Foundations For College Mathematics 2e

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Preface to Students and Instructors

This text contains terminology, content, and algorithms that may not be found in a traditional textbook because it is the author's intention to break from tradition and prepare students for the mathematics needed in a modern society. Further, as learning progresses, terminology may change to reflect new understandings. The author also recognizes that mathematics is learned by understanding, not by memorization. Developing mathematical ideas in the context of a situation helps students understand mathematics; at the same time, students understand that mathematics relates directly to the world.

The use of this text requires a graphing calculator with function notation; the calculator will be used as a tool to help us understand mathematical concepts and perform mathematical algorithms. Technology, which offers students and teachers a variety of methods for solving problems, is used in this text to explore mathematical ideas, and it changes what is considered important. Many traditional mathematical topics have diminished importance. This text offers content that is important to students directly entering the work force and to students continuing the study of mathematics and science. Many of the exercise sets offer questions that are engaging and demand higher level thinking.

Students have the responsibility to read and study the text in order to learn. Students have the responsibility to view the exercise sets as questions that are to stimulate thinking, not questions that are designed to encourage memorization of facts. Students have the responsibility to recognize that the exercise sets contain questions to help them formulate their own ideas about mathematical relationships.

There are limited references to calculator keystrokes; when they are included, they are for theTI-83 or TI-84. With slight modification, the keystrokes will also work on other calculators.

The teaching features of this text include:

- Technology integrated throughout to enhance the learning and teaching of mathematical concepts.
- Technology integrated throughout to provide options for performing mathematical algorithms.
- Mathematical concepts introduced in the context of real-world situations.
- Thorough analysis of applications.
- Guided discovery exercises.
- Reduced emphasis on the use of symbol manipulation and increased emphasis on the use of function as a central theme.
- Three methods (numeric, graphic, and algebraic) of representing a function.

- Distributed learning has been incorporated in this text. For example, the idea of function is introduced in Section 2.1 as a data relationship. The intuitive idea of function is further developed in Chapter Three by looking at algebraic expressions that model data relationships introduced in Chapter Two. Chapter Four continues with formal function notation, and functions are used in equation and inequality solving in Chapter Six. Chapters Seven, Eight, Nine, Ten, Twelve, and Fourteen offer analysis of individual function types.
- A variety of methods for solving problems are encouraged. Students are encouraged to explore on their own.
- Higher level thinking skills are encouraged through projects, open ended questions, and concept questions.
- Sample problems are checked numerically or graphically and students are encouraged to do likewise.
- Exercise sets contain low-level difficulty problems that prepare students for future topics.
- Exercise sets contain high-level difficulty problems that review topics previously taught.
- Exercise sets contain writing questions.
- Exercise sets contain concept questions.
- Exercise sets contain exploration problems. Many of the explorations can be used for group work. Many can be used as portfolio exercises.
- Exercise sets contain open ended questions.
- Mathematical words are defined by bold text.
- Extended laboratory projects on modeling are included with the text.

Neuro and cognitive science research has provided considerable information about how the brain functions. This textbook capitalizes on this research through the implementation of the cognitive processes of associations, pattern recognition, attention, visualizations, priming, meaning, and the enriched teaching/learning environment.

- We remember algebra longer and have better memory by using associations made through function permeating the content. That is, we are more likely to remember the mathematics taught because we capitalize on associations made through using a function approach.
- Learning is made simpler, faster, and more understandable by using pattern building as a teaching tool. In the function approach used in this text, almost all of the pencil and paper activities, e-teaching activities, and class discussions use pattern building to reach a generalization about a concept or skill.
- We cannot learn if we are not paying attention. The graphing calculator is used to draw attention to the mathematics through its basic functionalities **including**, various app software.
- Without visualizations, we do not understand or remember the mathematics as well. In the function approach visualizations are used <u>first</u> before any symbolic development. This greatly increases the likelihood that we will remember the mathematical concept being taught.

- Considerable brain processing takes place in the subconscious side of the brain, including a learning module. To make this processing possible for our purposes, the brain must be primed. The function implementation module (Chapters Two and Three) and early e-learning activities prime the brain for all the algebra that follows.
- The enriched teaching/learning environment promotes <u>correct</u> memory of math content. The wide variety of teaching activities facilitated by the function approach provides the enriched environment.
- Contextual situations (often represented as relationships) provide meaning to the algebra learned. Algebra taught without meaning creates memories without meaning that are quickly forgotten.

Algebra approached through function, as in this textbook, has reordered the content and capitalized on function concepts to <u>develop understanding</u>, long-term memory, and skills.

This text has been published as a preliminary edition, as a revised preliminary edition, a first edition test, and now as a second edition. It has been class tested over a ten year period and has been revised and edited no less than twelve times. It has been reviewed by over twenty-five reviewers.

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