Decision Directed Impulse Noise Mitigation for OFDM in Frequency Selective Fading Channels

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***Abstract*—Impulse noise is a significant problem in some OFDM applications including digital television broadcasting. In this paper we study a novel decision directed impulse mitigation algorithm both analytically and with simulations for the DVB-T parameters. In this algorithm the noise component in each received input sample is estimated based on preliminary decisions on the transmitted data. When the estimate is large enough to indicate that impulse noise is present in the sample, the estimated noise component is subtracted from the input sample before final demodulation. This technique has been shown to be extremely effective in flat fading channels. In this paper its application in frequency selective fading channels is analyzed. The optimum weighting factors for combining noise estimates from subcarriers subject to different fading are calculated. Simulation results are presented for Rayleigh and Ricean channels which show that the technique can reduce the error rates due to impulsive noise by an order of magnitude.**

Keywords-OFDM, impulse noise mitigation, digital television broadcasting, fading channel, decision directed estimation, error probability.

I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) technology is used in many digital broadband communication systems. One of the advantages of OFDM compared to single carrier systems is that it is more resistant to the effects of impulse noise because of the spreading effect of the FFT. However impulse noise can still cause significant problems in OFDM systems. This is a major practical problem in digital video broadcasting (DVB) [1-5]. Because of its practical importance, field measurements of the effect of impulse noise from different sources have been carried out. The ‘noise bucket’ concept has been developed to give a simple description of the effect of impulsive noise [1-2]. In DVB there is no interleaving across symbols, so the error rate in a symbol is determined by the received signal within that symbol. This has in turn been shown to be related to the total noise energy received during the symbol period, and largely independent of the precise structure of this noise [2].

The theoretical effects of impulse noise in multicarrier systems have also been analyzed [6], and a number of techniques for mitigating the effect of impulse noise have been described. One approach is to identify peaks in the received time domain signal and reduce these by either clipping or nulling the sample [5, 8, 9]. This is effective only for impulse noise with peaks larger than the wanted OFDM signal. This will be true only in very extreme cases. In high signal to noise environments such as broadcast television, the impulse noise can be well above the background Gaussian noise, yet well below the OFDM signal.

Several authors have used techniques that operate on the signal in the frequency domain [7, 10, 11]. Häring and Han Vinck [7] describe an iterative process in which information is exchanged between estimators operating in the time and frequency domains. The simulation results they present are for extreme cases with very large noise impulses. In [10], impulses are detected in the frequency domain by identifying subcarriers with extreme values. In [11] the positions of noise impulses are identified using pilot tones. This allows the corrupted samples to be nulled but no estimate of the actual value of the noise is made.

Very recently, decision directed impulse mitigation has been developed separately and independently by two groups [4, 12, 13]. Some details of the techniques are different, but both use a decision directed approach. Preliminary decisions are made about the transmitted data and from these an estimate is made of the noise in the received signal. The estimated noise is subtracted from the original signal before final demodulation. When the input noise is impulsive, the technique substantially reduces the noise power. The technique depends on the fact that the signal appears random in the time domain and highly structured in the discrete frequency domain whereas for the impulse noise the converse is true. In [13], a more theoretical approach is taken, and an analysis of the decision and noise estimation processes are presented. Whereas [4] has a more practical emphasis with results being presented for noise captured from a real world interference source.

In this paper we extend the theoretical analysis in [13] to include frequency selective fading channels, and derive optimum weighting factors for the combination of noise estimates from subcarriers subject to different fading levels. Simulation results for the symbol error rate (SER) for frequency selective channels are presented. These show that decision directed noise mitigation is very effective in this case.

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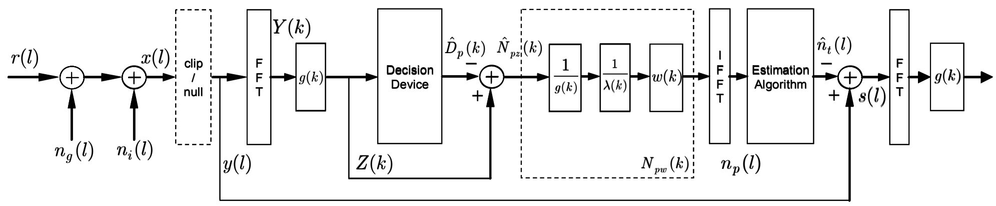

Figure 1. Receiver with decision-directed impulse mitigation.

