

Resonant, and Non-linear, Faraday Rotation in Rubidium Vapor

As a user of TeachSpin's 'Diode Laser Spectroscopy', you've seen the interaction of light and matter, through your ability to 'tune to resonance' by adjusting the laser frequency to match an energy-level difference in an atom. If you've performed basic experiments with the DLS system, you'll know that tuning the diode-laser wavelength into the right region near 780 nm, around a laser frequency near 384,000 GHz, will match the resonant wavelength(s) corresponding to the D_2 transition in atomic rubidium, the $S_{1/2} - P_{3/2}$ transition from the ground state to a low-lying excited state, in this alkali atom.

Once you've achieved such a matching, you have probably seen direct evidence of absorption of light by the atoms, and the fluorescence (or re-emission of light) from the atoms. But there are *other* effects of the light-matter interaction, and this write-up describes some of the effects that can be detected by adding a static and uniform *magnetic field* to the experimental arena. These 'magneto-optical' effects might be seen in atomic fluorescence (you can look up the 'Hanle effect'), but it is even easier to detect them 'in absorption' of a light beam, through their effect on the polarization of the transmitted light. Making such experiments even easier is the fact that the diode laser's output light is already highly, and linearly, polarized.

The very first magneto-optical effect ever discovered is the Faraday effect, found by Michael Faraday in 1845 in his pursuit of experimental evidence of a connection between light and magnetism. His experiments exploited (as a sample) some special heavy glass he himself had made years earlier. His effect involved linearly-polarized light, passing through a sample immersed in a static and uniform magnetic field having the same direction as the propagation of the light. Under these circumstances, he found that the plane of polarization of the light rotated systematically in the course of its propagation through the sample, emerging with a 'twist' by angle

$$\Delta\phi = V B L .$$

Here B is the component of magnetic field along the light-propagation direction, L is the length of the sample, and the 'Verdet constant' V depends on the material (and the wavelength of light) used.

Users of TeachSpin's 'Faraday Rotation' experiment in fact measure V for a particular glass (a 'special flint' glass labelled SF-57) and a particular wavelength (near 650 nm), and find a value near 20 rad/T·m.

Exercise: Convert that number to old-fashioned units, in (minutes of arc) per (Oersted of field) \times (centimeter of length); and also compute the Faraday rotation (in degrees) to be expected for such a glass sample, of $L = 0.10$ m, in a field of $B = 30$ mT.

You can see that Faraday rotation is not a huge effect in a typical laboratory demonstration. Now think of substituting, for 10 cm of solid glass, a sample consisting of only 25 mm of a very dilute vapor of free-flying rubidium atoms, and imagine how much *smaller* an effect might result.

Exercise:

Find a reference that gives the vapor pressure of rubidium as a function of temperature, and use it to get the pressure expected at (say) $T = 45\text{ }^{\circ}\text{C}$. Now use the ideal-gas law to compute the *number density* of atoms in the vapor (first, the number of moles per unit volume, then the number of atoms per unit volume). Finally, compute the *mass density* of rubidium vapor at this temperature. Compare that density to the mass density of SF-57 glass.

The enormous ‘dilution’ of mass density in atomic vapor (relative to glass) might be expected to give a similarly huge dilution of the Verdet constant. And that is indeed the case – *except* that the Verdet constant has a dependence on wavelength, and might be conjectured to have an enhancement for wavelengths of light that match a resonant frequency of the atoms in the vapor. In fact, there is so enormous an enhancement that it becomes *easy* to detect the Faraday rotation of 780-nm light in a rubidium-vapor sample – and also to show, in detail, how it depends on the tuning (or de-tuning) of the laser from the atomic resonance frequency.

This write-up, then, describes how to demonstrate Faraday rotation in rubidium vapor. We divide the treatment into two parts, which differ only in the intensity of light used – but the experimental techniques needed differ hardly at all. We use ‘resonant Faraday rotation’ to describe the use of low-power light, in which saturation of the atomic transition can be neglected; by contrast, it has become conventional to use the term ‘nonlinear Faraday rotation’ to describe the use of light strong enough to create *optical* non-linearity in the light-atom interaction. The level at which nonlinear effects become important is called the ‘saturation intensity’ (properly, an irradiance level). For the D_2 transition in atomic rubidium, the saturation intensity is about 3 mW/cm^2 .

