Essential Components of Mathematics Intervention

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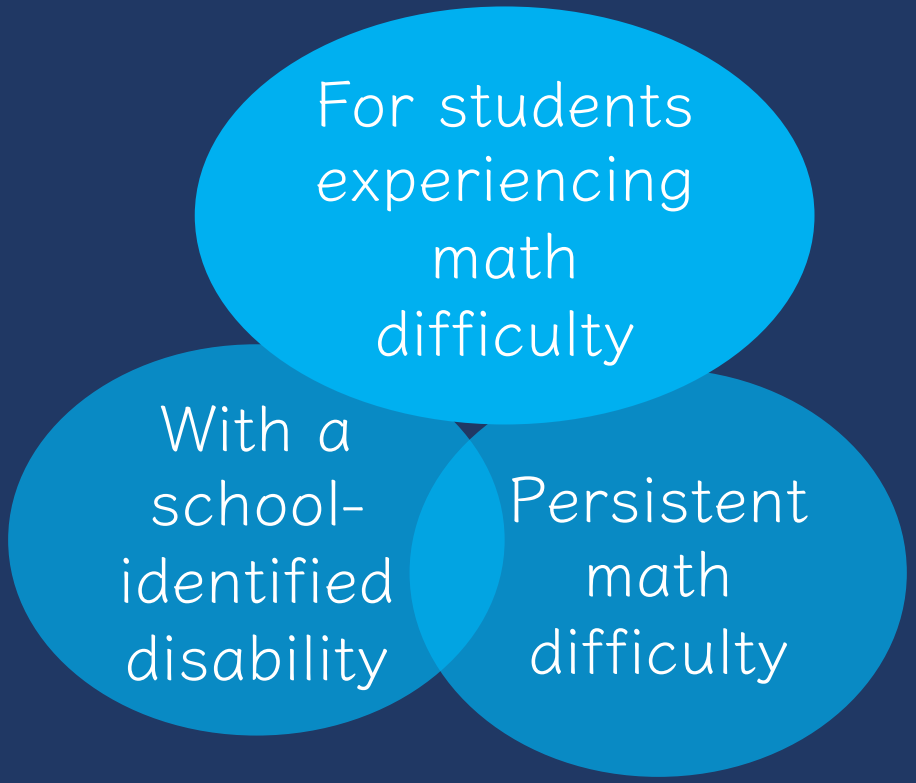
Introduce yourself.
Describe your role as an educator.
Describe the mathematics you support.

Share your Twitter handle!

Schedule for Today

9:00-10:25	- Necessity for providing mathematics intervention - Focusing on critical mathematics content - Designing your instructional platform - Explicit instruction
10:15-10:20	BREAK
10:20-11:30	- Precise mathematical language - Multiple representations
11:30-1:00	LUNCH
1:00-1:55	- Fluency and computation - Word-problem solving (attack strategies)
1:55-2:00	BREAK
2:00-3:00	- Word-problem solving (schemas) - Wrap-up and questions

MATH INTERVENTION

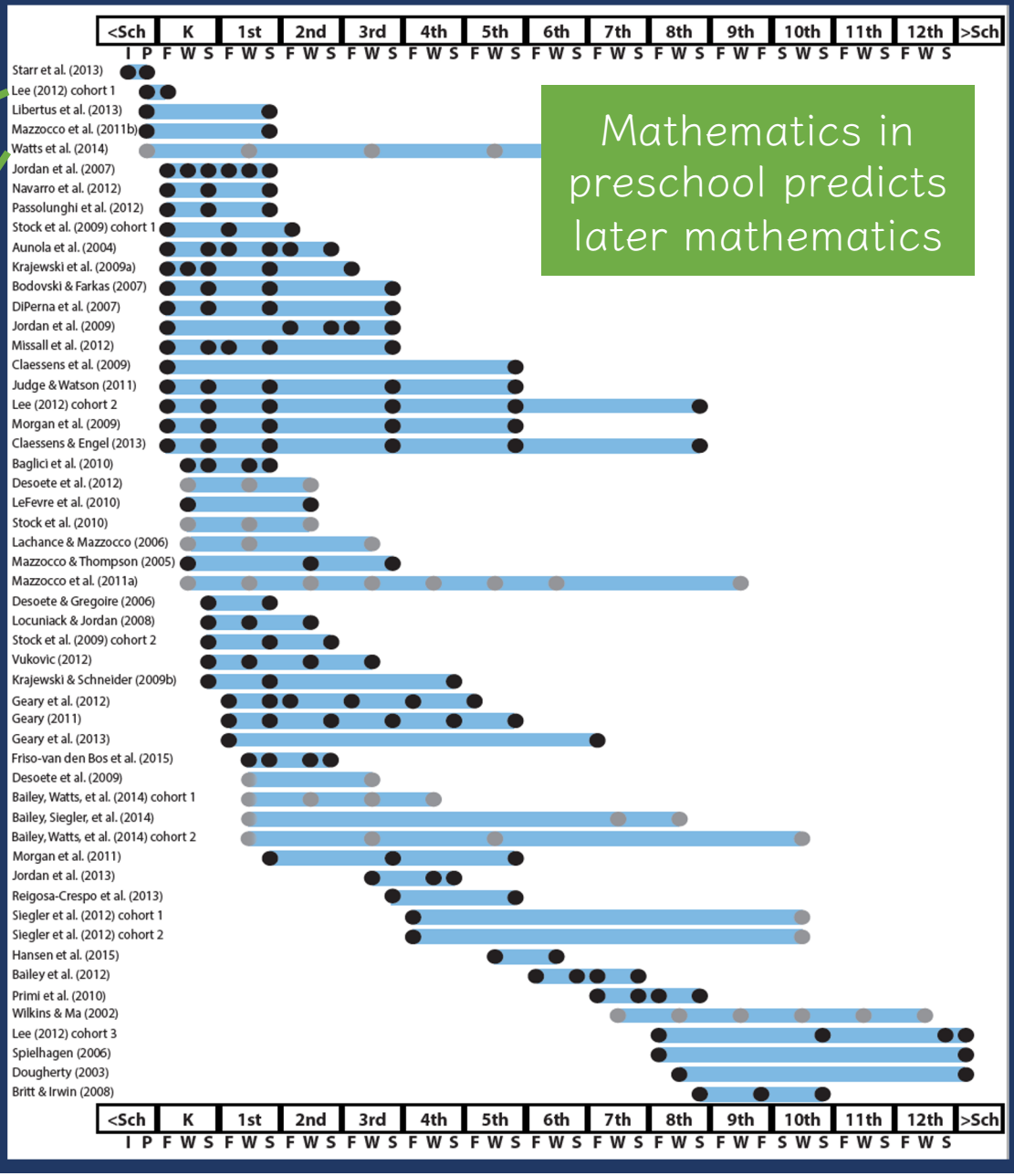


- Tier 2
- Tier 3
- Secondary
- Targeted
- Intensive
- Special Education

Why is mathematics intervention necessary?

Broad math in preK predicted K broad math

Broad math in preK predicted grade 10 broad math



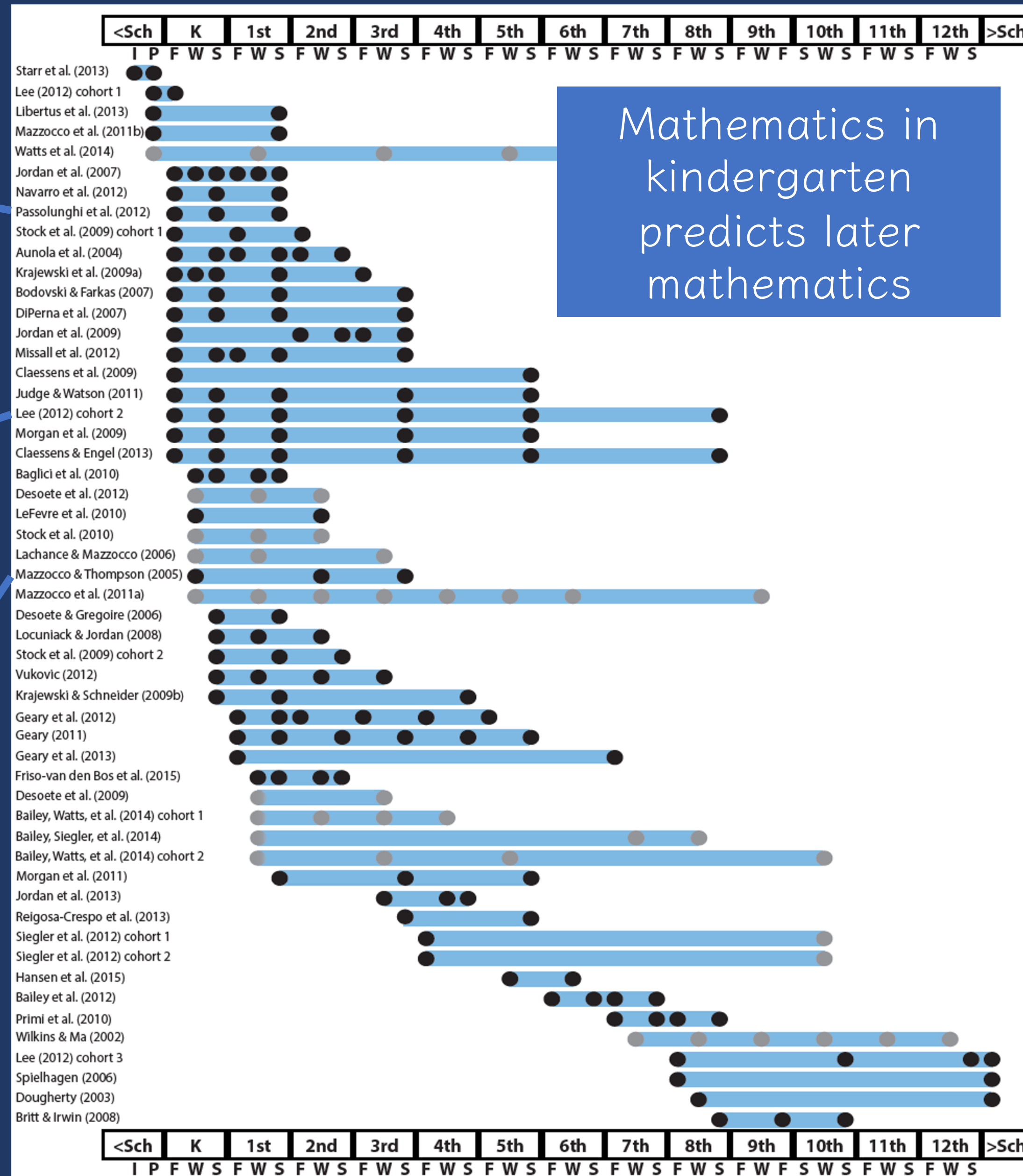
Mathematics in preschool predicts later mathematics

http://www.greatertexasfoundation.org/trajectories-of-mathematics-performance/

Counting in K predicted grade 1 broad math

Broad math in K predicted grade 8 broad math

K math accurately predicted math performance below 10th percentile in grades 2 and 3 with 84% correct classification



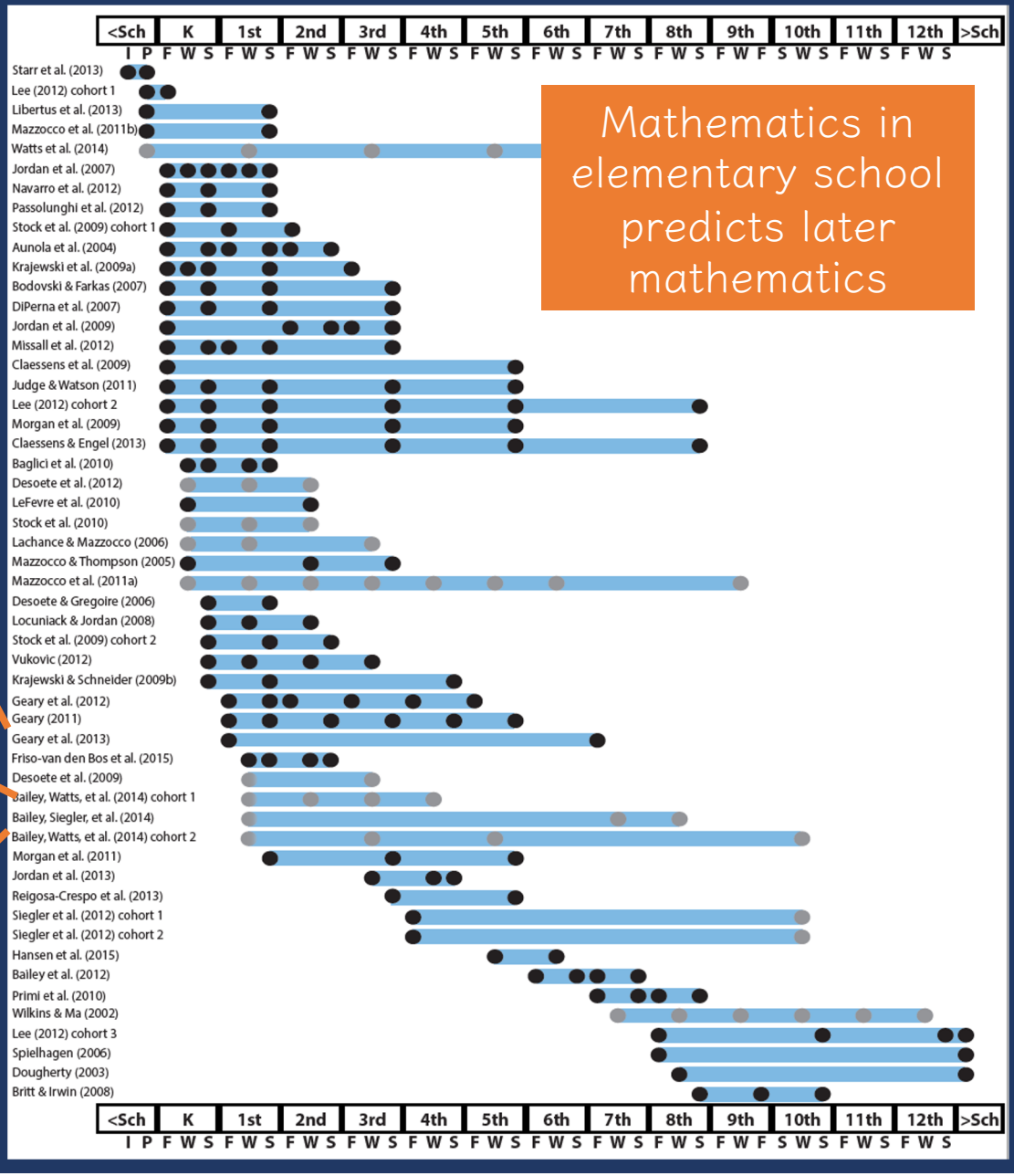
Mathematics in kindergarten predicts later mathematics

http://www.greatertexasfoundation.org/trajectories-of-mathematics-performance/

Addition influenced arithmetic with increasing importance from grades 1 to 5

Grade 1 arithmetic predicted arithmetic at grades 2, 3, and 4

Grade 1 broad math predicted broad math at grades 3, 5, and 10



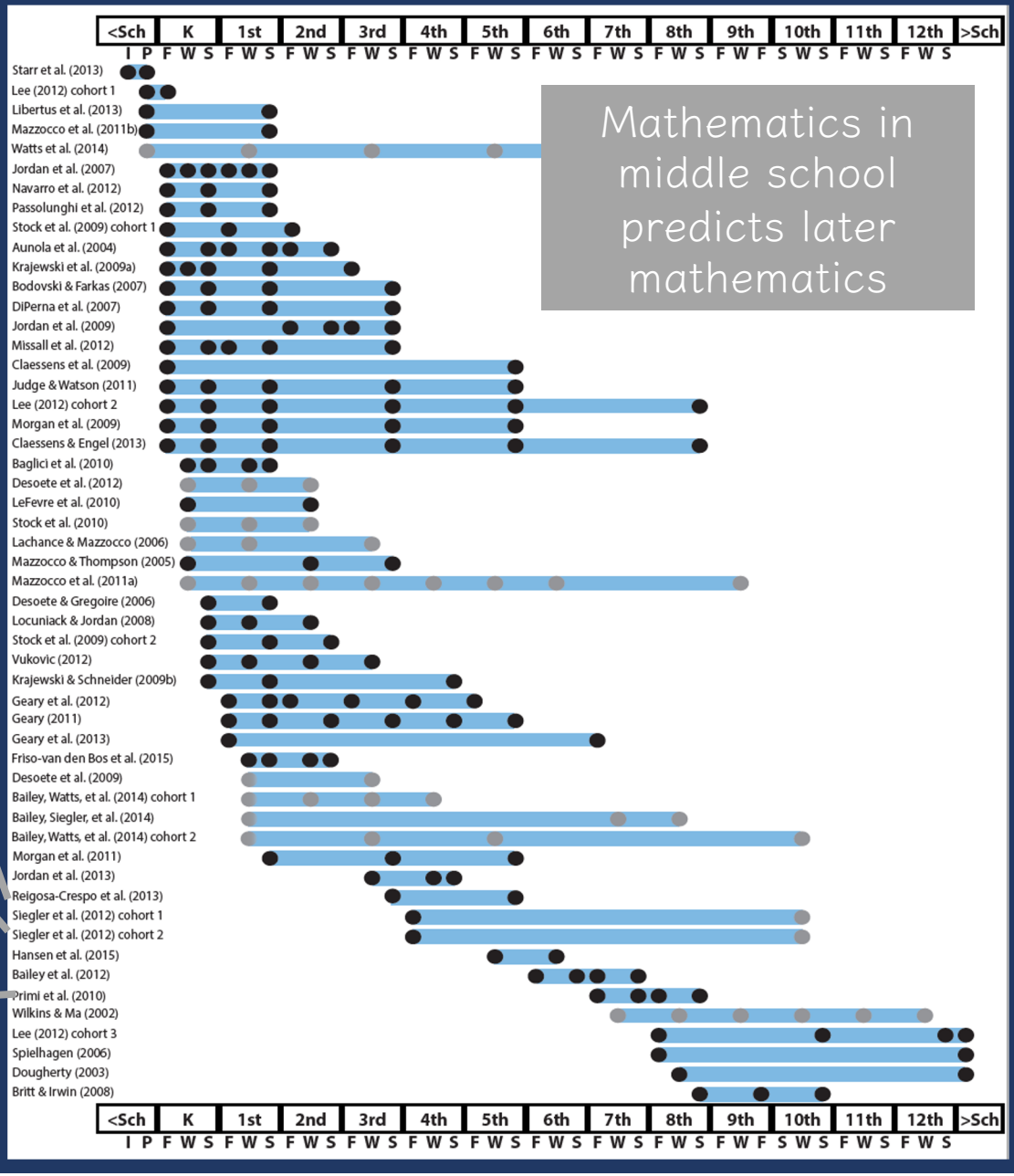
Mathematics in elementary school predicts later mathematics

http://www.greatertexasfoundation.org/trajectories-of-mathematics-performance/

Counting and comparison in grades 2 or 4 predicted broad math 1 year later

Fractions at 10-12 years old predicted broad math 5 years later

Broad math in grade 7 predicted broad math in grade 8

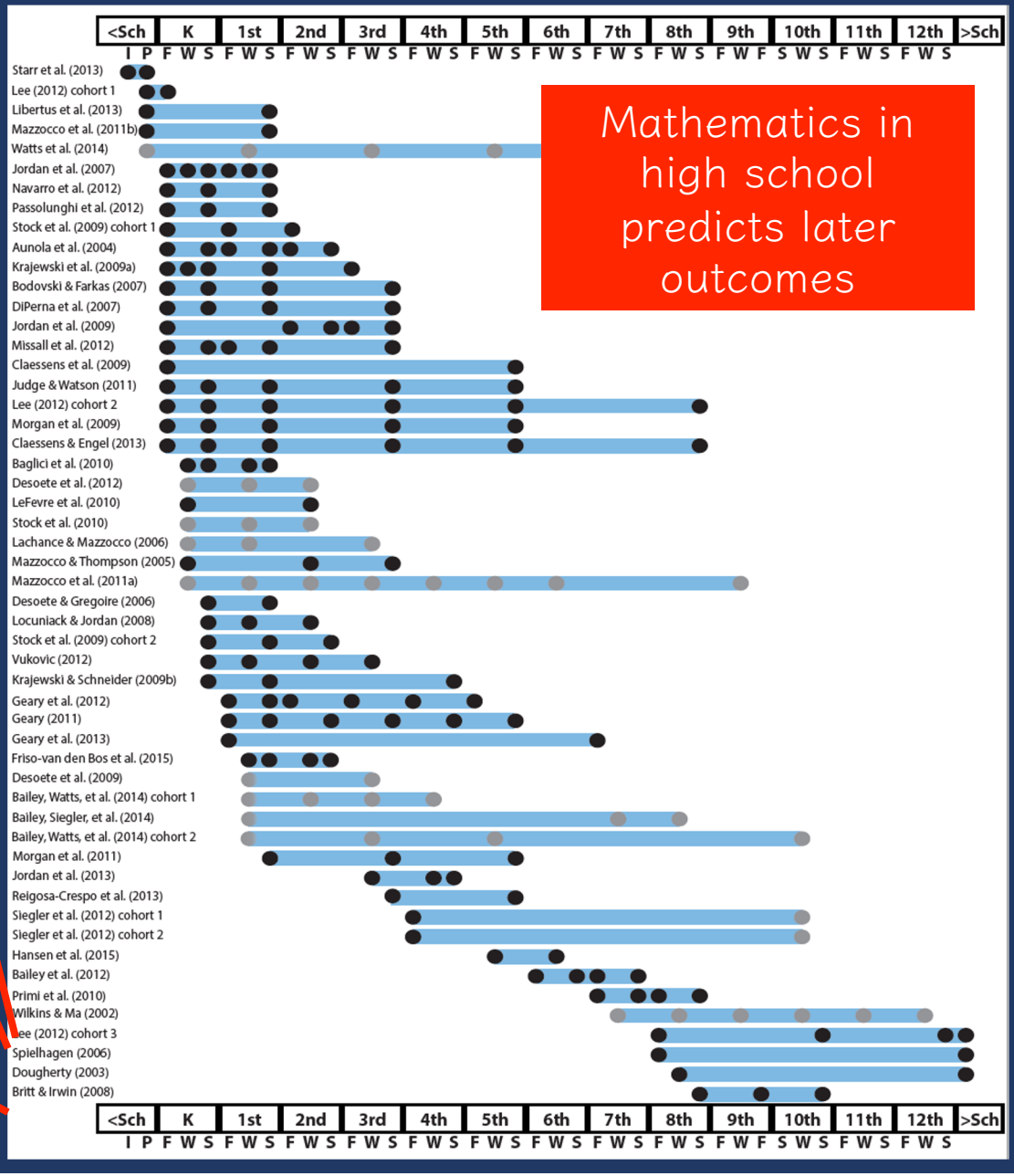


http://www.greatertexasfoundation.org/trajectories-of-mathematics-performance/

Broad math in grade 8 predicted completion of 4-year college degree

Students who took algebra in grades 8 took more advanced math courses and enrolled in 4-year colleges more often than students who took algebra in grade 9

Numeracy measured in adolescence impacted hourly earnings 7 to 15 years later



http://www.greatertexasfoundation.org/trajectories-of-mathematics-performance/

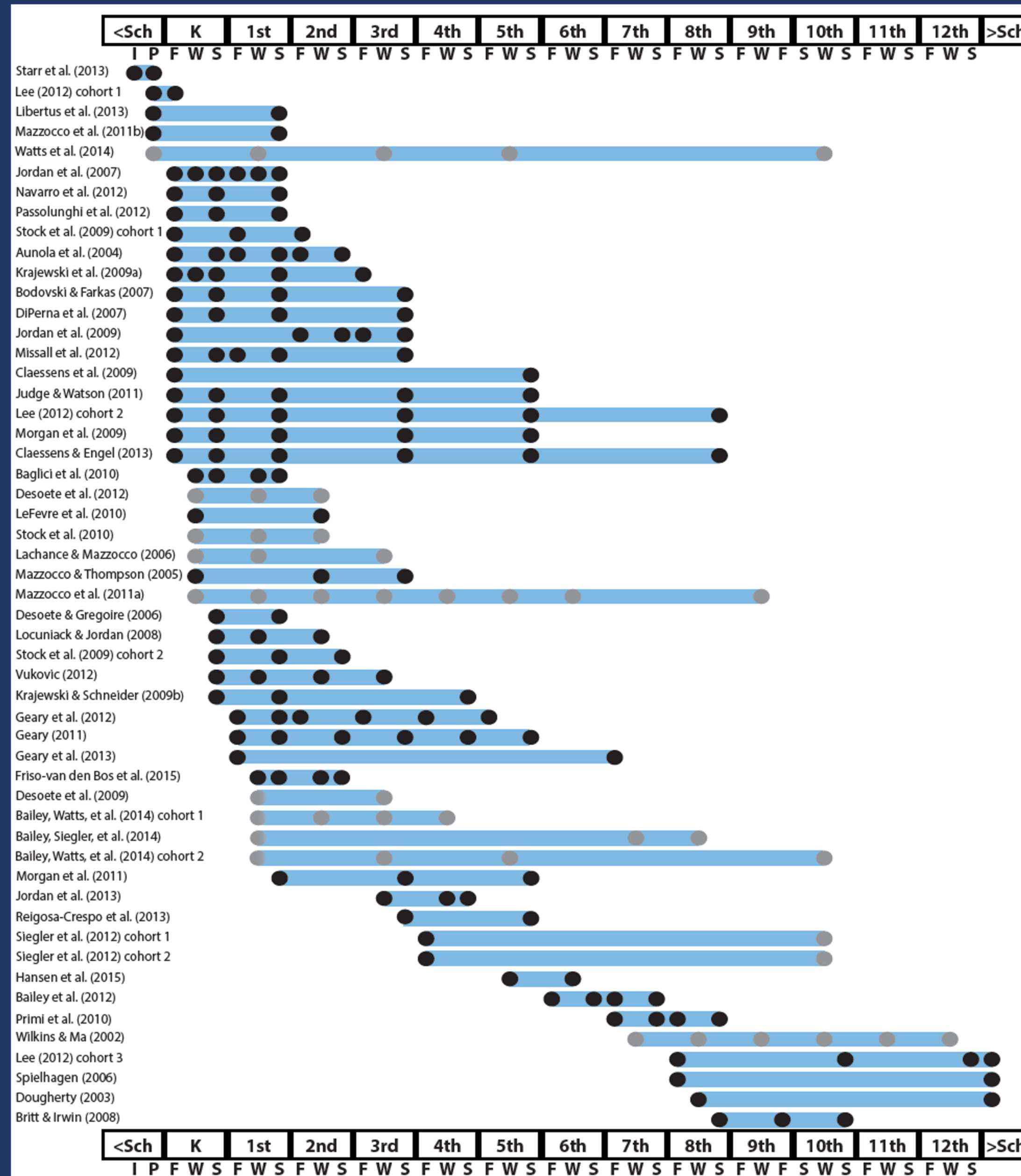
Mathematics in preschool predicts later mathematics

Mathematics in kindergarten predicts later mathematics

Mathematics in elementary school predicts later mathematics

Mathematics in middle school predicts later mathematics

Mathematics in high school predicts later outcomes



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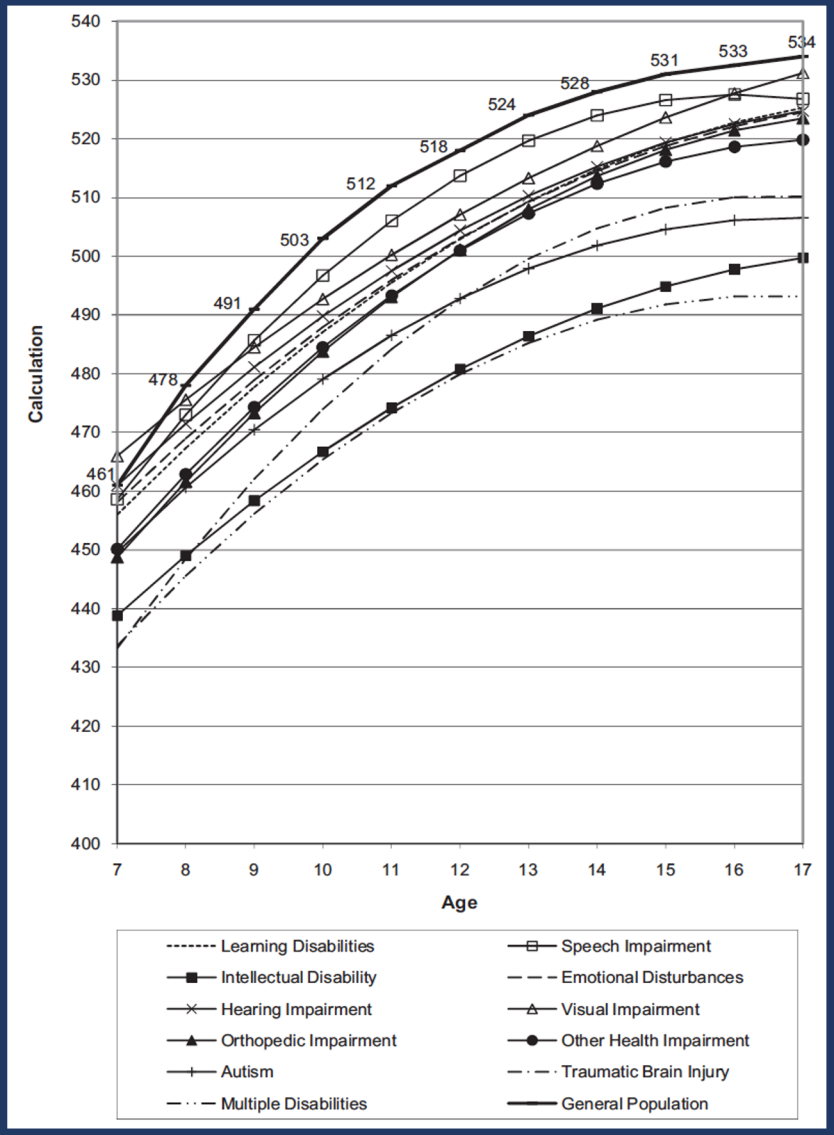
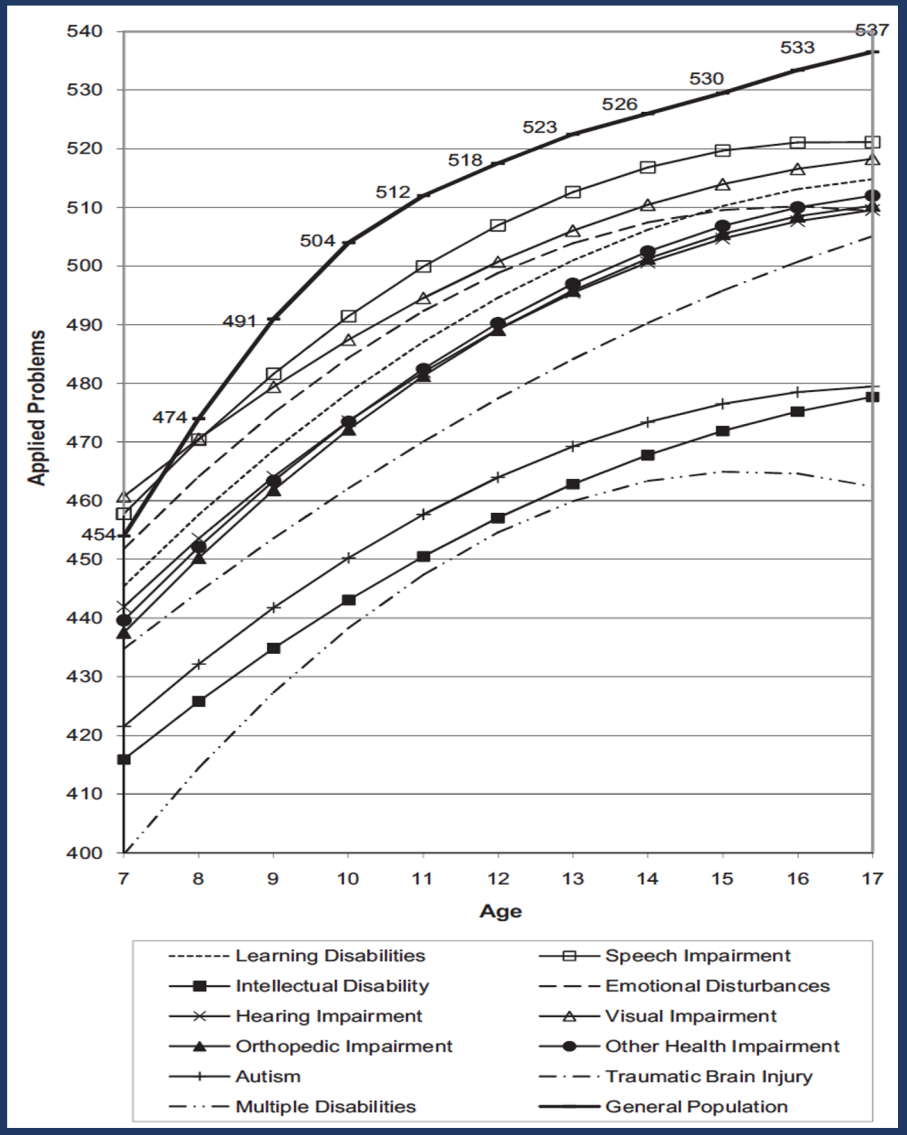


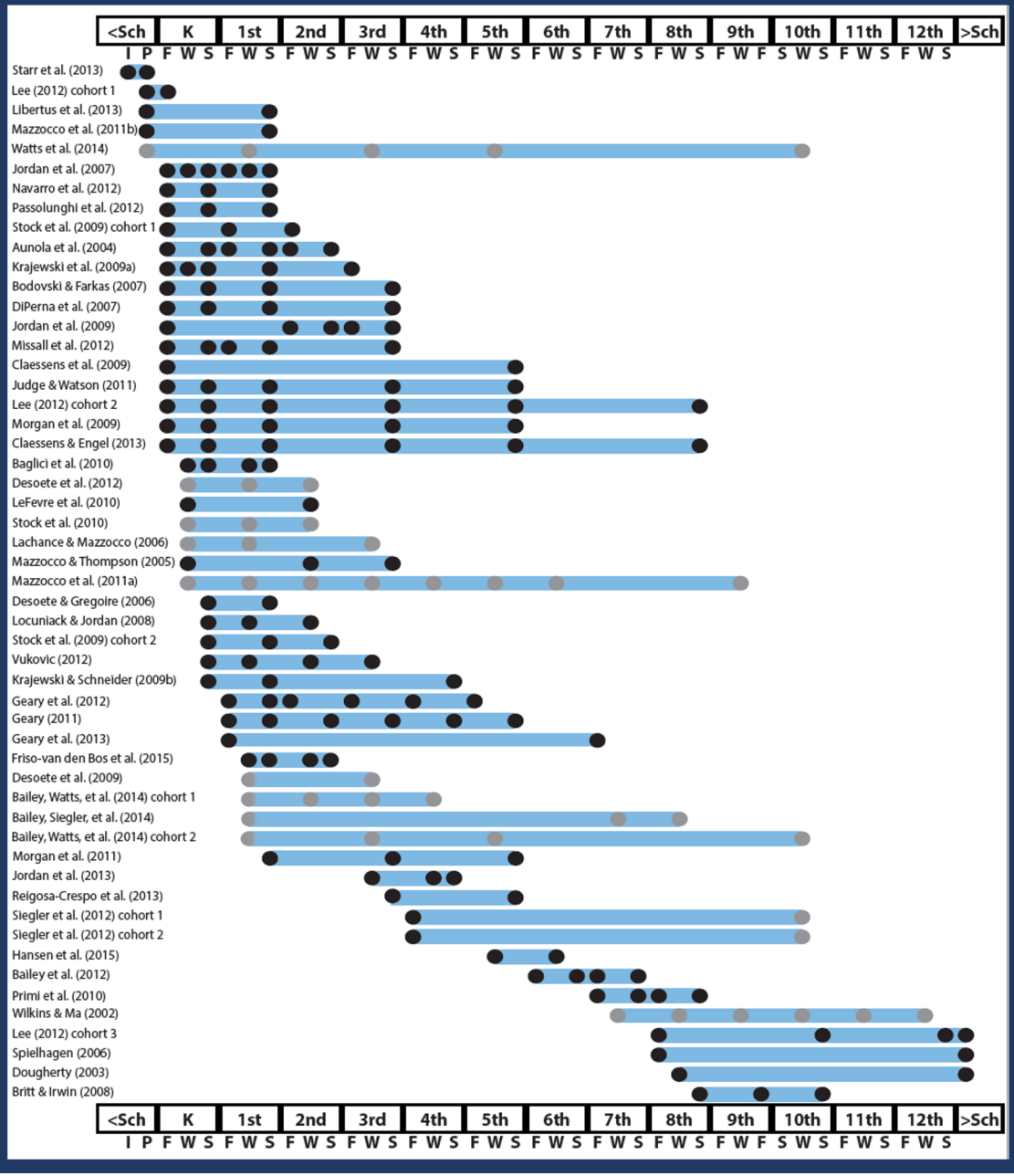
Computation



Problem Solving

Wei, Lenz, & Blackorby (2013)

How do you see earlier math relating to later math?



