Survey of Applications of Price of Anarchy

CAP 5993

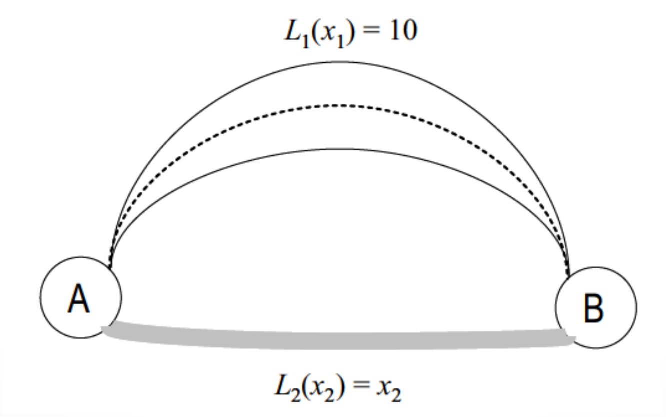
Mario Carvajal

Thesis

- Price of Anarchy. Predominantly a theoretical construct?
- Present situations that have been modeled in the recent literature to demonstrate the validity of the concept.
- Try to find real world applications to show the concept in action.

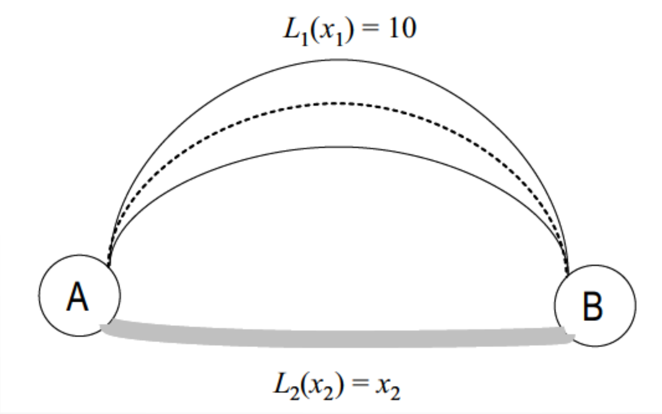
What is Price of Anarchy?

- The Price of Anarchy (PoA) is a concept in game theory that measures how the efficiency of a system degrades due to selfish behavior of its agents.
- Two paths traffic can travel from A to B:Highway (top), and narrow Alley.



Price of Anarchy

- Imagine 10 cars are trying to get from A to B.
- Highway takes 10 min to traverse regardless of how many cars are in it.
- Alley is more direct but gets congested.
- The duration of the commute on Alley depends on how many cars are traveling through it. 1 car takes one min. Two cars, 2 min, and so on…



Price of Anarchy

- No reason for anyone to take the highway!
- At worst, driving through Alley will take ten minutes
- Highway always takes 10 minutes.
- It is therefore in the best interest of all ten drivers to take Alley.
- This results in an average commute of 10 min, the NE.
- NE is not the best possible outcome. The social optimum has 5 cars 10 min commutes, and 5 others getting 5 min commutes.

Price of Anarchy

- But this is not stable. The 5 people driving on the highway have an incentive to switch to the Alley.
- They could reduce from 10 minutes to 6.
- For social optimum to be maintained, the drivers must be “coached", because it is against their immediate best interest.
- The difference between the “selfish" NE and the social optimum is the “price of anarchy."
- In this ex. it is 2.5 minutes, or 33%

Basketball

- Similar network model where each possession flows from inbound to basket.
- The analysis suggests that there may be a significant difference between taking the highest% shot each time and playing the most efficient game.
- TS% = True shooting percentage = field goal % adjusted for free throws and three-point shots.
- The team's optimum strategy is for their best player (player A) to shoot almost the same fraction of shots as his teammates - about 20%.
- Player A would then have a TS% of 0.625, far above the rest of his teammates who are shooting with a TS% of 0.5. It would seem to be obvious that player A should be getting more shots than his teammates.

Basketball

- But the result of limiting player A's shots, and keeping the defense from focusing too much on him, pays off.
- In this case it improves the team's TS% by 2.5%.
- For reference, in the 2008-2009 NBA season the standard deviation among teams in TS% was only 1.6%, so a difference of 2.5% may be quite significant.
- To achieve it, the team just needs to consciously choose not to have player A shoot on ~ 80% of possessions, even when he is their best option.

Offload Decision Models in Mobile Cloud Application Ecosystems

- In a mobile cloud computing scenario, applications have the option to either execute locally on the device or offload and execute remotely on a cloud platform.
- The application must estimate the cost and benefit of an offload prior to making a decision.
- Focus is on the competition between mobile cloud applications residing on the same device
- Depending on how much information this application has of other applications running on the same device and the device's bandwidth, this decision making process can have different results.