Exercise: Find a reference that quotes this value; also find a reference that *derives* such a value. Now suppose your diode laser system can deliver a power of 10 mW in a beam of area $(1\text{ mm})^2$. Compute its ‘intensity’ in these units, and compare to the ‘saturation intensity’, to see why non-linear atom-light interactions are accessible to you.

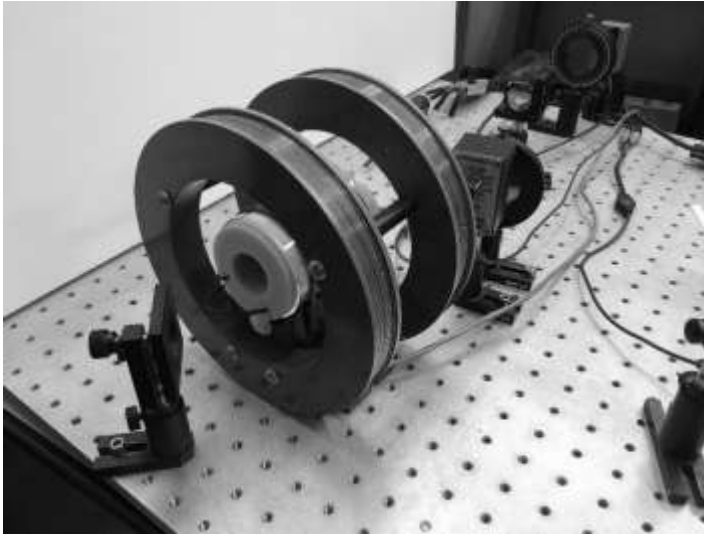
Resonant Faraday Rotation

This section describes how to set up a Diode Laser Spectroscopy system so that you can view resonant Faraday rotation, at 780 nm, in rubidium vapor. It assumes that you already know how to tune the laser to, and through, the 780-nm resonances in Rb, as you will have done for a resonant-absorption or resonance-fluorescence experiment. There are two new requirements for this new experiment.

The Helmholtz-coil system

The first of them is easy – you want the Rb vapor cell, inside its temperature-controlled environment, to be immersed in a static and uniform magnetic field.

The photo below shows how TeachSpin's vapor-cell assembly fits inside the TeachSpin set of Helmholtz coils, which create a field which is parallel to the axis of symmetry of the cylindrical cell, and thus parallel to the typical direction for propagating a laser beam through the cell.



Exercise:

The coils have turns of wire spread out in space, but the 'average turn' of each of two coils is of 3.5" radius, and the two coils are at 3.5" separation along the axis. Given that each coil has 320 turns, calculate the magnetic field at the center of the coil assembly, per unit current in the coils. You should find a 'coil constant' near 3.2 mT/A.

Note that the two coils are wired in series, and note the system has a dc resistance near 10Ω , and may be safely used with a continuous current up to 3 A. Under these conditions, the coils will dissipate 90 W of power, and they will get warm.

Building a 'balanced polarimeter'

The second new task is to become sensitive to the *polarization* of the light emerging from the rubidium cell.

You might have found the direction of polarization of a light beam by using the 'extinction' method – this requires the mechanical rotation of a Polaroid until you achieve a zero-transmission condition. The difficulty is that this method assumes purely linearly-polarized light (if there is elliptical polarization, you'll get a minimum, but not a null, upon rotation of the Polaroid); and secondly, this method requires an actual mechanical rotation, and is thus is not well suited to following the real-time state of polarization of a beam whose properties might be changing on a *millisecond* time scale. So this section describes a system with no moving parts, using electronic detectors, that gives a real-time indication of the state of polarization of a beam.