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DESIGN

DELIVERY

DESIGN

Determine critical content

Identify evidence-based practices

Create the instructional platform

Essential Components of Mathematics Intervention

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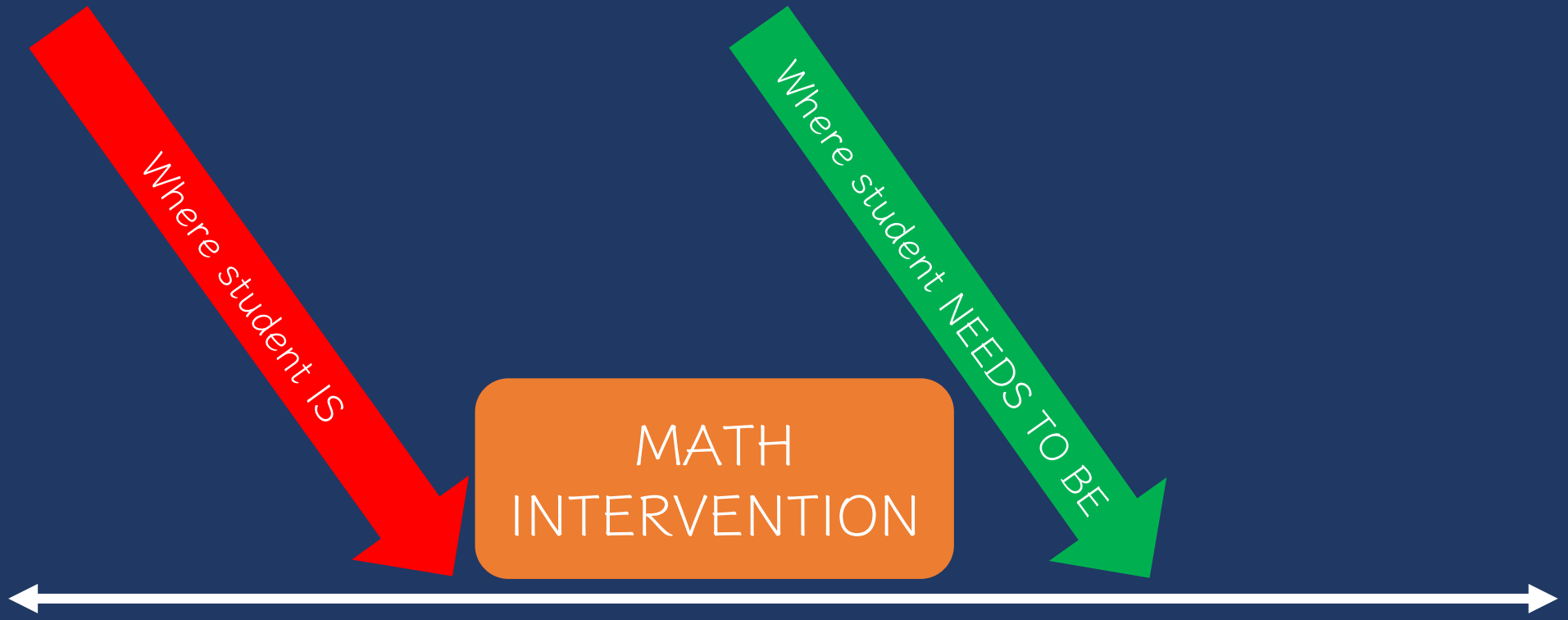
Mathematics Intervention

Critical Content

Determine critical content



Fluency

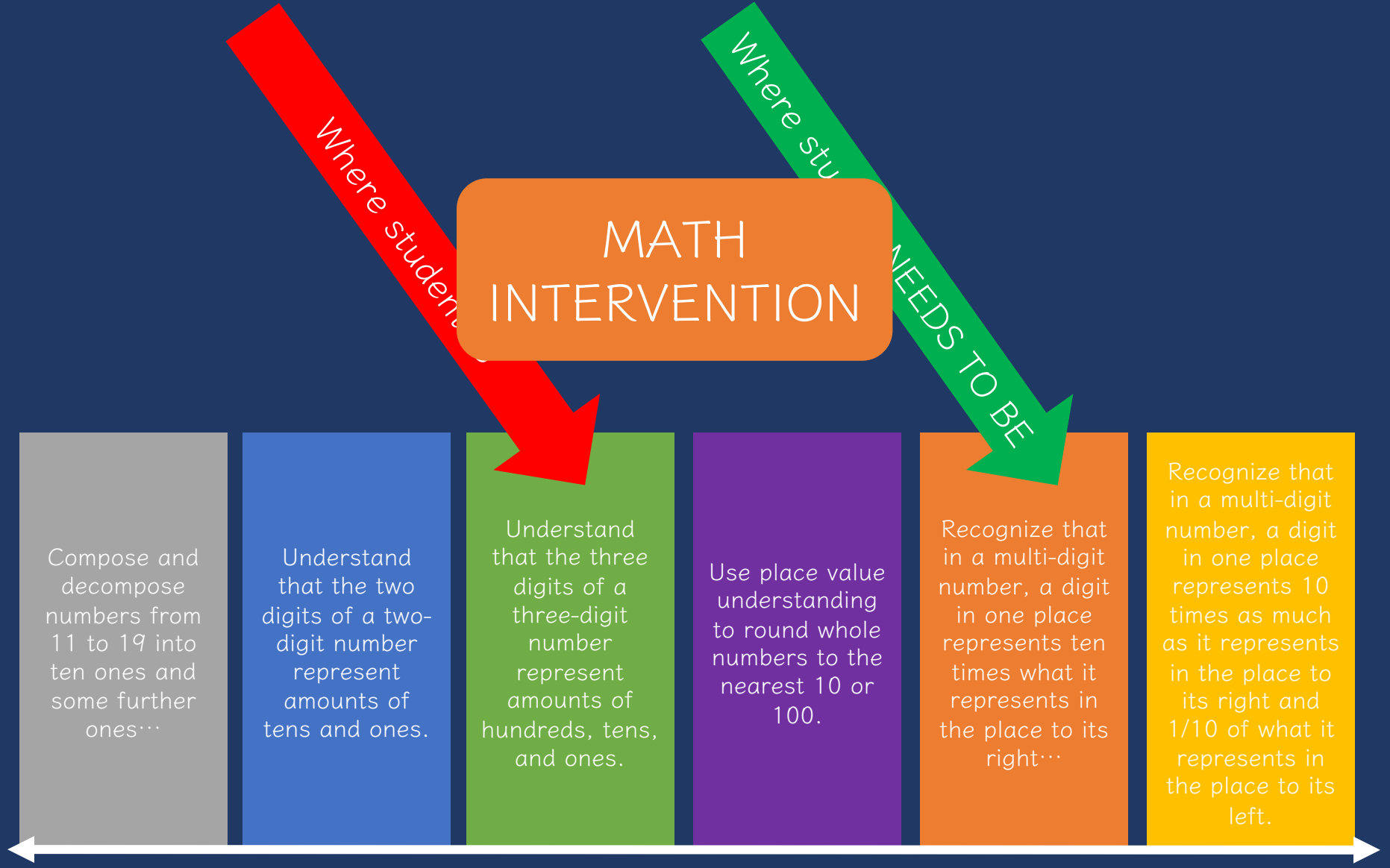
MATH INTERVENTION

Where student IS

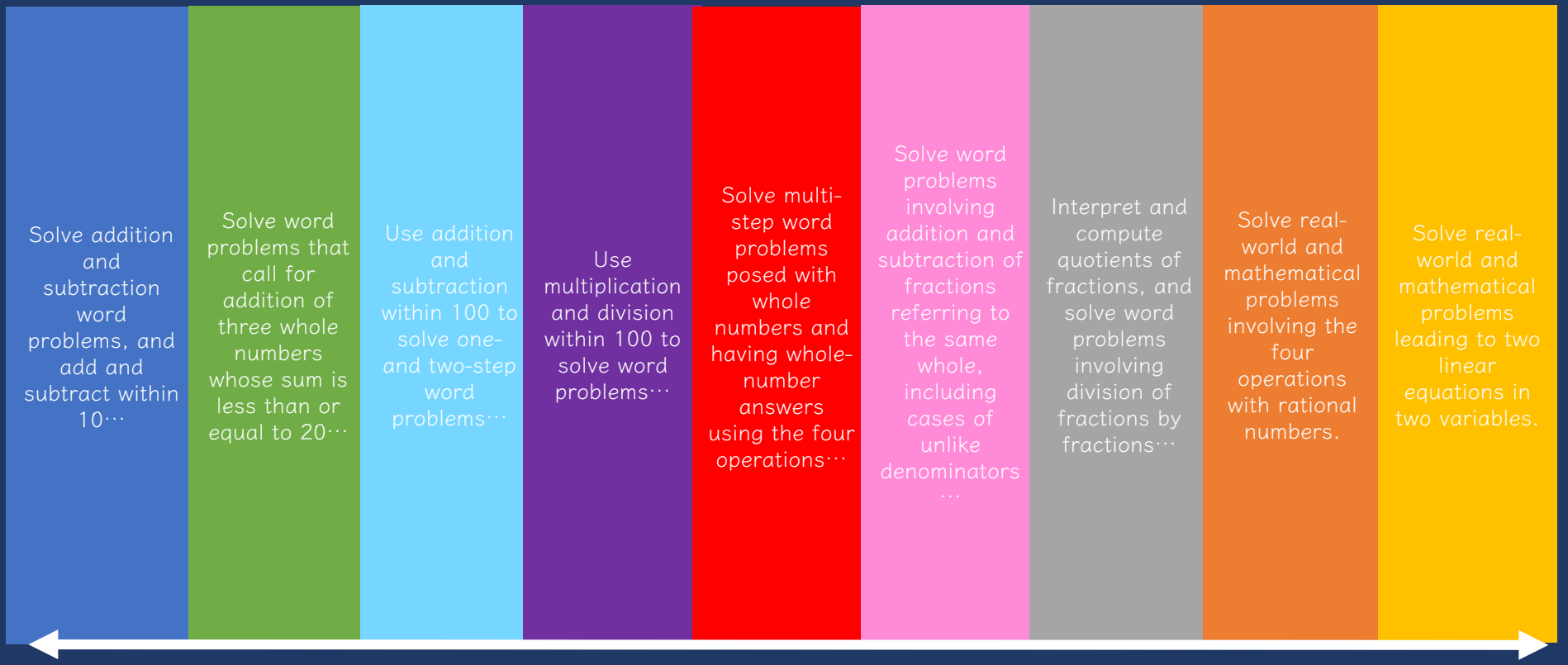
Where student NEEDS TO BE

Fluently add and subtract within 5.	Add and subtract within 20, demonstrating fluency for addition and subtraction within 10.	Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or relationships.	Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division…	Fluently add and subtract multi-digit whole numbers using the standard algorithm.	Fluently multiply multi-digit whole numbers using the standard algorithm.	Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm.

Place Value



Problem Solving



Kindergarten	Grade 1	Grade 2	Grade 3
Comparing and Ordering Numbers			
(2) Number and operations. The student applies mathematical process standards to understand how to represent and compare whole numbers, the relative position and magnitude of whole numbers, and relationships within the numeration system. The student is expected to:	(2) Number and operations. The student applies mathematical process standards to represent and compare whole numbers, the relative position and magnitude of whole numbers, and relationships within the numeration system related to place value. The student is expected to:	(2) Number and operations. The student applies mathematical process standards to understand how to represent and compare whole numbers, the relative position and magnitude of whole numbers, and relationships within the numeration system related to place value. The student is expected to:	(2) Number and operations. The student applies mathematical process standards to represent and compare whole numbers and understand relationships related to place value. The student is expected to:
(G) compare sets of objects up to at least 20 in each set using comparative language.	(E) use place value to compare whole numbers up to 120 using comparative language.	(D) use place value to compare and order whole numbers up to 1,200 using comparative language, numbers, and symbols (>, <, or =).	(D) compare and order whole numbers up to 100,000 and represent comparisons using the symbols >, <, or =.
(H) use comparative language to describe two numbers up to 20 presented as written numerals.			
	(F) order whole numbers up to 120 using place value and open number lines.		
	(G) represent the comparison of two numbers to 100 using the symbols >, <, or =.		

https://www.texasgateway.org/resource/vertical-alignment-charts-revised-mathematics-teks

An important subset of the major work in grades K–8 is the progression that leads toward middle school algebra.

K	1	2	3	4	5	6	7	8
Know number names and the count sequence Count to tell the number of objects Compare numbers Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from Work with numbers 11-19 to gain foundations for place value	Represent and solve problems involving addition and subtraction Understand and apply properties of operations and the relationship between addition and subtraction Add and subtract within 20 Work with addition and subtraction equations Extend the counting sequence Understand place value Use place value understanding and properties of operations to add and subtract Measure lengths indirectly and by iterating length units	Represent and solve problems involving addition and subtraction Add and subtract within 20 Understand place value Use place value understanding and properties of operations to add and subtract Measure and estimate lengths in standard units Relate addition and subtraction to length	Represent & solve problems involving multiplication and division Understand properties of multiplication and the relationship between multiplication and division Multiply & divide within 100 Solve problems involving the four operations, and identify & explain patterns in arithmetic Develop understanding of fractions as numbers Solve problems involving measurement and estimation of intervals of time, liquid volumes, & masses of objects Geometric measurement: understand concepts of area and relate area to multiplication and to addition	Use the four operations with whole numbers to solve problems Generalize place value understanding for multi-digit whole numbers Use place value understanding and properties of operations to perform multidigit arithmetic Extend understanding of fraction equivalence and ordering Build fractions from unit fractions by applying and extending previous understandings of operations Understand decimal notation for fractions, and compare decimal fractions	Understand the place value system Perform operations with multi-digit whole numbers and decimals to hundredths Use equivalent fractions as a strategy to add and subtract fractions Apply and extend previous understandings of multiplication and division to multiply and divide fractions Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition Graph points in the coordinate plane to solve real-world and mathematical problems*	Apply and extend previous understandings of multiplication and division to divide fractions by fractions Apply and extend previous understandings of numbers to the system of rational numbers Understand ratio concepts and use ratio reasoning to solve problems Apply and extend previous understandings of arithmetic to algebraic expressions Reason about and solve one-variable equations and inequalities Represent and analyze quantitative relationships between dependent and independent variables	Apply and extend previous understanding of operations with fractions to add, subtract, multiply, and divide rational numbers Analyze proportional relationships and use them to solve real-world and mathematical problems Use properties of operations to generate equivalent expressions Solve real-life and mathematical problems using numerical and algebraic expressions and equations	Work with radical and integer exponents Understand the connections between proportional relationships, lines, and linear equations** Analyze and solve linear equations and pairs of simultaneous linear equations Define, evaluate, and compare functions Use functions to model relationships between quantities

* Indicates a cluster that is well thought of as a part of a student's progress to algebra, but that is currently not designated as major by the assessment consortia in their draft materials. Apart from the one asterisked exception, the clusters listed here are a subset of those designated as major in the assessment consortia's draft documents.
** Depends on similarity ideas from geometry to show that slope can be defined and then used to show that a linear equation has a graph which is a straight line and conversely.

https://achievethecore.org/category/774/mathematics-focus-by-grade-level

Table A.3. Grades 6–8 Curriculum Focal Points and Connections Compared with the Expectations of the Content Standards in *Principles and Standards for School Mathematics*

Grade 6 · Grade 7 · Grade 8 · Outside grades 6–8 · Not identified at any level, pre-K–8

Curriculum Focal Points and Connections	Expectations of the Content Standards
Grade 6 Curriculum Focal Points	
***Number and Operations:* Developing an understanding of and fluency with multiplication and division of fractions and decimals** Students use the meanings of fractions, multiplication and division, and the inverse relationship between multiplication and division to make sense of procedures for multiplying and dividing fractions and explain why they work. They use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain the procedures for multiplying and dividing decimals. Students use common procedures to multiply and divide fractions and decimals efficiently and accurately. They multiply and divide fractions and decimals to solve problems, including multistep problems and problems involving measurement.	***Number and Operations, Grades 6–8***
	Work flexibly with fractions, decimals, and percents to solve problems
	Compare and order fractions, decimals, and percents efficiently and find their approximate locations on a number line
	Develop meaning for percents greater than 100 and less than 1
	Understand and use ratios and proportions to represent quantitative relationships
	Develop an understanding of large numbers [identified in Grades 4 and 5 Curriculum Focal Points] and recognize and appropriately use exponential, scientific, and calculator notation
	Use factors, multiples, prime factorization, and relatively prime numbers to solve problems
	Develop meaning for integers and represent and compare quantities with them
	Understand the meaning and effects of arithmetic operations with fractions, decimals, and integers
	Use the associative and commutative properties of addition and multiplication and the distributive property of multiplication over addition to simplify computations with integers, fractions, and decimals
	Understand and use the inverse relationships of addition and subtraction, multiplication and division, and squaring and finding square roots to simplify computations and solve problems
	Select appropriate methods and tools for computing with fractions and decimals from among mental computation, estimation, calculators or computers, and paper and pencil, depending on the situation, and apply the selected methods
***Number and Operations:* Connecting ratio and rate to multiplication and division** Students use simple reasoning about multiplication and division to solve ratio and rate problems (e.g., "If 5 items cost $3.75 and all items are the same price, then I can find the cost of 12 items by first dividing $3.75 by 5 to find out how much one item costs and then multiplying the cost of a single item by 12"). By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative sizes of quantities, students extend whole number multiplication and division to ratios and rates. Thus, they expand the repertoire of problems that they can solve by using multiplication and division, and they build on their understanding of fractions to understand ratios. Students solve a wide variety of problems involving ratios and rates.	
***Algebra:* Writing, interpreting, and using mathematical expressions and equations** Students write mathematical expressions and equations that correspond to given situations, they evaluate expressions, and they use expressions and formulas to solve problems. They understand that variables represent numbers whose exact values are not yet specified, and they use variables appropriately. Students understand that expressions in different forms can be equivalent, and they can rewrite an expression to represent a quantity in a different way (e.g., to make it more compact or to feature different information). Students know that the solutions of an equation are the values of the variables that	

https://www.nctm.org/curriculumfocalpoints/

DESIGN

What is the critical content for your students?

DESIGN

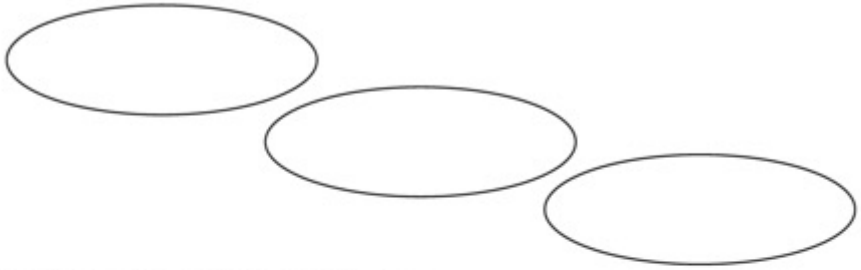
Determine critical content

Identify evidence-based practices

Evidence-Based Practices

Instructional Platform

Instructional Delivery



Instructional Strategies

evidence-based practice

A practice that has shown **consistent and positive** results

An intervention (i.e., packaged program) that has shown **consistent and positive** results

A method or strategy that has shown **consistent and positive** results

A method or strategy that has shown a **positive** result

Assessment data to show results

Improvement from before intervention

Improvement compared to no treatment students

Replication

Multiple researchers

Multiple students

Multiple times

Setting and students similar to your own

DESIGN

Which evidence-based practices do you rely on for mathematics intervention?

DESIGN

Determine critical content

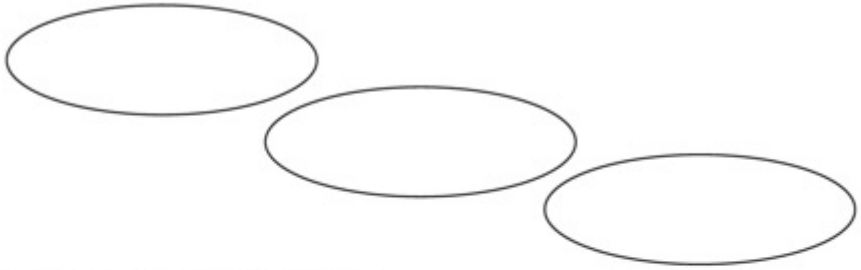
Identify evidence-based practices

Create the instructional platform

Evidence-Based Practices

Instructional Platform

Instructional Delivery



Instructional Strategies

1 Systematic Instruction: Provide systematic instruction during intervention to develop student understanding of mathematical ideas.

STRONG EVIDENCE

▾ Show More

2 Mathematical Language: Teach clear and concise mathematical language and support students' use of the language to help students effectively communicate their understanding of mathematical concepts.

STRONG EVIDENCE

▾ Show More

3 Representations: Use a well-chosen set of concrete and semi-concrete representations to support students' learning of mathematical concepts and procedures.

STRONG EVIDENCE

▾ Show More

4 Number Lines: Use the number line to facilitate the learning of mathematical concepts and procedures, build understanding of grade-level material, and prepare students for advanced mathematics.

STRONG EVIDENCE

▾ Show More

5 Word Problems: Provide deliberate instruction on word problems to deepen students' mathematical understanding and support their capacity to apply mathematical ideas.

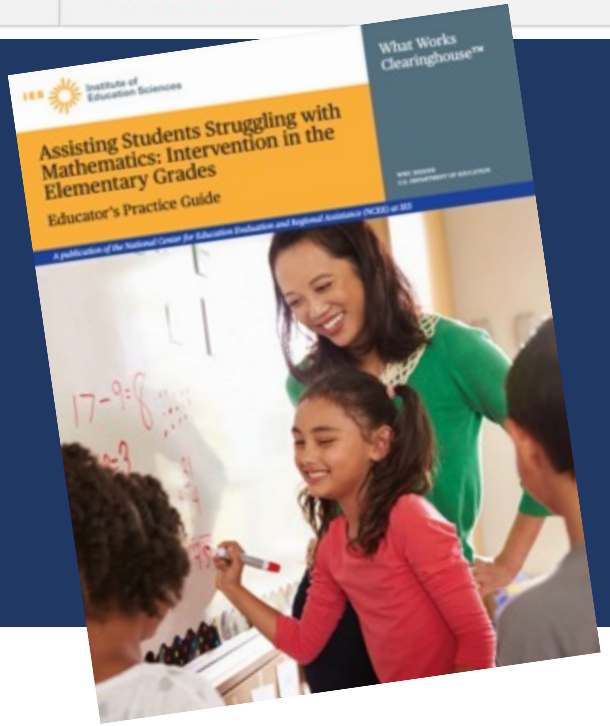
STRONG EVIDENCE

▾ Show More

6 Timed Activities: Regularly include timed activities as one way to build fluency in mathematics.

STRONG EVIDENCE

▾ Show More



https://ies.ed.gov/ncee/wwc/PracticeGuides

Instructional Platform

INSTRUCTIONAL DELIVERY

Explicit instruction

Precise language

Multiple representations

INSTRUCTIONAL STRATEGIES

Fluency building

Problem solving instruction

DESIGN

Determine critical content
Identify evidence-based practices
Create the instructional platform

For your mathematics intervention:

(1) How will you determine critical content?

(2) How will you determine evidence-based practices?

(3) What do you plan to place into your instructional platform?

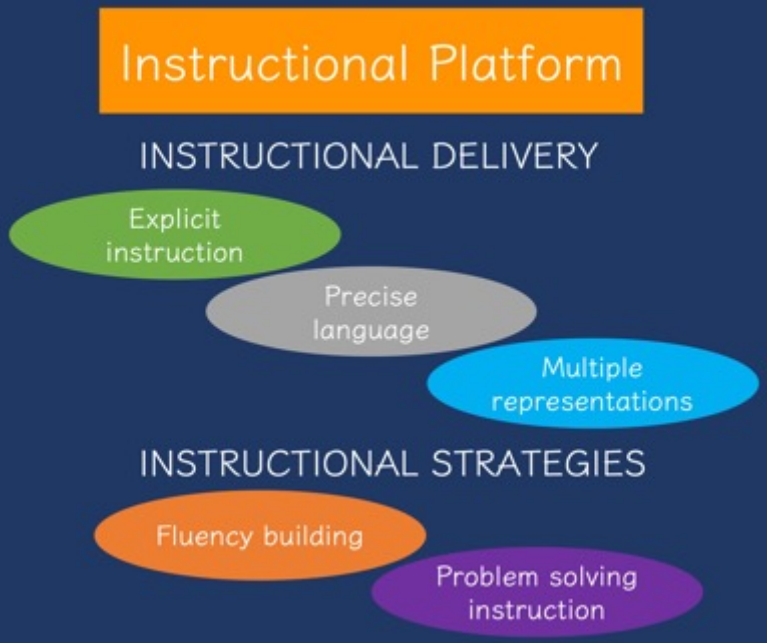
DESIGN

Determine critical content

Identify evidence-based practices

Create the instructional platform

DELIVERY

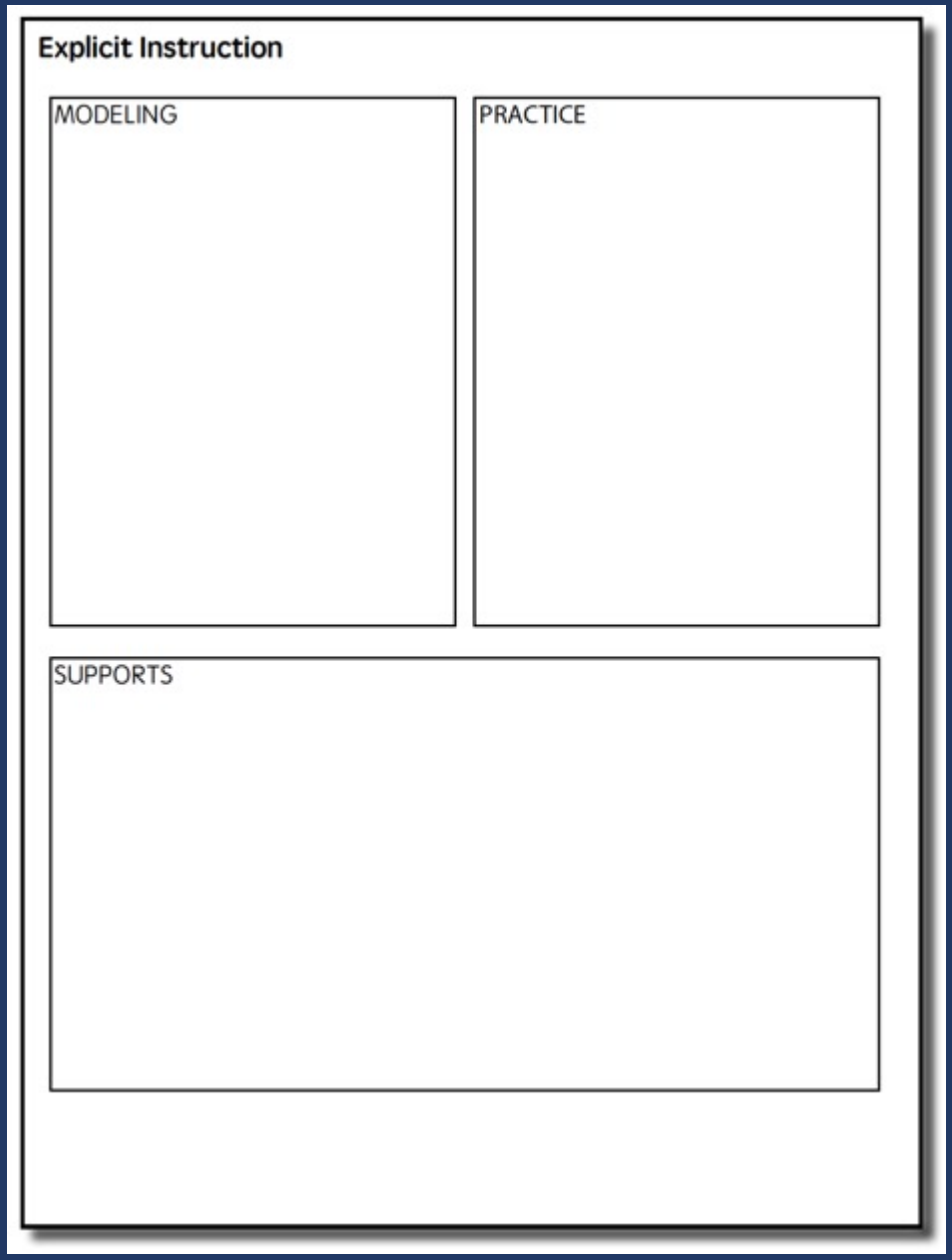


Instructional Platform

INSTRUCTIONAL DELIVERY

Explicit instruction

INSTRUCTIONAL STRATEGIES



Explicit Instruction

MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

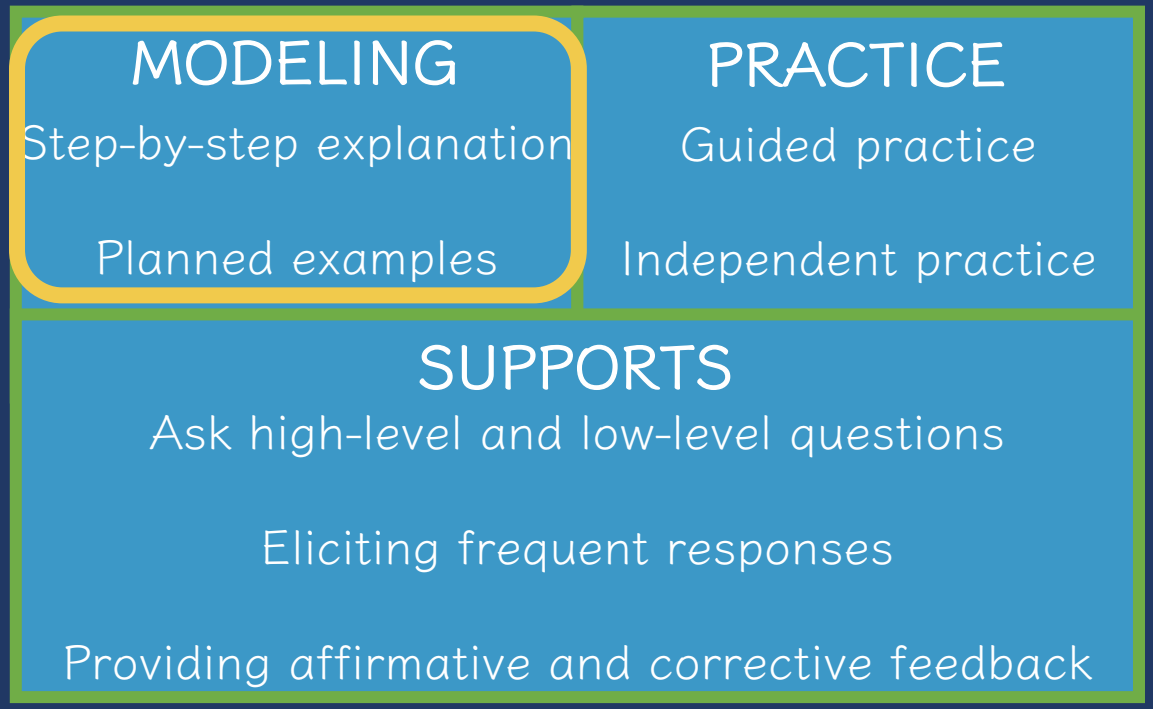
Independent practice

SUPPORTS

Ask high-level and low-level questions

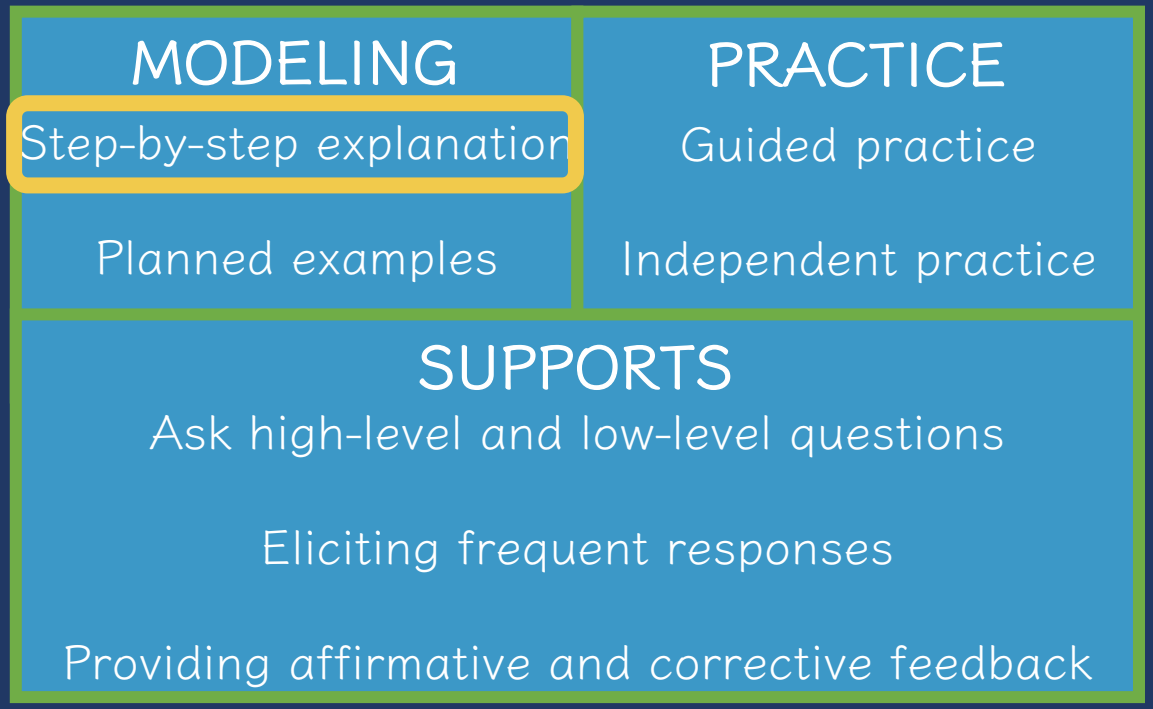
Eliciting frequent responses

Providing affirmative and corrective feedback



Modeling is a dialogue between the teacher and students.

In **Modeling,** a teacher introduces or reviews mathematical content.



Modeling includes a step-by-step explanation of how to do a mathematical problem.

A teacher may do 1 modeled problem or several.

"Today, we are learning about division. This is important because sometimes you have to share objects or things with your friends."

$$\begin{array}{r} 26 \\ +\ 79 \\ \hline \end{array}$$

"Let's solve this problem. What's the problem?"

"26 plus 79."

"To solve 26 plus 79, first decide about the operation. Should we add, subtract, multiply, or divide?"

"Add."

"How did you know we want to add?"

"There's a plus sign."

$$\begin{array}{r} 26 \\ +\ 79 \\ \hline \end{array}$$

"The plus sign tells us we want to add. To add, let's use the partial sums strategy. What strategy?"

"Partial sums."

"With the partial sums strategy, we start adding in the greatest place value. What's the greatest place value in this problem?"

"The tens."

"So, let's add the tens. What's 20 plus 70?"

"90."

$$\begin{array}{r} 26 \\ +\ 79 \\ \hline \end{array}$$

"20 plus 70 equals 90. Let's write 90 right here below the equal line. What will we write?"

"90."

"90 is the partial sum when you add the tens. What does 90 represent?"

"It's the partial sum of adding 20 plus 70."

"Now, let's add the ones. What should we add?"

"6 plus 9."

$$\begin{array}{r} 26 \\ +\ 79 \\ \hline \end{array}$$

"6 plus 9 equals what?"

"15."

"Let's write 15 below the 90. Where do we write the 15?"

"Below the 90."

"15 is the partial sum when you add the ones. Now, let's add the partial sums together. What will we add?"

"90 plus 15."

$$\begin{array}{r} 26 \\ +\ 79 \\ \hline \end{array}$$

"What's 90 plus 15?"

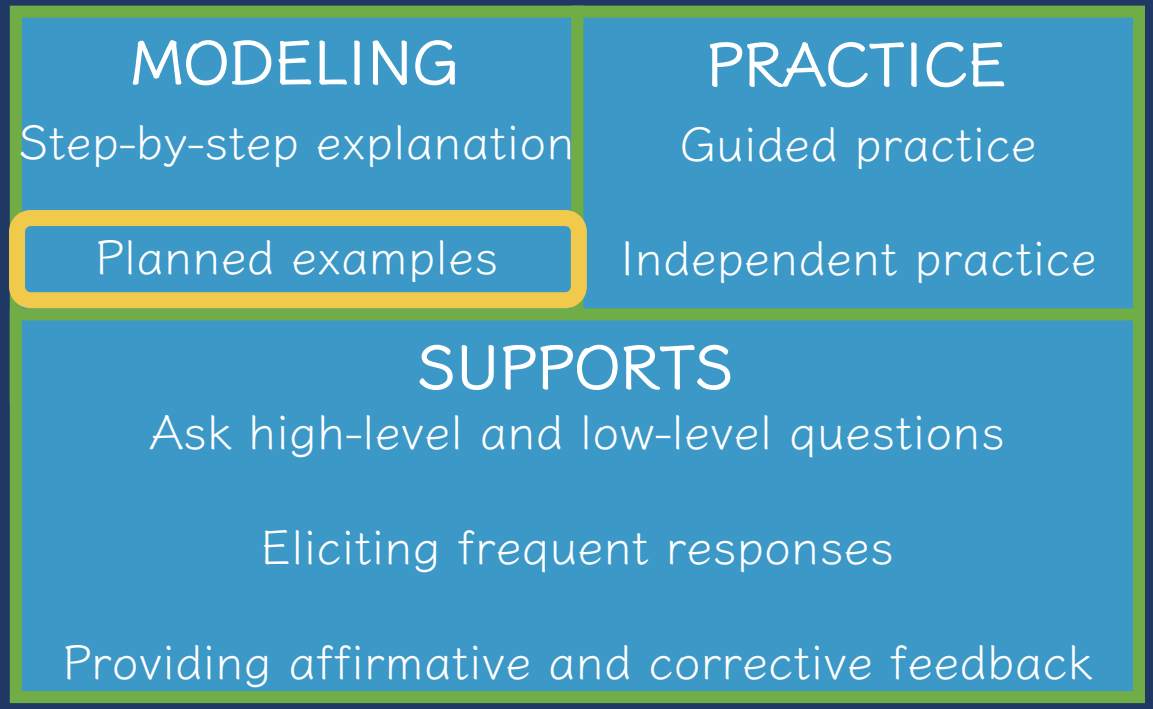
"105."

"How did you add those numbers?"

"I added 90 plus 10 then added 5 more."

"So, when you add 26 plus 79, the sum is 105. Who can share how we solved this problem?"

"We used the partial sums strategy. We added the tens then added the ones. Then we added the partial sums."



Modeling needs to include planned examples. These examples should be sequenced so easier skills lead to more difficult skills.

Planned examples in **Modeling** may also include worked examples – both correct and incorrect worked examples.

MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

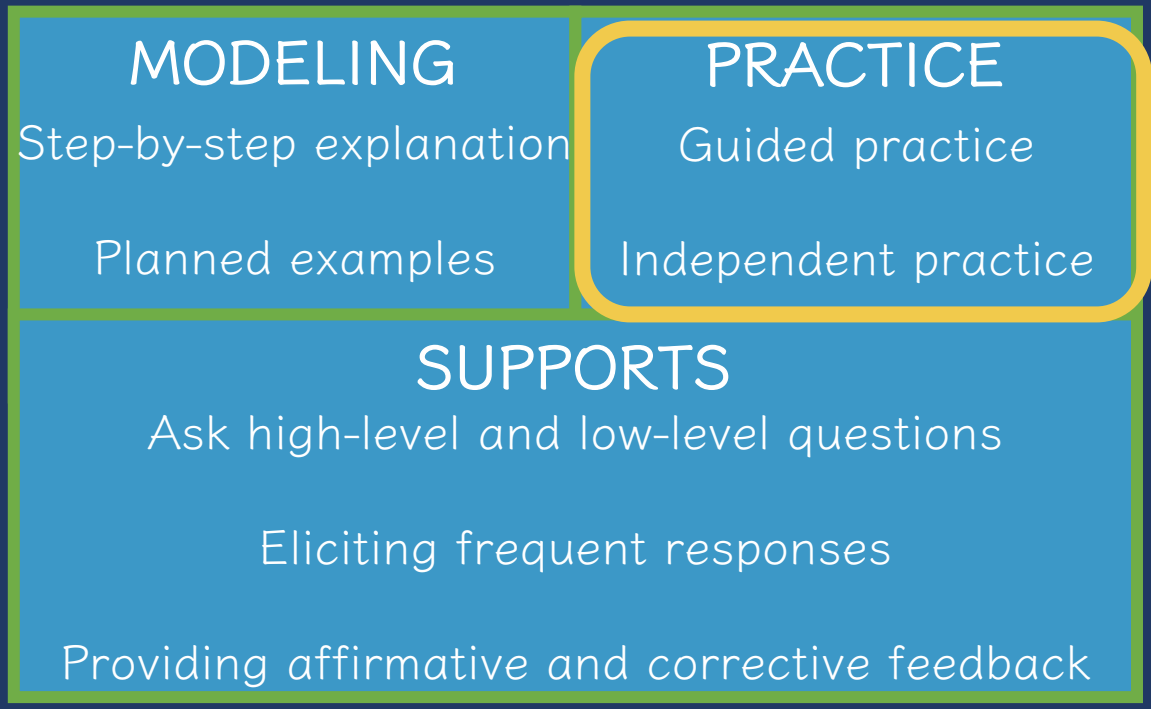
SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

Sometimes, people refer to the modeling as "I Do." I (Sarah) think that misrepresents modeling. What do you think?



Practice continues as a dialogue between the teacher and students.

During **Practice,** students have multiple opportunities to practice problems with varying levels of teacher support.

MODELING	PRACTICE
Step-by-step explanation	Guided practice
Planned examples	Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

"Let's work on a problem together."

Guided practice is practice in which the teacher and students practice problems together.

MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

"Now, you'll practice a problem on your own. Don't forget to use your attack strategy."

Independent practice is practice in which the students practice independently with teacher support.

MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

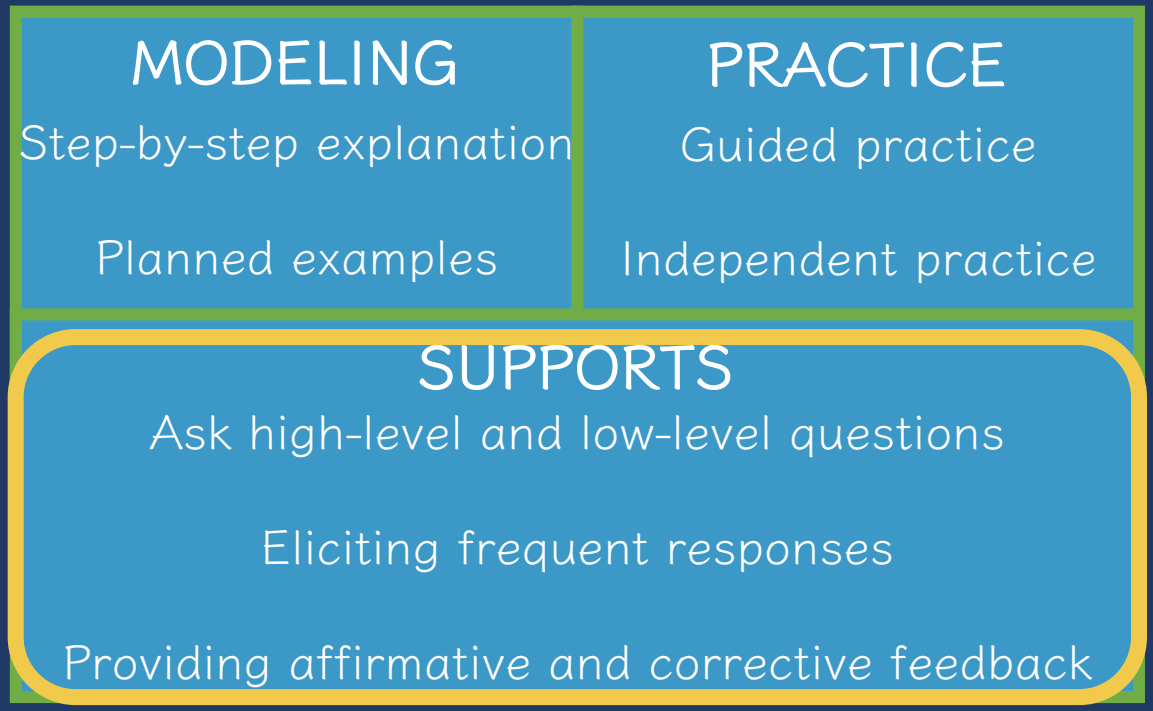
SUPPORTS

Ask high-level and low-level questions

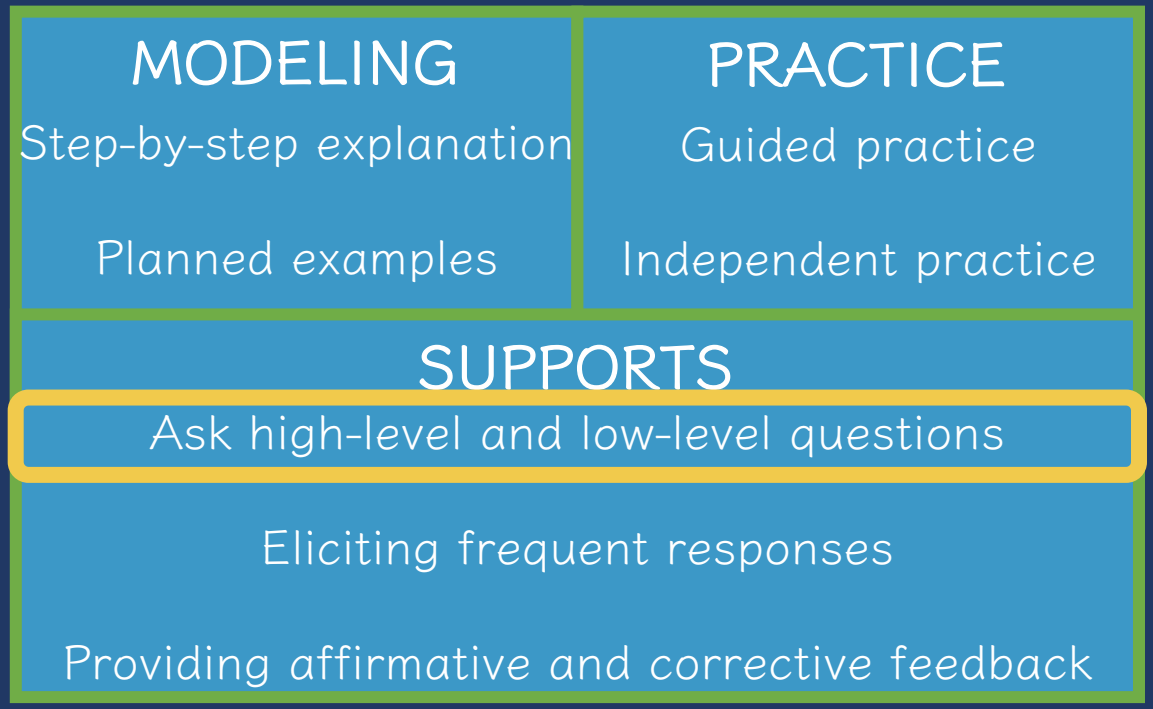
Eliciting frequent responses

Providing affirmative and corrective feedback

I (Sarah) believe that guided practice is essential as a scaffold for students. Would you say your students participate in enough guided practice?



These **Supports** should be used in both **Modeling** and **Practice.**



During **Modeling** and **Practice**, it is essential to engage students and check for understanding.

Ask a combination of high-level and low-level questions.

MODELING	PRACTICE
Step-by-step explanation	Guided practice
Planned examples	Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

"What is 7 times 9?"

"63."

MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

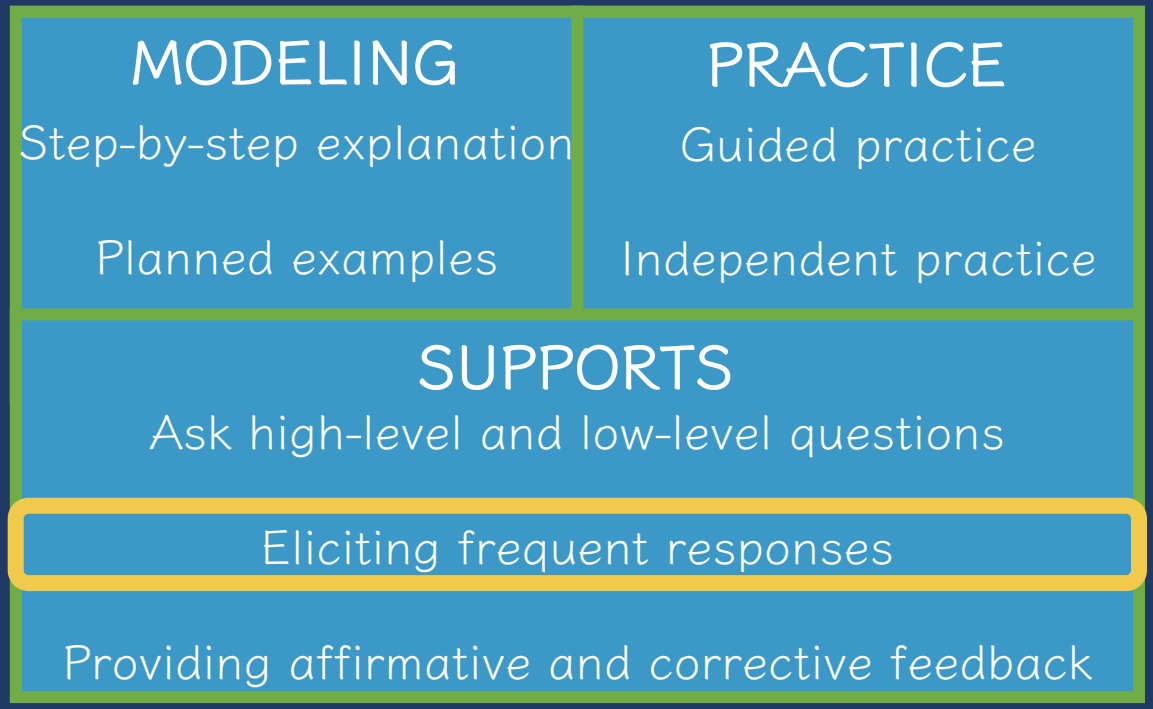
Ask high-level and low-level questions

Eliciting frequent responses

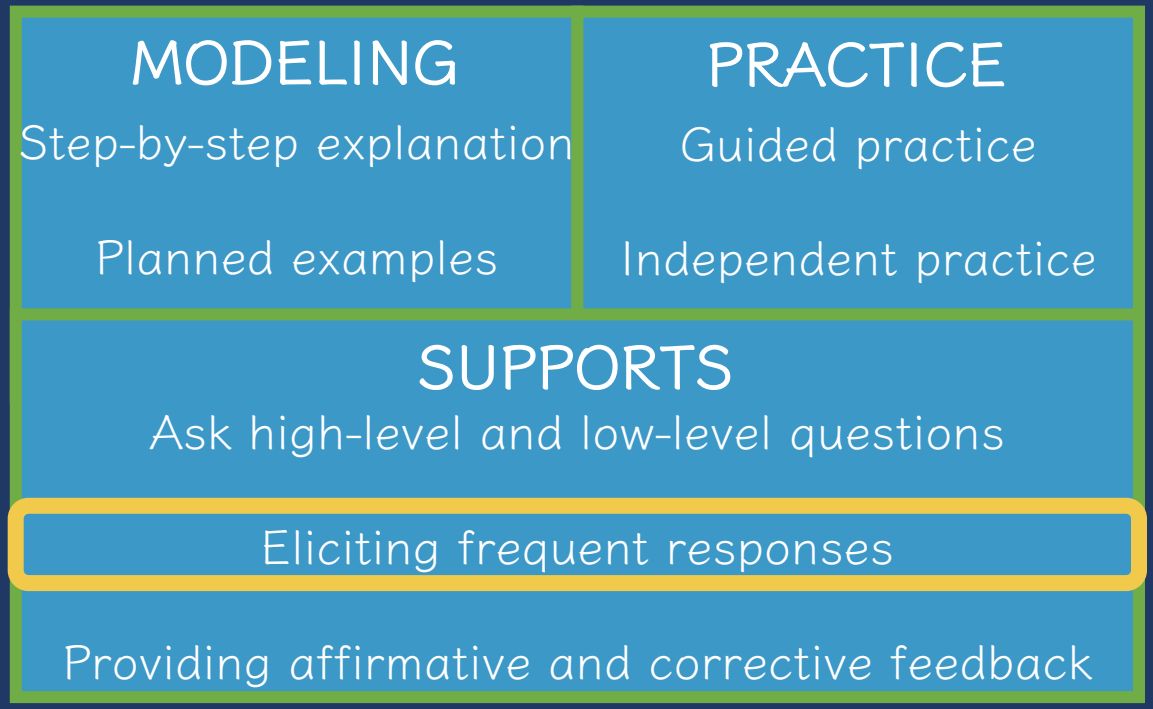
Providing affirmative and corrective feedback

"Why do you use zero pairs?"

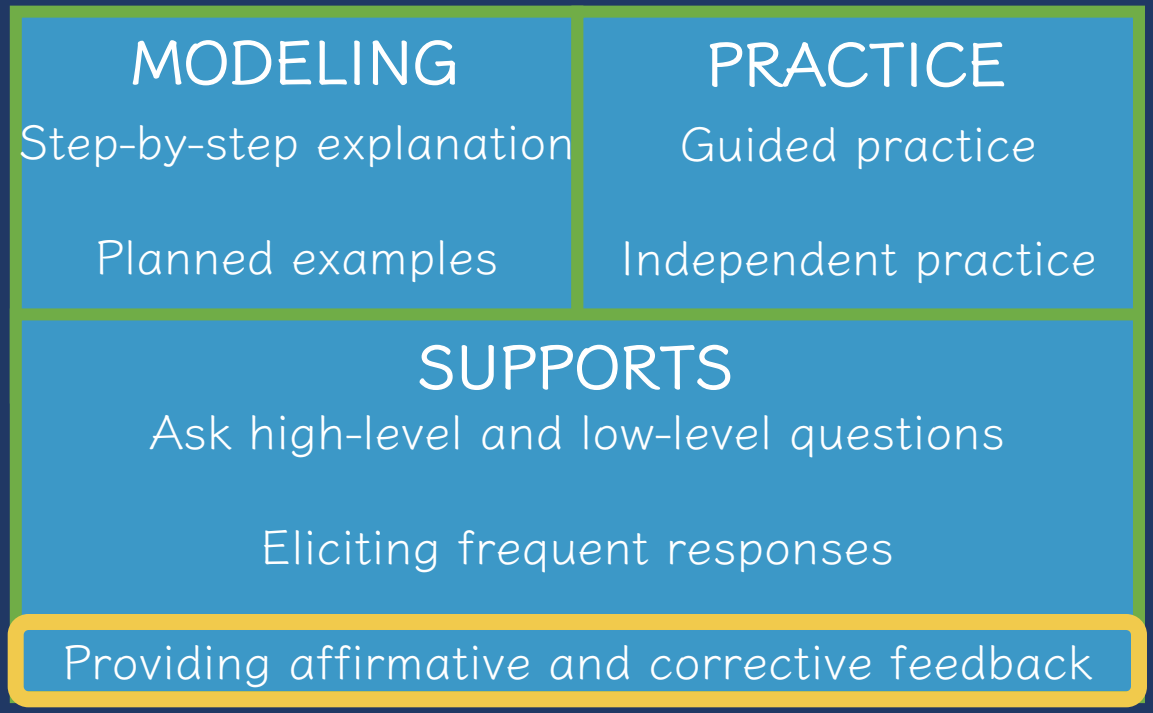
"Because a positive 1 and a negative 1 equal 0. I use the zero pair to help me subtract."



During **Modeling** and **Practice**, students should frequently respond. The frequent responses keeps student attention and keeps student learning active.



- Oral
- Written
- With manipulatives
- With drawings
- With gestures



During **Modeling** and **Practice**, students should receive immediate feedback on their responses.

Students should receive affirmative and (when necessary) corrective feedback.

MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

"Nice work using your word problem attack strategy."

MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

"Let's look at that again. Tell me how you added in the hundreds column."

MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

Introduction of material

Review of material

MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

What's your strength with modeling? What could you improve?

What's your strength with practice? What would you improve?

Which support is a strength? Provide an example.

Which support could you improve? Why would this be important?

Explicit Instruction

MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

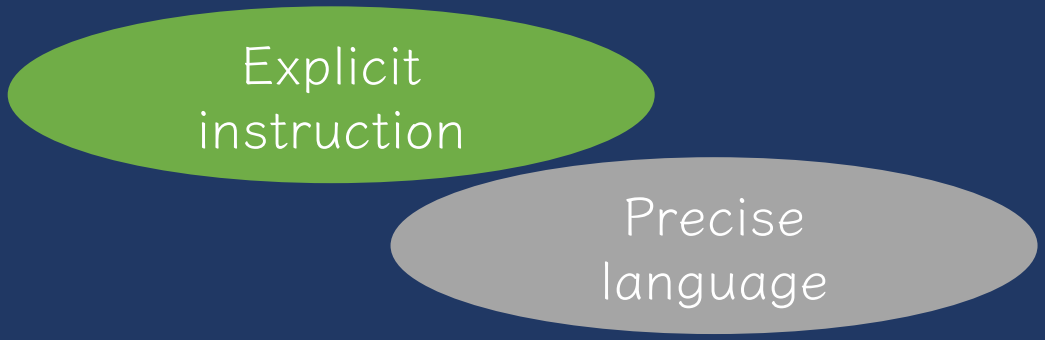
Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

Instructional Platform

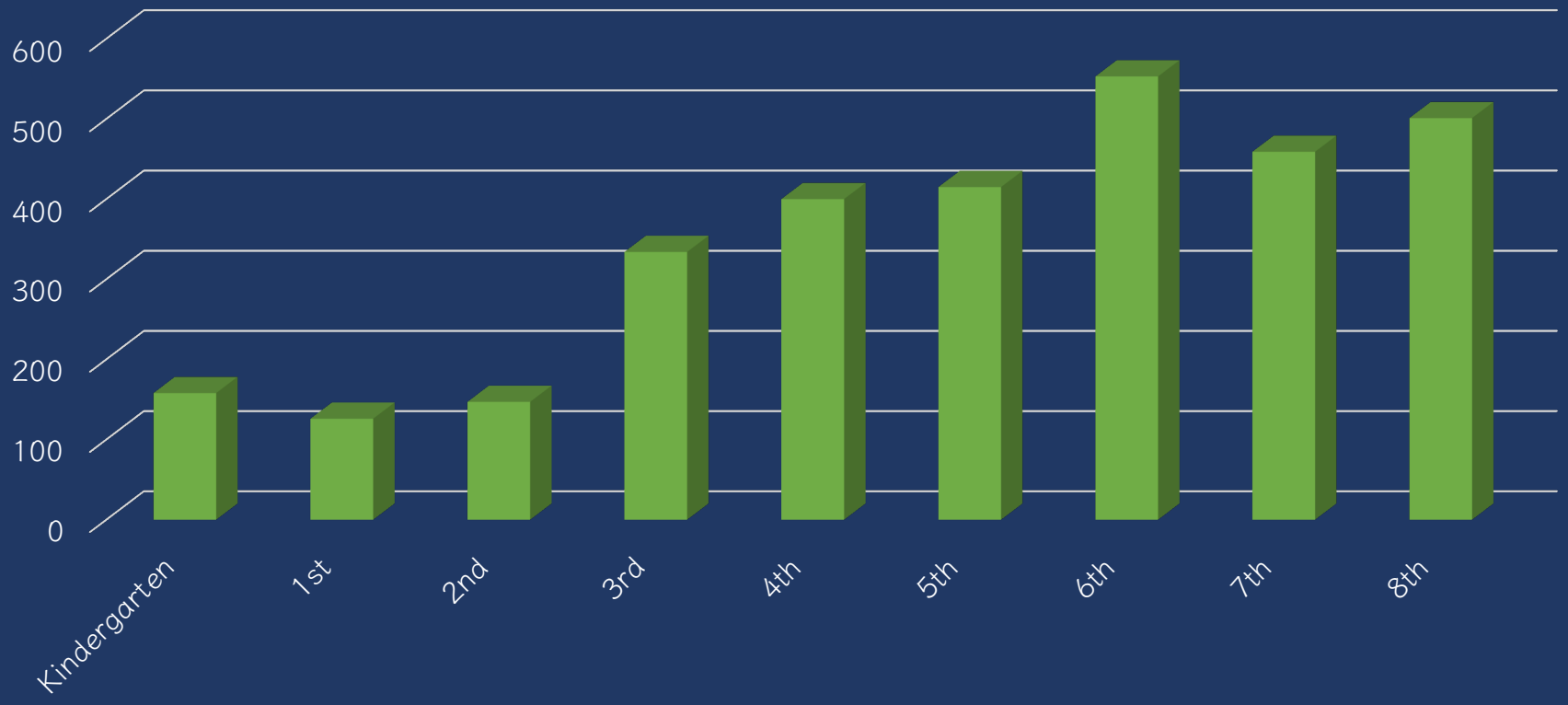
INSTRUCTIONAL DELIVERY



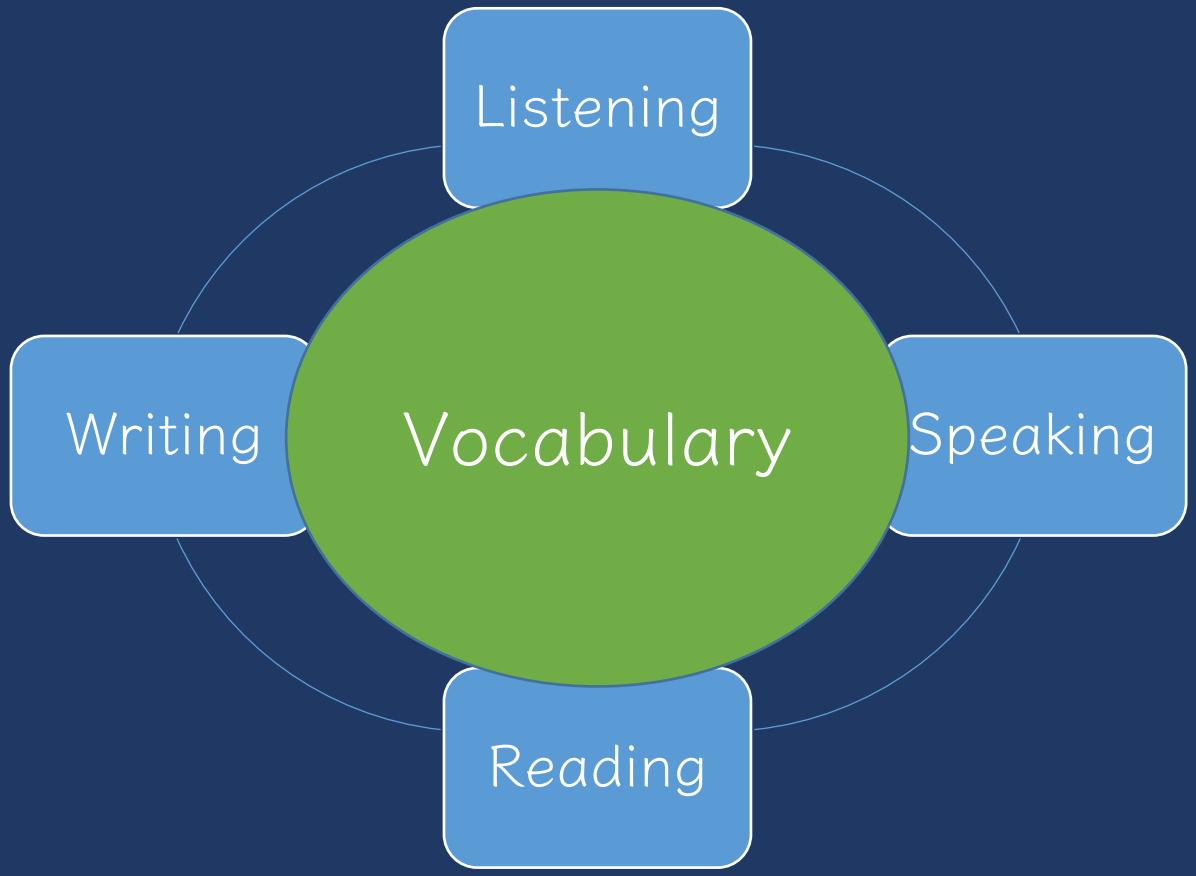
INSTRUCTIONAL STRATEGIES

Mathematical Language

Instead of that...	Say this...



Powell, Bos, & Lin (2019)



1. Some math terms are shared with English but have different meanings

base

right

degree

Rubenstein & Thompson (2002)

1. Some math terms are shared with English but have different meanings

2. Some math words are shared with English with similar meanings
   (but a more precise math meaning)

difference

even

Rubenstein & Thompson (2002)

1. Some math terms are shared with English but have different meanings

2. Some math words are shared with English with similar meanings
   (but a more precise math meaning)

3. Some math terms are only used in math

trapezoid

numerator

parallelogram

Rubenstein & Thompson (2002)

1. Some math terms are shared with English but have different meanings

2. Some math words are shared with English with similar meanings
   (but a more precise math meaning)

3. Some math terms are only used in math

4. Some math terms have more than one meaning

round

square

second

base

Rubenstein & Thompson (2002)

1. Some math terms are shared with English but have different meanings

2. Some math words are shared with English with similar meanings (but a more precise math meaning)

3. Some math terms are only used in math

4. Some math terms have more than one meaning

5. Some math terms are similar to other content-area terms with different meanings

divide vs. Continental Divide

variable vs. variably cloudy

Rubenstein & Thompson (2002)

1. Some math terms are shared with English but have different meanings

2. Some math words are shared with English with similar meanings (but a more precise math meaning)

3. Some math terms are only used in math

4. Some math terms have more than one meaning

5. Some math terms are similar to other content-area terms with different meanings

6. Some math terms are homographs

eight vs. ate

sum vs. some

rows vs. rose

base vs. bass

Rubenstein & Thompson (2002)

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6. Some math terms are homographs

7. Some math terms are related but have distinct meanings

factor vs. multiple

hundreds vs. hundredths

numerators vs. denominator

Rubenstein & Thompson (2002)

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5. Some math terms are similar to other content-area terms with different meanings

6. Some math terms are homographs

7. Some math terms are related but have distinct meanings

8. An English math term may translate into another language with different meanings

mesa vs. tabla

Rubenstein & Thompson (2002)

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2. Some math words are shared with English with similar meanings (but a more precise math meaning)

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6. Some math terms are homographs

7. Some math terms are related but have distinct meanings

8. An English math term may translate into another language with different meanings

9. English spelling and usage may have irregularities

four vs. forty

Rubenstein & Thompson (2002)

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2. Some math words are shared with English with similar meanings (but a more precise math meaning)

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5. Some math terms are similar to other content-area terms with different meanings

6. Some math terms are homographs

7. Some math terms are related but have distinct meanings

8. An English math term may translate into another language with different meanings

9. English spelling and usage may have irregularities

10. Some math concepts are verbalized in more than one way

skip count vs. multiples

one-fourth vs. one quarter

1. Some math terms are shared with English but have different meanings

2. Some math words are shared with English with similar meanings (but a more precise math meaning)

3. Some math terms are only used in math

4. Some math terms have more than one meaning

5. Some math terms are similar to other content-area terms with different meanings

6. Some math terms are homographs

7. Some math terms are related but have distinct meanings

8. An English math term may translate into another language with different meanings

9. English spelling and usage may have irregularities

rhombus vs. diamond

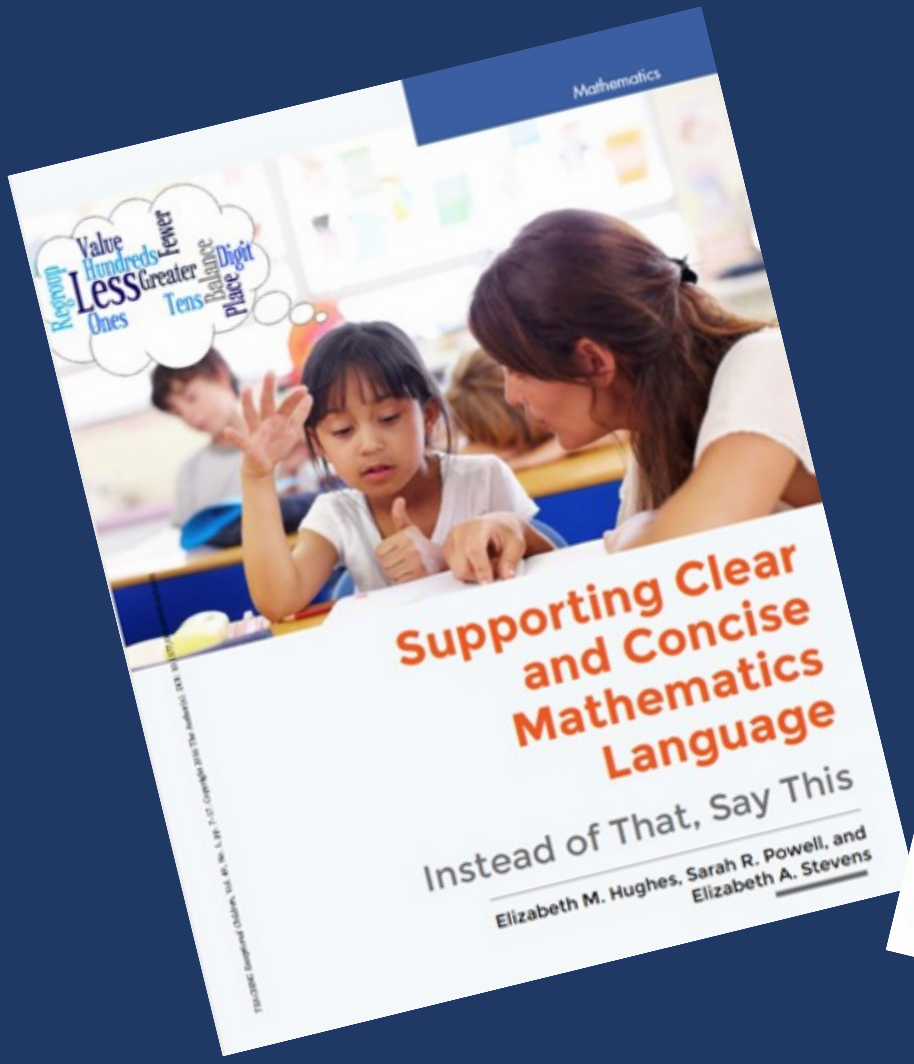
10. Some math concepts are verbalized in more than one way

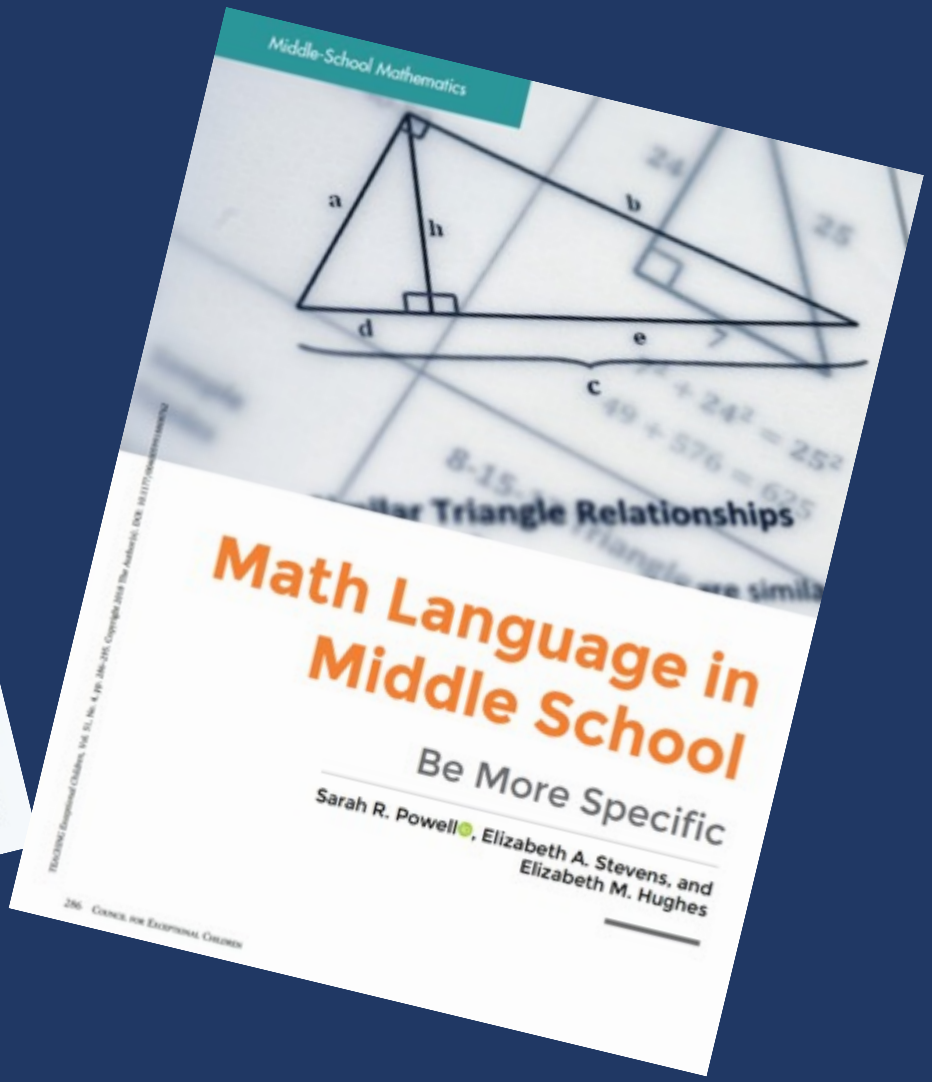
vertex vs. corner

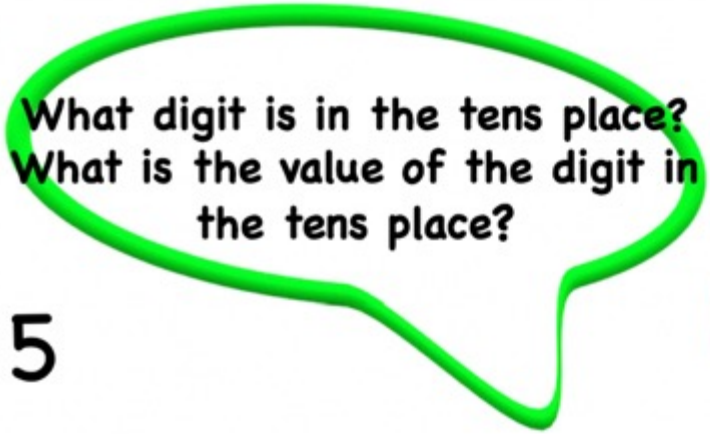
11. Informal terms may be used for formal math terms

Use formal math language

Use terms precisely







135

Why this is important...

- A number refers to the entire amount.
- The 3 in the tens place value is not a number, but rather a digit in the number 135.
- Reinforces conceptual understanding of place value.
- Emphasizes that 3 is part of the number 135 with a value of 30.

The alligator eats the bigger number

is less than
OR
is greater than

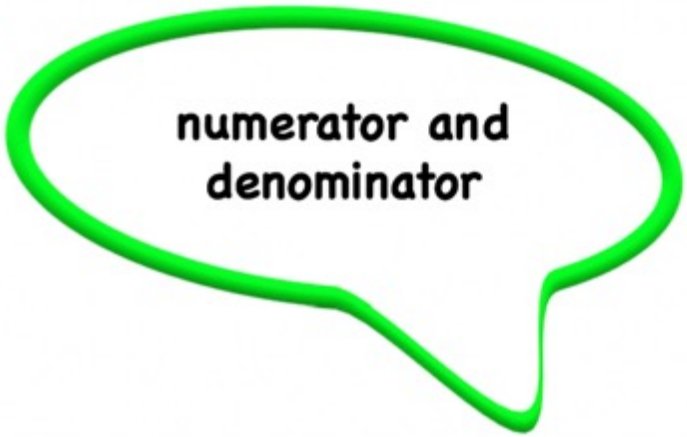
Why this is important...

- Students must learn how to read and write the inequality symbols.
- Students must learn to read equations correctly from left to right because < and > are two distinct symbols.

$$\begin{array}{r} 167 \\ +\ 294 \\ \hline \end{array}$$

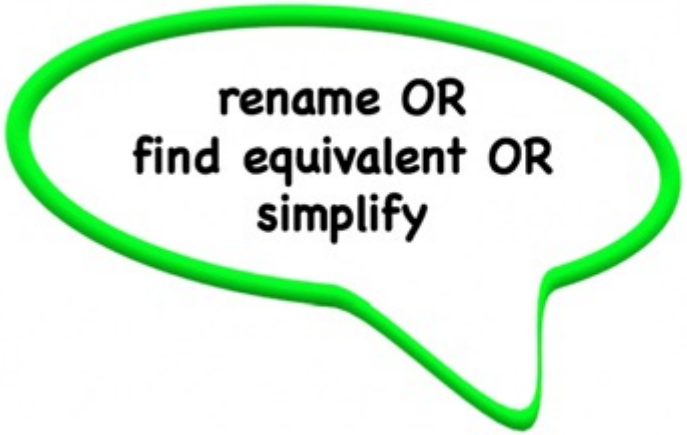
Why this is important...

- "Carry" or "borrow" is procedural.
- The other terms reinforce the conceptual understanding or regrouping ones into tens, tens into hundreds, and so on (i.e., the total amount does not change) *or* ungrouping hundreds into tens, tens into ones, and so on.



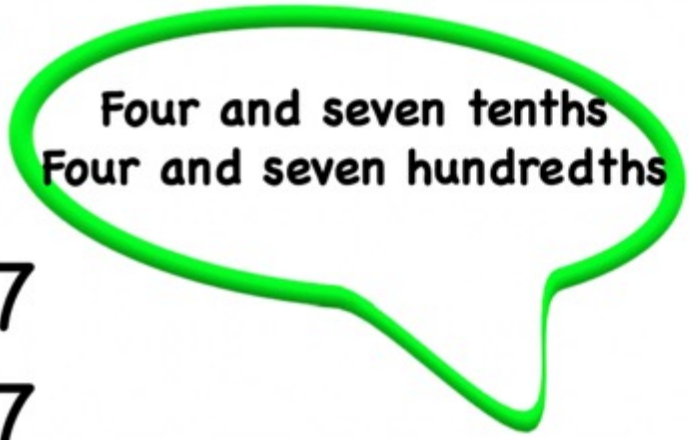
Why this is important...

- Identifying that there are two separate (whole) numbers suggests that whole number properties can be applied to fractions.
- Emphasizing that a fraction is ONE number with ONE magnitude on a number line that is communicated with a numerator and denominator is important.



Why this is important...

- Reducing suggests that the quantity or magnitude of the new number will be less than the original number.

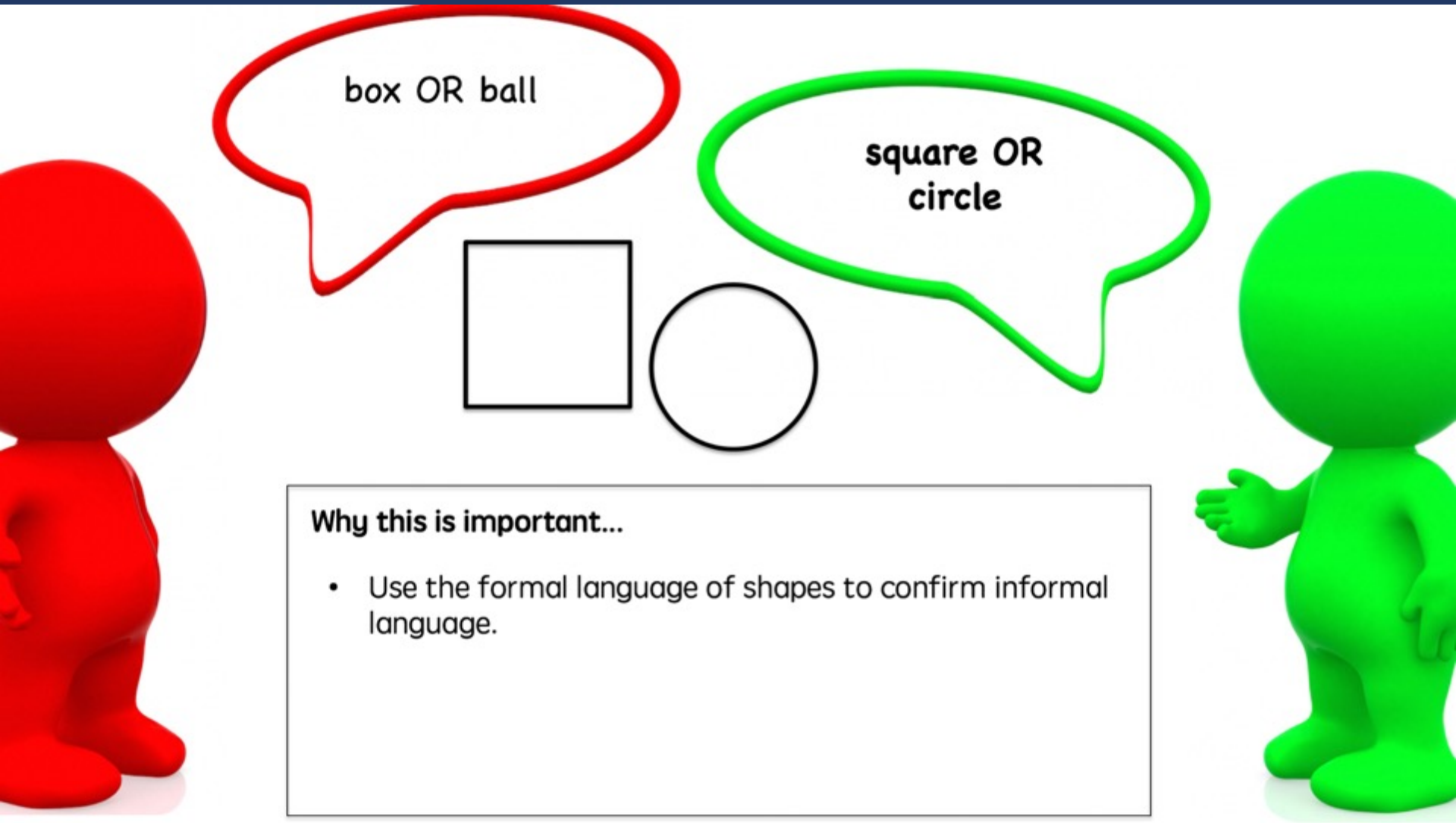


4.7

4.07

Why this is important...