Offload Decision Models

- Counterintuitive strategy decisions were made by applications in the complete information game when no global control was applied.
- The authors showed that significant reduction in social cost can be obtained in a cooperative setting.
- The following example assumes 3 scenarios where 2 hybrid applications share a device.

Offload Decision Models

- Each number in the table represents the amount of time it takes the application to run on a platform assuming exclusive usage of the device's resources, in seconds.

	Scenario A		Scenario B		Scenario C	
$i \in [s]^{\mathbb{H}}$	$\mathbb{N}$	$\mathbb{R}$	$\mathbb{N}$	$\mathbb{R}$	$\mathbb{N}$	$\mathbb{R}$
$i = 1$	15	(10)	(15)	10	15	(10)
$i = 2$	18	(15)	18	(15)	(18)	15
Cost/Platform	0	(25)	15	15	(18)	15
Social Cost	25		15		18	

Offload Decision Models

- A circle represents the decision made by the application.
- Scenario A is a typical example of applications making uninformed decisions. Both applications assume that it is the only application running on the device and the cost comparison means both applications prefer to execute on the cloud. This makes R congested while leaving the N vacant.

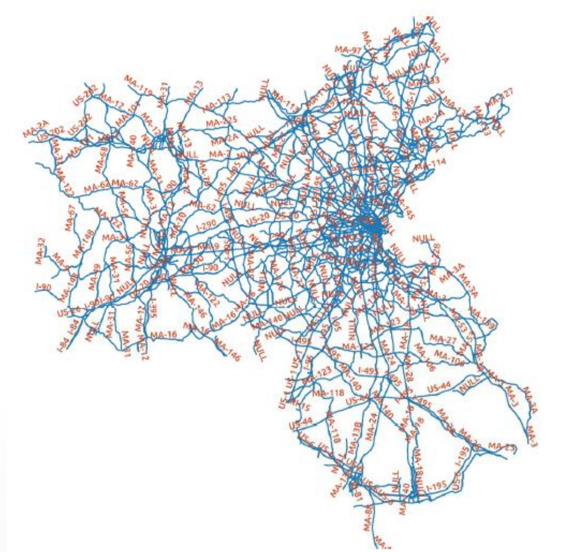
	Scenario A		Scenario B		Scenario C	
$i \in [s]^{\mathbb{H}}$	$\mathbb{N}$	$\mathbb{R}$	$\mathbb{N}$	$\mathbb{R}$	$\mathbb{N}$	$\mathbb{R}$
$i = 1$	15	(10)	(15)	10	15	(10)
$i = 2$	18	(15)	18	(15)	(18)	15
Cost/Platform	0	(25)	15	15	(18)	15
Social Cost	25		15		18	

Offload Decision Models

- The total cost on R is 25 seconds compared to the cost of 0 on N.
- The social cost is 25 seconds. This is higher than when choice of platform is split between N and R.
- Significant reduction in social cost can be obtained in a cooperative setting.
- The dependencies between price of anarchy and various system parameters are also investigated.
- High deviation in application weights encourages high price of anarchy in non-cooperative scenarios.

The Price of Anarchy in Transportation Networks from Actual Traffic Data

- A large scale traffic network of eastern Massachusetts was modeled using real traffic data from 2012.
- Using spatial average speeds and flow capacity for each segment of the network. System optimum problem was formulated in order to find socially optimal flows of the network.



Transportation Networks

- Analyzed network performance, in terms of the total latency, under a user-optimal policy versus a system-optimal policy.
- The dataset includes traffic data for more than 13,000 road segments with the average distance of 0.7 miles, covering the average speed for every minute of the year 2012.

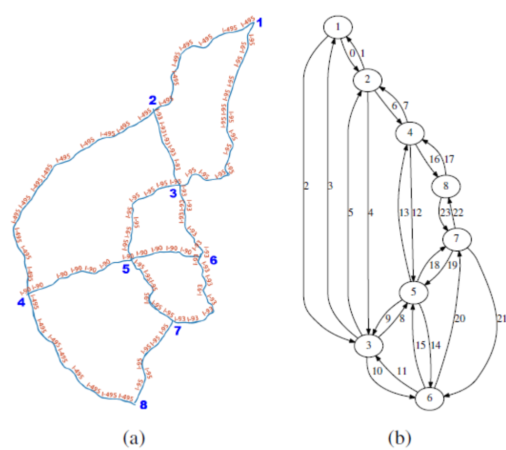


Fig. 2. (a) An interstate highway sub-network of Eastern Massachusetts (a sub-map of Fig. 1; the blue numbers indicate node indices); (b) The topology of the sub-network (the numbers beside arrows are link indices, and the numbers inside ellipses are node indices).