Here we adapt the method to the parts you already have in your DLS kit; you'll need the so-called 50:50 beam-splitter, two Polaroids in rotating mounts, and the two photodetector/amplifiers you may have already used. Here's the logic of the method, illustrated geometrically; below, you'll find the experimental realization of the idea.

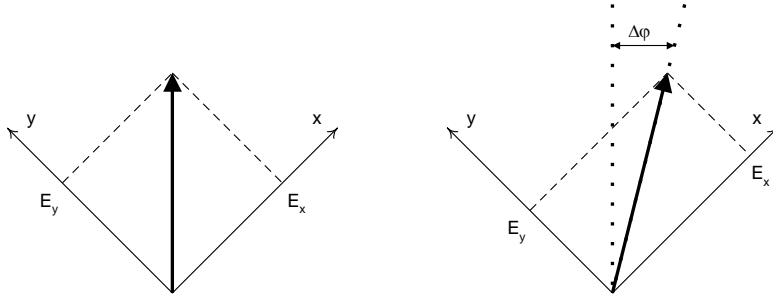


Fig. 1: Two \mathbf{E} -field vectors, with their projections onto the x - and y -axes.

Suppose you have a linearly-polarized light beam propagating in the z -direction, with \mathbf{E} -field direction lying halfway between the x - and y -axes. This field can be described by

$$E_x = E \cos(45^\circ) , E_y = E \sin(45^\circ) .$$

Now if the light changes not at all in intensity, but only in that its direction of its polarization rotates by angle $\Delta\phi$ toward the x -axis, the field components become

$$E_x = E \cos(45^\circ - \Delta\phi) , E_y = E \sin(45^\circ - \Delta\phi) .$$

The drawing below shows that the projections of the \mathbf{E} -vector onto the x - and y -axes, which were originally equal in magnitude, now both change: one gets bigger, and the other gets smaller. If we compute the squares of the E -field projections, we find the *difference* of those squares gives us

$$E_x^2 - E_y^2 = E^2 [\cos^2(45^\circ - \Delta\phi) - \sin^2(45^\circ - \Delta\phi)] = E^2 \sin(2\Delta\phi) .$$

Exercise:

Work out the trigonometry which establishes the last step of this result.

Notice that the E -field projections are just what Polaroids are good at creating; notice that squares of E -fields are related to intensities, just what photodetectors are good at sensing; and finally notice that forming the real-time difference between two electronic signals is also easy. The result is *first-order* sensitivity, via the sine-function, to the magnitude *and sign* of the polarization rotation.

Optional Exercise:

What if the light also encounters *absorption* in the sample? Write the incoming light, with $\mathbf{E} \propto (\mathbf{x} + \mathbf{y}) \exp i(kz - \omega t)$, as an explicit superposition of the two circular-polarization eigenstates, written as $\mathbf{E}_\pm \propto (\mathbf{x} \pm i\mathbf{y}) \exp i(kz - \omega t)$. Now assume that each circular polarization eigenstate propagates through the sample with its own index of refraction, n_\pm , and its own attenuation constant, β_\pm . Write the light field that emerges from a sample of length L , under these assumptions. (This will, in general, be a case of *elliptical* polarization.) Yet its x and y components are easy to read off; so find the intensity of the light that will reach the two detectors, sensitive as they are to x - and y -polarized light respectively. Finally, write the *difference* of the two intensities, and show that it reveals a $\sin(2\Delta\phi)$ -sensitivity to a polarization-rotation angle, where $\Delta\phi = (2\pi/\lambda)(n_+ - n_-)(L/2)$, but with an accompanying attenuation factor that can be written $\exp [-(\beta_+ + \beta_-)L]$.

Now here's how we build a 'balanced polarimeter' in practice:

First, we need to form two ‘clones’ out of a single light beam. For this, we use the nominally 50:50 beam splitter, but we use it at near-normal incidence. In this configuration, it is *not* guaranteed that it creates a transmitted and a reflected beam, each with half of the input beam’s power. But in this configuration, the beamsplitter will transmit some fixed fraction, and reflect another fixed fraction, of the incoming light, and those fractions will *not* depend on the state of polarization of the light.