- Accurately shares the magnitude of the decimal.
- Emphasizes place value.

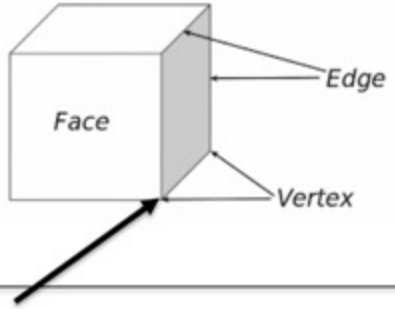


Why this is important...

- Use the formal language of shapes to confirm informal language.

point

vertex



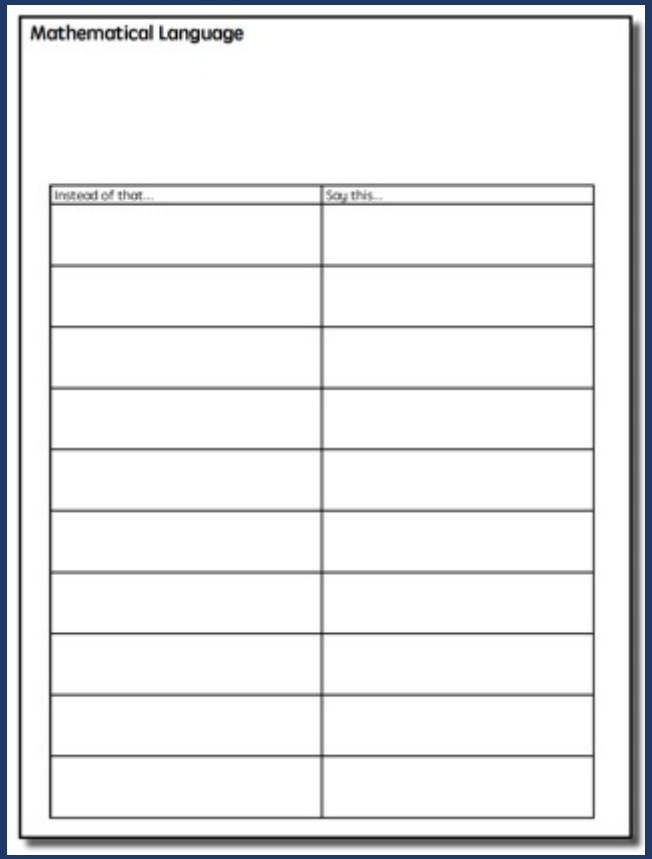
Why this is important...

- This is the endpoint where two or more line segments or rays meet.

Why this is important...

- The informal language helps children remember the actions, but this vocabulary is not used on assessments.
- Use the formal mathematical terms.

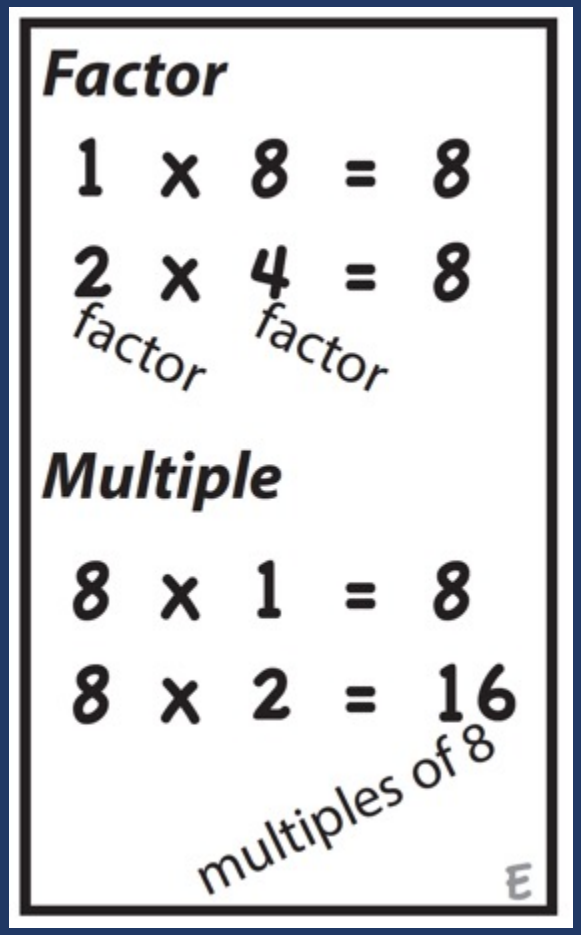
What are examples of, "Instead of ___, Say ___?"


Mathematical Language

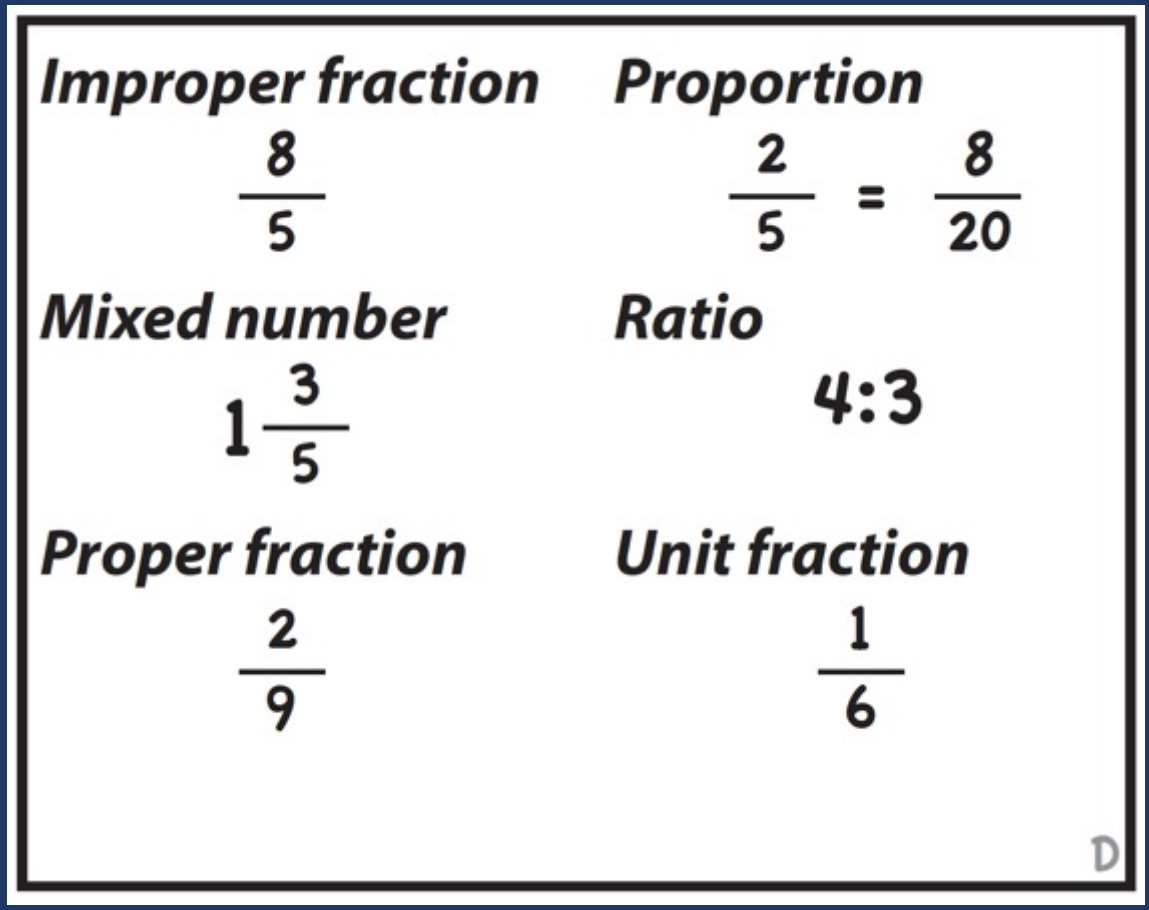
Instead of that...	Say this...

Use formal math language

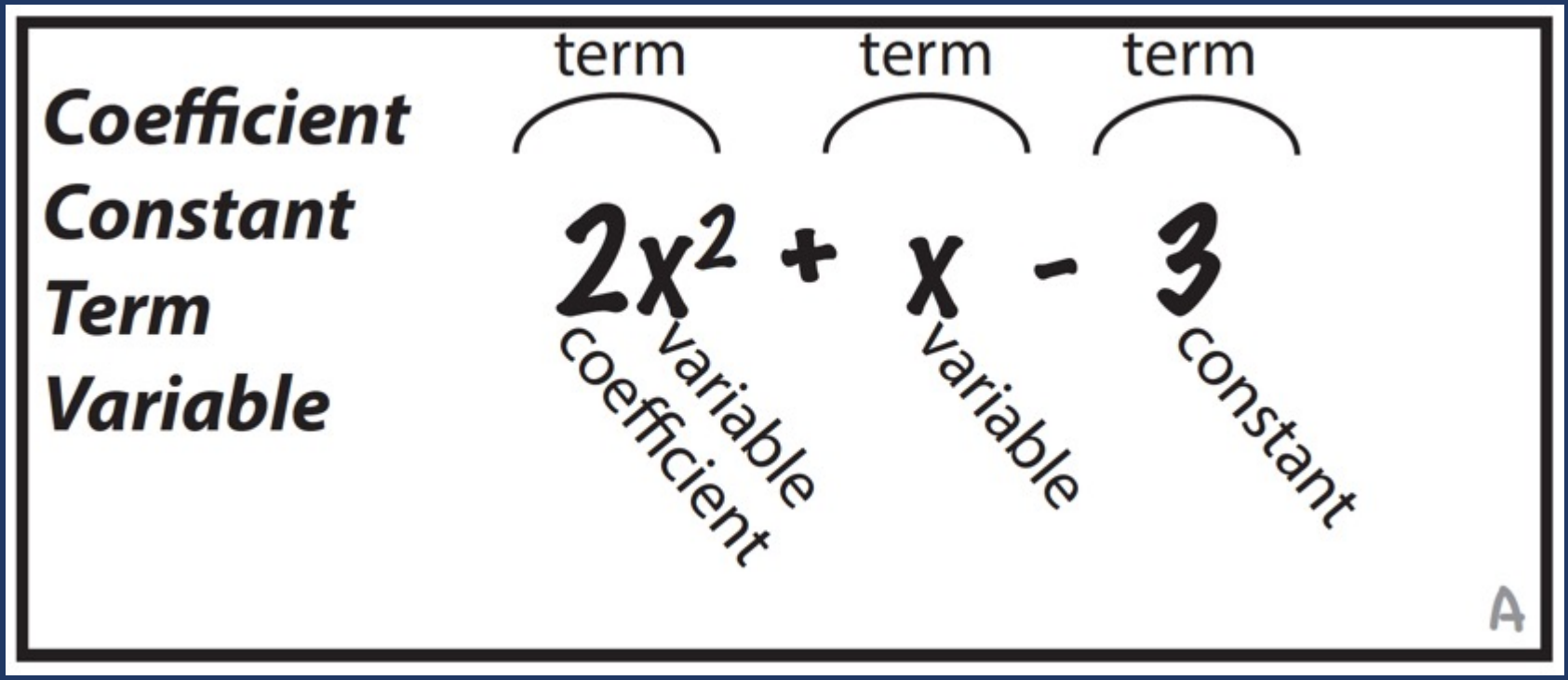
Use terms precisely



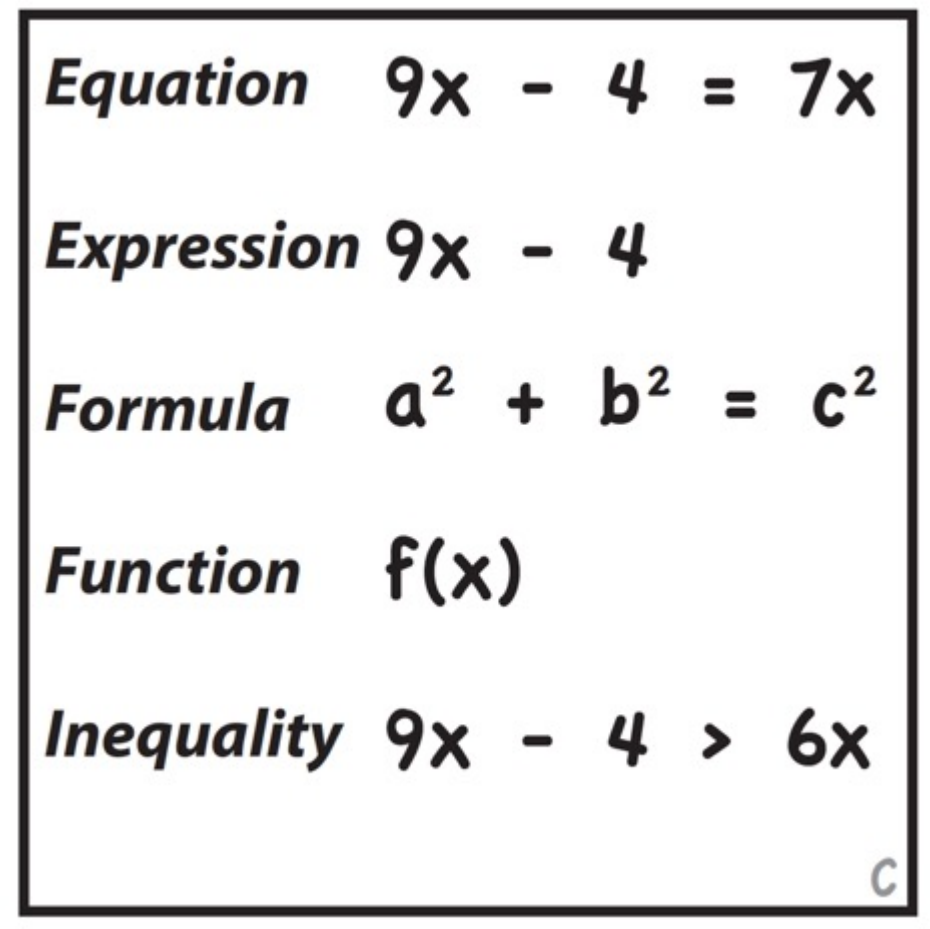
Rubenstein & Thompson (2002)



Rubenstein & Thompson (2002)



Rubenstein & Thompson (2002)



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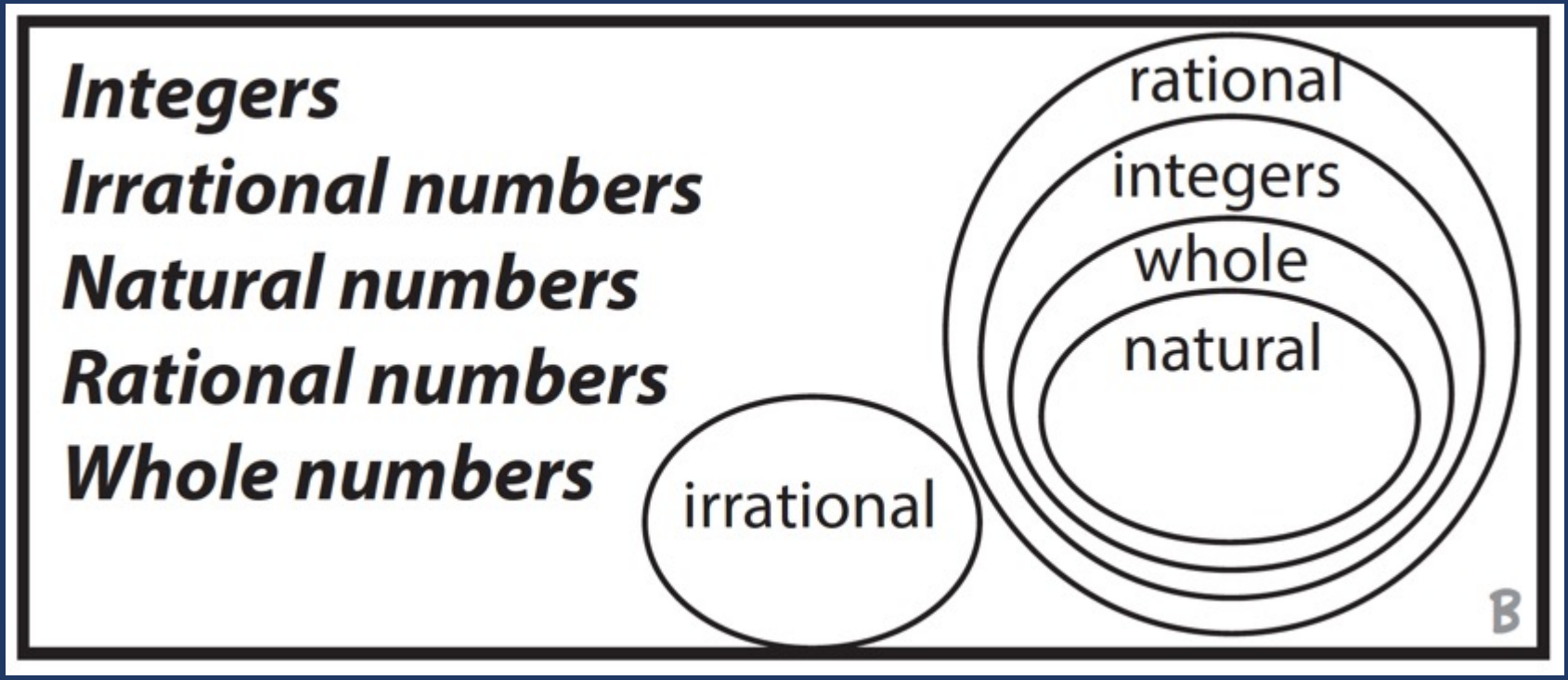
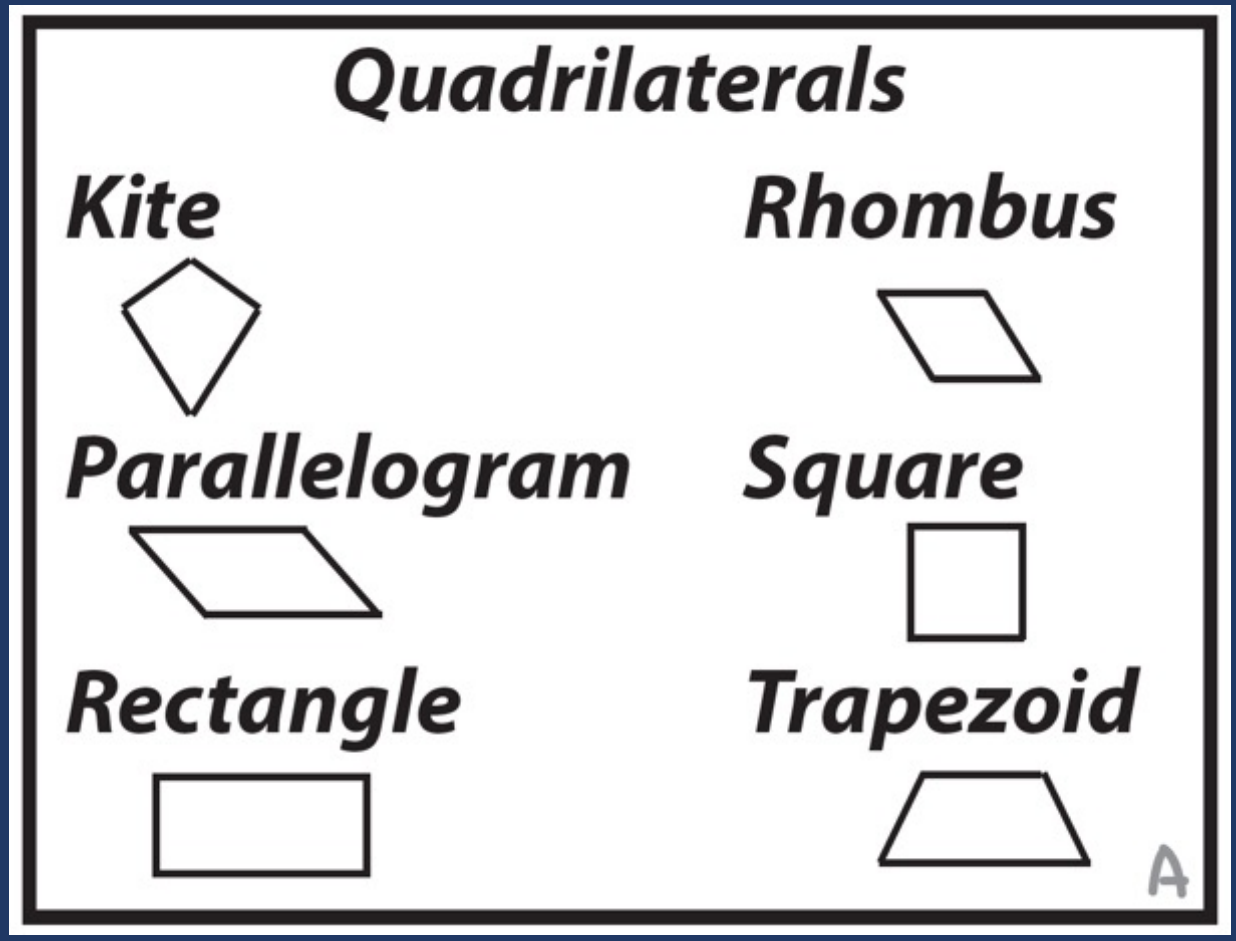
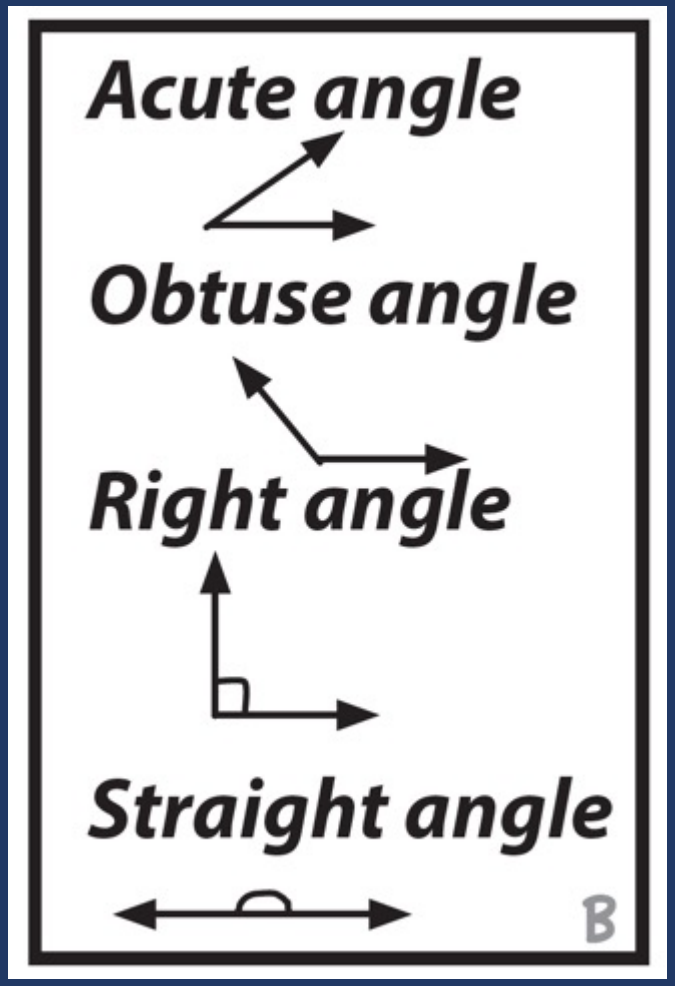


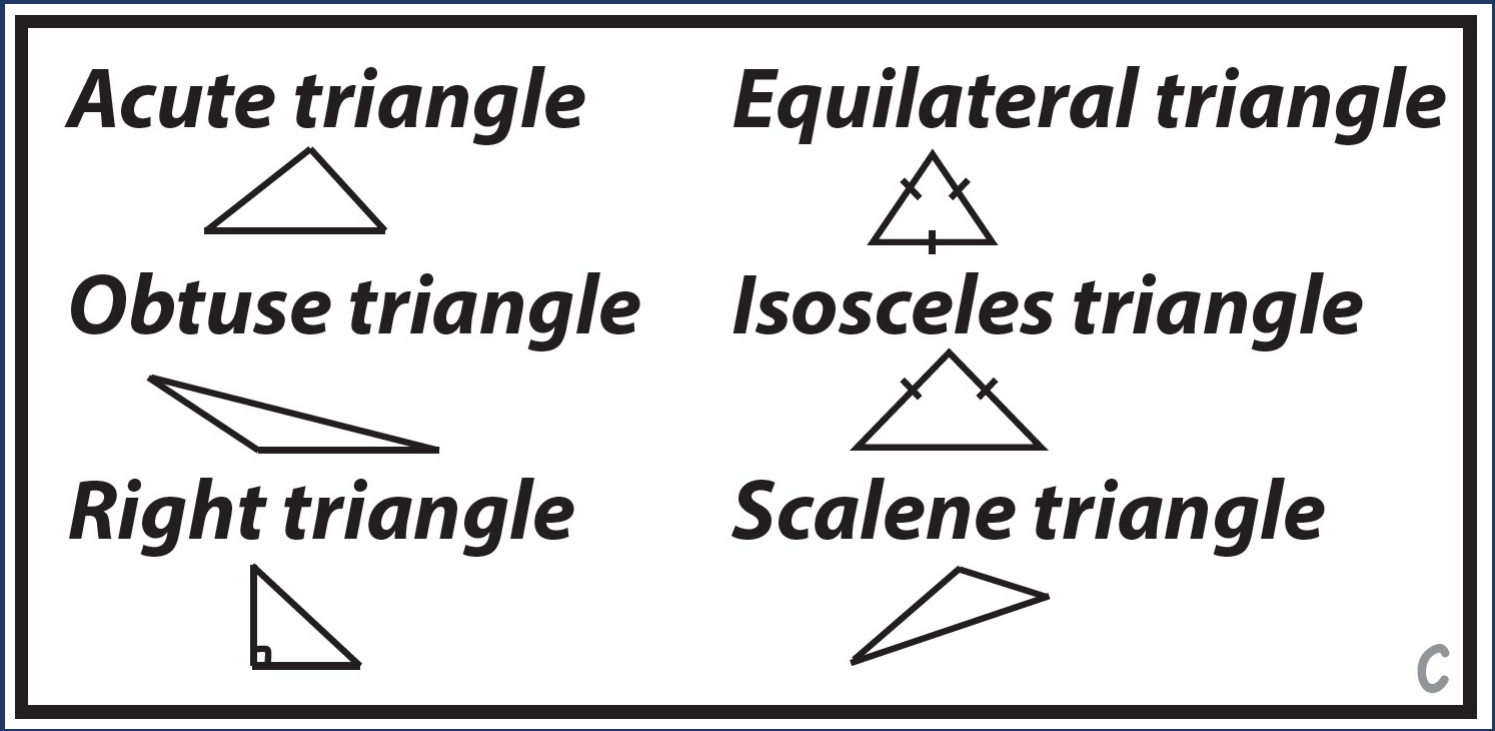
Rubenstein & Thompson (2002)



Rubenstein & Thompson (2002)



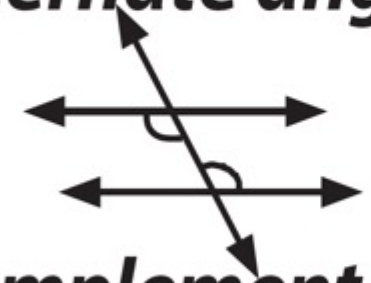
Rubenstein & Thompson (2002)



Rubenstein & Thompson (2002)

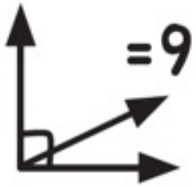
Adjacent angles

Alternate angles

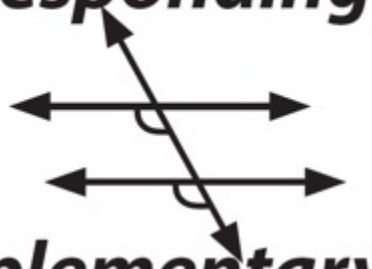


Complementary angles

=90°



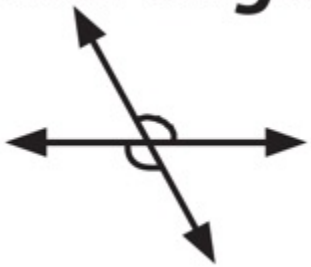
Corresponding angles



Supplementary angles

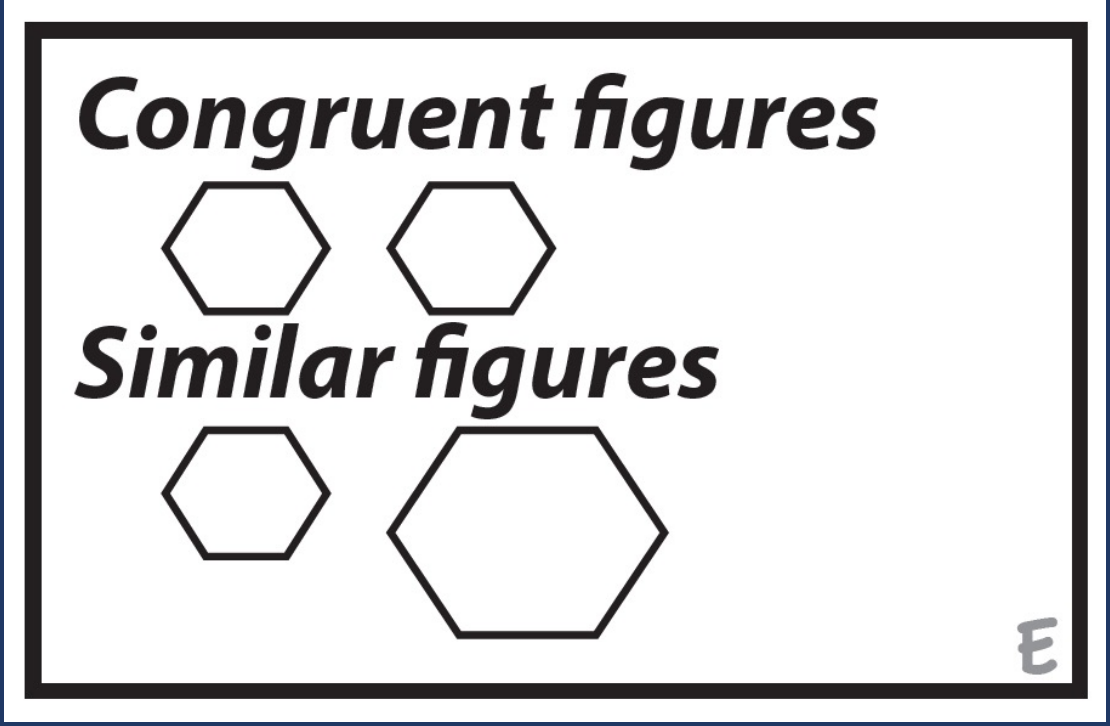
=180°

Vertical angles

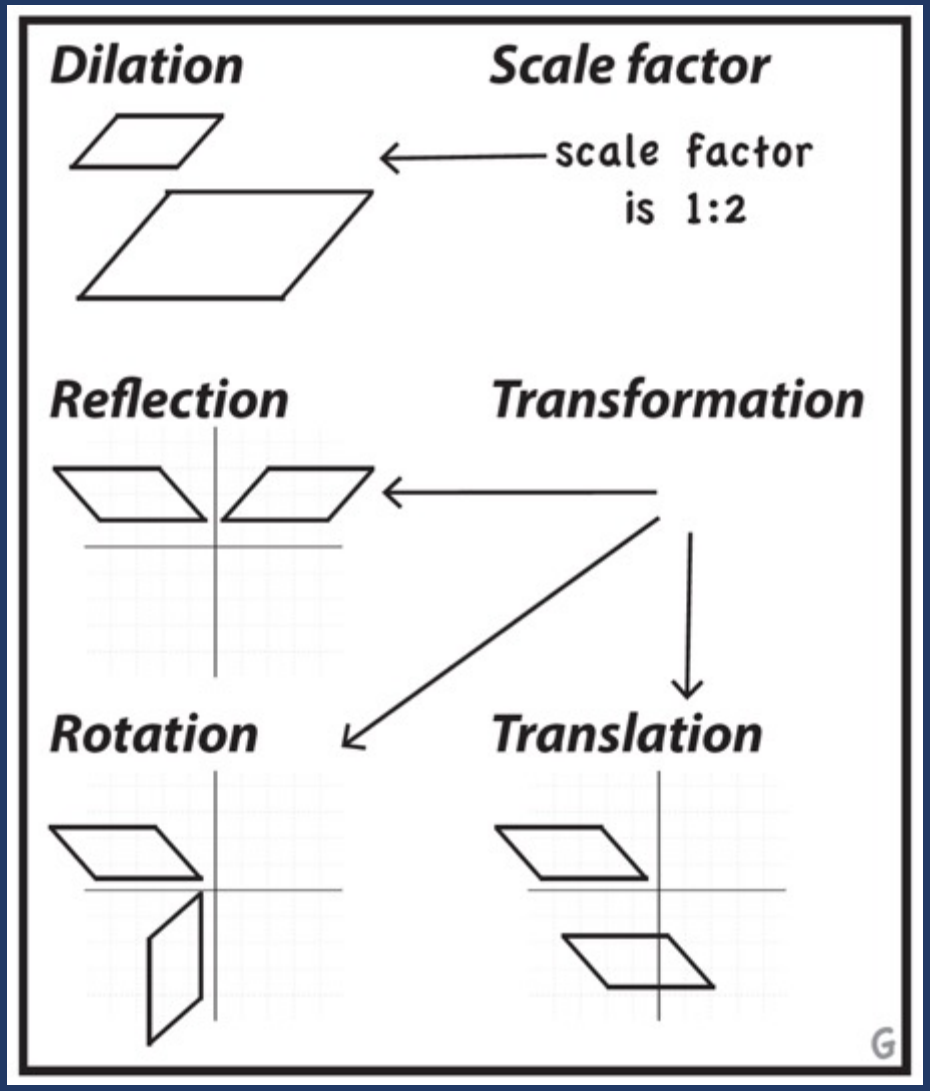


D

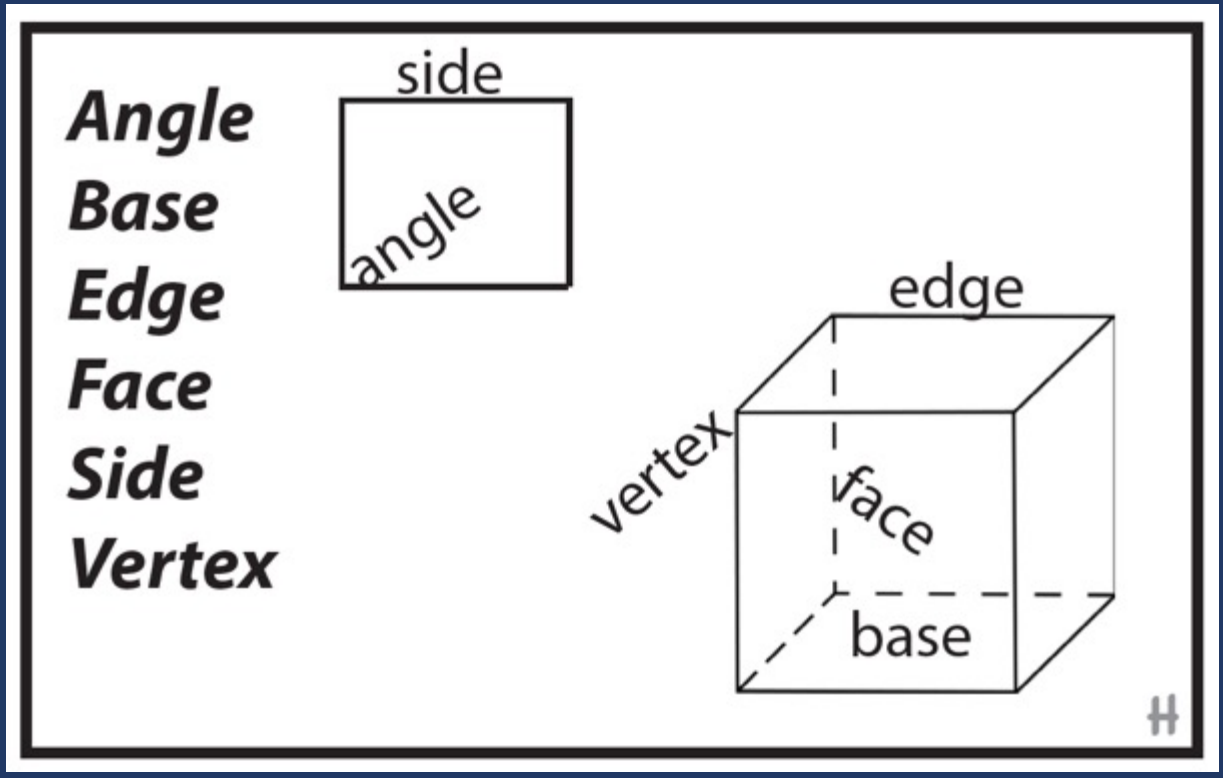
Rubenstein & Thompson (2002)



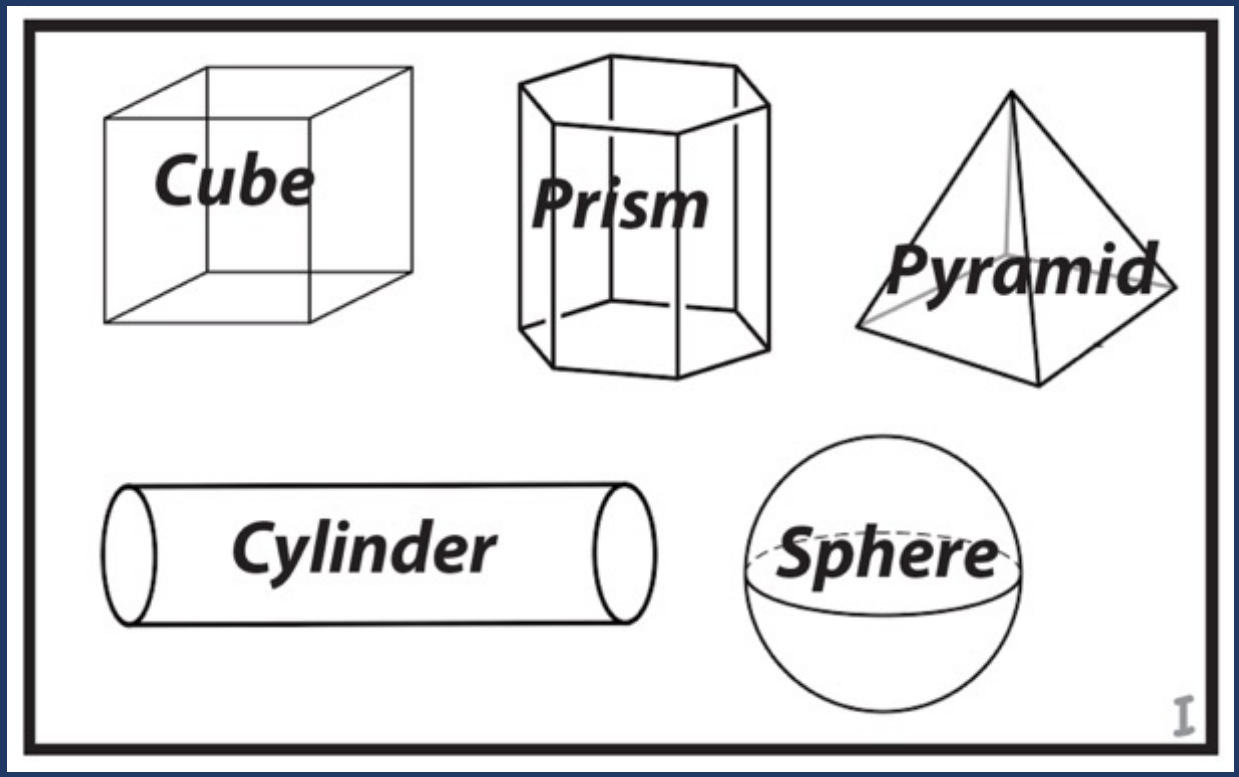
Rubenstein & Thompson (2002)



