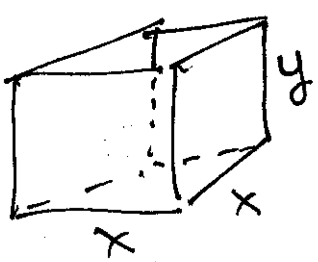
Math 1496 - Calc I

Section 4.7 Applied Min/Max Problems

So often we are required to max. profit or minimize cost. Here calculus plays an important role. We will consider several examples

Ex 1 Suppose we are to construct a square base box that is to hold 8 ft^3. What dimensions should the box be to min cost (i.e. surface area)



$$V = x^2 y = 8 \Rightarrow y = 8/x^2$$

$$A = 2x^2 + 4xy$$

$$= 2x^2 + 4x \cdot \frac{8}{x^2} = 2x^2 + \frac{32}{x}$$

$$A' = 4x - \frac{32}{x^2} \qquad A' = 0 \text{ when } 4x - \frac{32}{x^2} = 0$$

$$x^3 = \frac{32}{4} = 8 \qquad x = \sqrt[3]{8} = 2$$

2nd Derivative Test

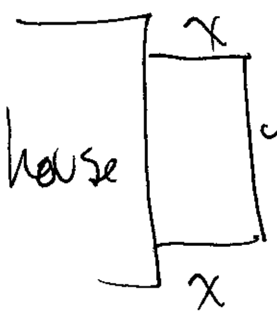
$A'' = 4 + \frac{64}{x^3}$ when $x = 2$ $A'' > 0$

So we have a min

$y = \frac{8}{x^2}$ & if $x = 2$ $y = \frac{8}{4} = 2$

dimension $2' \times 2' \times 2'$

Ex 2 Suppose we wish to build a dog pen at the side of a house. We have 32 ft of fence. How should we build the pen to max area



So 1st $P = 2x + y = 32 \Rightarrow y = 32 - 2x$

Next $A = xy$

$= x(32 - 2x)$

$= 32x - 2x^2$

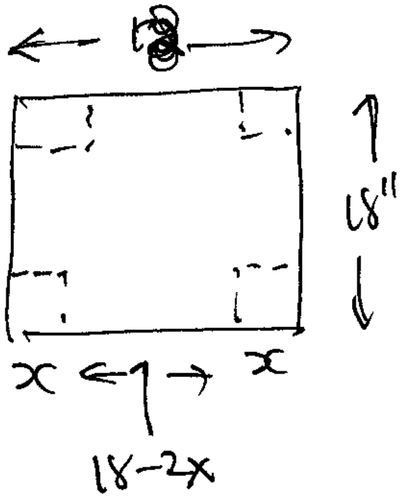
$A' = 32\cancel{x} - 4x$ $A' = 0$ $32 - 4x = 0$ $x = 8$

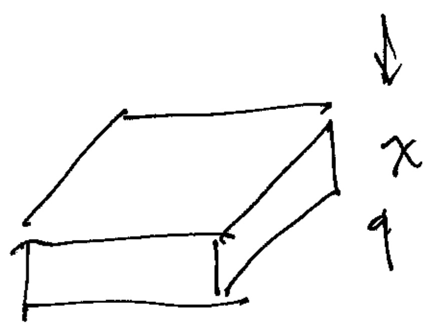
$A'' = -4 < 0$ $\frown$ so a max.

Now if $x = 8$ $y = 32 - 2(8) = 16$

so the pen is $8' \times 16'$

<u>ex3</u> An open box is made from a $18'' \times 18''$ rectangular piece of cardboard by cutting squares from each corner. & turning up the sides. Find the volume of the largest box.





$$V = x(18-2x)^2$$

$$V' = (18-2x)^2 + 2x(18-2x)(-2)$$

$$= (18-2x)(18-2x-4x) = (18-2x)(18-6x)$$

$$= 12(6-x)(3-x)$$

$V' = 0$ when $x = 3, 6$

Note $x = 6$ gives $V = 0$ (so prob $x = 3$)

$$V'' = 12\{-1(3-x) - (6-x)\}$$

$$= 12\{-9 + 2x\}$$

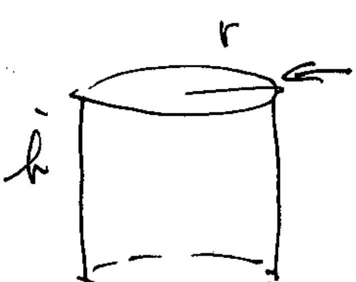
$x = 6$ $V'' < 0$ ⇒ max

so $V = 6(18-12)^2$

$= 6 \cdot 6^2 = 216$ cubic inches

Ex 4 Suppose we ~~have~~ soup can that must hold $1L = 1000 mL$

Find the dimensions at minimize surface area



$V = \pi r^2 h = 100$ $\qquad h = \dfrac{1000}{\pi r^2}$

$$A = 2\pi r^2 + 2\pi r h$$

$$= 2\pi r^2 + \frac{2000}{r}$$

$$A' = 4\pi r - \frac{2000}{r^2}$$

$A' = 0$ when $4\pi r = \frac{2000}{r^2} \Rightarrow r^3 = \frac{500}{\pi}$

$$r = \sqrt[3]{\frac{500}{\pi}}$$

$A'' = 4\pi + \frac{4000}{r^3} > 0$ when $r = \sqrt[3]{\frac{500}{\pi}}$

so a min

$$h = \frac{1000}{\pi\left(\frac{500}{\pi}\right)^{2/3}} = \frac{2\,\frac{500}{\pi}}{\left(\frac{500}{\pi}\right)^{2/3}} = 2\sqrt[3]{\frac{500}{\pi}}$$

so $h = 2r = d$

So the height = diameter