Now in the two beams emerging, we place Polaroids in their rotatable mounts, and we set the two Polaroids’ pass-directions to lie at $+45^\circ$ and -45° relative to the original state of linear polarization of the light. (These projection-directions thus match the odd choice of the directions of x - and y -axes we’ve drawn above.)

Next, we let each of the two beams fall upon its own photodetector, to give an electronic signal proportional to the intensity of the beam coming through the Polaroid. The difference between these two signals could be formed and monitored using the “ch.1 – ch.2” function built into an oscilloscope, but you’ll eventually be using the electronics dedicated to this purpose that’s already built into your diode-laser controller.

Setting up an optical table for Resonant Faraday Rotation

Here’s a block diagram of a typical layout that might be used for the study of resonant Faraday rotation. We assume you’ve already learned to mount objects on a standard optical table, and that you’re familiar with many of the optical elements in your Diode Laser Spectroscopy system. We further assume that you know the **safety rules** of wearing protective eyewear, and that you know the use of an IR viewing card, and/or the CCD camera, for following a laser beam around your optical table.

Here’s our suggestions for a layout. Start with the laser on a bare optical table, aimed as shown to send a beam parallel to the long edge of the table. Next put the two steering mirrors in place, ensuring that the laser beam reflects off the approximate centers of their reflective faces. Now use the four steering knobs at your disposal to get a beam which flies along the length of the optical table, at a fixed height (4” or 10 cm), and parallel to the long edges of the table.

Now center the Rb vapor-cell assembly on this aimed laser beam. This might be a good time for tuning the laser; using either a transmission method or a fluorescence method to confirm you can reach the vicinity of the Rb resonances.

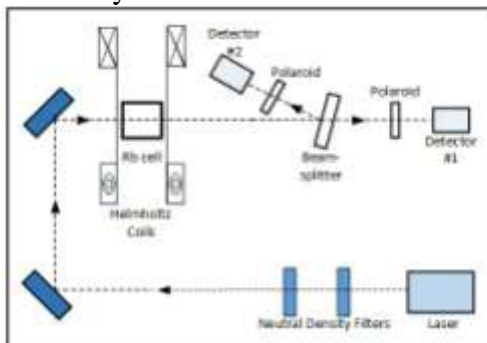


Fig. 2: One possible arrangement for an optical table for demonstrating resonant Faraday rotation.

Next, you might deploy the Helmholtz-coil assembly, putting it into position so that the Rb vapor cell takes up the desired location, centered on the laser beam. For now, it would be just as well to leave the coils un-powered.

Then it's time to position the '50:50' beamsplitter into the beam emerging from the vapor cell, at approximately the orientation shown. (You want the reflected beam to 'have room' around it for the components you are about to place in the beam, so you can't use the beamsplitter *exactly* at normal-incidence). Notice that the mount for the beamsplitter has two knobs, giving you a tip-tilt fine adjustment of where that reflected beam goes.

Now it's time for coarse, and for fine, optical alignment:

For *coarse* alignment, you can use the laser beam at full intensity, and follow the beam around the optical table with a viewing card. Confirm that the beam hits both steering mirrors, and the entrance and exit faces of the vapor cell, and the beamsplitter, near their respective centers. Also, use the table-mounting screws to get the two photodetectors approximately centered on the transmitted, and the reflected, beams from the beamsplitter.

For *fine* alignment, put one neutral-density filter into the path of the laser beam, near the laser head's output, to dim the beam (nominally by 10-fold in power). Now your goal is to display the electrical signals emerging from the photodetectors, and use those signals as a guide to alignment. To get the signals, remember you need to supply the photodetectors with power (for their internal current-to-voltage converters). You can get the necessary power from the 4-pin connectors on the Diode-Laser controller's front panel. To see the signals, we suggest you use an oscilloscope, in the 'roll' or 'scroll' mode, to draw a real-time continuous trace, in which it's easy to see changes.