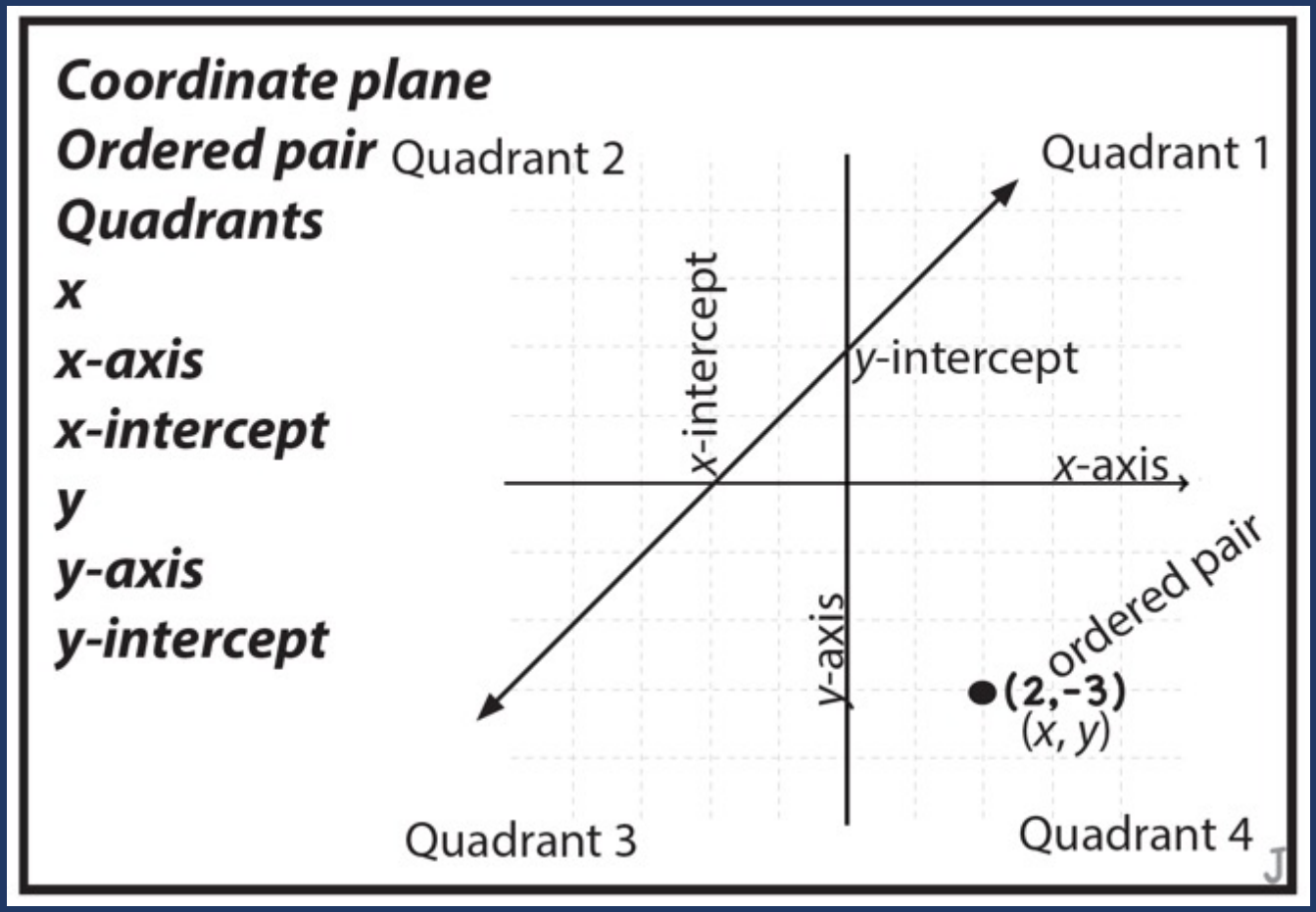
Rubenstein & Thompson (2002)



Rubenstein & Thompson (2002)



Rubenstein & Thompson (2002)



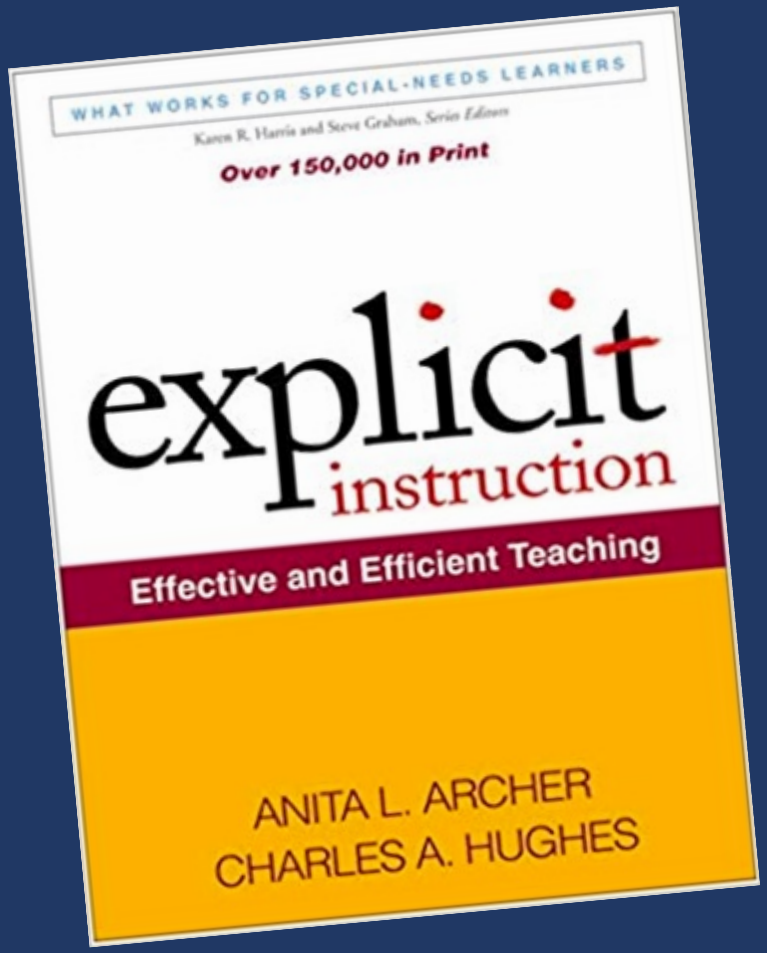
Rubenstein & Thompson (2002)

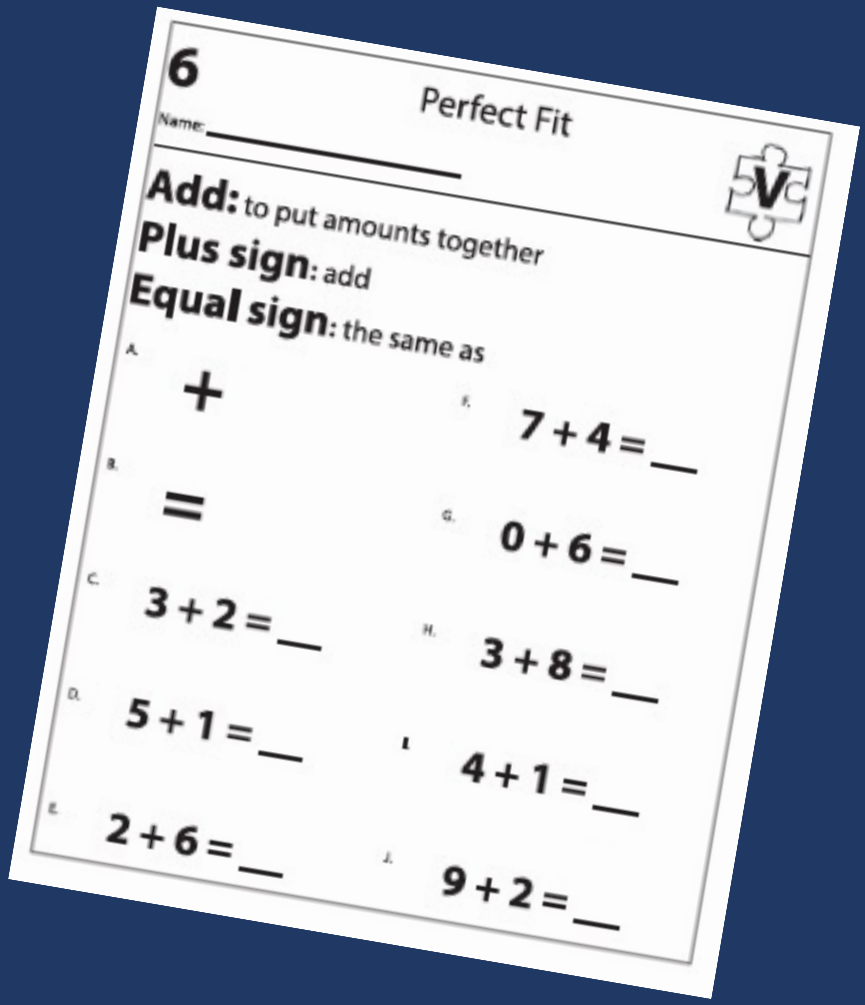
Which terms do your students not use precisely?

Use formal math language

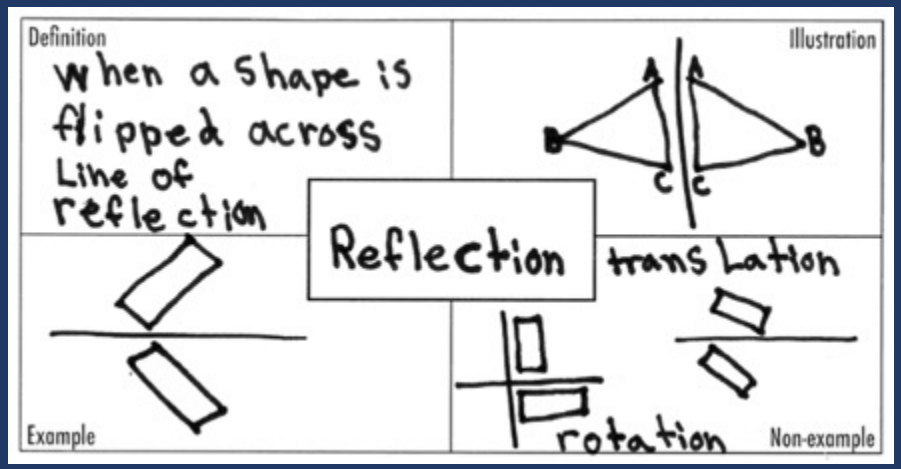
Use terms precisely

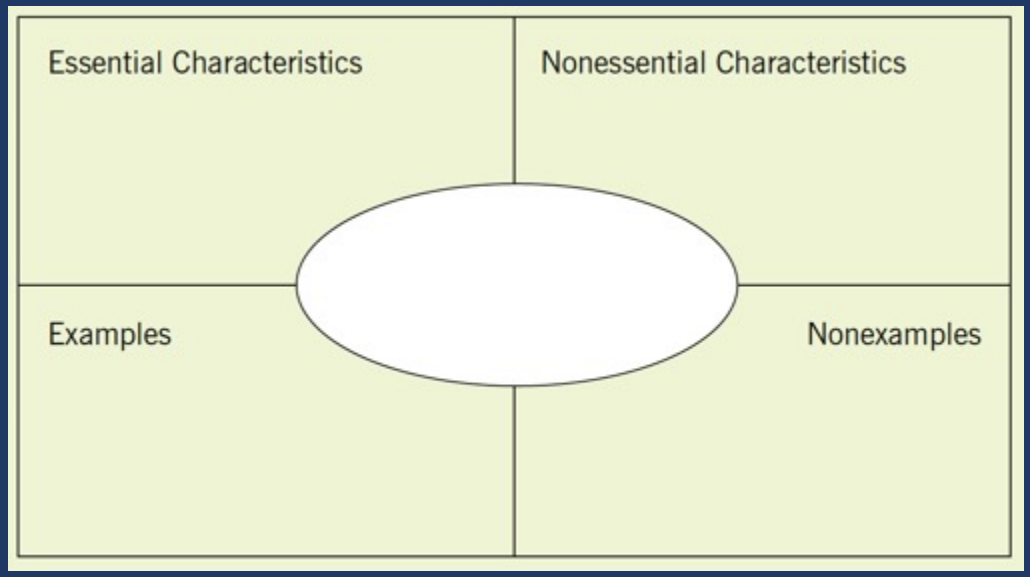
1. Use explicit instruction





2. Use graphic organizers





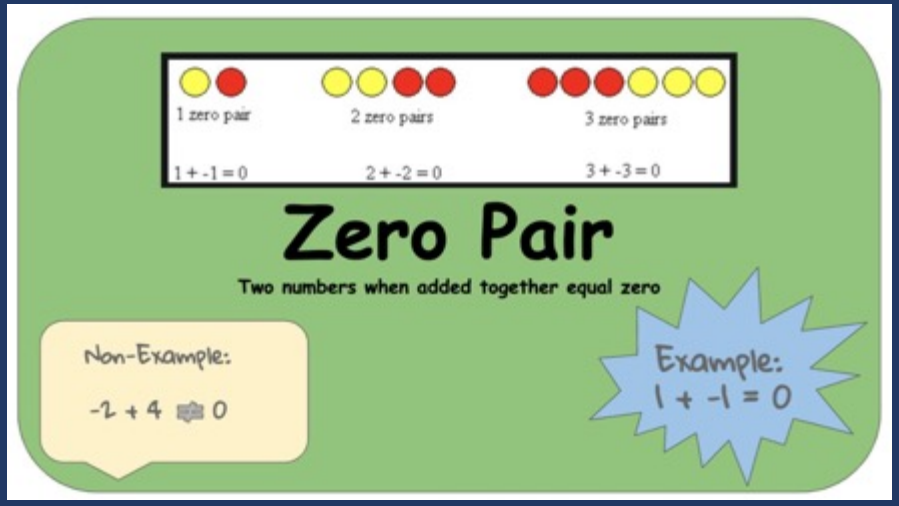
Dunston & Tyminski (2013)

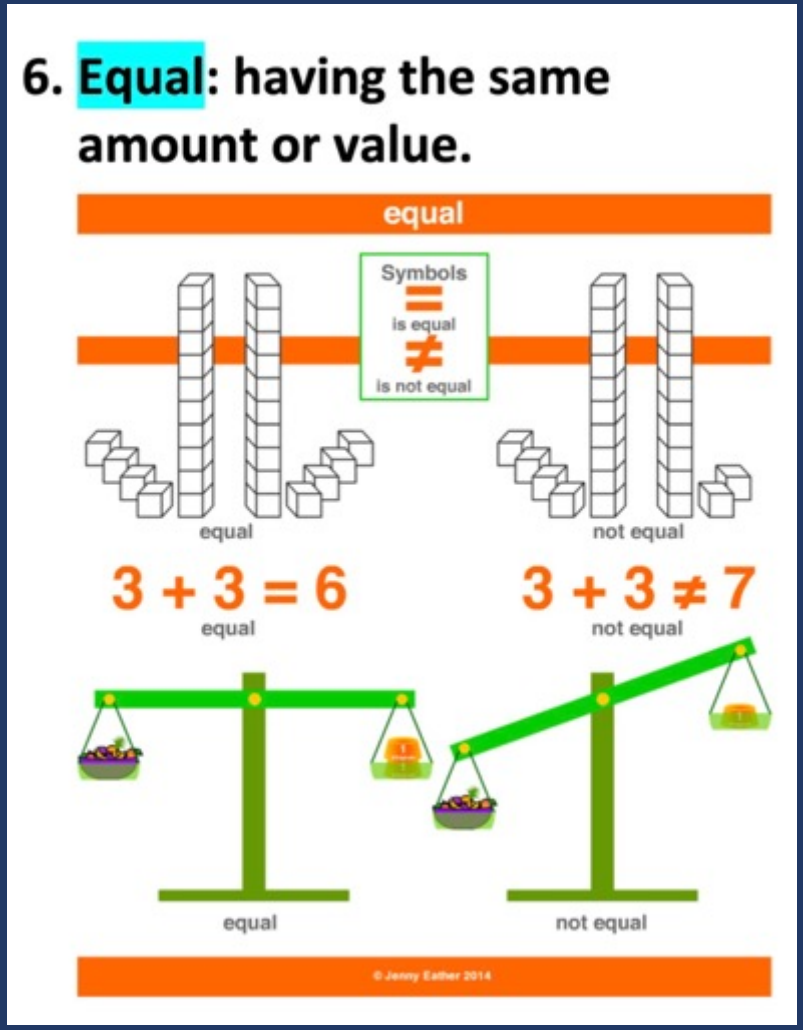
2. Use graphic organizers

Word	Lightbulb Word
Definition	Picture

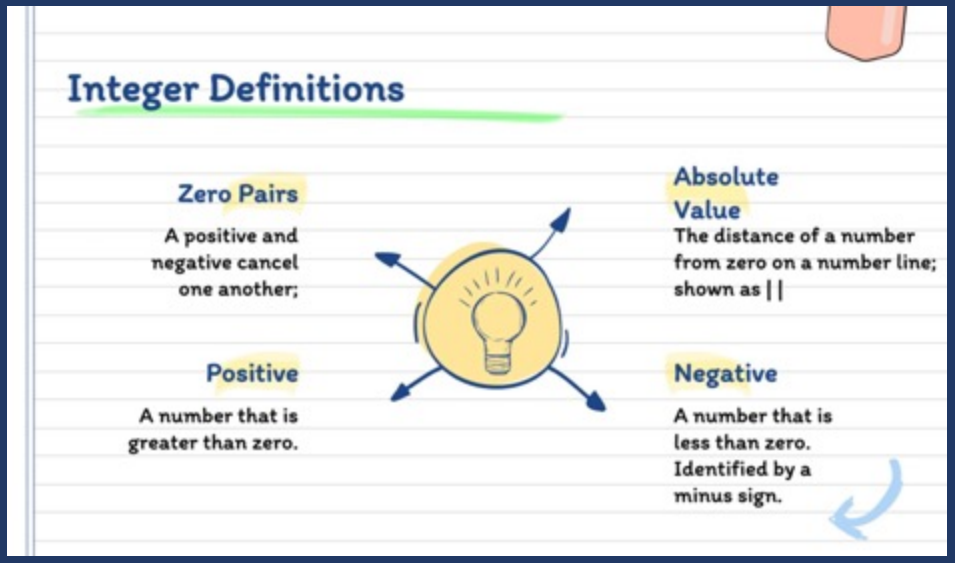
Dunston & Tyminski (2013)

3. Have students create vocabulary cards





4. Have students create glossaries



Numerator: how many parts of the whole

- Ex. $\frac{4}{10}$

Odd number: a number not divided evenly by 2

- Ex. 1, 3, 5, 7, 9....

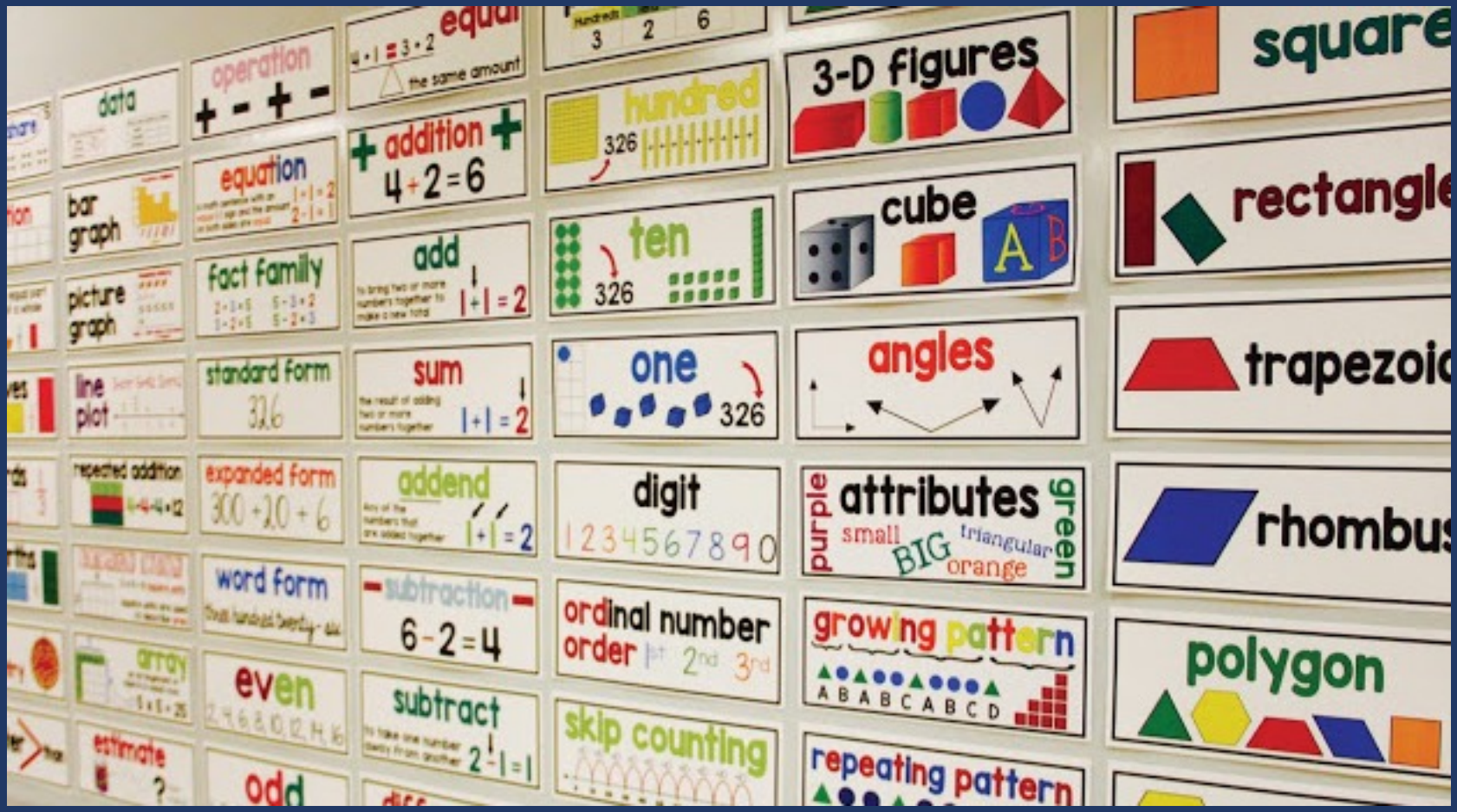
Percent: a specific number in comparison to 100

- 74%

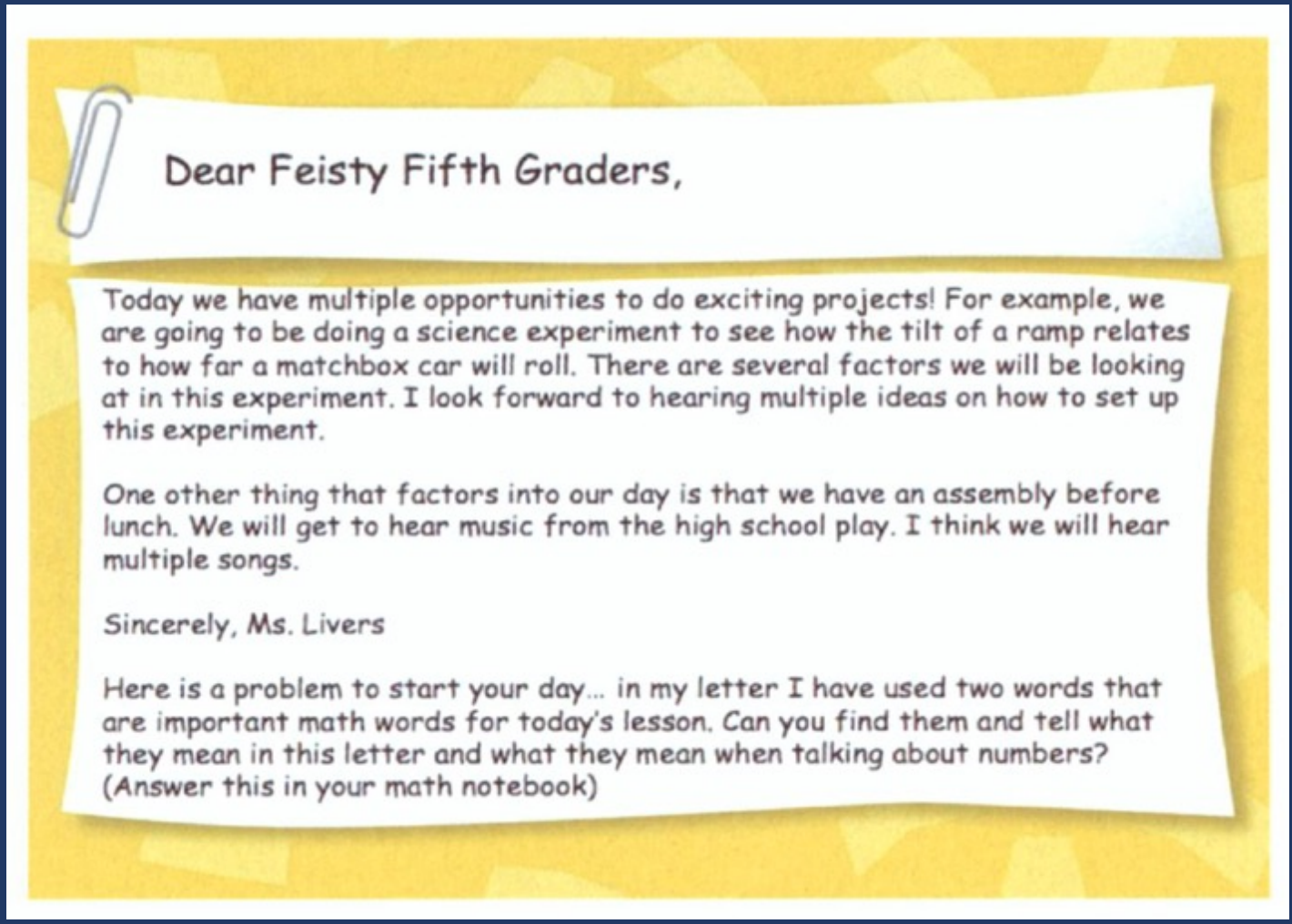
Polygon: any enclosed shape that is made up of 3 or more straight lines

- Ex.

5. Create a word wall



6. Preview vocabulary



Dear Feisty Fifth Graders,

Today we have multiple opportunities to do exciting projects! For example, we are going to be doing a science experiment to see how the tilt of a ramp relates to how far a matchbox car will roll. There are several factors we will be looking at in this experiment. I look forward to hearing multiple ideas on how to set up this experiment.

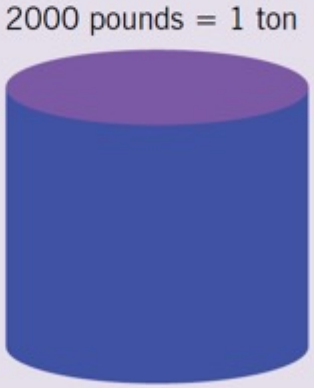
One other thing that factors into our day is that we have an assembly before lunch. We will get to hear music from the high school play. I think we will hear multiple songs.

Sincerely, Ms. Livers

Here is a problem to start your day... in my letter I have used two words that are important math words for today's lesson. Can you find them and tell what they mean in this letter and what they mean when talking about numbers? (Answer this in your math notebook)

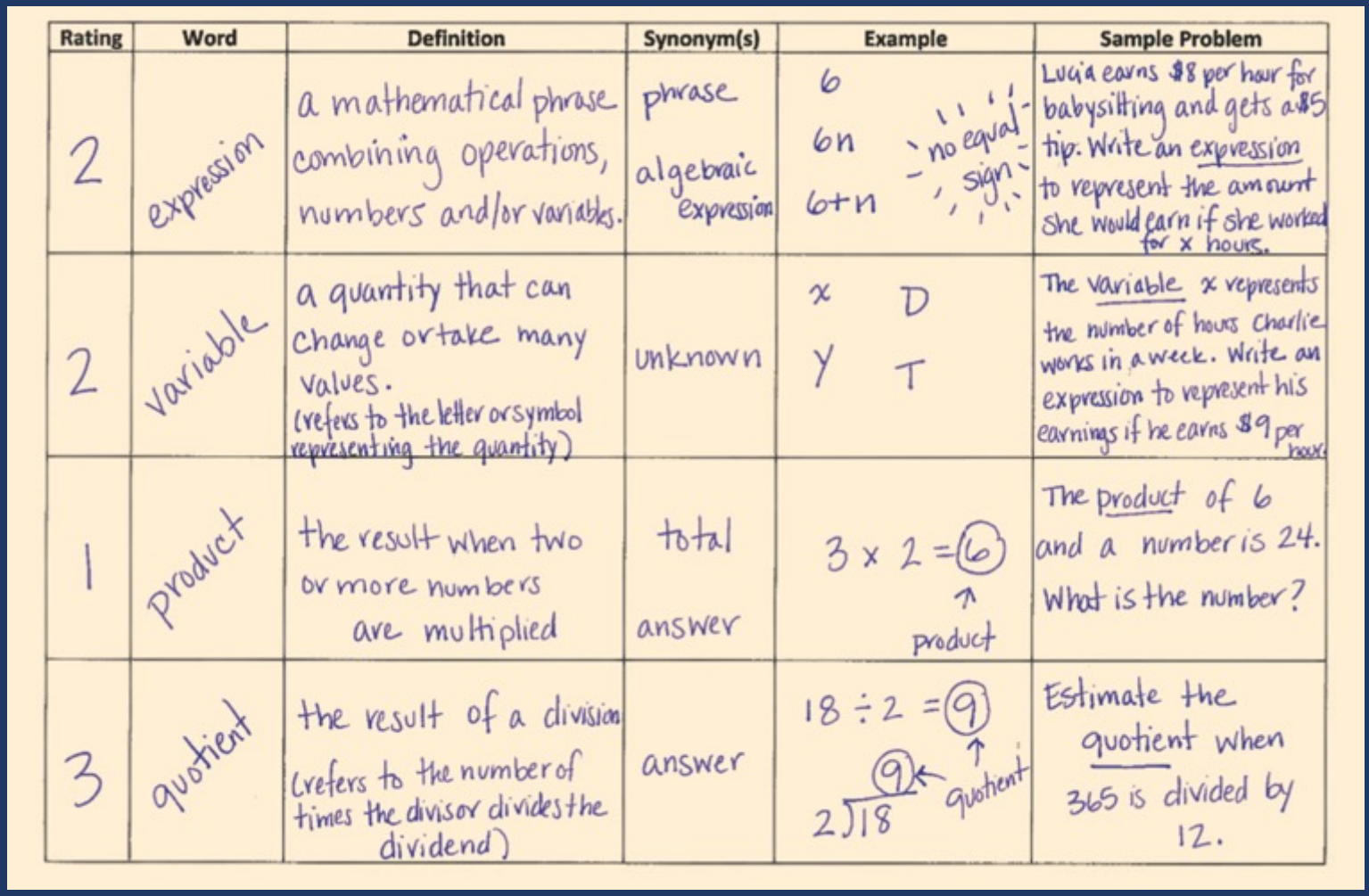
Bay-Williams & Livers (2009)

7. Cluster vocabulary

	Length	Weight
Meaning	How long something is	How heavy something is
Visual		

Livers & Bay-Williams (2014)

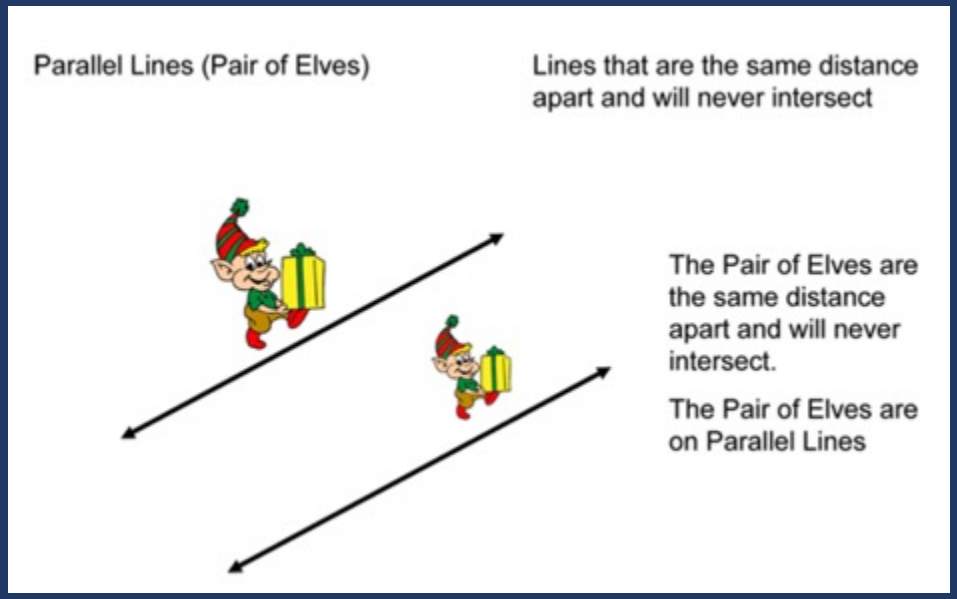
7. Cluster vocabulary

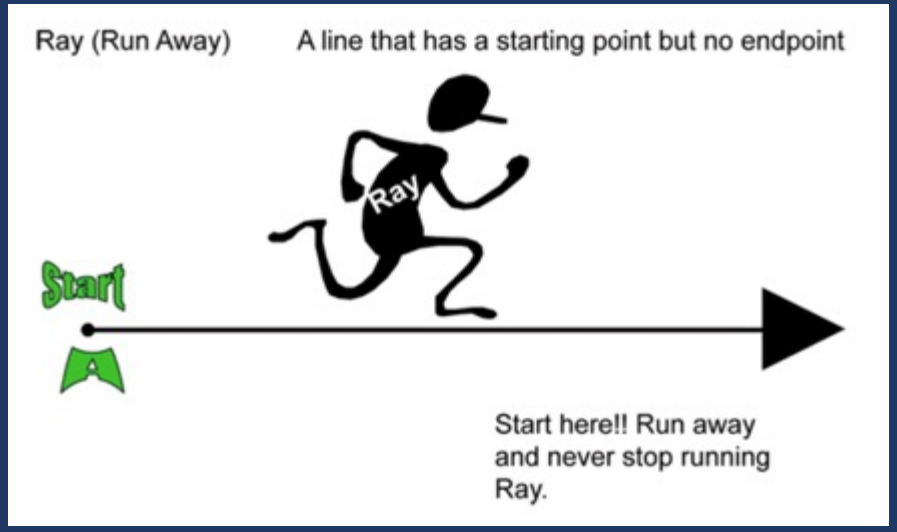


Rating	Word	Definition	Synonym(s)	Example	Sample Problem
2	expression	a mathematical phrase combining operations, numbers and/or variables.	phrase algebraic expression	6 6n 6+n (no equal sign)	Lucia earns $8 per hour for babysitting and gets a $5 tip. Write an expression to represent the amount she would earn if she worked for x hours.
2	variable	a quantity that can change or take many values. (refers to the letter or symbol representing the quantity)	unknown	x D y T	The variable x represents the number of hours Charlie works in a week. Write an expression to represent his earnings if he earns $9 per hour.
1	product	the result when two or more numbers are multiplied	total answer	3 x 2 = 6 (product)	The product of 6 and a number is 24. What is the number?
3	quotient	the result of a division (refers to the number of times the divisor divides the dividend)	answer	18 ÷ 2 = 9 (quotient) 9 ← quotient 2⟌18	Estimate the quotient when 365 is divided by 12.

