

Elmwood Press
Core Mathematics C4
Paper G
(Mark Scheme)

**All exam papers are issued free to students for education purpose only.
Mr.S.V.Swarnaraja (Marking Examiner, Team Leader & Author)
www.swanash.com, Mobile: +94777304755 , email: swa@swanash.com**

Worked Solutions

Edexcel C4 Paper G

1. (a) $\frac{dy}{dx} = \frac{2 \cos t}{-\sin t}$
 when $t = \frac{\pi}{2}$, gradient = 0 (2)

(b) using $\cos^2 t + \sin^2 t = 1$,
 $x^2 + \left(\frac{y}{2}\right)^2 = 1$
 $x^2 + \frac{y^2}{4} = 1$ (3)

2. (a) $\frac{1}{x-2} + \frac{3}{2x+1} + \frac{1}{x+2}$ (using 'cover up' rule) (4)

(b) $\int_3^4 \left(\frac{1}{x-2} + \frac{3}{2x+1} + \frac{1}{x+2} \right) dx$
 $= \left[\ln(x-2) + \frac{3}{2} \ln(2x+1) + \ln(x+2) \right]_3^4$
 $= \ln 2 + \frac{3}{2} \ln 9 + \ln 6$
 $- \left(\ln 1 + \frac{3}{2} \ln 7 + \ln 5 \right)$
 $= \ln \left(\frac{2 \times 6}{5} \right) + \frac{3}{2} \ln \left(\frac{9}{7} \right)$
 $= \ln \left(\frac{12}{5} \right) + \frac{3}{2} \ln \frac{9}{7}$ (5)

3. (a) $(1+ax)^6 = 1 + 6ax + 15a^2x^2$ (3)

(b) $(1+bx)(1+6ax+15a^2x^2) = 1 + 6ax + 15a^2x^2 + bx + 6abx^2$

we have $6a + b = -9$...[A]

$15a^2 + 6ab = 24$...[B]

from equation [A] $b = -9 - 6a$

substitute in [B] $15a^2 + 6a(-9 - 6a) = 24$

Hence $a = -2$, $b = 3$ (7)

4. (a) $= \int x \frac{d}{dx}(\tan x) dx$ [By parts] (4)
 $= x \tan x - \int \tan x dx = x \tan x + \ln \cos x + c$

(b) $\int y^{-\frac{1}{2}} dy = \int x \sec^2 x dx$ $2y^{\frac{1}{2}} = x \tan x + \ln \cos x + c$

$y = 4$, $x = 0$: $2\sqrt{4} = 0 + \ln 1 + c$

$c = 4$

$2\sqrt{y} = x \tan x + \ln \cos x + 4$

when $x = \frac{\pi}{4}$, $2\sqrt{y} = \frac{\pi}{4} \cdot \tan \frac{\pi}{4} + \ln \frac{1}{\sqrt{2}} + 4$

$2\sqrt{y} = \frac{\pi}{4} + \ln 2^{-\frac{1}{2}} + 4$

$2\sqrt{y} = \frac{\pi}{4} - \frac{1}{2} \ln 2 + 4$

$\sqrt{y} = \frac{\pi}{8} - \frac{1}{4} \ln 2 + 2$

$y = \left(\frac{\pi}{8} - \frac{1}{4} \ln 2 + 2 \right)^2$ (6)

5. (a)

| | | | | | |
|----------------|----|----------------|---|----------------|---|
| x | 1 | $1\frac{1}{2}$ | 2 | $2\frac{1}{2}$ | 3 |
| $\frac{12}{x}$ | 12 | 8 | 6 | 4.8 | 4 |

(i) using two trapeziums, $I = \frac{1}{2} [12 + 4 + (2 \times 6)] = 14$ (2)