Recall that these photodetectors' output signals are *negative-going* – the output is below 0 V, and gets farther below zero for increasing power in the incoming laser beam. If you see an output around -12 V, you are saturating the electronics in the detector, and should change to a lower 'gain' setting on the photodetector.

Start with the transmitted beam – find the signal from its photodetector, and use the two knobs on the second, or 'downstream' steering mirror to maximize this signal. You are fine-adjusting the direction of the transmitted beam to fall onto the center of the photodiode in this detector.

Once you have that optimized, leave both steering mirrors alone, and work on the reflected beam from the beamsplitter. Find the signal from its photodetector, and use the two knobs on the beamsplitter's tip-tilt mount to maximize this signal. Without disturbing the transmitted beam, you are fine-adjusting the reflected beam to fall onto the center of the photodiode in this detector.

When you have visible signals from both photodetectors, arrange your 'scope to display them both at once, using its dual-channel capability. It is neither necessary nor expected that the signals will be the same size – the beamsplitter is not guaranteed to be 50:50 at near-normal incidence.

Exercise:

What power do you have in each beam? Recall that optical power is converted, in the photodiode, to electrical current at a ‘rate’ near 0.6 A/W; and then this current is converted to a voltage, in an op-amp circuit, at a ‘rate’ set by the ‘gain’ setting, which has a label in units of resistance. The output voltage is thus $V_{\text{out}} = (-) R_{\text{set}} \cdot I_{\text{diode}} = (-) R_{\text{set}} \cdot [P_{\text{opt}} \cdot (0.6 \text{ A/W})]$. Choose at least one setting of R , read the output voltage, and infer the optical power P_{opt} .

Now put a Polaroid into each of the two beams whose intensity you are monitoring. Separately for each beam, rotate the Polaroid until you get maximal transmission of light. You’re finding the ‘pass direction’ of each Polaroid, which might occur near the 0° -marking on the device’s protractor scale. The signal will *not* be as large as it was in the Polaroid’s absence, since a Polaroid’s transmission is not as large as 100% even when its pass direction matches the light’s polarization direction.

Next, it’s time to rotate each Polaroid, nominally by 45 degrees. In practice, it’s easier to use the fact that $\cos^2(45^\circ) = 0.5$, and just to turn the Polaroid until the ‘scope-visible output signal drops to *half* its former value. The trick is to turn the two Polaroids in the right *relative* directions, so that when there are future rotations $\Delta\phi$ of the polarization-angle of the light beam, the new \mathbf{E} -vector will come *closer* to the pass direction for one Polaroid, and *farther* from it for the other. Use your capacity for 3-space thinking, and perhaps a ‘prop’ such as a short length of stick to depict an \mathbf{E} -field vector, to achieve this.

Optional exercise:

If you have access to a third Polaroid, in a rotatable mount, that will work at 780 nm, there is a direct way to make sure you have set up your polarimeter correctly. Put that third Polaroid into the laser beam just *upstream* of the vapor cell, and rotate it to pass the maximal amount of light. (You might first find its ‘extinction’ orientation, and then rotate it 90° from this orientation.) Now recall something about a polarizer that is lined up with the direction of the pre-existing linear polarization of light: if you rotate that Polaroid by angle θ , then

- to first order in θ , there is *no* change in the intensity of the output beam;
- but the output beam *is* rotated in polarization, by exactly angle θ .

So if you rotate that ‘third Polaroid’ by $\pm 5^\circ$, you are creating a sort of ‘calibration signal’ to test your polarimeter downstream. Watch the two photodiode signals under a 5° -change in that third-Polaroid’s orientation, and check that to see that one of them rises, and the other falls.

Once you have completed all these tests, you are ready for forming the ‘difference’ that will balance your polarimeter. Assuming the diode-laser beam emerging from the vapor cell is linearly polarized, and has its polarization vertical, and further assuming that you have the two detectors’ Polaroids rotated by $\pm 45^\circ$, you should have approximately equal optical power falling on both photodiodes. So you can probably use the same gain setting on both photodiodes, and you should see approximately equal voltages from the two detectors. But if the signals differ by a factor of up to 2, the signals can still be balanced – here’s how.