Marin (2018)

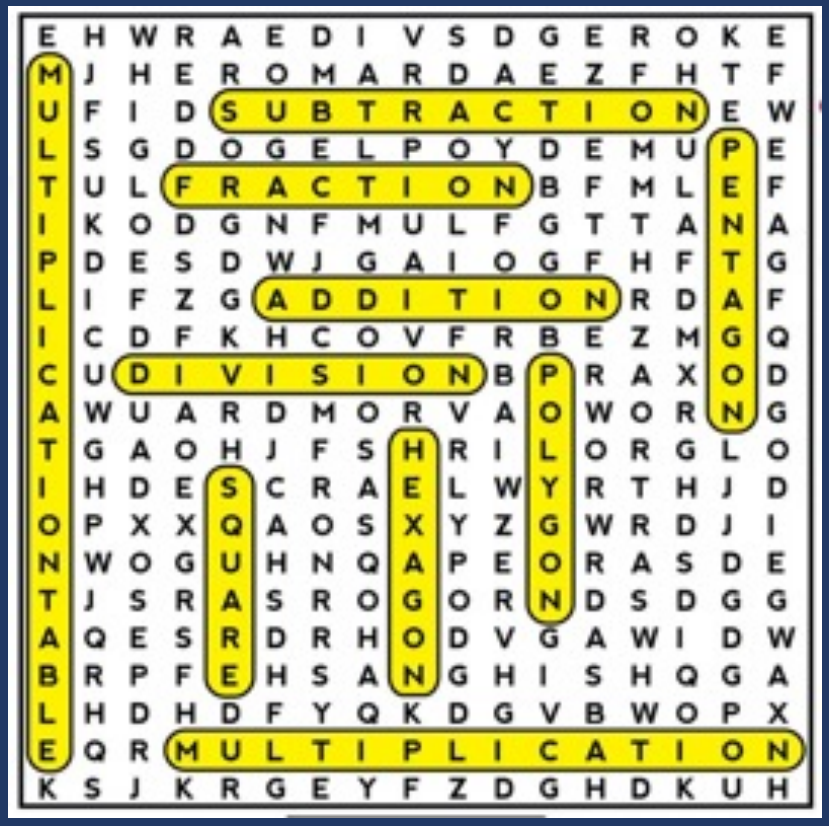
8. Use mnemonics

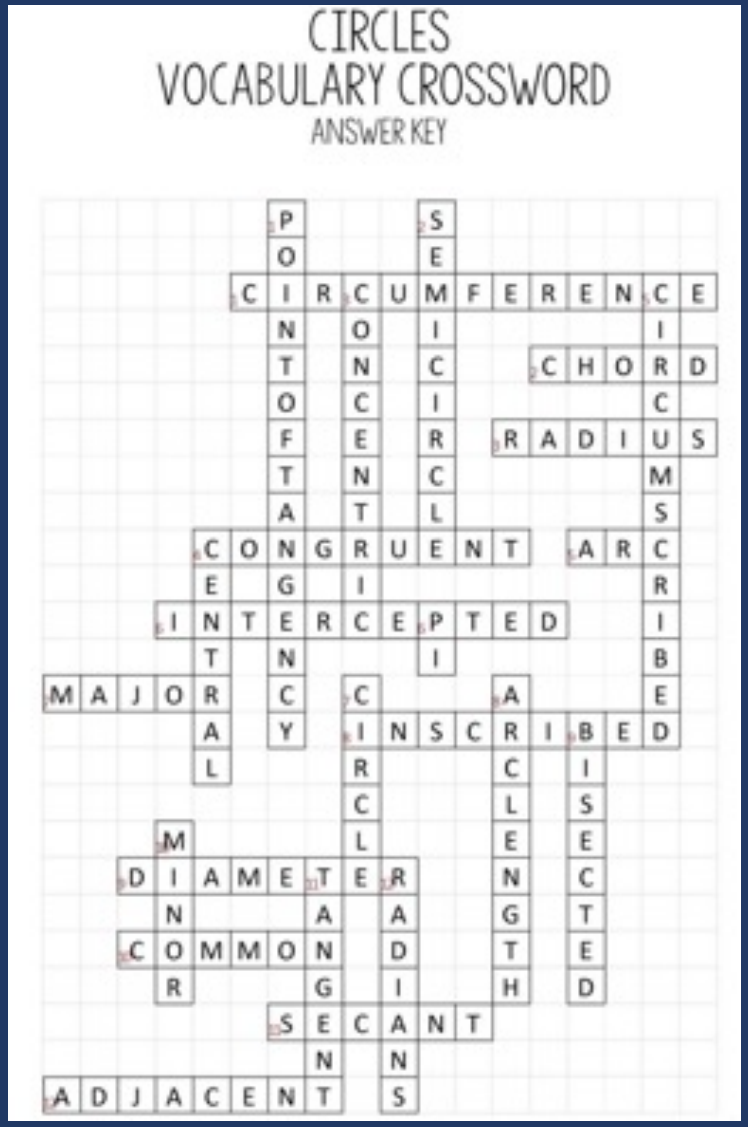




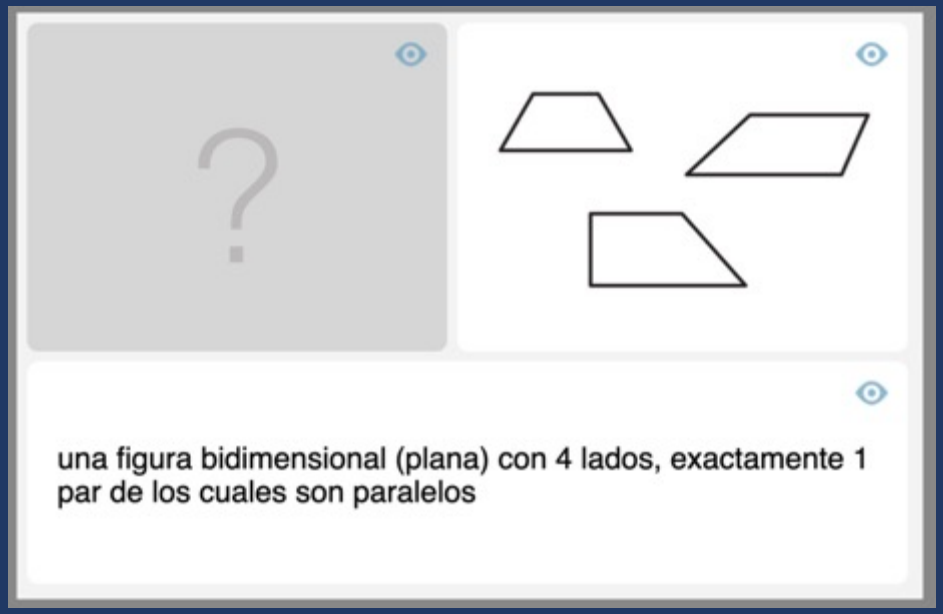
Riccomini et al. (2015)

9. Do word games





10. Use technology



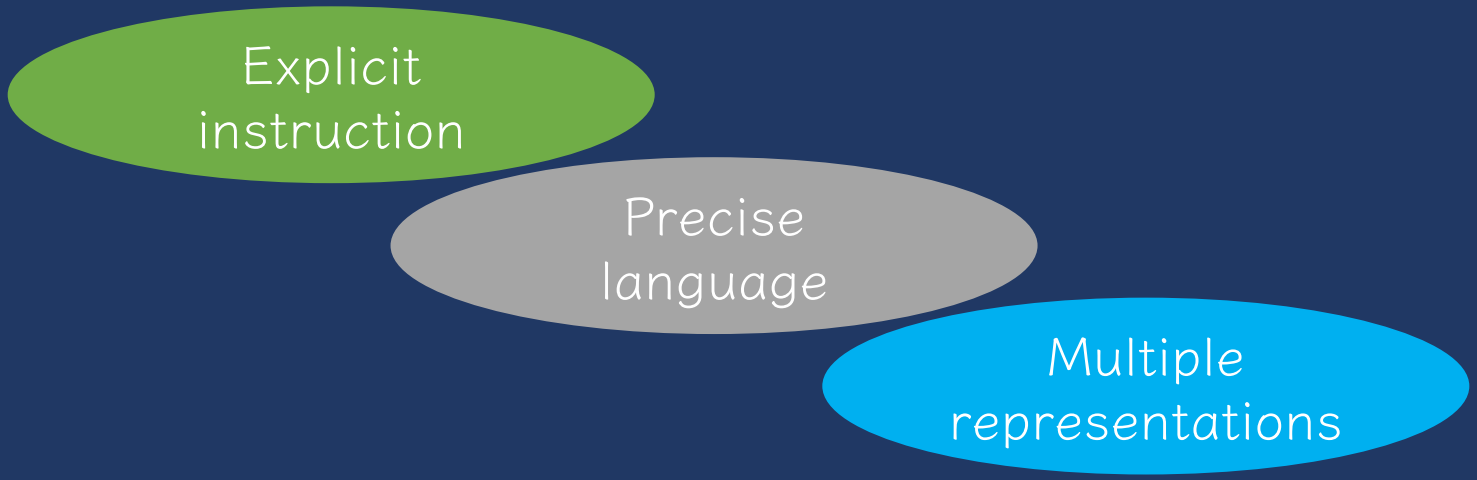
Math Learning Center

Math Lingo

What are other ways to support learning mathematics vocabulary?

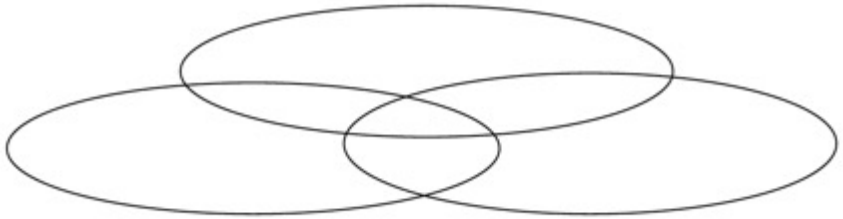
Instructional Platform

INSTRUCTIONAL DELIVERY

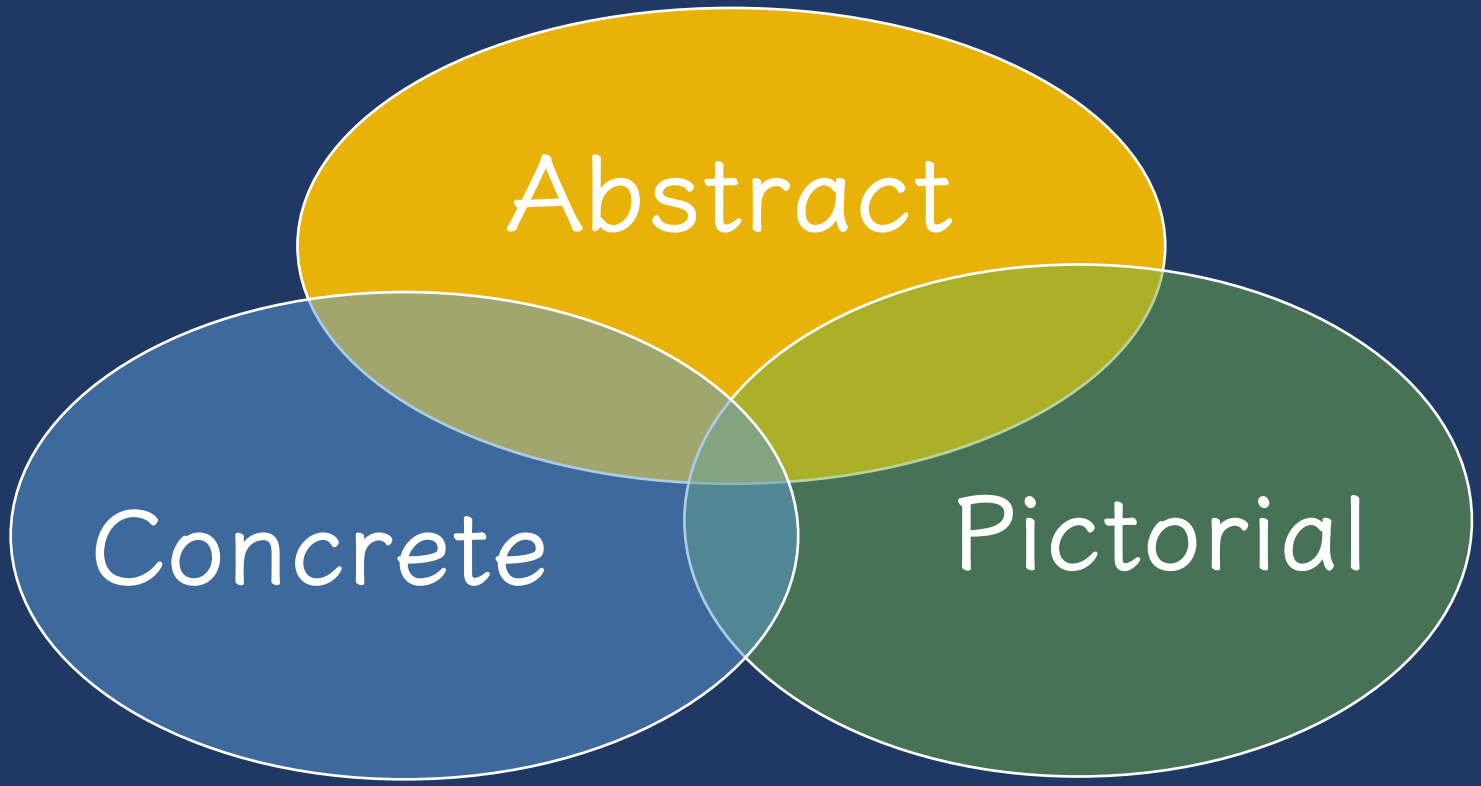


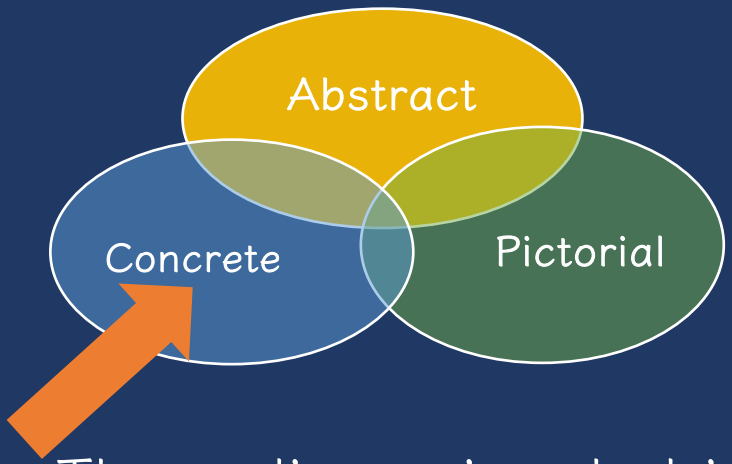
INSTRUCTIONAL STRATEGIES

Multiple Representations

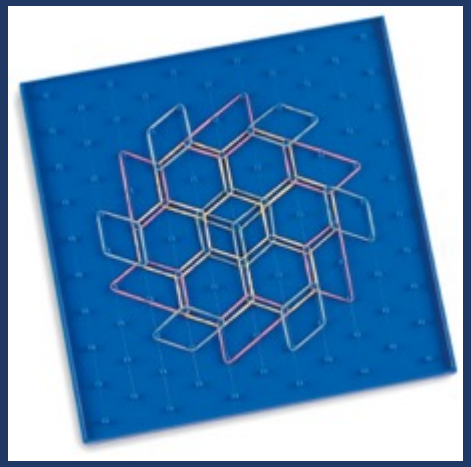


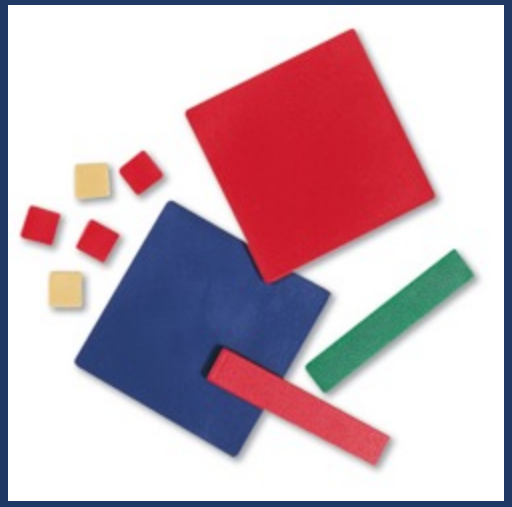
Multiple Representations

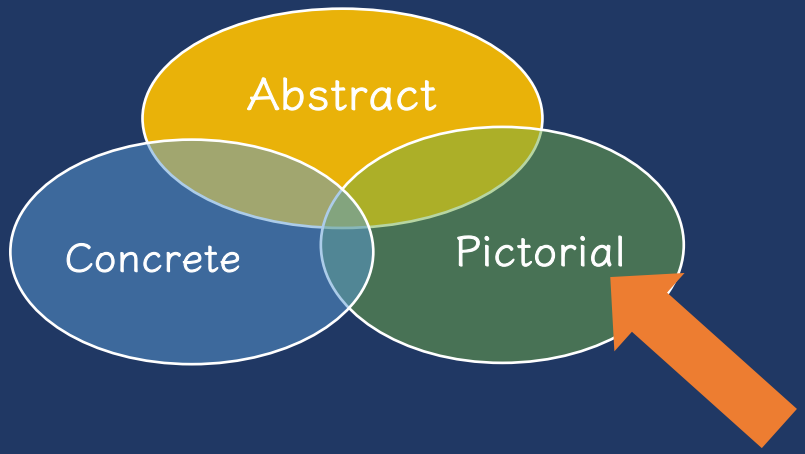


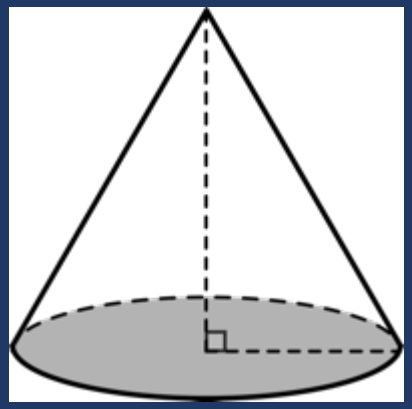


Three-dimensional objects

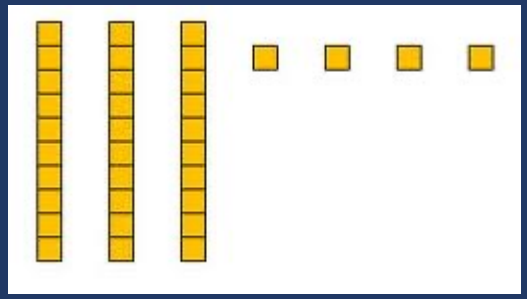


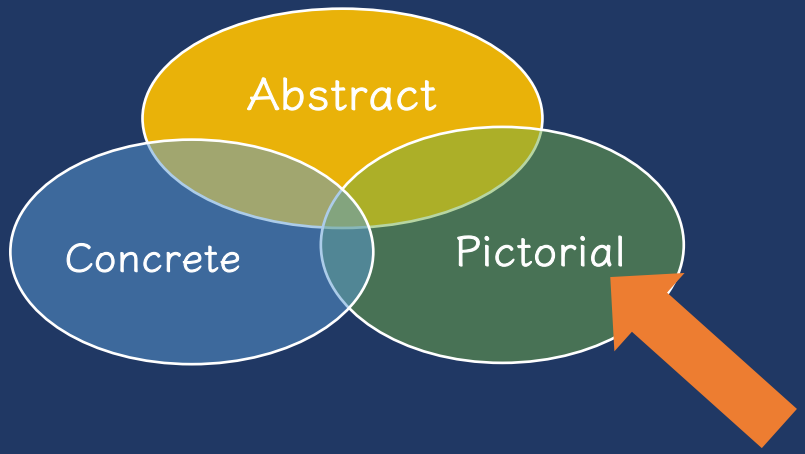




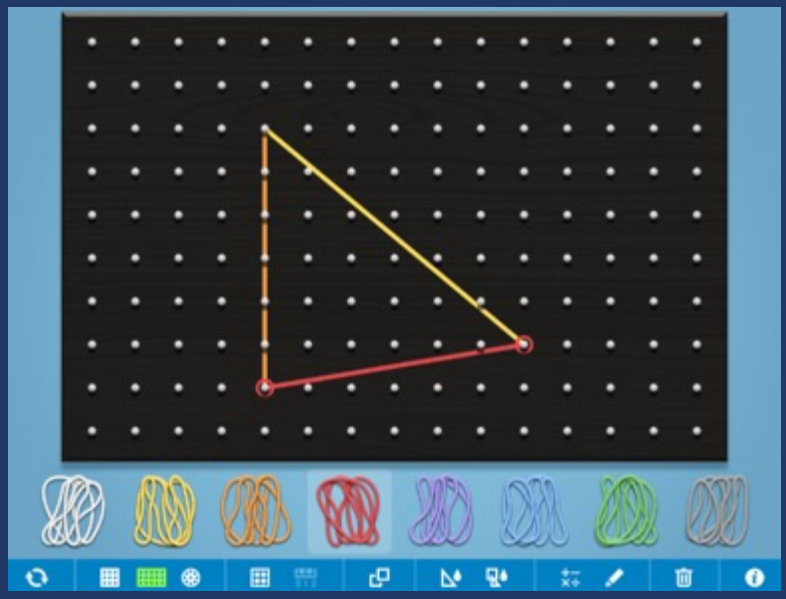


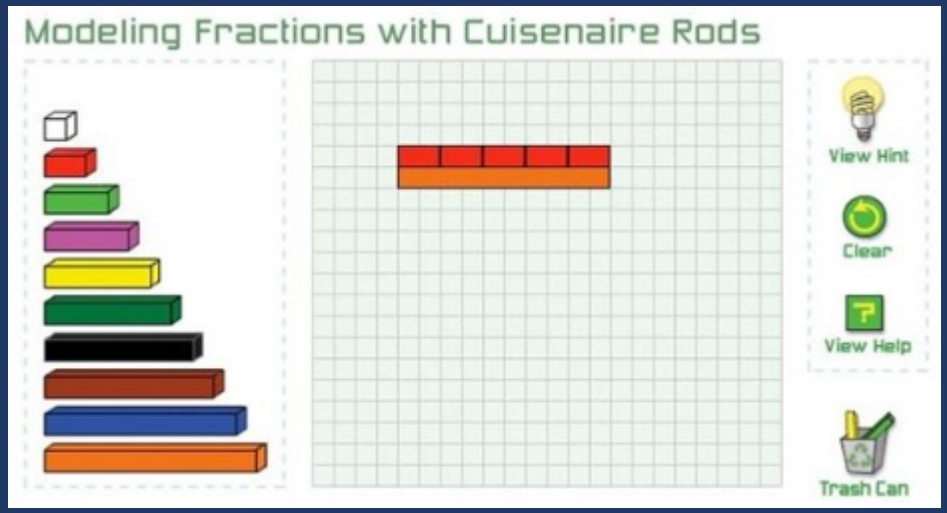
Two-dimensional images

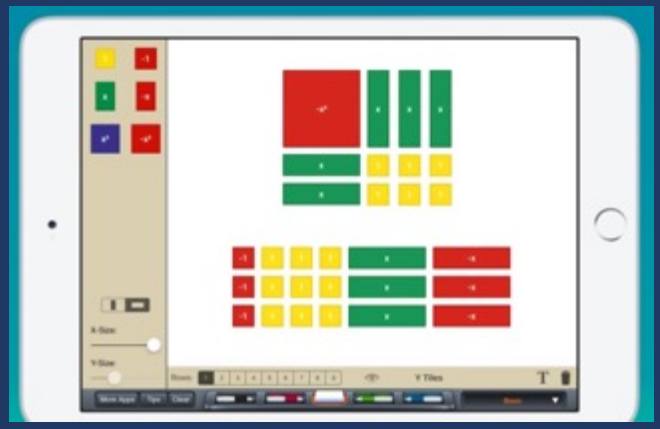


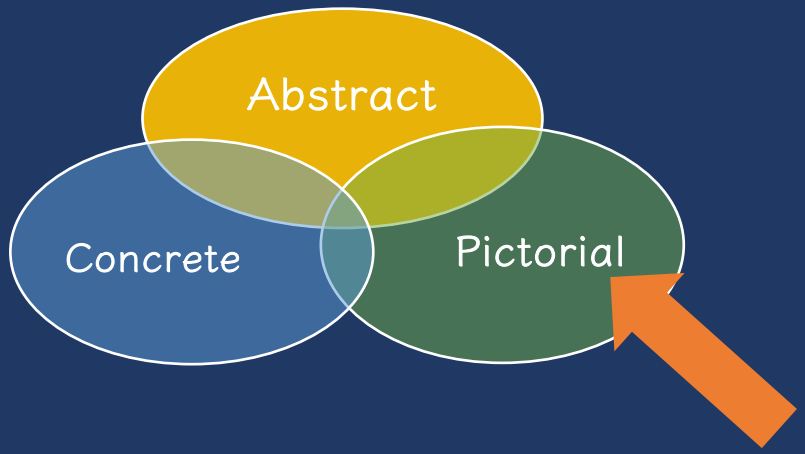


Two-dimensional images

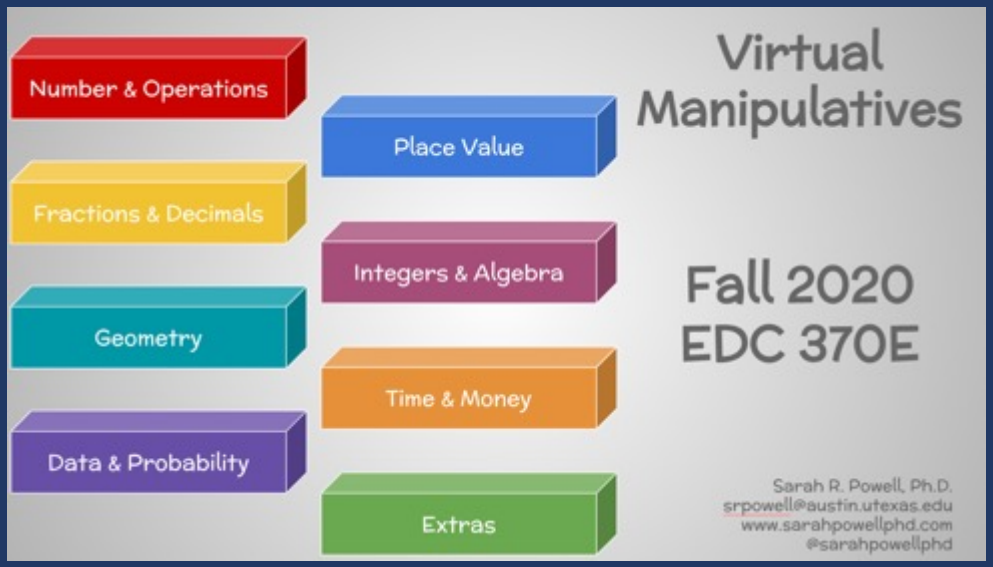


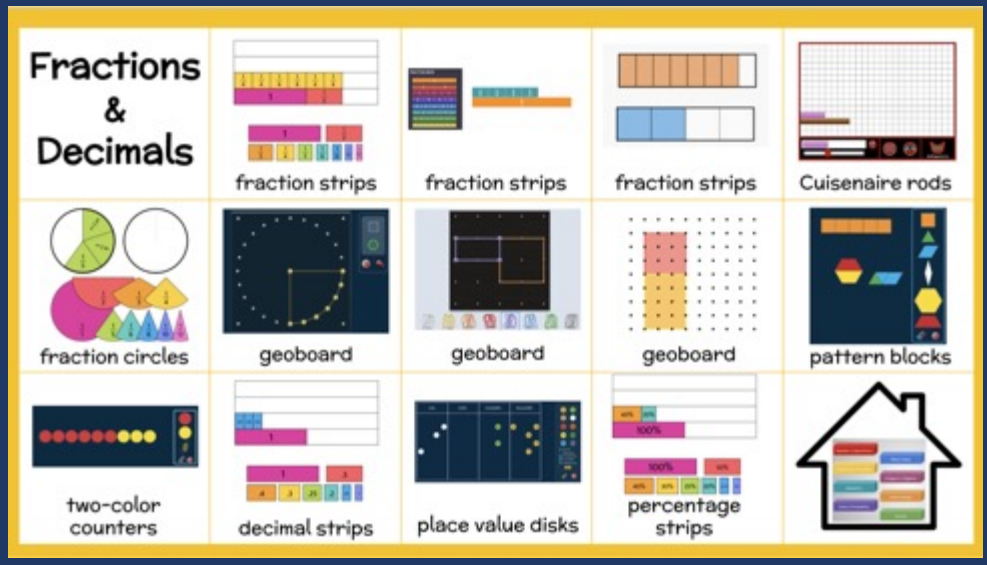


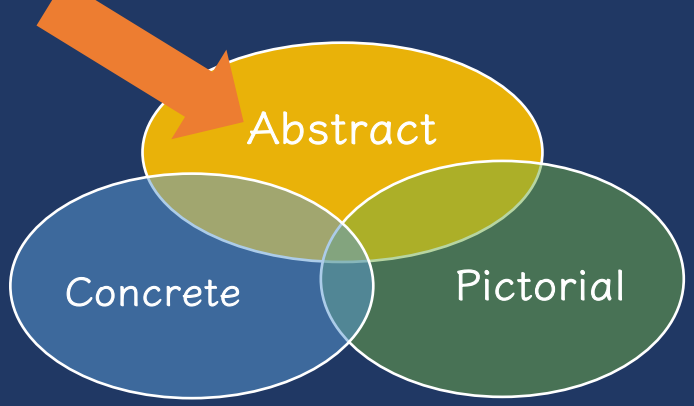




Two-dimensional images







Numerals and symbols and words

$2 + 8 = 10$

34 = 3 tens and 4 ones

$x - 6 = 8$

$$\begin{array}{r} 4{,}179 \\ +\quad 569 \\ \hline \end{array}$$

If you are left handed:

What's one of your favorite hands-on manipulatives?

If you are right handed:

What's one of your favorite virtual manipulatives?

Instructional Platform

INSTRUCTIONAL DELIVERY

Explicit instruction

Precise language

Multiple representations

INSTRUCTIONAL STRATEGIES

Fluency building

Fluency

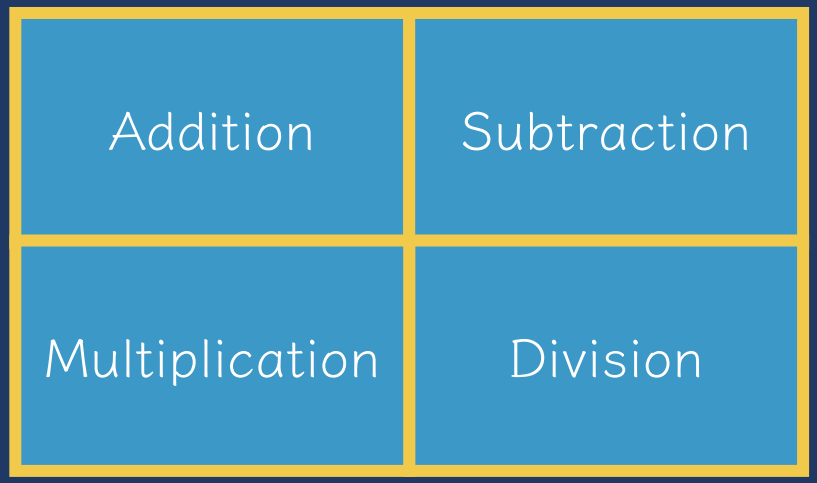
Addition

Subtraction

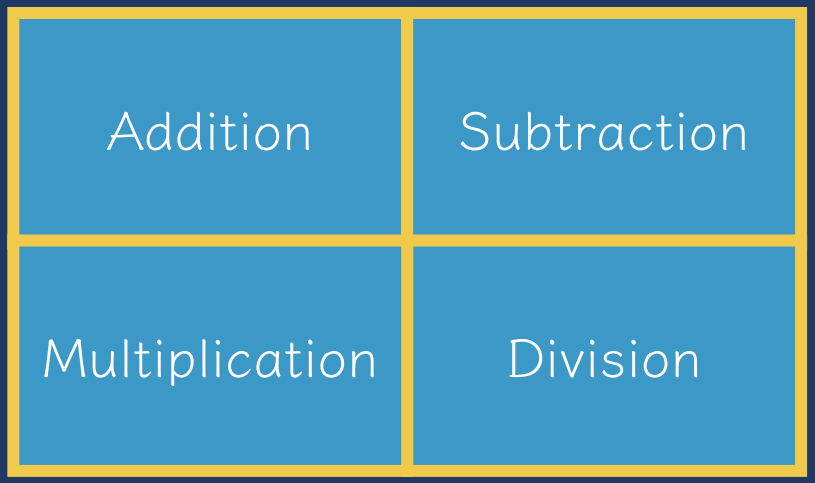
Multiplication

Division

Building Fluency



- Fluency is doing mathematics *easily* and *accurately*.
- Fluency in mathematics makes mathematics *easier*.
- Fluency provides less stress on working memory.
- Fluency helps students build confidence with mathematics.



- With fluency, it is important to emphasize both conceptual learning and procedural learning.
- Fluency is not strictly procedural!

Addition

100 addition basic facts

- Single-digit addends sum to a single- or double-digit number

$$\begin{array}{rl} 5 & \text{(addend)} \\ +\ 4 & \text{(addend)} \\ \hline 9 & \text{(sum)} \end{array}$$

Addition: Total (Part-Part-Whole, Combine)

Count one set, count another set, put sets together, count sum

$$2 + 3 = 5$$

Addition: Join (Change Increase)

Start with a set, add the other set, count sum

$$2 + 3 = 5$$

Total

Parts put together into a total

- Karly saw 4 cardinals and 5 blue jays. How many birds did Karly see?

  4 + 5 = ?

Change

An amount that **increases** or decreases

- Premila had $4. Then they earned $3 for cleaning their room. How much money does Premila have now?

  $4 + 3 = ?$

Total Versus Change (Join)

$$3 + 9 = \_\_$$

If you have brown *eyes*:

What's a Total story to show addition?

If you don't have brown *eyes*:

What's a Change/Join story to show addition?

Subtraction

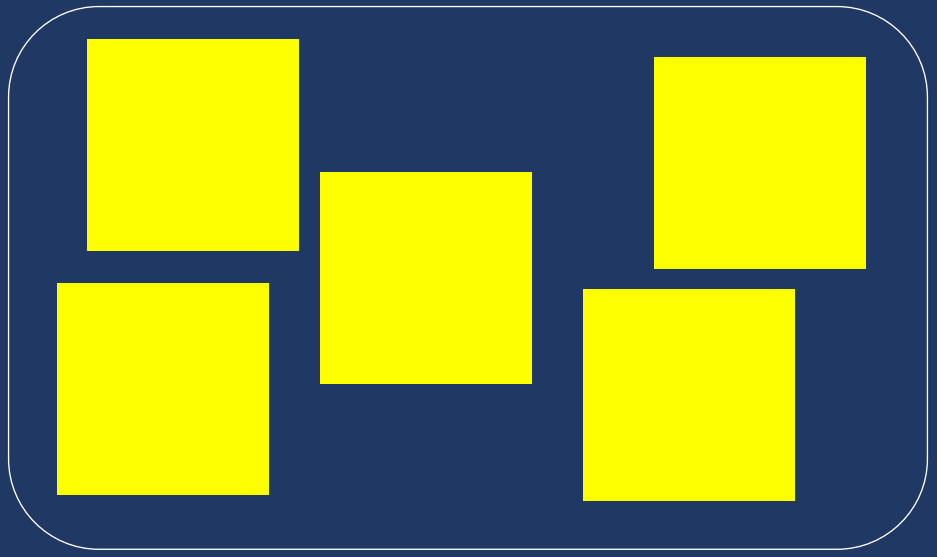
100 subtraction basic facts

- Subtrahend and difference are single-digit numbers and minuend is single- or double-digit number

```
   16      (minuend)
 −  8      (subtrahend)
 ----
    8      (difference)
```

Subtraction: Separate (Change Decrease)

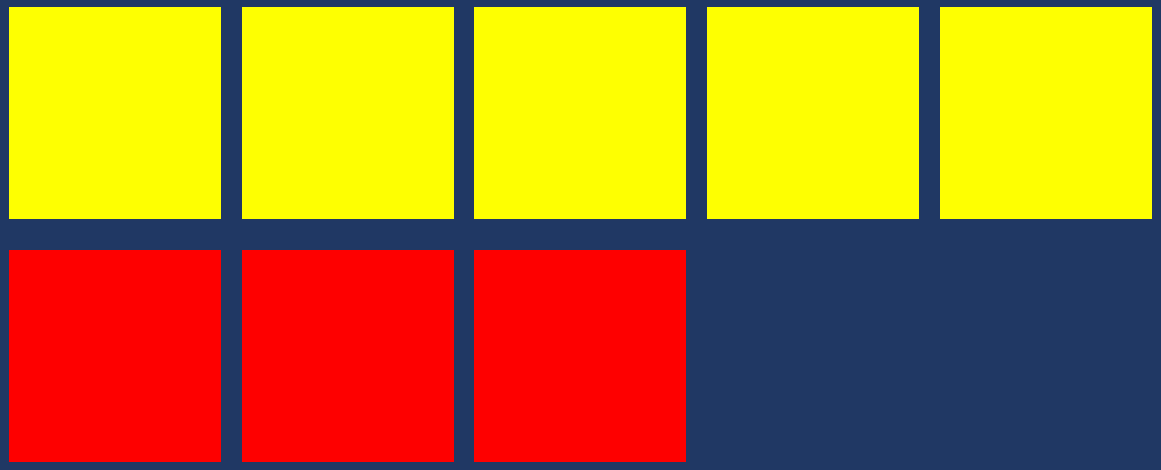
Start with a set, take away from that set, count difference



$$5 - 3 = 2$$

Subtraction: Difference (Compare)

Compare two sets, count difference



$$5 - 3 = 2$$

Change

An amount that increases or **decreases**

- Bronwyn had q cookies. Then they ate 2 of the cookies. How many cookies does Bronwyn have now?

  $q - 2 = ?$

Difference

Greater and less amounts compared for a difference

- Rachel has 9 apples. Jodie has 4 apples. How many more apples does Rachel have? (How many fewer does Jodie have?)

  $9 - 4 = ?$

Change (Separate) versus Difference

$$9 - 5 = \_\_$$

If you weren't born in Texas:

What's a Change/Separate story to show subtraction?

If you were born in Texas:

What's a Difference story to show subtraction?

Multiplication

100 multiplication basic facts

- Multiplication of single-digit factors results in a single- or double-digit product

$$\begin{array}{rl} 2 & \text{(factor)} \\ \times \quad 3 & \text{(factor)} \\ \hline 6 & \text{(product)} \end{array}$$

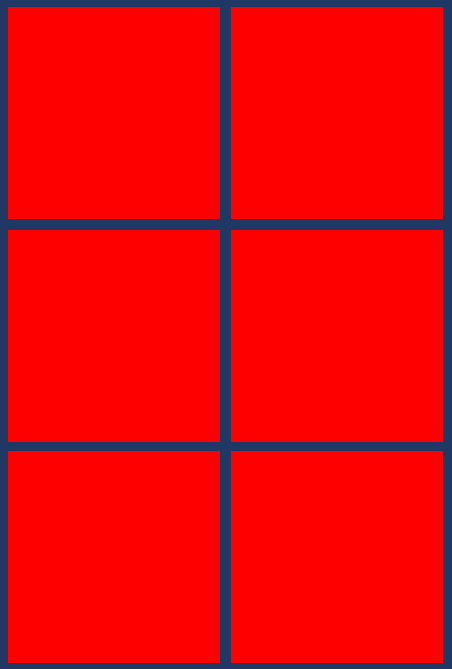
Multiplication: Equal Groups

Show the groups, show the amount for each group, count product

$$3 \times 2 = 6$$

Multiplication: Equal Groups

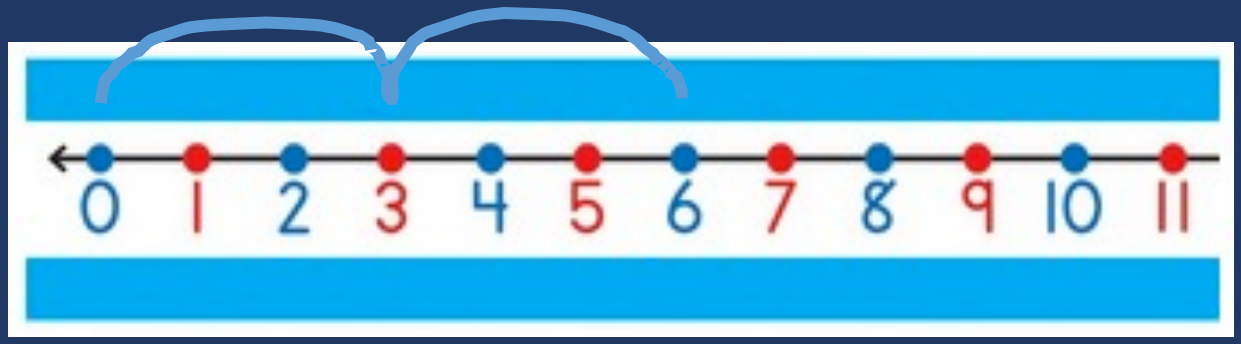
Show the groups, show the amount for each group, count product



$$3 \times 2 = 6$$

Multiplication: Comparison

Show a set, then multiply the set



$$3 \times 2 = 6$$

Equal Groups

Groups multiplied by number in each group for a product

- Rhiannon has 2 boxes of crayons. There are 12 crayons in each box. How many crayons does Rhiannon have altogether?

$$2 \times 6 = ?$$

Comparison

Set multiplied by a number of times for a product

- Vivienne picked 6 apples. Jessica picked 2 times as many apples as Vivienne. How many apples did Jessica pick?

$$6 \times 2 = ?$$

Equal Groups versus Comparison

$$2 \times 5 = \_\_$$

If you have glasses on:

What's an Equal Groups story to show multiplication?

If you don't have glasses on:

What's a Comparison story to show multiplication?

