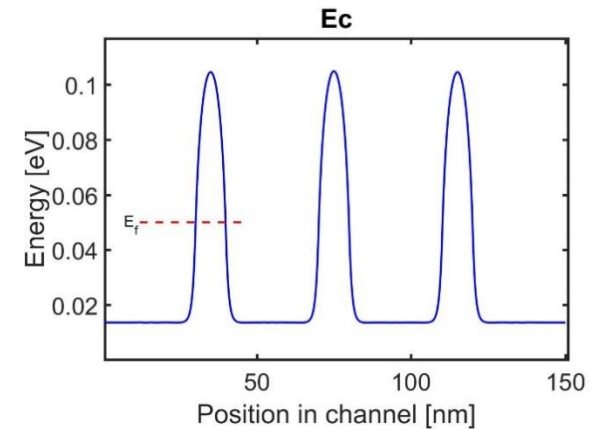
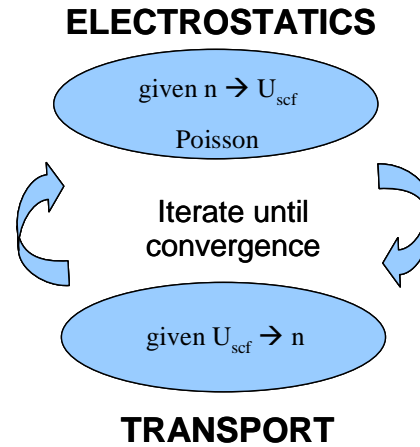
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Studies on superlattices and nanocomposites*
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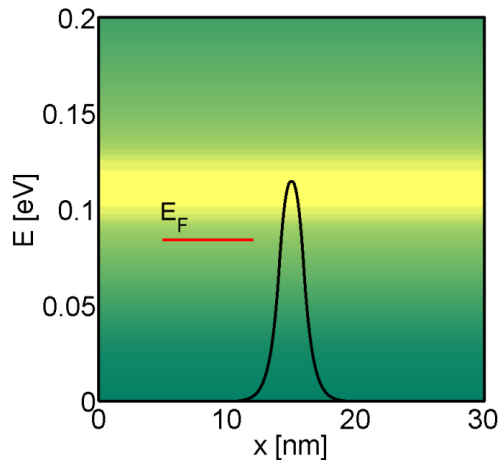
# Infrastructure development – multi-physics approach



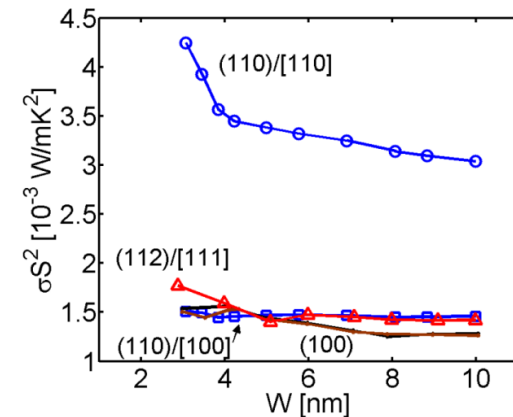
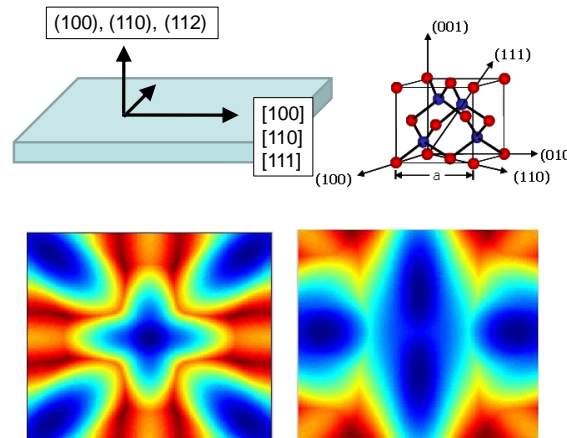
*Electronic Monte Carlo*



*Self-consistent electrostatics*



*Quantum tunneling*

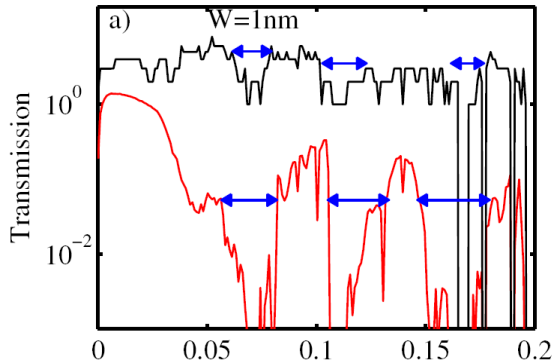
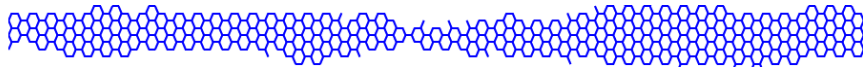


*Bandstructure effects*



# Phonon transport treatment in nanocomposites

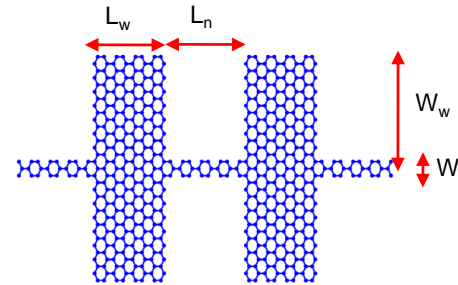
## Phonon NEGF – coherent effects:



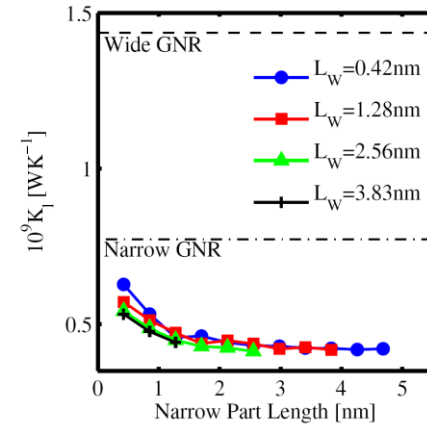
'effective transmission gaps formed'

Karamitaheri et al, *Phys. Rev. B*, 91, 165410, 2015

## Going below the Casimir limit:

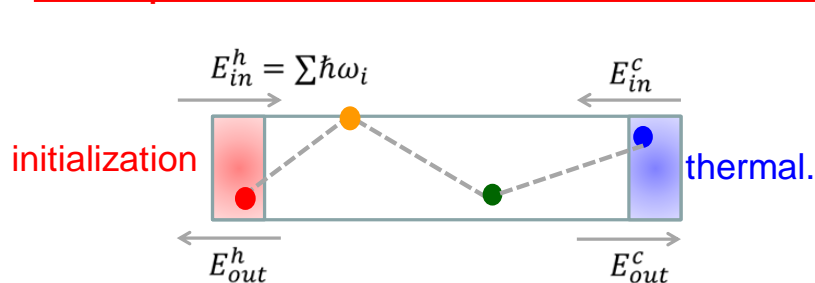


Corrugated structures



Karamitaheri et al, *J. Appl. Phys.* 119, 244302, 2016

## Nanoporous Si: Monte Carlo calculations:



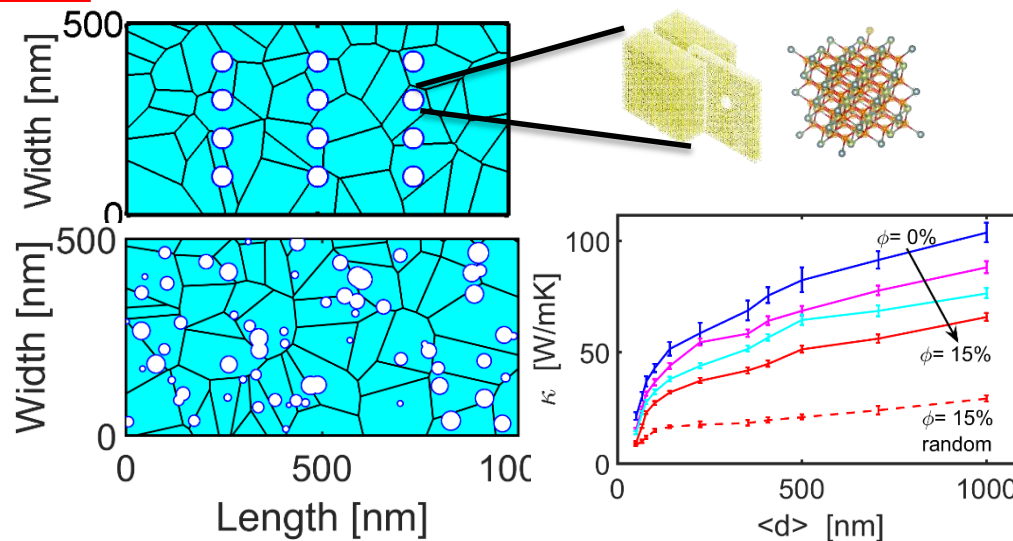
Phonon flux

$$\phi = N_{tot} \frac{(E_{in}^h - E_{out}^h) - (E_{in}^c - E_{out}^c)}{N < t_{TOF} >}$$

Fourier's law

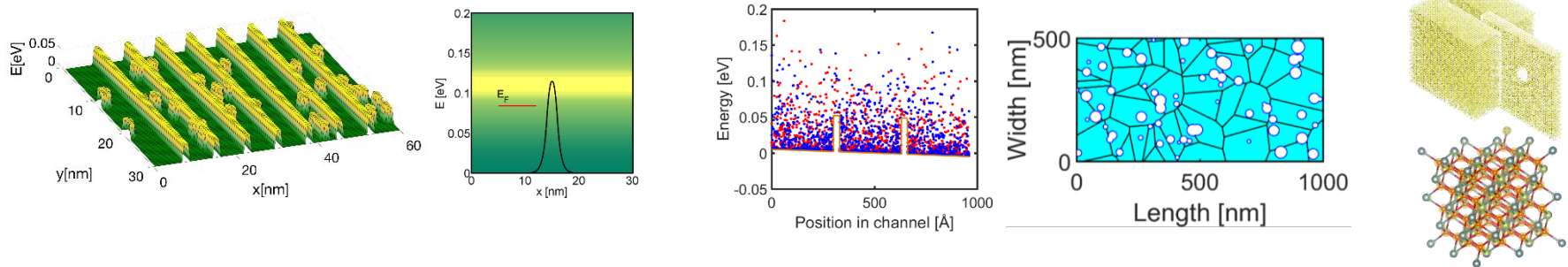
$$\kappa = -\phi / \nabla T$$

Wolf et al, *J. Appl. Phys.*, 115, 204306, 2014



# Conclusions

- NEGF transport in nanocomposites
- Importance of multi-physics, multi-scale needs for TE transport



## Acknowledgements:

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(Madrid), Nick Bennett (Edinburg)



ERC StG: NANOthermMA