Remove those signal cables from the two channels of your ‘scope, and bring them to the two BNC connectors labelled Detector Inputs in the Detector section of your diode-laser controller. The panel graphics there illustrate the difference-amplifier you are about to use. Set the gain of that amplifier to 1, and set both Balance controls fully clockwise. Now monitor, with one channel of your ‘scope, the output of the difference amplifier. You won’t (yet) be getting zero. So find out which (of the two) Balance knobs needs to have its setting reduced-from-maximal in order to bring this difference signal closer to zero. Leaving the *other* Balance control at its maximum, use the control you’ve found to bring the output right to zero. Once you’ve done so, you’ve ‘balanced your polarimeter’. You can now raise the gain of the difference amplifier (if you want more sensitivity), and/or use the low-pass filter in the signal-flow chain (if you want to remove high-frequency noise).

If you have that ‘third Polaroid’ still in place, you can now confirm that your single-cable output to the ‘scope does retain polarization sensitivity. In fact you can confirm that rotation of that third Polaroid by angles $\pm\epsilon$ does give output-voltage changes proportional to $\pm\epsilon$; this exercise can also tell you the sensitivity you have attained, a ‘calibration constant’ in (say) Volts of output change per degree of polarization change.

Scanning over rubidium resonances

Finally, you’re ready to do some atomic physics, in which the independent variable will be the frequency of your laser, and the dependent variable will be your polarization-rotation signal from the balanced polarimeter you have just learned to build, align, adjust, and calibrate. Since the atomic physics is simplest for dimmer light, we suggest that you first add another neutral-density filter into the beam emerging from the laser (attenuating it ten-fold in power), and perhaps add a further ‘optical density’ of 0.5 or 0.7 to the beam, using the slide-mount plastic neutral-density filters (thus attenuating power by a further factor of $10^{-0.5} \approx 0.32$ or $10^{-0.7} \approx 0.20$). Now that your light beams are dimmer, you will likely want to use a higher ‘gain’ setting on both your photodiodes; and once you’ve changed those settings, you may need to re-balance your differencing amplifier. (Alas, you will have changed your polarimeter’s calibration constant as well.)

Now set your diode-laser controller to scan repeatedly over a useful range of optical frequency, for instance covering all four Rb transitions. You might not see the Rb cell fluoresce, even with the CCD camera (now that the beam intensity is so low), and you will not see any polarization-rotation signal (since there is as yet no magnetic field to create Faraday rotations!). If you want to confirm that you are scanning over the four absorption resonances, temporarily remove the cable from the photodetector in the ‘reflected beam’ from its Detector Input BNC, so your signal will be derived wholly from the ‘transmitted beam’ and its photodetector. You should recognize the pattern of four absorption dips that serve as a fingerprint for the D_2 transitions.

When you’ve seen this, reconnect that second detector cable, see the now nearly-null signal, and (finally!) send a steady direct current of order 1 Ampere through the Helmholtz coils. You should see the near-null baseline suddenly sprout four features, each of which has a new and characteristic shape – with a central lobe of one sign, with adjoining lobes of the opposite sign.

Your signals should grow, approximately linearly, with coil current. If you dial down the current to zero, it is safe to interchange its two leads to the power supply; then dialing up the current should give a field \mathbf{B} in the opposite direction, and Faraday-rotation signals of the opposite sign. You are seeing Faraday rotation, confirming that it's resonant (ie. dependent on the tuning of the laser to atomic resonance frequencies), and confirming that it's proportional to magnetic field (in sign as well as magnitude).

That completes the set-up of the optics, the magnetics, and the electronics for detecting resonant Faraday rotation. There is a vast amount that you can do with the signals you have obtained, in particular in comparing the details of their surprising shapes with the predictions of (relatively) simple theory. Remember, the theory is simple only in the limit of really small optical intensities, so you might want again to measure the power in the beam you're sending into the Rb cell, estimate its area, and compare its irradiance to 3 mW/cm^2 .