Division

90 division basic facts

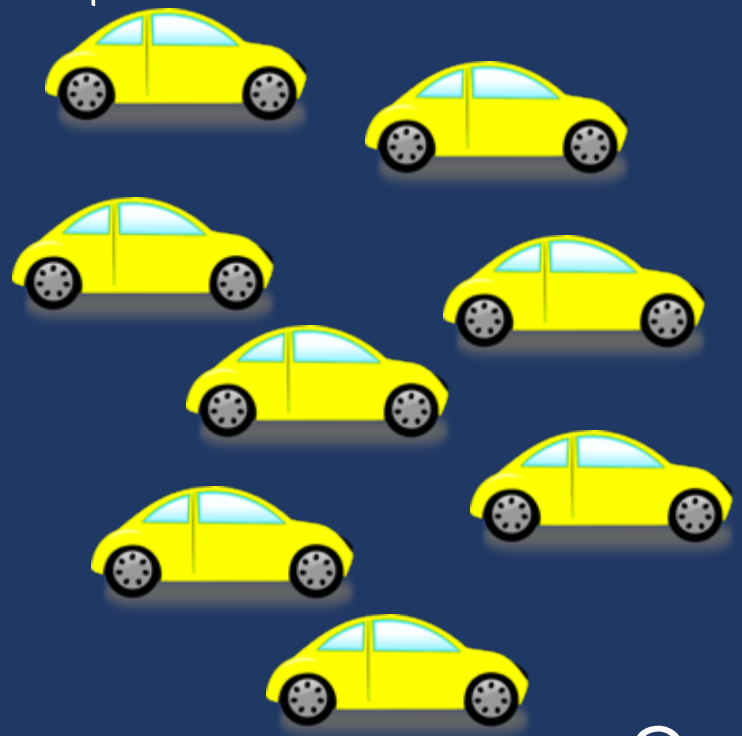
- Divisor and quotient are single-digit numbers and dividend is single- or double-digit number

$$8 \div 4 = 2$$

(dividend) (divisor) (quotient)

Division: Equal Groups (Partitive Division)

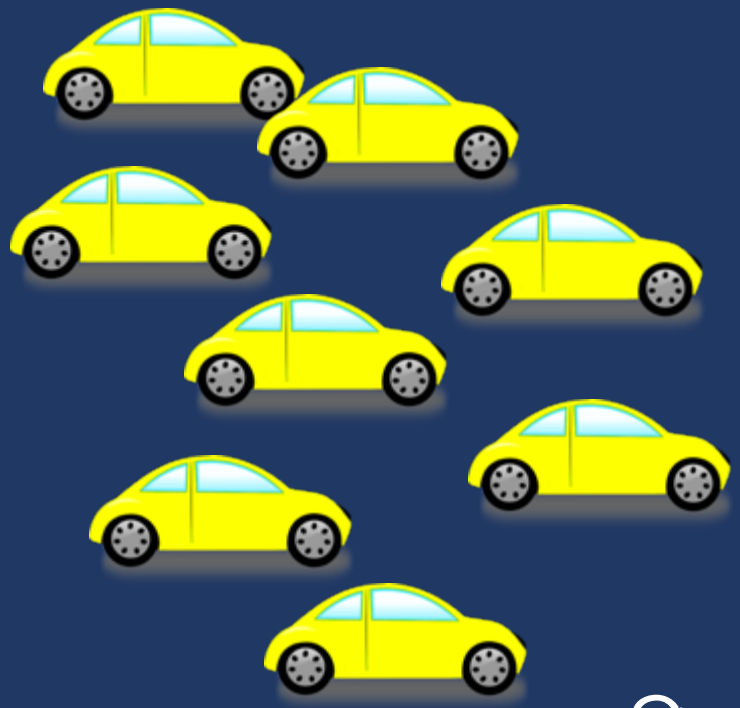
Show the dividend, divide equally among divisor, count quotient



$$8 \div 2 = 4$$

Division: Equal Groups (Quotative Division)

Show the dividend, make groups of the divisor, count groups



$$8 \div 2 = 4$$

Equal Groups

Groups multiplied by number in *each* group for a product

- Stefanie has 12 apples. She wants to share them *equally* among her 2 friends. How many apples will *each* friend receive?

  $2 \times ? = 12$

- Nicole has 12 apples. She put them into bags containing 6 apples *each*. How many bags did Nicole use?

  $? \times 6 = 12$

Partitive versus Quotative

$$12 \div 4 = \_\_$$

If you watched *Friends*:

What's a Partitive story to show division?

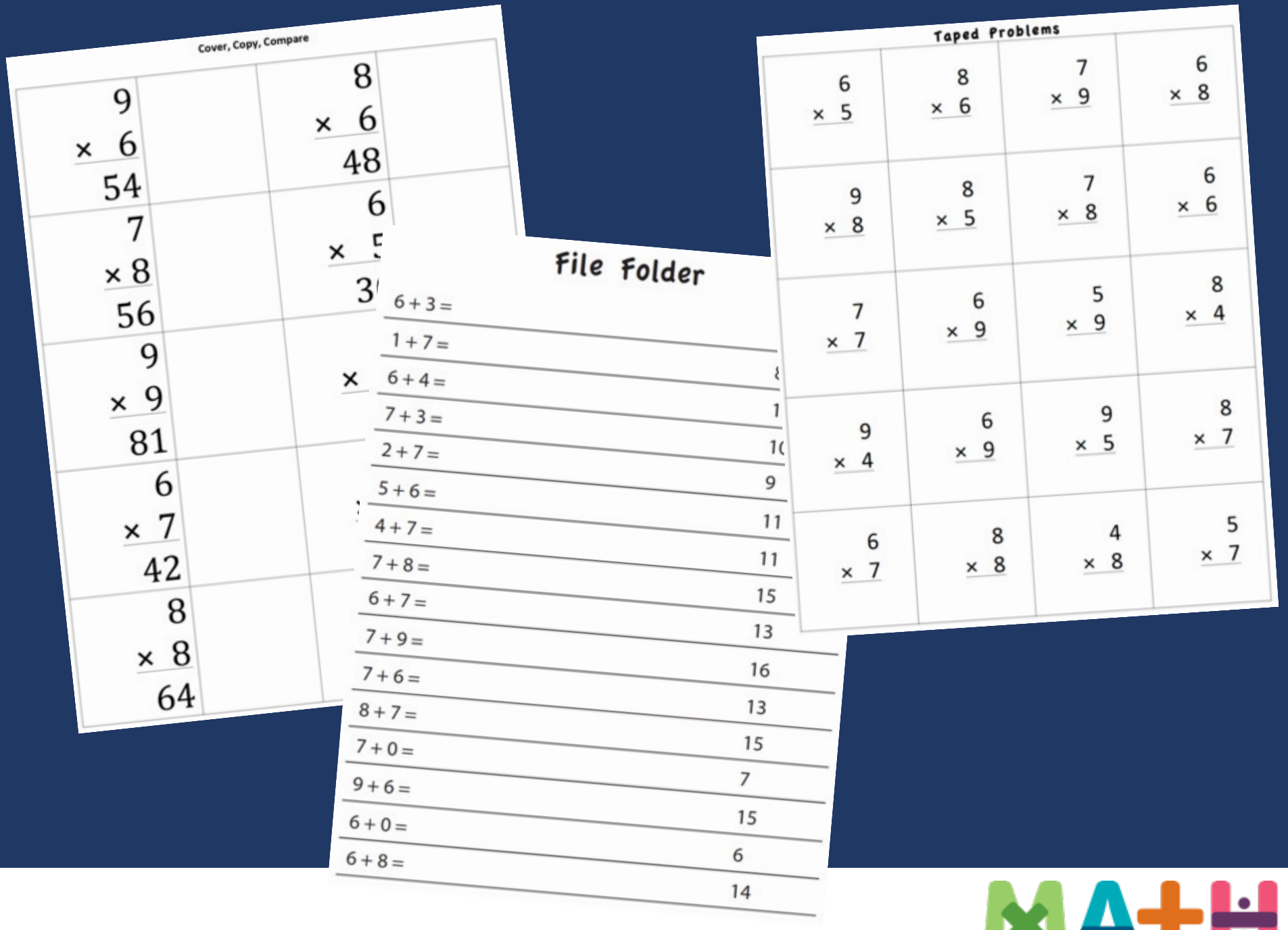
If you watched *Seinfeld*:

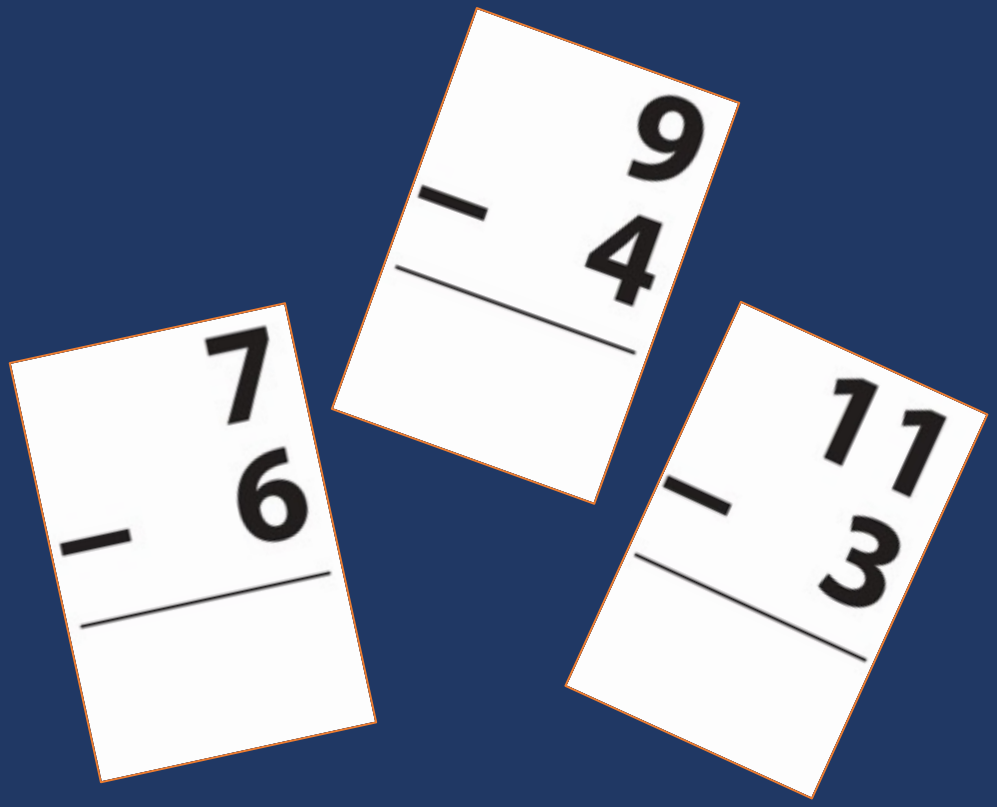
What's a Quotative story to show division?

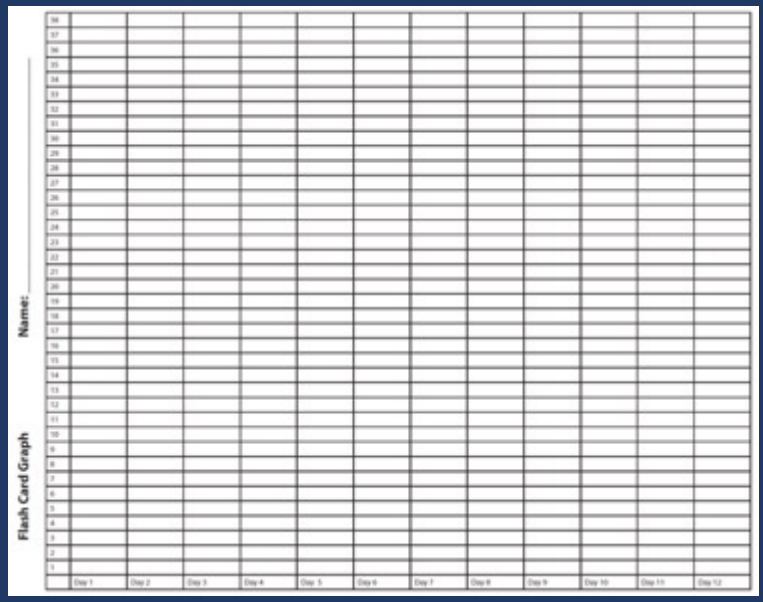
Addition	Subtraction
Multiplication	Division

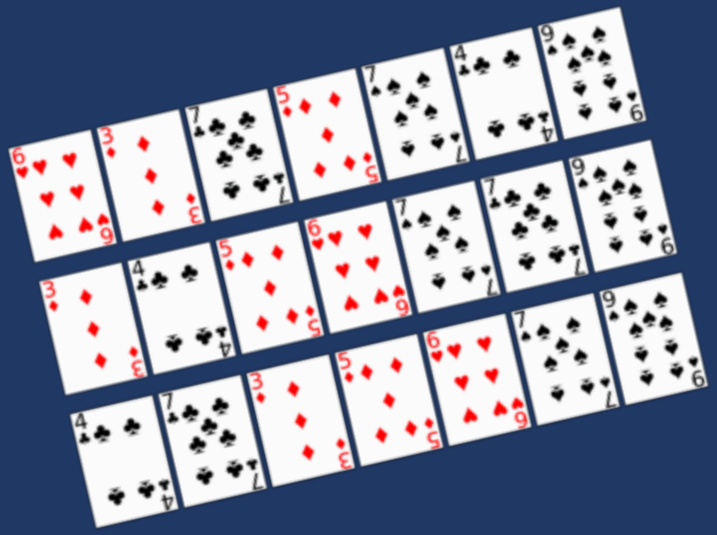
- Build fluency with math facts.
  - Addition: single-digit addends
  - Subtraction: single-digit subtrahend
  - Multiplication: single-digit factors
  - Division: single-digit divisor

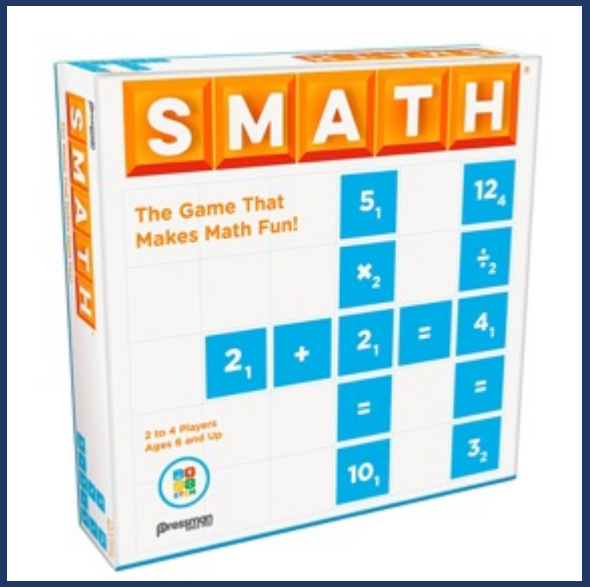
$$\begin{array}{cccc} 5 & 9 & 6 & 56 \\ +\ 8 & -\ 4 & \times\ 7 & \div\ 8 \end{array}$$

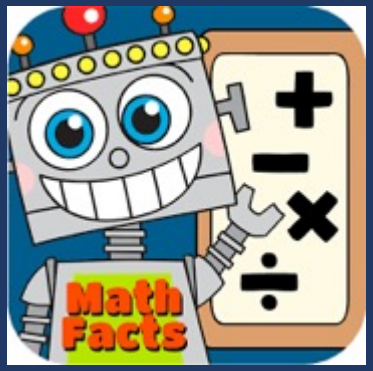












Addition	Subtraction
Multiplication	Division

What are other ways to practice fluency?

Addition	Subtraction
Multiplication	Division

- Build fluency with whole-number computation

$$\begin{array}{r} 15 \\ +\ 28 \\ \hline \end{array} \qquad \begin{array}{r} 1009 \\ -\ 724 \\ \hline \end{array}$$

$$\begin{array}{r} 23 \\ \times\ 9 \\ \hline \end{array} \qquad \begin{array}{r} 7250 \\ \div\ 15 \\ \hline \end{array}$$

Addition	Subtraction
Multiplication	Division

- Build fluency with rational-number computation

$$\begin{array}{r} 1.4 \\ +\ 3.9 \\ \hline \end{array} \qquad \begin{array}{r} 7.892 \\ \div\ 0.14 \\ \hline \end{array}$$

$$\frac{2}{3} \times \frac{3}{4} \qquad \frac{9}{4} - \frac{3}{8}$$

Addition	Subtraction
Multiplication	Division

- Build fluency with integer computation

$-135 \div 2 =$

$$\begin{array}{r} 6 \\ \times\ -12 \\ \hline \end{array}$$

$-14 - (-7) =$

$$\begin{array}{r} 1.4 \\ +\ -3.9 \\ \hline \end{array}$$

Addition	Subtraction
Multiplication	Division

How will your support math language in your intervention?

How will you use multiple representations in your math intervention?

How will you build fluency in your intervention?

Instructional Platform

INSTRUCTIONAL DELIVERY

Explicit instruction

Precise language

Multiple representations

INSTRUCTIONAL STRATEGIES

Fluency building

Problem solving instruction

Word-Problem Solving

Maya has 120 caramel apples to sell. Each caramel apple is covered with one topping.

- $\frac{1}{5}$ of the caramel apples are covered with peanuts.
- $\frac{1}{3}$ are covered with chocolate chips.
- $\frac{3}{10}$ are covered with coconut.
- The rest are covered with sprinkles.

How many caramel apples are covered with sprinkles?

A 100

B 33

C 25

D 20

Solve the problem

What skills are necessary to solve this problem?

Maya has 120 caramel apples to sell. Each caramel apple is covered with one topping.

- $\frac{1}{5}$ of the caramel apples are covered with peanuts.
- $\frac{1}{3}$ are covered with chocolate chips.
- $\frac{3}{10}$ are covered with coconut.
- The rest are covered with sprinkles.

How many caramel apples are covered with sprinkles?

A 100

B 33

C 25

D 20

How would you solve this problem? What skills are necessary to solve this problem?

Problem-Solving Difficulties

Teaching Problem Solving

Problem-Solving Difficulties

- Reading the problem
- Understanding the vocabulary
- Identifying relevant information
- Ignoring irrelevant information
- Interpreting charts and graphs
- Identifying appropriate operation(s)
- Performing the computation(s)

Maya has 120 caramel apples to sell. Each caramel apple is covered with one topping.

- $\frac{1}{5}$ of the caramel apples are covered with peanuts.
- $\frac{1}{3}$ are covered with chocolate chips.
- $\frac{3}{10}$ are covered with coconut.
- The rest are covered with sprinkles.

How many caramel apples are covered with sprinkles?

A 100

B 33

C 25

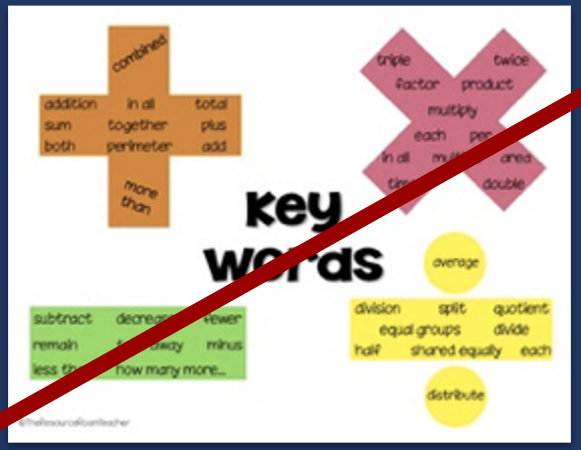
D 20

What are additional areas of difficulty that we should add to this list?

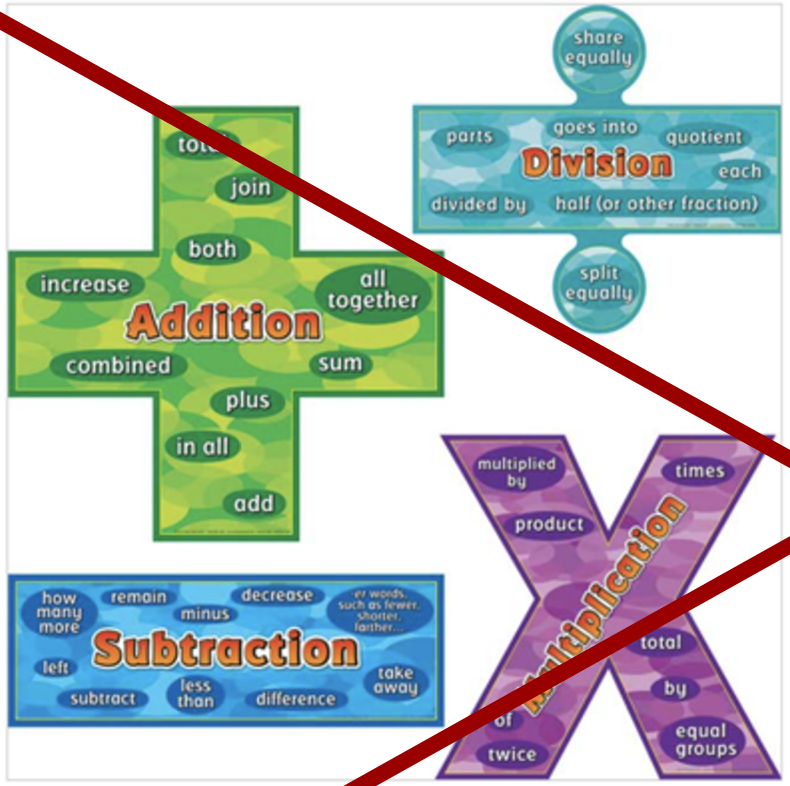
~~1. Keywords tied to operations~~

Lincoln had 8 pencils **fewer** than Roscoe. If Roscoe had 18 pencils, how many pencils did Lincoln have?

Lincoln had 8 pencils **fewer** than Roscoe. If Lincoln had 18 pencils, how many pencils did Roscoe have?



Word-Problem Words Poster Set



Item #162978

(4.1)

75%

of respondents would recommend this to a friend

7 Reviews

Write a Review

What do you want to know about this product?

Expect an answer in about 48 hours

$12.99

Pay with 3 monthly payments of just $4.33. No fees.

Learn more

1 poster set

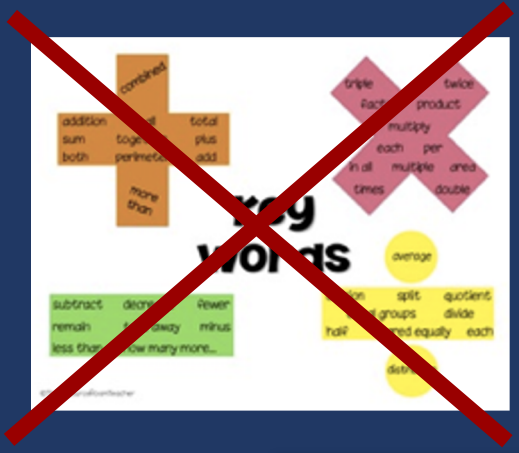
Quantity

– 1 +

Ready to Ship

Add to Cart

Add to Wish list



Description of Single-Step Word Problems (n = 132)

Schema	Occurrence of schema		Any keyword		Schema-specific keywords[a]		Multiple keywords[a]		Keyword(s) led to correct solution[a]	
	n	%	*n*	%	*n*	%	*n*	%	*n*	%
Total	27	20.5	26	96.3	23	88.5	5	19.2	21	80.8
Difference	17	12.9	17	100.0	14	82.4	2	11.8	12	70.6
Change	11	8.3	7	63.6	5	71.4	5	71.4	2	28.6
Equal groups	29	22.0	26	89.7	22	84.6	18	69.2	8	30.8
Comparison	10	7.6	9	90.0	9	100.0	4	44.4	5	55.6
Ratios or proportions	29	22.0	23	79.3	9	39.1	9	39.1	6	26.1
Product of measures	9	6.8	9	100.0	8	88.9	1	11.1	5	55.6

[a]When a problem featured a keyword.

Description of Multi-Step Word Problems (n = 84)

	Occurrence of schema[a]		Any keyword		Keyword(s) led to correct solution[b]	
Schema	*n*	%	*n*	%	*n*	%
Total	40	47.6	39	97.5	3	7.7
Difference	11	13.1	11	100.0	1	9.1
Change	21	23.8	19	95.0	1	5.3
Equal groups	49	58.3	48	98.0	1	2.1
Comparison	7	8.3	7	100.0	0	0.0
Ratios or proportions	22	25.0	16	76.2	1	6.3
Product of measures	7	8.3	7	100.0	2	28.6

[a]Sum across schemas does not equal 100 because each word problem featured more than one schema.

[b]When a problem featured a keyword.

Mr. Rivera's taxable income is $20 each hour before taxes are taken out. Mr. Rivera worked a total of 40 hours each week for 50 weeks.

What is the dollar amount, to the nearest dollar, taken out for taxes based on Mr. Rivera's taxable income?

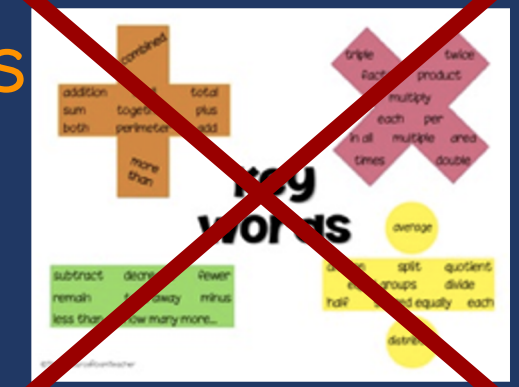
Jessica rented 1 video game and 3 movies for a total of $11.50.

- The video game cost $4.75 to rent.
- The movies cost the same amount each to rent.

What amount, in dollars, did Jessica pay to rent each movie?

The temperature of a substance decreased by 24°C per minute for 3 minutes. What was the overall change of the temperature of the substance?

Important notes about keywords



Keywords are important to identify and understand

Keywords are often the mathematical vocabulary that help an students understand what the story is about and what they need to do

Talk about keywords ("What does *more than* tell you about?")

But, *do not* tie a keyword to a specific operation!

~~2. Presenting problems by operation~~

Name: _____ Date: _____

Addition Word Problems

Solve the word problems. Show your work.

1. Noah had 12 books. He got 5 more books. How many books did Noah have in all?
2. Bonnie found 8 rocks on the sidewalk and 7 rocks in her backyard. How many rocks did Bonnie find in all?
3. Edward had 5 toy cars. He got 8 more toy cars. How many toy cars did Edward have in all?
4. Mariela collected 11 feathers. Then she found 3 more feathers. How many feathers did Mariela have in all?
5. LaMonte made 14 cookies. Then he made 5 more cookies. How many cookies did LaMonte have in all?

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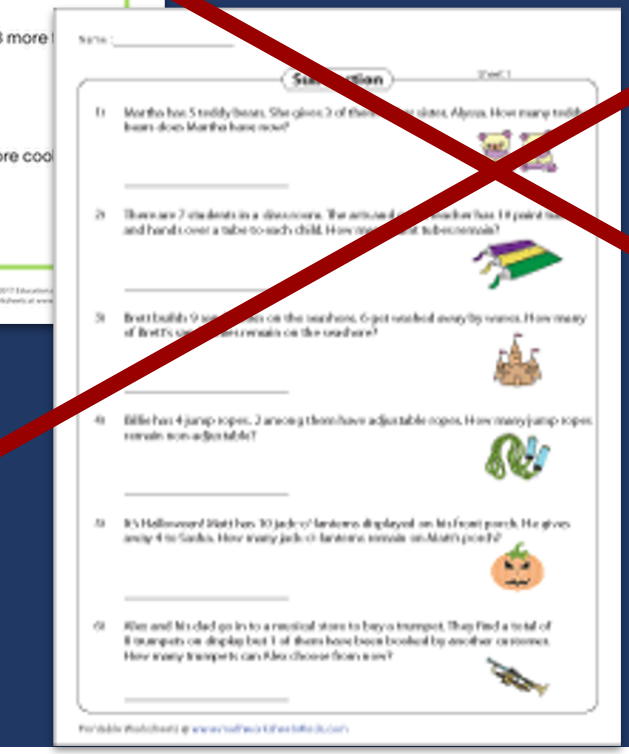

LONG DIVISION
WORD PROBLEMS

1. Zookeeper Al wants to give each monkey at the zoo an equal number of bananas. There are 37 monkeys in the zoo and 567 bananas. How many bananas does each monkey get? And How many are left over for him to eat himself?
2. Betty has 345 oranges and needs to pack them up equally in 23 boxes. How many oranges go in each box and how much does she have left over?
3. Miss King has 1376 pages of scrap paper. She wants to make them into scrap paper packets for her 32 students. How many pages will each packet have? How many extra pages will she have left over?
4. Mr. Chong has 1,440 pages of scrap paper. He instead wants to make packets of 40 pages each but forgets to check if that will be enough for his 37 students. Will there be enough packets per student? If not how much more scrap paper does he need?

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Teaching Problem Solving

Have an attack strategy

Teach word-problem schemas

Have an attack strategy

RIDE

Read the problem.

Identify the relevant information.

Determine the operation and unit for the answer.

Enter the correct numbers and calculate, then check the answer.

RIDGES

Read the problem.
I know statement.
Draw a picture.
Goal statement.
Equation development.
Solve the equation.

Have an attack strategy

RICE

Read and record the problem.
Illustrate your thinking.
Compute.
Explain your thinking.

STAR

Stop and read the problem carefully.
Think about your plan and the strategy you will use.
Act. Follow your plan and solve the problem.
Review your answer.

Have an attack strategy

SUPER

Slowly read the story problem twice.
Underline the question and circle the numbers you need.
Picture it. Draw the scenario to show what is happening.
Explain the problem with a number sentence.
Rewrite the answer in a sentence.

SHINES

Slowly and carefully read the problem.
Highlight or underline key information.
Identify the question by drawing a circle around it.
Now solve the problem. Show your work.
Examine your work for precision, accuracy, and clarity.
Share your answer by writing a sentence.

Have an attack strategy

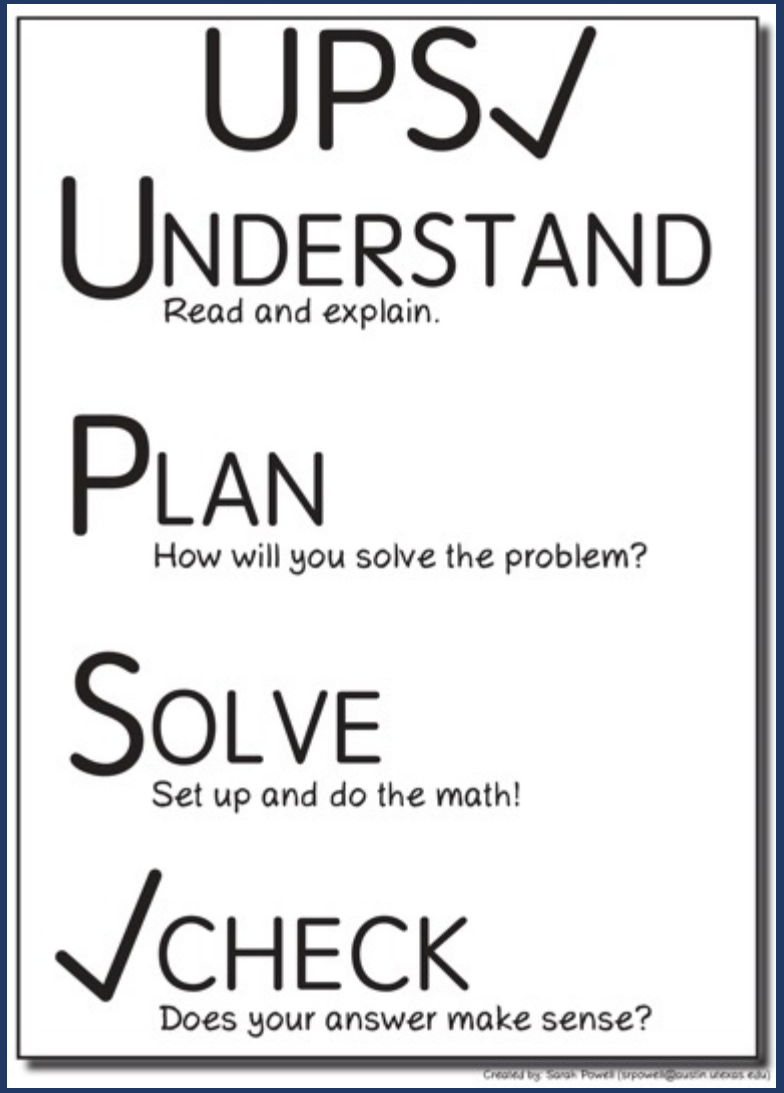
R-CUBES

Read the problem.
Circle key numbers.
Underline the question.
Box action words.
Evaluate steps.
Solve and check.

SOLVE

Study the problem.
Organize the facts.
Line up the plan.
Verify the plan with computation.
Examine the answer.

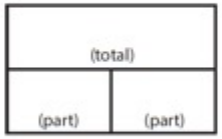
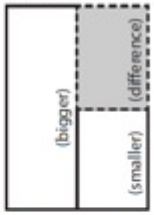
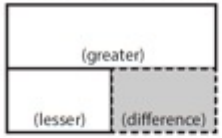
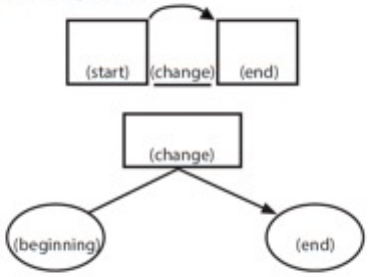
Have an attack strategy



Share your favorite attack strategy.

Teach word-problem schemas

- Total
- Difference
- Change
- Equal Groups
- Comparison
- Ratios/Proportions

Schema and Definition	Equations and Graphic Organizers	Examples			Variations
Total (Combine; Part-part-whole) Parts combined for a sum	P1 + P2 = T (part + part = total)  	Sum unknown: Lyle has 11 red apples and 18 green apples. How many apples does Lyle have altogether?	Part unknown: Lyle has 29 red and green apples. If 11 of the apples are red, how many green apples does Lyle have?		More than two parts: Lyle has 34 apples. Of the apples, 11 are red, 18 are green, and the rest are yellow. How many yellow apples does Lyle have?
Difference (Compare) Sets compared for a difference	B − s = D (bigger − smaller = difference)  G − L = D (greater − less = difference)  	Difference unknown: Sasha wrote 85 words in her essay, and Tabitha wrote 110 words. How many fewer words did Sasha write than Tabitha?	Bigger/greater unknown: Tabitha wrote 25 more words than Sasha. If Sasha wrote 85 words, how many words did Tabitha write?	Smaller/lesser unknown: Tabitha wrote 110 words in her essay. Sasha wrote 25 words fewer than Tabitha. How many words did Sasha write?	(None)
Change (Join; Separate) An amount that increases or decreases	ST +/ − C = E (start +/ − change = end)  	End (increase) unknown: Jorge had $52. Then, he earned $16 babysitting. How much money does Jorge have now?	Change (increase) unknown: Jorge had $52. Then, he earned some money babysitting. Now, Jorge has $68. How much did Jorge earn babysitting?	Start (increase) unknown: Jorge has some money, and then he earned $16 for babysitting. Now, Jorge has $68. How much money did he have to start with?	Multiple changes: Jorge had $78. He stopped and bought a pair of shoes for $42 and then he spent $12 at the grocery. How much money does Jorge have now?
		End (decrease) unknown: Jorge had $52. Then, he spent $29 at the ballpark. How much money does Jorge have now?	Change (decrease) unknown: Jorge had $52 but spent some money when he went to the ballpark. Now, Jorge has $23. How much did Jorge spend at the ballpark?	Start (decrease) unknown: Jorge had some money. Then, he spent $29 at the ballpark and has $23 left. How much money did Jorge have before going to the ballpark?	

Total

Parts put together into a total

Daniela saw 3 canoes and 8 kayaks. How many boats did Daniela see?

Daniela saw 11 boats. If 3 of the boats were canoes, how many were kayaks?

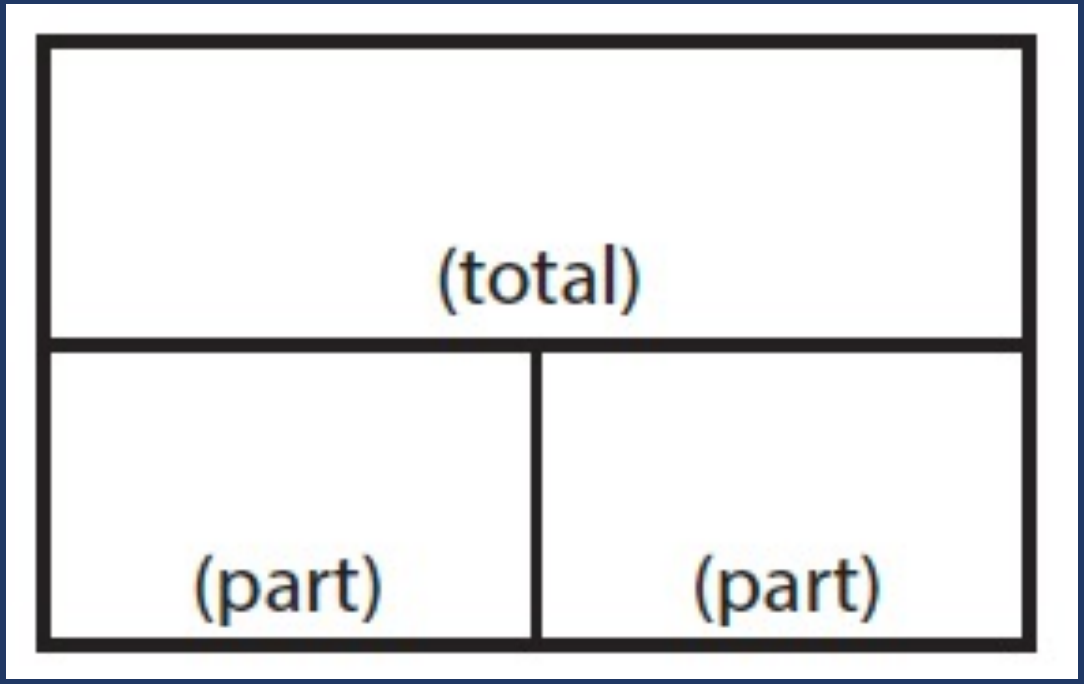
Daniela saw 11 boats. 8 of the boats were kayaks, how many were canoes?

Total

"Are parts put together for a total?"

Total

P1 + P2 = T



Fuchs et al. (2008); Griffin & Jitendra (2009)

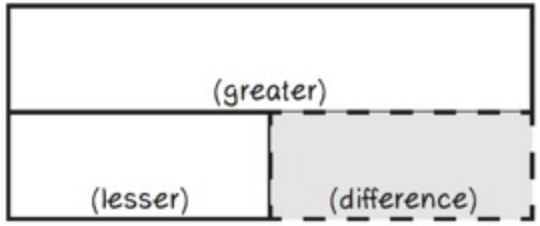
TOTAL

Are parts put together for a total?

P1 + P2 = T

DIFFERENCE

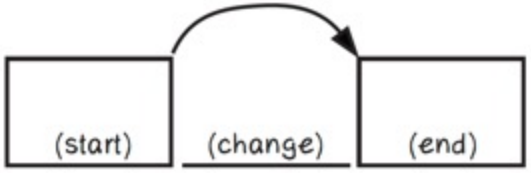
Are amounts compared for a difference?



G - L = D

CHANGE

Does an amount increase or decrease?

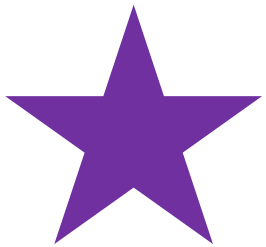


ST +/- C = E

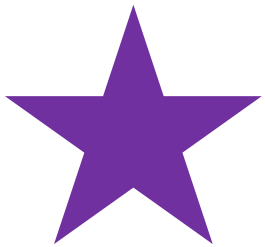
Total

Additive Word Problems

A.
Megan baked 38 sugar cookies and 24 chocolate chip cookies. Enter the total number of cookies Megan baked in all.



B.
In March and April, it rained a total of 11.4 inches. If it rained 3.9 inches in March, how many inches did it rain in April?



C.
Jana has 162 wooden beads and 95 glass beads. How many more wooden beads than glass beads does Jana have?

D.
The temperature in Norfolk was 12 degrees warmer than in Roanoke where the temperature was 79 degrees. It was 86 degrees in Marion. What was the temperature in Norfolk?

Total

What's a student-friendly definition of a Total problem?

What's an example Total problem?

Difference

Greater and lesser amounts compared for a difference

Adrianna has 10 pencils. Tracy has 4 pencils. How many more pencils does Adrianna have? (How many fewer?)

Adrianna has 6 more pencils than Tracy. If Tracy has 4 pencils, how many does Adrianna have?

Tracy has 6 fewer pencils than Adrianna. Adrianna has 10 pencils. How many pencils does Tracy have?