II. IMPULSE MITIGATION USING DECISION DIRECTED NOISE ESTIMATION FOR A FREQUENCY SELECTIVE FADING CHANNEL

Fig. 1 shows a receiver using decision directed noise mitigation in a frequency selective fading channel. The received OFDM baseband signal samples are given by

$$x(l) = r(l) + n_g(l) + n_i(l) = r(l) + n_t(l) \qquad (1)$$

where $r(l)$ is the wanted OFDM signal, $n_g(l)$ is the Gaussian noise and $n_i(l)$ is the impulse noise. $n_t(l) = n_g(l) + n_i(l)$ is the total noise at the input. The samples $x(l)$ are optionally passed through a clipping/nulling operation. The samples at the output of the clipping/nulling, $y(l)$, are serial-to-parallel converted to form the vector of *N* complex samples that are input to the *N*-point discrete Fourier transform (DFT). The output of the DFT is the *N*-point vector Y. When the channel is subject to frequency selective fading, provided that the cyclic prefix is greater than the delay spread, there is no intercarrier interference and the effect of fading can be corrected using a single tap equalizer which multiplies each element $Y(k)$ by a complex number $g(k)$ which depends on, $\hat{H}(k)$, the channel estimate for that subcarrier. For zero forcing equalization, $g(k) = 1/\hat{H}(k)$. Preliminary decisions, $\hat{D}_p(k)$, about the transmitted data are made based on $Z(k)$. From this the 'observed noise' is calculated to give

$$N_{pz}(k) = Z(k) - \hat{D}_p(k) \qquad (2)$$

Except for extreme cases, most of the received subcarrriers are correctly decoded and the observed noise gives accurate information about the received noise in that subcarrier. In the cases where the subcarrier is incorrectly decoded, 'decision noise' will be added to the observed value. For an additive white Gaussian noise (AWGN) channel, the values $N_{pz}(k)$ are directly input to the IFFT [13], for a frequency selective channel the subcarriers are weighted according the fading characteristics of that subcarrier. Fig. 1 shows a series of three multipliers: $1/g(k)$ compensates for the equalizer, $1/\lambda(k)$ compensates for the average attenuation of noise caused by the decision process, and $w(k)$ is a weighting factor chosen so that $n_p(l)$ is the linear MMSE estimate of $n_t(l)$, given that the noise estimates from deeply faded subcarriers are less reliable than noise estimates from less faded subcarriers or from pilot tones. However the combined effect is one complex multiplication per subcarrier.

The vector of weighted values, $N_{pw}(k)$, is then converted back into the discrete time domain to give the time domain noise observations $n_p(l)$. If there are no decision errors, and the weighting factors reverse the effect of the equalizer $n_p(l) = n_t(l)$. However even in the presence of decision errors $n_p(l)$ contains some information about $n_t(l)$. $n_p(l)$ is then input to an estimation device to generate an estimate $\hat{n}_t(l)$ of the total input noise. This is subtracted from $z(l)$ to generate $s(l)$. The rest of the receiver is a standard OFDM receiver consisting of DFT etc. For the technique to be effective in reducing the overall bit error rate (BER), $n_t(l)$ must be impulsive (not stationary Gaussian) and the estimation algorithm must be non-linear. Fig. 1 shows an optional clipping or nulling function operating on the received baseband samples. This reduces the effect of very large noise impulses that are above the envelope of the OFDM signal. However simulations [9, 13] show that this improves the performance only in very extreme cases and this non-linearity will not be considered in this paper.

III. IMPULSE NOISE MODELS

A number of models for impulse noise have been presented in the literature [6, 14, 15]. Some characterize only the probability density function of the amplitude of the noise, whereas others also consider the time correlation of impulse events. Very recent research by the British Broadcasting Corporation (BBC), which measured a variety of impulse noise sources, has shown that many of the impulse noise sources of practical importance in OFDM applications can be modeled as gated Gaussian noise [1].

In this paper we use a particular form of gated Gaussian noise, where the noise is the sum of additive white Gaussian noise (AWGN) of variance σ_n^2 and a second higher variance Gaussian noise component which lasts for a fraction, μ, of the time duration of each OFDM symbol and which has variance σ_i^2 during this time. (i.e. the variance is calculated over only μT not over T). In general $\sigma_i^2 >> \sigma_n^2$. The total noise power is then $\sigma^2 = \mu\sigma_i^2 + \sigma_n^2$. Each of these variances is for the real and imaginary components taken separately. The impulsive samples are spread randomly throughout each OFDM symbol.

The gated Gaussian model is used because it gives a good indication of the performance of OFDM systems. Here the critical factor is whether the BER for each symbol is above the threshold at which the error correcting coding will reduce the final BER to an acceptable level, rather than the BER averaged over the entire received signal. It also allows the length and power of the impulse noise to be varied in a way that makes clear the practical implications of the technique.