Nonlinear Faraday Rotation

If you have done all of the above, it is trivially simple to investigate 'nonlinear' Faraday rotation. You want to get the Rb atoms exposed to a stronger light field, and to accomplish this, all that is needed is to remove one of the (glass) neutral-density filters from the laser beam. But we recommend that instead of just setting aside this neutral-density filter, that you instead put it into the laser beam leaving the Rb cell (but upstream of the beamsplitter of your polarimeter). Why is that a clever idea? For one thing, it means you won't need to re-adjust sensitivities at the photodetectors or your downstream electronics. But here's another atom-related insight:

You are used to a mental model for an optical element like a neutral-density filter, of the form

$$I_{\text{out}} = T_{\text{filter}} \cdot I_{\text{in}} .$$

Here I 's are optical intensities, or power levels, and T_{filter} is a transmission coefficient for the filter. A neutral-density filter might have a wavelength-independent value such as $T = 0.1$; a 'colored filter' is designed to have T be a certain function of wavelength. But the whole of 'linear optics' assumes that any such transmission coefficient has a value independent of the *strength* of the light used.

In this linear approximation, you'd have $T_{\text{filter}} = 0.1$ (say), independent of wavelength; and you'd have another transmission coefficient T_{Rb} for the Rb cell, with a T -value certainly depending on wavelength, and perhaps also depending on magnetic field. But the net effect of moving a filter from upstream of the Rb cell, to downstream of that cell, would merely be to change the time-order in which the light encounters two transmissive elements. So you'd expect the form of the result to change,

$$\text{from } I_{\text{out}} = T_{\text{Rb}} \cdot [T_{\text{filter}} \cdot I_{\text{in}}] \quad \text{to} \quad I_{\text{out}} = T_{\text{filter}} \cdot [T_{\text{Rb}} \cdot I_{\text{in}}] .$$

But since multiplication is commutative, you'd expect no difference at all.

But in *fact*, upon making this change-of-location of the neutral-density filter, you *will* see changes in the signals you can observe. Any such change is direct evidence against the 'optical linearity' model that's implicit in the assignment of a transmission coefficient to the Rb cell. In fact, any signal that stays *unchanged* under such a relocation of the neutral-density filter is sure

to be from an effect describable by linear optics, and so the changes you do see are direct evidence in favor of optical non-linearity.

With that point established, go ahead and work in the regime of moderately high intensity falling on the Rb cell. Now, what sort of systematic data ought you to take? The independent variables you have under your control still include laser frequency, and magnetic field strength, both of which you can vary continuously, as well as light intensity level, which you can set to various values.

The ‘resonant Faraday rotation’ is perhaps best encountered as a continuing scan over laser frequency, viewed as the magnetic field is slowly changed. But certain features on nonlinear Faraday rotation are best explored by fixing the laser frequency (perhaps to the center of one of the rubidium resonances) and then taking scans over magnetic field strength. (Scans passing continuously through zero field are very useful for understanding what’s going on.) Faraday rotation in the optically-linear regime is predicted to be a smooth function of magnetic field; but in the regime of optical non-linearity, there are Faraday rotation signals that are *not* linear, not even monotonic, in magnetic field strength. In fact, there are spectacular *departures* from linearity in magnetic field awaiting your discovery.

References

For a direct demonstration of ‘optical saturation’, using a move-the-filter method, see ‘Demonstrating optical saturation and velocity selection in rubidium vapor’ by K. Razdan and D. A. Van Baak, *Am. J. Phys.* **67**, 832-836 (1999).

For a treatment of the theory and practice of resonant Faraday rotation, see ‘Resonant Faraday rotation as a probe of atomic dispersion’ by D. A. Van Baak, *Am. J. Phys.* **64**, 724-735 (1996).

For an introduction to the theory and practice of nonlinear Faraday rotation, see ‘Nonlinear laser spectroscopy and magneto-optics’ by Dmitry Budker, Donald J. Orlando, and Veleriy Yashchuk, *Am. J. Phys.* **67**, 584-592 (1999).

Extensions

(any of which might form an entire senior project, or an undergraduate research project, and some of which are at *or beyond* the research frontier!)