Total

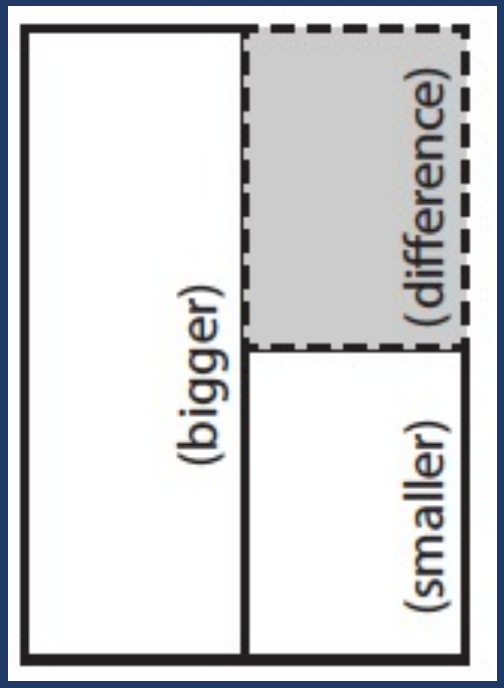
"Are parts put together for a total?"

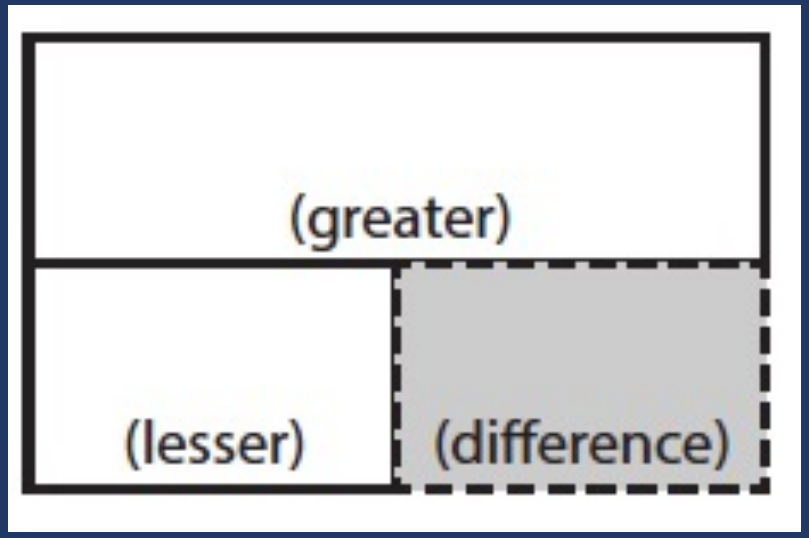
Difference

"Are amounts compared for a difference?"

Difference

G – L = D





Fuchs et al. (2008); Griffin & Jitendra (2009)

Difference

Additive Word Problems

A.
Megan baked 38 sugar cookies and 24 chocolate chip cookies. Enter the total number of cookies Megan baked in all.

B.
In March and April, it rained a total of 11.4 inches. If it rained 3.9 inches in March, how many inches did it rain in April?

C.
Jana has 162 wooden beads and 95 glass beads. How many more wooden beads than glass beads does Jana have?

D.
The temperature in Norfolk was 12 degrees warmer than in Roanoke where the temperature was 79 degrees. It was 86 degrees in Marion. What was the temperature in Norfolk?

Difference

What's a student-friendly definition of a Difference problem?

What's an example Difference problem?

Change

An amount that **increases** or decreases

Nickole had 6 notebooks. Then, she bought 3 notebooks. How many notebooks does Nickole have now?

Nickole had 6 notebooks. Then, she bought a few more notebooks. Now, Nickole has 9 notebooks. How many notebooks did she buy?

Nickole had some notebooks. Then, she bought 3 notebooks. Now, Nickole has 9 notebooks. How many notebooks did she have to start with?

Change

An amount that increases or **decreases**

Samantha baked 20 cookies. Then, she ate 3 of the cookies. How many cookies does Samantha have now?

Samantha baked 20 cookies. Then, she ate some of the cookies. Now, she has 17 cookies. How many cookies did Samantha eat?

Samantha baked some cookies. She ate 3 of the cookies and has 17 cookies left. How many cookies did Samantha bake?

Total

"Are parts put together for a total?"

Difference

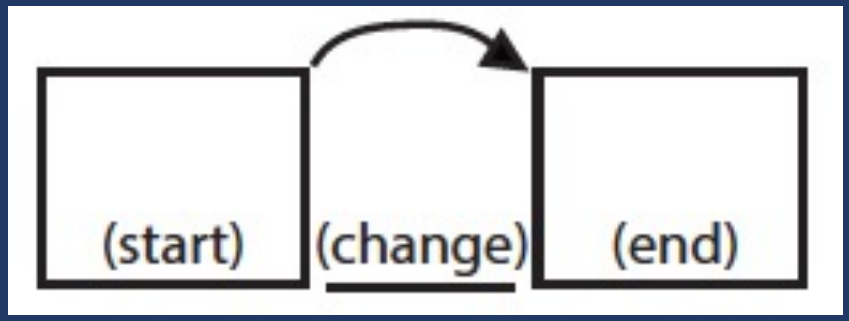
"Are amounts compared for a difference?"

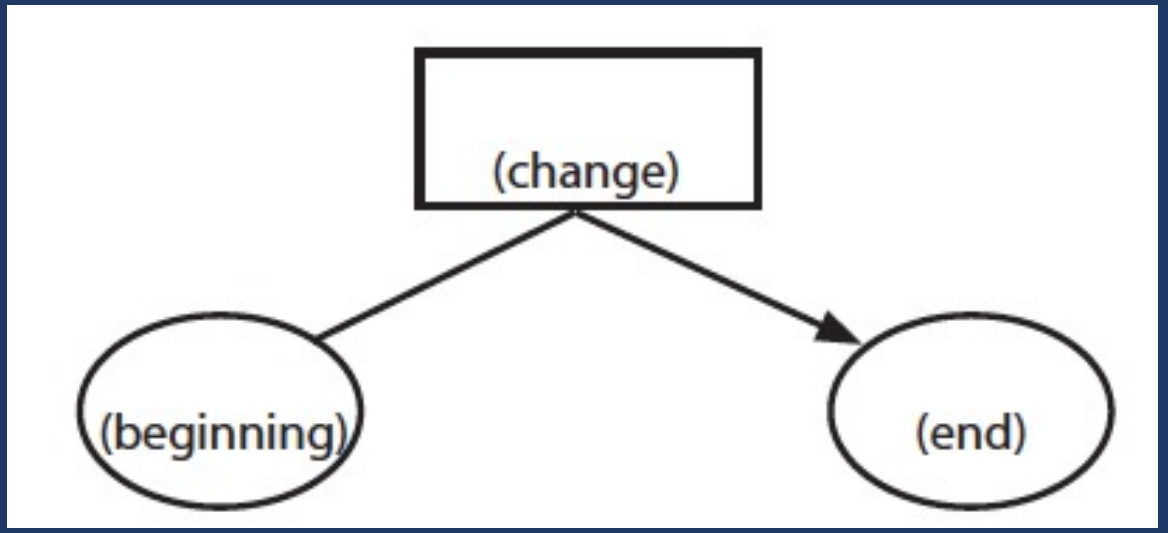
Change

"Does an amount increase or decrease?"

Change

ST +/− C = E





Fuchs et al. (2008); Griffin & Jitendra (2009)

Change

Additive Word Problems

E.
A plant was 3 3/4 inches tall at the beginning of June. By the end of July, the plant was 9 1/8 inches tall. How many inches did the plant grow in 2 months?

F.
Martina has some money in her bank account. Then, she spent $135.69 and has a balance of −$24.80. How much money did Martina have to begin with?

G.
Sam mows lawns and made $560 last week. She made $95 on Monday, $135 on Tuesday, and $70 on Wednesday. How much did Sam make on Thursday and Friday?

H.
Hui saved $70 in January. In February, she spent $64 of the money she saved. She saved $92 more in March. How much has Hui saved by the end of March?

Change

What's a student-friendly definition of a Change problem?

What's an example Change problem?

Additive Word Problems

E.
A plant was 3 3/4 inches tall at the beginning of June. By the end of July, the plant was 9 1/8 inches tall. How many inches did the plant grow in 2 months?

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Martina has some money in her bank account. Then, she spent $135.69 and has a balance of −$24.80. How much money did Martina have to begin with?

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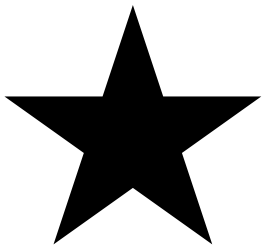


Total

Which schema?

G.

Sam mows lawns and made $560 last week. She made $95 on Monday, $135 on Tuesday, and $70 on Wednesday. How much did Sam make on Thursday and Friday?

P1 + P2 + P3 + P4 = T

Change

Which schema?

H.

Hui saved $70 in January. In February, she spent $64 of the money she saved. She saved $92 more in March. How much has Hui saved by the end of March?

$$ST - C + C = E$$

Schema Quiz Time!

Change

Pablo goes to a stamp show where he can share, buy, and sell stamps.

26. Part A

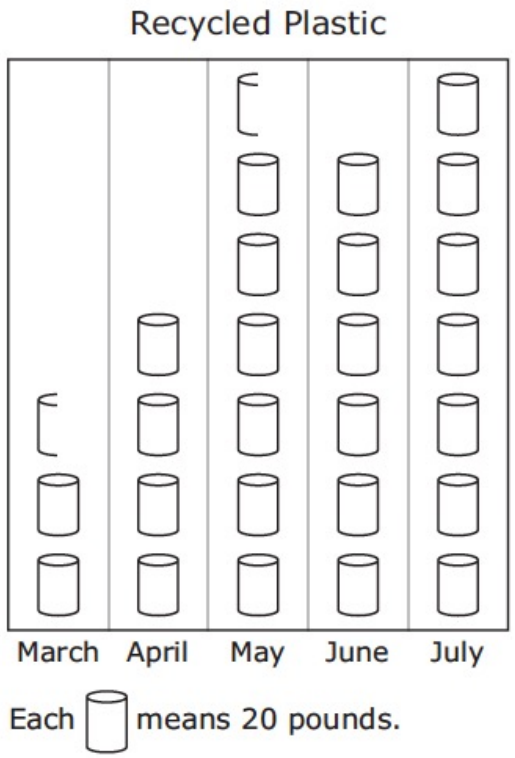
The first day, Pablo starts with 744 stamps. He buys 27 stamps from his friend. He then sells 139 stamps.

What is the total number of stamps that Pablo has after the first day of the stamp show?

Grade 3 PARCC

Difference

The graph below shows the number of pounds of plastic the Keller family recycled for five months.



Based on the graph, how many more pounds of plastic did the family recycle in July than in April?

Grade 3 STAAR

Total

Mr. Conley delivers packages. The bar graph shows the total number of packages he delivered on five days last week.

10. Part A

What is the total number of packages Mr. Conley delivered on Monday and Tuesday?

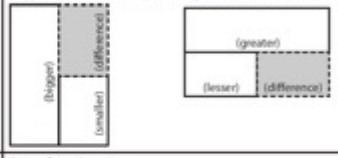
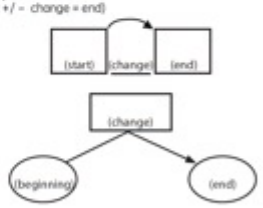
Ⓐ 300

Ⓑ 340

Ⓒ 350

Ⓓ 360

Grade 3 PARCC

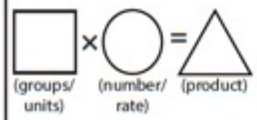
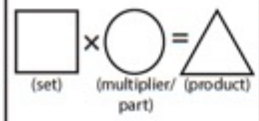
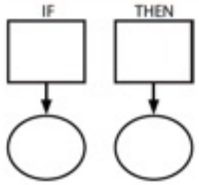
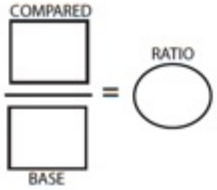
Schema and Definition	Equations and Graphic Organizers	Examples			Variations
Total (Combine; Part-part-whole) Parts combined for a sum	P1 + P2 = T (part + part = total) 	Sum unknown: Lyle has 11 red apples and 18 green apples. How many apples does Lyle have altogether?	Part unknown: Lyle has 29 red and green apples. If 11 of the apples are red, how many green apples does Lyle have?		More than two parts: Lyle has 34 apples. Of the apples, 11 are red, 18 are green, and the rest are yellow. How many yellow apples does Lyle have?
Difference (Compare) Sets compared for a difference	B − s = D (bigger − smaller = difference) G − L = D (greater − less = difference) 	Difference unknown: Sasha wrote 85 words in her essay, and Tabitha wrote 110 words. How many fewer words did Sasha write than Tabitha?	Bigger/greater unknown: Tabitha wrote 25 more words than Sasha. If Sasha wrote 85 words, how many words did Tabitha write?	Smaller/lesser unknown: Tabitha wrote 110 words in her essay. Sasha wrote 25 words fewer than Tabitha. How many words did Sasha write?	(None)
Change (Join; Separate) An amount that increases or decreases	ST +/− C = E (start +/− change = end) 	End (increase) unknown: Jorge had $52. Then, he earned $16 babysitting. How much money does Jorge have now? End (decrease) unknown: Jorge had $52. Then, he spent $29 at the ballpark. How much money does Jorge have now?	Change (increase) unknown: Jorge had $52. Then, he earned some money babysitting. Now, Jorge has $68. How much did Jorge earn babysitting? Change (decrease) unknown: Jorge had $52 but spent some money when he went to the ballpark. Now, Jorge has $23. How much did Jorge spend at the ballpark?	Start (increase) unknown: Jorge has some money, and then he earned $16 for babysitting. Now, Jorge has $68. How much money did he have to start with? Start (decrease) unknown: Jorge had some money. Then, he spent $29 at the ballpark and has $23 left. How much money did Jorge have before going to the ballpark?	Multiple changes: Jorge had $78. He stopped and bought a pair of shoes for $42 and then he spent $12 at the grocery. How much money does Jorge have now?

Which schema is *easiest* for your students? Why?

Which schema is *more difficult*? Why?

Teach word-problem schemas

- Total
- Difference
- Change
- Equal Groups
- Comparison
- Ratios/Proportions

Schema and Definition	Graphic Organizers	Examples			Variations
Equal Groups (Vary) A number of equal sets or units		*Product unknown:* Maria bought 5 cartons of eggs with 12 eggs in each carton. How many eggs did Maria buy?	*Groups unknown:* Maria bought 60 eggs. The eggs were sold in cartons with 12 eggs each. How many cartons of eggs did Maria buy?	*Number unknown:* Maria bought 5 cartons of eggs for a total of 60 eggs. How many eggs were in each carton?	*With rate:* Maria bought 5 cartons of eggs. Each carton cost $2.95. How much did Maria spend on eggs?
Comparison One set as a multiple or part of another set		*Product unknown:* Malik picked 7 flowers. Danica picked 3 times as many flowers. How many flowers did Danica pick?	*Set unknown:* Danica picked 3 times as many flowers as Malik. If Danica picked 21 flowers, how many flowers did Malik pick?	*Times unknown:* Malik picked 7 flowers. Danica picked 21 flowers. How many times more flowers did Danica pick?	*With fraction:* Malik picked 25 red and yellow flowers. If 1/5 of the flowers were yellow, how many were red?
Proportions (Percentages; Unit Rate) Relationships among quantities		*Subject unknown:* Sally typed 56 words in 2 minutes. How many words could Sally type in 7 minutes?	*Object unknown:* Sally typed 56 words in 2 minutes. How many minutes would it take Sally to type 192 words?		*With percentage:* Watson received an 80% on his science quiz. If the test had 40 questions, how many questions did Watson answer correctly?
Ratio		*Base unknown:* Justin baked cookies and brownies. The ratio of cookies to brownies was 3:5. If he baked 15 cookies, how many brownies did he bake?	*Compared unknown:* Justin baked cookies and brownies. The ratio of cookies to brownies was 3:5. If he baked 25 brownies, how many cookies did he bake?	*Ratio unknown:* Justin baked 15 cookies and 25 brownies. What's the ratio of cookies to brownies?	*With unit rate:* Paula bought 5 boxes of markers. She spent $9.75. What is the price of one box of markers?

Material collected from: Jitendra, DiPipi, & Perron-Jones, 2002; Jitendra & Star, 2011; Jitendra et al., 2009; Van de Walle et al., 2013; Xin, Jitendra, & Deatline-Buchman, 2005; Xin & Zhang, 2009.

Equal Groups

Groups multiplied by number in each group for a product

Toni has 2 bags of apples. There are 6 apples in each bag. How many apples does Toni have altogether?

Toni has 12 apples. They want to share them equally among their 2 friends. How many apples will each friend receive?

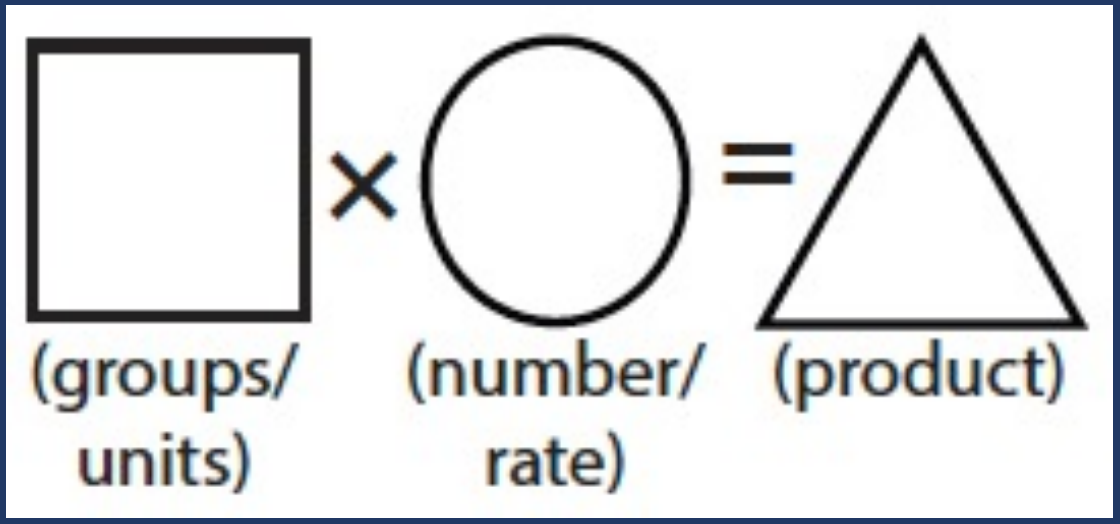
Toni has 12 apples. They put them into bags containing 6 apples each. How many bags did Toni use?

Equal Groups

"Are there groups with an equal number in each group?"

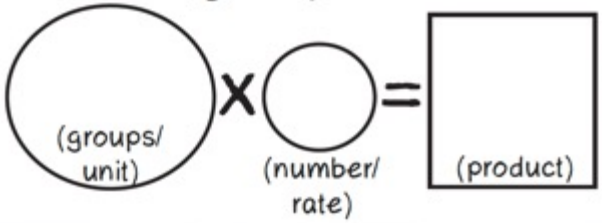
Equal Groups

GR × N = P



EQUAL GROUPS

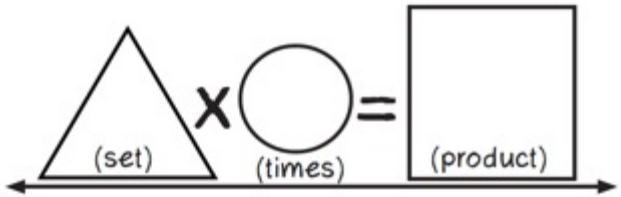
Are there groups with an equal number in each group?



GR x N = P

COMPARISON

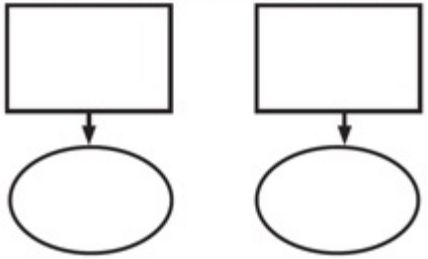
Is a set compared a number of times?



S x T = P

RATIOS/PROPORTIONS

Are there relationships among quantities?



$$\frac{Q_1}{Q_2} = \frac{Q_3}{Q_4}$$

Equal Groups

Multiplicative Word Problems

A.
Ms. Thompson sold 6 cartons of cherries at the Farmers' Market. Each carton holds 25 cherries. How many cherries did she sell?

B.
Jane bought 112 light bulbs. The light bulbs come in packs of 4. How many packs of light blubs did Jane buy?

C.
Enrique has 2 times as many pencils as Ava. Ava has 6 pencils. How many pencils does Enrique have?

D.
Susan has 7 times as many books as Mo. Mo has 18 books. How many books Susan has?

Equal Groups

What's a student-friendly definition of an Equal Groups problem?

What's an example Equal Groups problem?

Comparison

Set multiplied by a number of times for a product

Brooke picked 6 apples. Shaleeni picked 4 times as many apples as Brooke. How many apples did Shaleeni pick?

Equal Groups

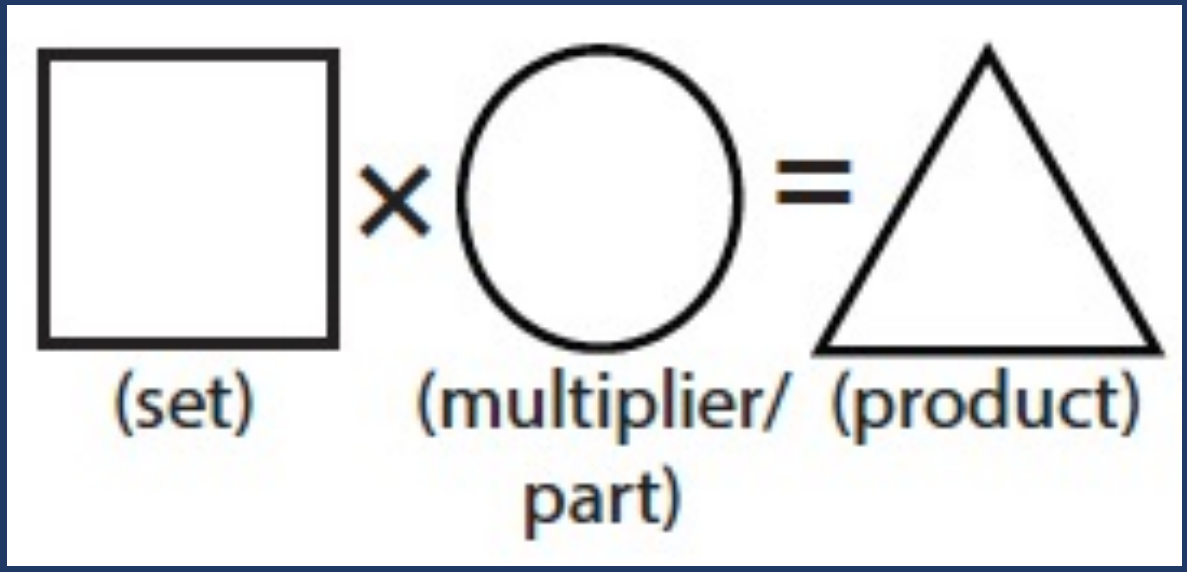
"Are there groups with an equal number in each group?"

Comparison

"Is a set compared a number of times?"

Comparison

S × T = P



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Comparison

What's a student-friendly definition of a Comparison problem?

What's an example Comparison problem?

Ratios/Proportions

Description of relationships among quantities

Emma typed 56 words in 2 minutes. At this rate, how many words could Emma type in 7 minutes?

Melissa baked cookies and brownies. The ratio of cookies to brownies was 3:5. If she baked 25 brownies, how many cookies did she bake?

Xin et al. (2005)

Equal Groups

"Are there groups with an equal number in each group?"

Comparison

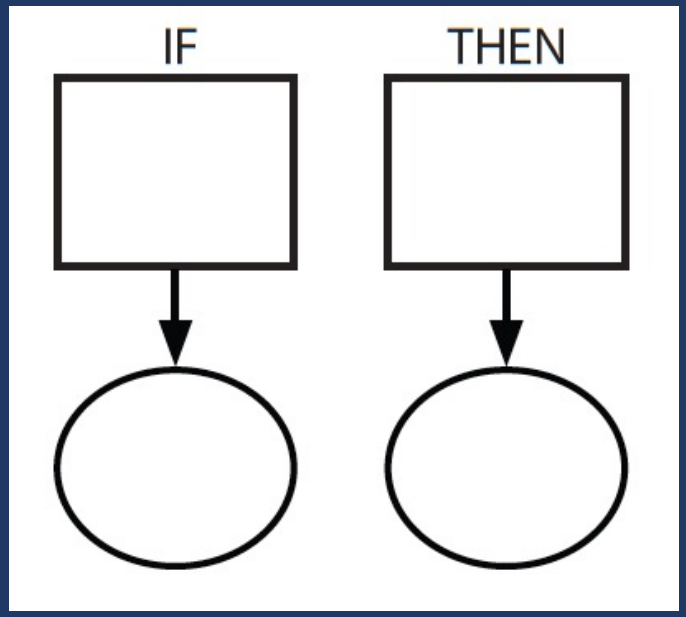
"Is a set compared a number of times?"

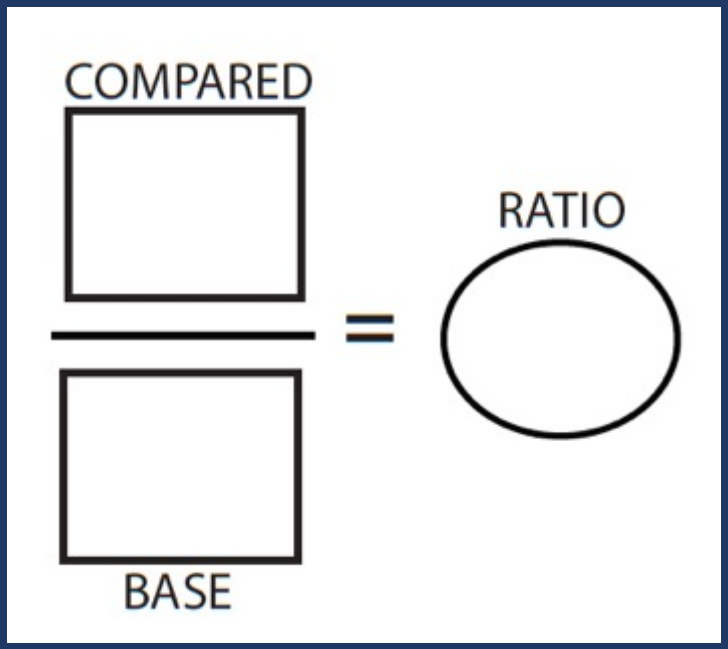
Ratios/Proportions

"Are there relationships among quantities - if this, then this?"

Ratios/Proportions

Description of relationships among quantities





Xin et al. (2005)

Ratios/Proportions

Multiplicative Word Problems

E.
The number of blueberry muffins that a baker makes each day is 40% of the total number of muffins she makes. On Monday, the baker makes 36 blueberry muffins. What is the total number of muffins that the baker makes on Monday?

F.
An airplane's altitude changed −378 feet over 7 minutes. What was the mean change of altitude in feet per minute?

G.
Sara buys a sweater at a department store. The sweater costs $30. The store is having a 25% off sale on everything in the store. Enter the amount of money, in dollars, Sara saves from the sale. Do not consider the sales tax.

H.
Sam's two new aquariums each hold exactly 200 gallons of water. One aquarium will hold small fish and the other will hold large fish. Now he needs new fish for his aquarium. He will buy 5 small fish for every 10 gallons of water in the aquarium. He will buy 8 large fish for every 40 gallons of water in the aquarium. What is the total number of fish Sam will have? What will be the ratio of Sam's small fish to large fish?

Ratios/Proportions

What's a student-friendly definition of a Ratios/Proportion problem?

What's an example Ratio/Proportion problem?

Schema Quiz Time!

Equal Groups

Mr. Kowolski ordered 35 boxes of granola bars. Each box contained 24 granola bars.

What is the total number of granola bars Mr. Kowolski ordered?

Grade 4 PARCC

Ratios/Proportions

A company makes 625 cell phone cases each day. How many cell phone cases does the company make in 31 days?

Grade 5 STAAR

Comparison

Danielle's full-grown dog weighs 10 times as much as her puppy. The puppy weighs 9 pounds.

Enter the number of pounds the full-grown dog weighs.

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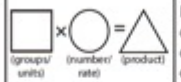
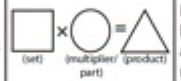
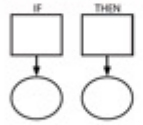
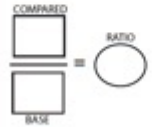
Equal Groups (Vary) A number of equal sets or units		*Product unknown:* Maria bought 5 cartons of eggs with 12 eggs in each carton. How many eggs did Maria buy?	*Groups unknown:* Maria bought 60 eggs. The eggs were sold in cartons with 12 eggs each. How many cartons of eggs did Maria buy?	*Number unknown:* Maria bought 5 cartons of eggs for a total of 60 eggs. How many eggs were in each carton?	*With rate:* Maria bought 5 cartons of eggs. Each carton cost $2.95. How much did Maria spend on eggs?
Comparison One set as a multiple or part of another set		*Product unknown:* Malik picked 7 flowers. Danica picked 3 times as many flowers. How many flowers did Danica pick?	*Set unknown:* Danica picked 3 times as many flowers as Malik. If Danica picked 21 flowers, how many flowers did Malik pick?	*Times unknown:* Malik picked 7 flowers. Danica picked 21 flowers. How many times more flowers did Danica pick?	*With fraction:* Malik picked 25 red and yellow flowers. If 1/5 of the flowers were yellow, how many were red?
Proportions (Percentages; Unit Rate) Relationships among quantities		*Subject unknown:* Sally typed 56 words in 2 minutes. How many words could Sally type in 7 minutes?	*Object unknown:* Sally typed 56 words in 2 minutes. How many minutes would it take Sally to type 192 words?		*With percentage:* Watson received an 80% on his science quiz. If the test had 40 questions, how many questions did Watson answer correctly?
Ratio		*Base unknown:* Justin baked cookies and brownies. The ratio of cookies to brownies was 3:5. If he baked 15 cookies, how many brownies did he bake?	*Compared unknown:* Justin baked cookies and brownies. The ratio of cookies to brownies was 3:5. If he baked 25 brownies, how many cookies did he bake?	*Ratio unknown:* Justin baked 15 cookies and 25 brownies. What's the ratio of cookies to brownies?	*With unit rate:* Paula bought 5 boxes of markers. She spent $9.75. What is the price of one box of markers?

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Which schema is *easiest* for your students? Why?

Which schema is *more difficult*? Why?

Teach word-problem schemas

- Total
- Difference
- Change
- Equal Groups
- Comparison
- Ratios/Proportions

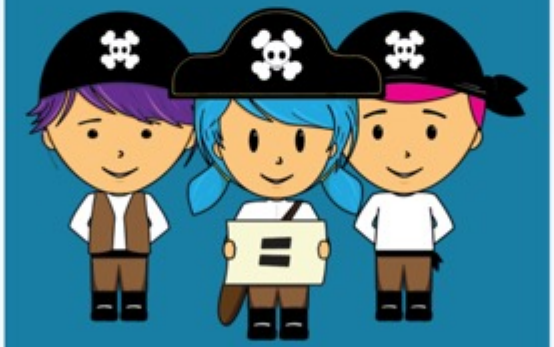
Pirate Math Equation Quest

About | Research | Individual | Small Group | STAAR | Videos

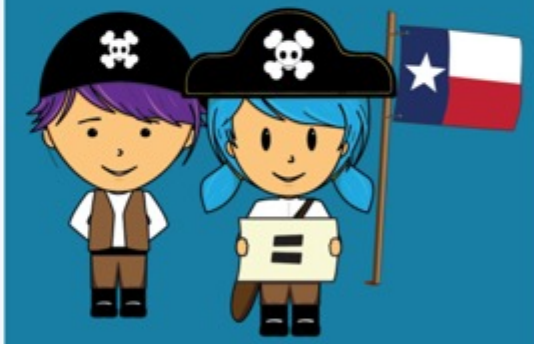
Welcome to Pirate Math Equation Quest!

Individual Word-Problem Intervention

Small-Group Word-Problem Intervention



Small-Group Word-Problem Intervention for STAAR



Instructional Platform

INSTRUCTIONAL DELIVERY

Explicit instruction

Precise language

Multiple representations

INSTRUCTIONAL STRATEGIES

Fluency building

Problem solving instruction

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https://intensiveintervention.org/intensive-intervention-math-course

National Center on
INTENSIVE INTERVENTION
at American Institutes for Research

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Intensive Intervention in Mathematics Course Content

NCII, through a collaboration with the University of Connecticut, developed a set of course content focused on developing educators' skills in designing and delivering intensive mathematics instruction. This content is designed to support faculty and professional development providers with instructing pre-service and in-service educators who are developing and/or refining their implementation of intensive mathematics intervention.

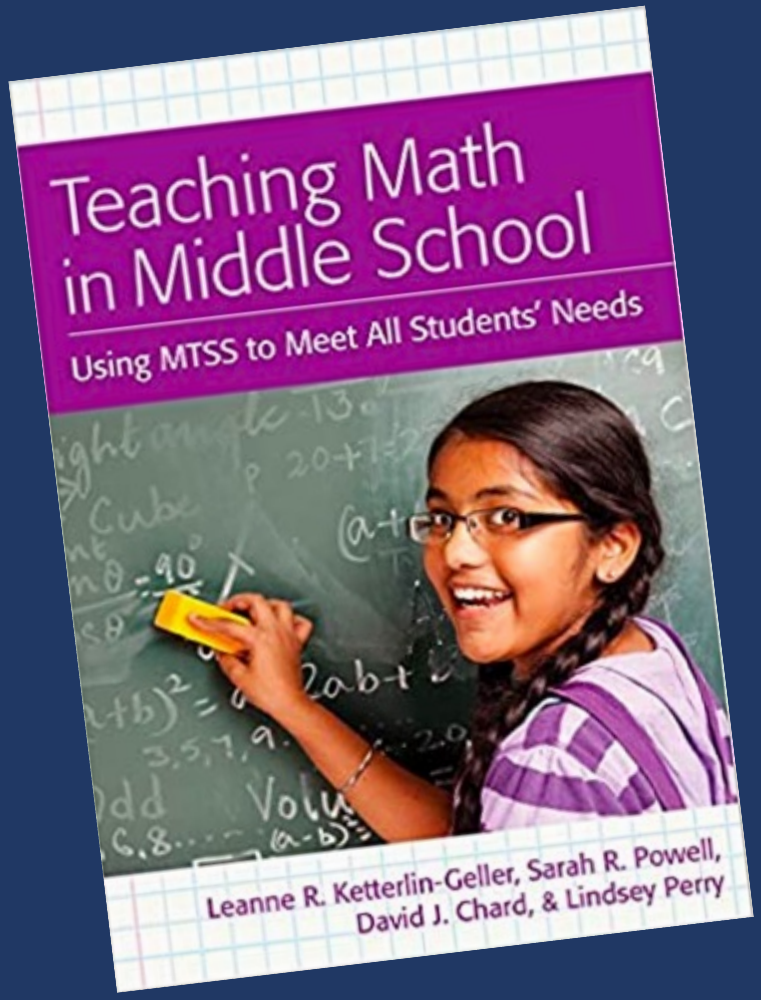
Intensive instruction was recently identified as a high-leverage practice in special education , and DBI is a research based approach to delivering intensive instruction across content areas (NCII, 2013). This course provides learners with an opportunity to extend their understanding of intensive instruction through in-depth exposure to DBI in mathematics, complete with exemplars from actual classroom teachers.

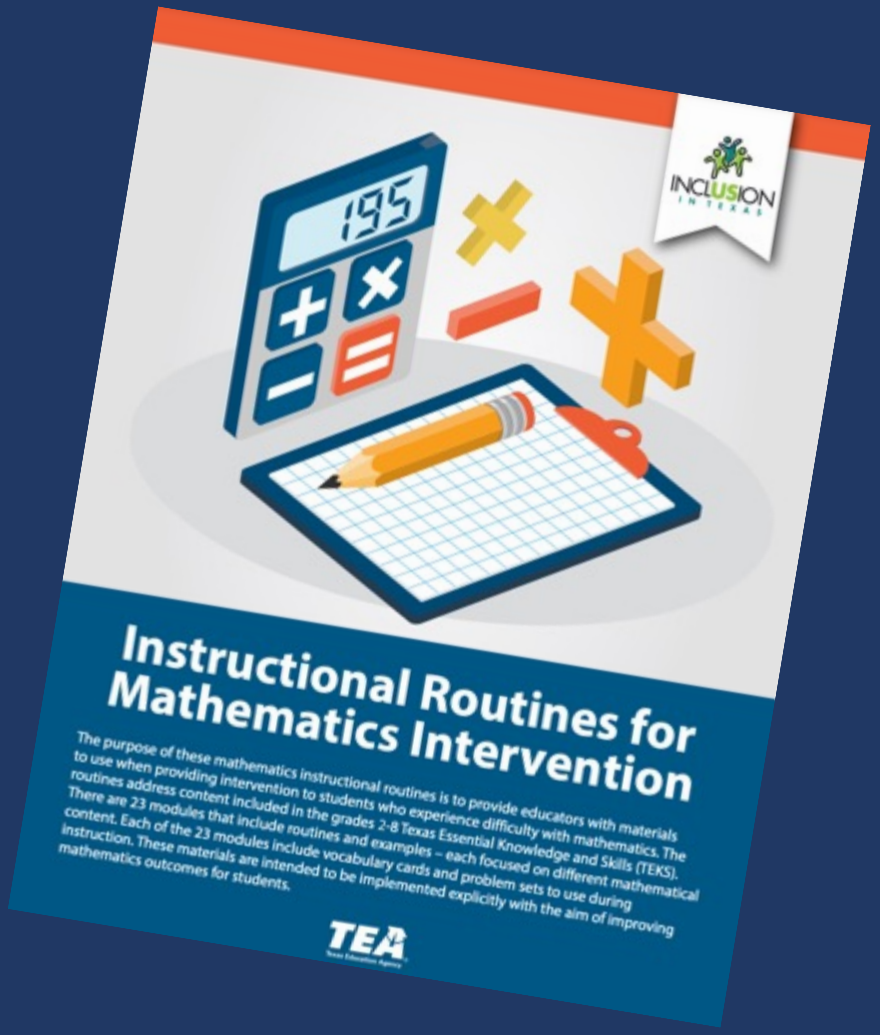
NCII, through a collaboration with the University of Connecticut and the National Center on Leadership in Intensive Intervention and with support from the CEEDAR Center , developed course content focused on enhancing educators' skills in intensive mathematics intervention. The course includes eight modules that can support faculty and professional development providers with instructing pre-service and in-service educators who are learning to implement intensive mathematics intervention through data-based individualization (DBI). The content in this course complements concepts covered in the Features of Explicit Instruction Course and so we suggest that users complete both courses.

MODULE 4: INTENSIVE MATHEMATICS INTERVENTION: INSTRUCTIONAL DELIVERY

MODULE 5: INTENSIVE MATHEMATICS INTERVENTION: INSTRUCTIONAL STRATEGIES

https://www.amazon.com/Teaching-Math-Middle-School-Students/dp/1598572741





https://www.inclusionintexas.org/apps/pages/index.jsp?uREC_ID=2155039&type=d&pREC_ID=2169859

PAC Pre-Algebra Competence

- Students struggle with pre-algebra not only because of math but also weaknesses in other skills (executive functions, language, attention)
- Goal: Pinpoint underlying important skills to develop more effective, targeted interventions and assessments
- We'd like to recruit 7th grade math teachers ($100 stipend) to help us:
  - Recruit 7th grade students (distributing consent forms)
  - Schedule some testing sessions (initial screening, 2 in fall, 1 in spring)
    - Flexible scheduling
    - Will offer free tutoring for struggling 6th graders on algebraic equation solving
- Contact information: Dawn Filer dawn.filer@austin.utexas.edu

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