IV. ANALYSIS OF NOISE ESTIMATION

We will now analyze the system for a frequency selective fading channel and calculate the optimum weighting factors for the frequency domain noise observations. Consider the effect of the system in Fig. 1 on the noise (rather than the signal) components. The time domain noise samples at the input to the first FFT are $n_t(l)$. Let the noise component of the *k*th output of the FFT be denoted by $N_t(k)$. The noise after the equalizer is therefore $g(k)N_t(k) = N_{tz}(k)$.

In [13] it was shown that the effect of the decision process on the noise (rather than the signal) could be analyzed by considering it as two non-linearities operating separately on the real and imaginary components of the noise in each subcarrier. If there are enough noise impulses in each OFDM symbol for the central limit theorem to apply, the frequency domain noise is Gaussian and the effect of the nonlinearity can be calculated using Bussgang's theorem [12]. Using this, it was shown that [13] for an AWGN channel,

$$\Re/\Im\{N_{pz}(k)\} = \lambda\Re/\Im\{N_{tz}(k)\} + \Re/\Im\{N_{dz}(k)\} \quad (3)$$

where $\Re/\Im\{\ \}$ represents the real or imaginary component of the variable, λ is a constant which depends on the constellation and the signal to noise ratio, $N_{tz}(k)$ is the actual noise component of the subcarrier and $N_{dz}(k)$ is an uncorrelated distortion component. Fig. 2 shows λ as a function of d/σ, where $2d$ is the distance between constellation points in the received constellation after equalization. Theoretical values of λ are derived in [13].

For the frequency selective channel, λ is replaced by $\lambda(k)$ as the value depends on the fading of the particular subcarrier. Thus, in the frequency selective case

$$N_{pz}(k) = \lambda(k)g(k)N_t(k) + N_d(k) \quad (4)$$

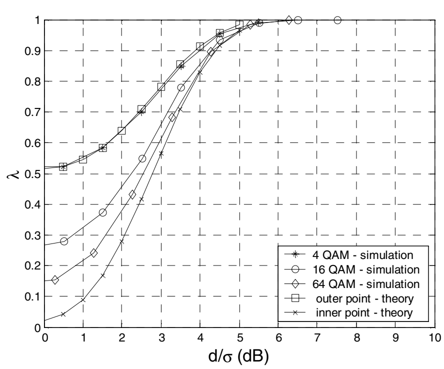

Figure 2. λ versus d/σ.

The analysis of the next section depends on which combination of the three possible multiplying factors is used. The simplest case to analyze is where the multipliers $1/\lambda(k)$ and $1/g(k)$ are used but $w(k) = 1$. Then the input to the IFFT is

$$N_{pw}(k) = \frac{N_{pz}(k)}{\lambda(k)g(k)} = N_t(k) + \frac{N_d(k)}{\lambda(k)g(k)} \quad (5)$$

And the output of the FFT is given by

$$\begin{aligned} n_p(l) &= \sum_{k=0}^{N-1}\left(N_t(k) + \frac{N_d(k)}{\lambda(k)g(k)}\right)\exp\left(\frac{-j2\pi kl}{N}\right) \\ &= n_t(l) + n_d(l) \end{aligned} \quad (6)$$

Note that although the total and distortion noise component in the frequency domain (3) are uncorrelated, that may not be true in the time domain (6), if the product terms which result from the FFT operation are correlated.

If the average number of decision errors in an OFDM block is sufficient for the central limit theorem to apply $n_d(l)$ is a Gaussian random variable with variance

$$E\left[|n_d(l)|^2\right] = \frac{1}{N}\sum_{k=0}^{N-1}\frac{E\left[|N_d(k)|^2\right]}{|\lambda(k)|^2|g(k)|^2} \quad (7)$$

The variance of the estimates in (7) is independent of the subscript l. Note that here the ***decision noise*** is structured in the frequency domain, but if there are enough decision errors in one symbol the resulting decision noise in the time domain is Gaussian. Earlier we were considering ***impulse noise*** which is structured in the time domain, but appears Gaussian in the frequency domain if there are enough impulses in a symbol period. For the other combinations of multiplying factors, the expression for $n_p(l)$ cannot be simplified in this way. However some more insight into the optimum weighting factors can be gained if we consider $n_t(l)$ in terms of its discrete Fourier components. Let

$$n_t(l) = \sum_{k=0}^{N-1} n_t(l,k) \quad (8)$$

where

$$n_t(l,k) = \frac{N_t(k)}{\sqrt{N}}\exp\left(\frac{j2\pi kl}{N}\right) \quad (9)$$