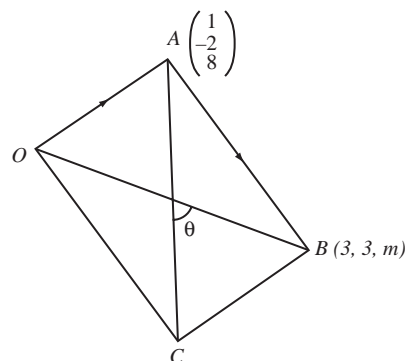
(ii) using four trapeziums, $I = \frac{1}{4} [12 + 4 + 2(8 + 6 + 4.8)] = 13.4$ (2)

(b) $\int_1^3 \frac{12}{x} dx = [12 \ln x]_1^3 = 12 \ln 3 = 13.1833\dots$

in (i) % error = $\frac{14 - 13.1833}{13.1833} \times 100 = 6.2\%$

in (ii) % error = $\frac{13.4 - 13.1833}{13.1833} \times 100 = 1.6\%$ (5)

6. (a) $\vec{AB} = \begin{pmatrix} 2 \\ 5 \\ m-8 \end{pmatrix}$



$$\vec{AO} \cdot \vec{AB} = 0$$

$$\begin{pmatrix} -1 \\ 2 \\ -8 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 5 \\ m-8 \end{pmatrix} = -2 + 10 - 8m + 64 = 0$$

$$m = 9 \quad (4)$$

(b) $\vec{OC} = \vec{AB}$ (OABC is a rectangle)

$$= \begin{pmatrix} 2 \\ 5 \\ 1 \end{pmatrix} \quad (1)$$

(c) $\vec{AC} = \begin{pmatrix} 1 \\ 7 \\ -7 \end{pmatrix}$

equation of line AC is $r = \begin{pmatrix} 1 \\ -2 \\ 8 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 7 \\ -7 \end{pmatrix}$ (2)

(d) $\vec{OB} = \begin{pmatrix} 3 \\ 3 \\ 9 \end{pmatrix}$

let angle between diagonals be θ

$$\vec{AC} \cdot \vec{OB} = |\vec{AC}| \times |\vec{OB}| \cos \theta$$

$$\begin{pmatrix} 1 \\ 7 \\ -7 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 3 \\ 9 \end{pmatrix} = \sqrt{99} \sqrt{99} \cos \theta$$

$$3 + 21 - 63 = 99 \cos \theta$$

$$\theta = 113.2^\circ$$

acute angle between diagonals = 67° (nearest degree) (3)

7. (a) $y = 12x - e^{\frac{1}{2}x}$

$$\frac{dy}{dx} = 12 - \frac{1}{2} e^{\frac{1}{2}x}$$

$$\frac{dy}{dx} = 0, \quad 24 = e^{\frac{1}{2}x}$$

$$x = 2 \ln 24$$

(3)

(b) $y = 12 \times 2 \ln 24 - 24 = 24 \ln 24 - 24$

(1)

(c) $\frac{d^2y}{dx^2} = -\frac{1}{4}e^{\frac{1}{2}x}$

so $\frac{d^2y}{dx^2} < 0$ and the stationary point is a maximum.

(1)

(d) area = $\int_2^4 \left(12x - e^{\frac{1}{2}x}\right) dx = \left[6x^2 - 2e^{\frac{1}{2}x}\right]_2^4$

$$= 96 - 2e^2 - (24 - 2e) = 72 + 2e(1 - e)$$

(5)

8. (a) $4x - \left(x \frac{dy}{dx} + y \cdot 1\right) + 2y \frac{dy}{dx} = 0$

$$\frac{dy}{dx}(2y - x) = y - 4x$$

$$\frac{dy}{dx} = \frac{y - 4x}{2y - x}$$

(4)

(b) at (2, -6) gradient of curve = $\frac{-6 - 8}{-12 - 2} = 1$

\therefore gradient of normal = -1

equation of normal is $y + 6 = -1(x - 2)$

$$x + y + 4 = 0$$

(3)

(c) gradient = 0 where $y = 4x$

substitute $y = 4x$ into equation of curve.

$$2x^2 - x \cdot 4x + 16x^2 = 56 \Rightarrow x = \pm 2$$

gradient = 0 at (2, 8) and (-2, -8)

(5)