There are a host of investigations you can conduct with resonant, and with nonlinear, Faraday rotation.

In the limit of low light intensities, you can compare the lineshape, under laser-frequency scan, with the theoretically-expected lineshape. For such a task, you’ll need (in the experiment) to get

a quantitative laser-frequency scale (using an interferometer, or Fabry-Perot cavity), and you'll need (for the theory) to understand Doppler broadening and its effect on resonant Faraday rotation.

In the same limit, you can try to measure the *size* of Faraday rotation empirically – what Verdet constant can you extract from your data? Does this rotation (per unit field and length of sample) scale with the density of the sample? (You can change that density, a lot, by changing the temperature of your Rb cell.) Does your Verdet constant agree with the semi-classical theory's prediction? Can you find in the literature, and test experimentally, the claim that a quantum-mechanical prediction is greater by factor 7/6? Is this true for all four of the optical resonances?

With nonlinear Faraday rotation, there are even more things to investigate. Supposing that you fix the laser frequency to lie at the center of (one of the four) optical resonances, you can extract Faraday-rotation signals as a function of field. What dispersive features do you see, centered near $B = 0$? What features do you see that are *not* centered at $B = 0$? Are such features common to all four resonances; or if not, how do they differ?

Suppose you look only at the narrowest of the dispersive features, centered at $B = 0$. How wide is your feature, measured from peak-to-valley? How does this width depend on light intensity? Can you make the width decrease farther by reducing the components of magnetic field perpendicular to the direction of light propagation? Can you reduce it still farther by widening the laser beam transversely? (You might use a lens well upstream of the Rb cell to spread the beam, and another lens downstream of it to get the beam to re-converge to small spots at the location of the photodetectors.)

More spectacular yet – suppose that upstream of the cell, you use a 90:10 beamsplitter to divide the laser light into a brighter 'pump' beam, and a dimmer 'probe' beam, and that you arrange for these two beams to pass through the same cell, *not* overlapping at all, but propagating parallel to each other, about 5-10 mm apart. Now suppose that downstream of the cell, you entirely block the pump beam, and you analyze the polarization-rotation of the probe beam using the balanced-polarimeter technique. Finally, suppose you modulate the pump beam, upstream of the cell, with an optical chopper, and that you use a lock-in amplifier on the polarization-rotation signal to isolate the effect that the chopped-pump beam has on the probe-beam's signal. You should see even more spectacular dispersive-shaped signals, as you scan the longitudinal static field. They can only be attributed to ground-state coherences, created within Rb atoms flying through the pump beam, and then continuing across empty space inside the Rb cell and interacting with the probe beam. (What's a typical thermal velocity of a Rb atom in your cell? How far, in space, would a Rb atom fly during an *excited*-state lifetime? How does this compare to your pump-probe beam spacing?)

And more speculative yet, but easy to investigate even in a single-optical-beam experiment – if ground-state coherences are superpositions of ground-state sublevels differing in magnetic quantum number m by 2 units, then all the effects you've seen in magnetic-field scans depend on an energy difference, of size $\Delta E = 2 \mu_B B$. Convert this to a 'frequency difference' $\Delta f = \Delta E/h$,

and show that $\Delta f/\Delta B$ is about 28 GHz/T, ie. 28 kHz/ μ T. Suppose you reach a peak (or a valley) of a dispersive signal at field $B = 5 \mu$ T, and you infer the existence of a pair of levels separated by frequency difference 140 kHz. What would happen if you were to bathe the Rb cell with a radio-frequency magnetic field with frequency of this order? Guidance from the parallel effects in optical-pumping experiments suggests that you try r.f. magnetic fields of transverse direction, and that you try frequency 70 kHz, hoping for a double-quantum transition to be driven between the two magnetic sublevels. Would this have any visible effect on your polarization-rotation signal? If you find such a signal, you'll be the first person ever to see this kind of 'double-resonance' effect. There's a master's-thesis worth of work in either the experiment, or the theory, of this effect (if it exists!).