If we consider the case where there are enough impulsive samples in the symbol period for the central limit theorem to apply then $N_t(k)$ are independent identically distributed (iid) Gaussian random variables and $n_t(l,k)$ are also iid Gaussian random variables. When weighting factors are used

$$n_p(l) = \sum_{k=0}^{N-1} w(k)\left(N_t(k) + \frac{N_d(k)}{\lambda(k)g(k)}\right)\exp\left(\frac{-j2\pi kl}{N}\right) \quad (10)$$

Combining (9) and (10) and rearranging gives

$$n_p(l) = \sum_{k=0}^{N-1} w(k)(n_t(l,k) + n_d(l,k)) \quad (11)$$

Where $n_d(l,k)$ is a Gaussian random variable with

$$E\left[|n_d(l,k)|^2\right] = \frac{E\left[|N_d(k)|^2\right]}{\lambda^2(k)|g(k)|^2 N} \quad (12)$$

and

$$E\left[|n_t(l,k)|^2\right] = \frac{E\left[|N_t(k)|^2\right]}{N} \quad (13)$$

It can be shown that the weighting factors which give the linear MMSE estimate in (11) if all of the terms in (11) are uncorrelated are

$$w(k) = \left(\frac{E\left[|n_t(l,k)|^2\right]}{E\left[|n_d(l,k)|^2\right]}\right) \Big/ \left(\frac{E\left[|n_t(l,k)|^2\right]}{E\left[|n_d(l,k)|^2\right]} + 1\right) \quad (14)$$

Combining (11), (12) ,(13) and (14) gives

$$w(k) = \frac{\lambda^2(k)E\left[|N_t(k)|^2\right]}{\lambda^2(k)|g(k)|^2 E\left[|N_t(k)|^2\right] + E\left[|N_d(k)|^2\right]} \quad (15)$$

The structure in Fig. 1 also allows the estimation process to include information contained in the pilot tones. For pilot tones, the noise is calculated by subtracting the received signal in the subcarrier from the known transmitted signal in that subcarrier, rather than the estimated data. For pilot tones the weighting factors are set to $\lambda(k) = w(k) = 1$.

The values $n_p(l)$ are input to the estimation algorithm which derives the estimate $\hat{n}_t(l)$. A number of algorithms are possible [13]. In this paper, a threshold operating on the real and imaginary components separately was used in the simulations. Noise components above a certain threshold are subtracted out and those below a certain threshold are ignored. Represent $\Re(n_p(l))$ as r_p, $\Re(n_t(l))$ as r_t and $\Re(n_d(l))$ as r_d Thus the estimate of r_t is given by

$$\begin{aligned} \hat{r}_t &= ar_p \quad \text{for } |r_p| > \alpha \\ &= 0 \quad \text{for } |r_p| < \alpha \end{aligned} \quad (16)$$

where a is a weighting factor and α is the threshold value. An analysis of the threshold operation has been presented in [12].

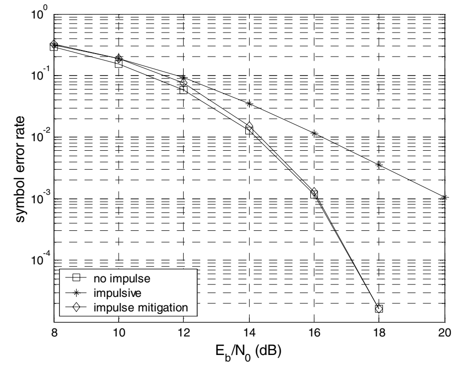

Figure 3. AWGN channel, $\sigma_i^2 = -3$dB , $\mu = 0.005$.

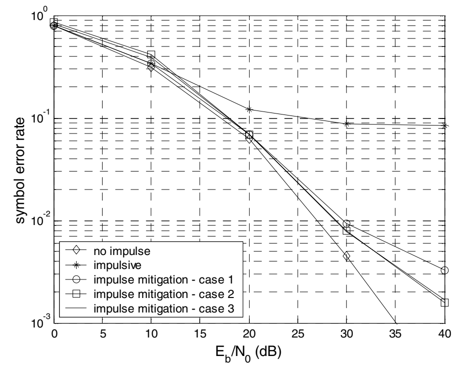

Figure 4. Ricean channel, $\sigma_i^2 = -3$dB , $\mu = 0.005$.