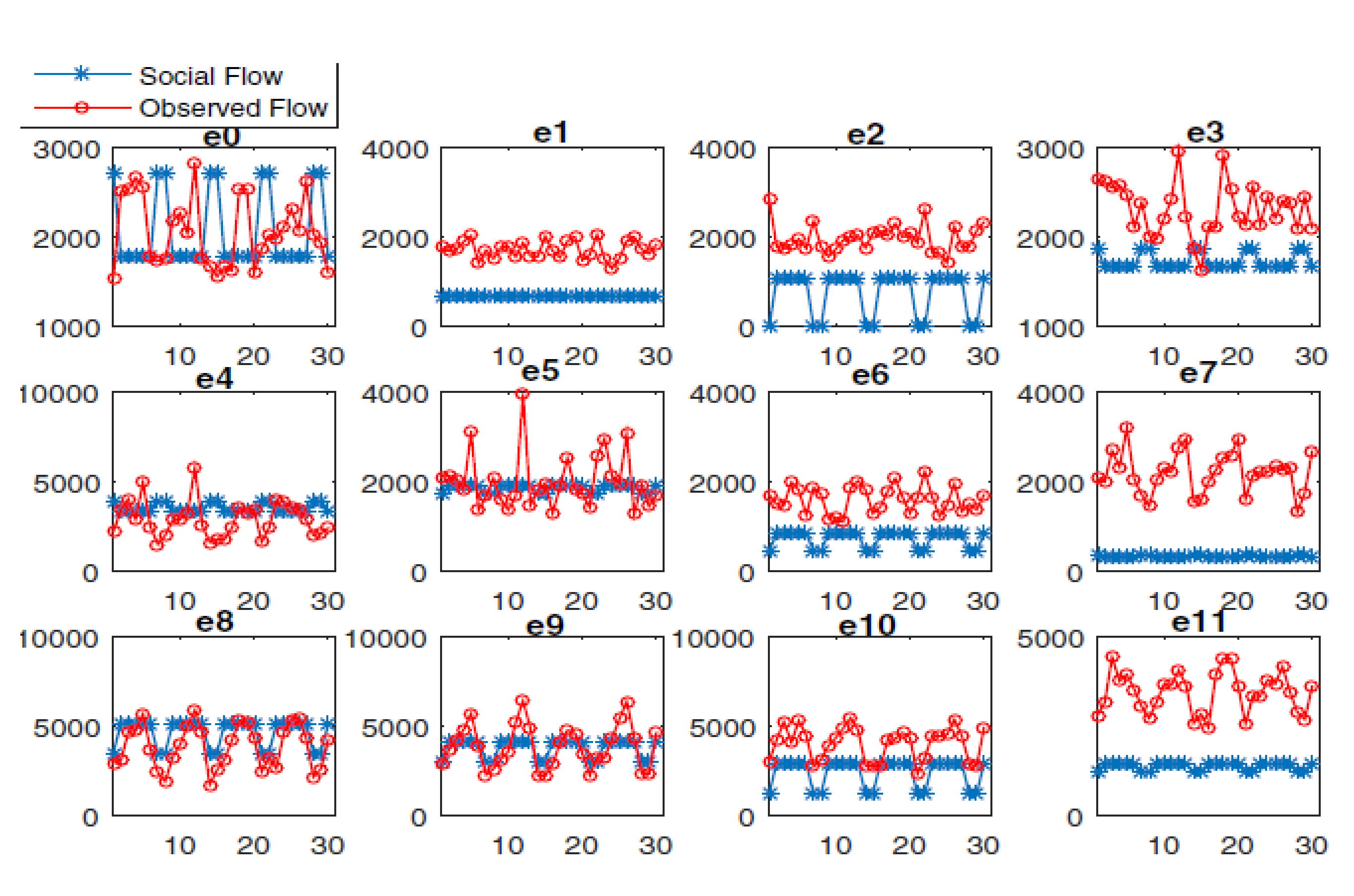
Transportation Networks

- Capacity data is given for 4 different time periods in a day.
- The total roadway capacity for all available lanes for that time period is given.
- These values are calculated based on the share of daily traffic counts in each hour of that time period. For each time period there exists a period capacity factor applied to represent peak hour conditions.

Transportation Networks

- Results: the system is considerably inefficient under selfish driving.
- PoA is 1.23 in the best case showing that we can reduce the total latency in the network by at least 23% if drivers follow socially optimal paths.

Transportation Networks



Conclusion

- It is very difficult to find real applications of PoA.
- Probably because modeling complex behavior is very difficult.

Thank you