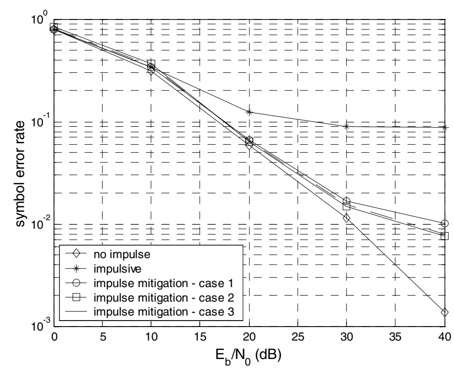

Figure 5. Rayleigh channel, $\sigma_i^2 = -3$dB , $\mu = 0.005$.

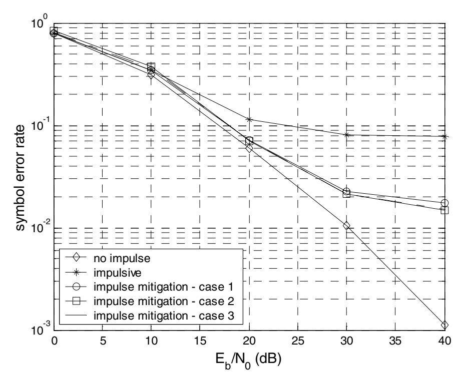

Figure 6. Rayleigh channel, $\sigma_i^2 = -9$dB , $\mu = 0.02$.

V. SIMULATION RESULTS

Matlab simulations were used to examine the performance of decision directed noise mitigation in a frequency selective fading channel. The Ricean and Rayleigh fading channel models described in the DVB standard were used [17]. For each simulation, the average power in each of the real and imaginary components of the wanted OFDM signal is unity. Figs. 3-6 show the resulting SER as a function of E_b/N_0 where N_0 is the single sided spectral density of the white Gaussian (non impulse component). In other words, for each plot, the impulse noise is kept constant and the effect of varying E_b/N_0 is measured. 64QAM modulation and 2048 subcarriers were used. The DVB standard specifies a range of cyclic prefix lengths. In these simulations a cyclic prefix of 64 was used and the calculation of E_b/N_0 included the energy in the cyclic prefix. In the simulations it was assumed that all subcarriers were carrying data and no pilot tones were used. A threshold $\alpha = 0.3$ (standardized in terms of the standard deviation of the wanted OFDM signal) was used for real and imaginary impulse detection. The weighting factor was set at $a = 1$. Zero forcing equalization was used in the simulations. However the technique and the analysis are applicable to other forms of equalization such as MMSE equalization. Perfect channel state information was assumed. Each graph shows the performance with no impulse noise, with impulse noise but no noise mitigation, and of three cases of impulse mitigation. Case 1 using only the multiplying factor $1/g(k)$. Case 2 used $1/g(k)$ and $1/\lambda(k)$ and in case 3 all three multiplying factors shown in Fig. 1 are used.

Figs. 3 – 5 show the performance in an AWGN, a Ricean fading channel and a Rayleigh fading channel for the case where $\sigma_i^2 = -3$ dB and $\mu = 0.005$. The impulse mitigation technique is very effective in all cases. It is most effective in the AWGN channel and performs better in the Ricean than the Rayleigh. However even in the Rayleigh case it reduces the SER by approximately an order of magnitude. Fig. 6 shows the performance with a Rayleigh fading channel and different impulsive noise parameters, $\sigma_i^2 = -9$ dB and $\mu = 0.02$. The energy of the impulse noise is the same, but in Fig. 5 the results are for higher levels of impulse noise for a shorter fraction of the symbol. While slightly less effective, the noise mitigation still gives significant improvements when the impulse noise is 9dB below the signal power.

Noise mitigation is clearly very effective. It reduces the SER by about an order of magnitude for practical values of E_b/N_0. Case 2 is significantly better than case 1, but adding the third multiplier (case 3) gives no improvement or makes the performance slightly worse.

Further simulations (result not shown) show that the technique is also effective for a wide range of other impulse noise parameters and for 4QAM and 16QAM modulation.

VI. CONCLUSIONS

The performance of decision directed impulse noise mitigation in OFDM has been analyzed for the case of a frequency selective fading channel. It has been shown that to obtain the best performance the noise component from each subcarrier should be multiplied by two factors, one to compensate for the effect of the single tap equalizer and one to compensate for the decision non-linearity.

Simulation results are presented for DVB-T parameters and for the Rayleigh and Ricean fading channel models given in the DVB standard. For these cases it is shown that impulse noise mitigation can reduce the symbol error rate by an order of magnitude.

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