Forecast:
Detectable waves

The Relaxation Times

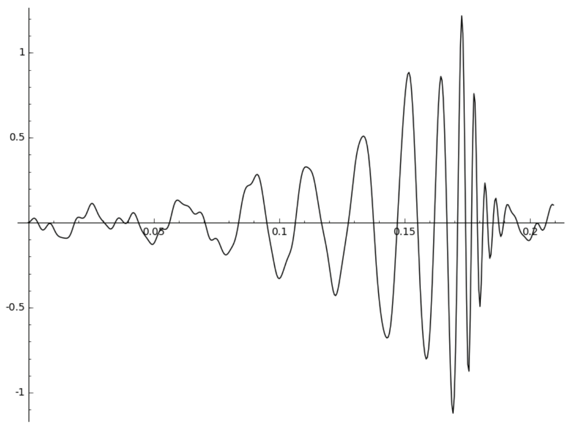
2495 Main Street
Buffalo, NY 14214
716-885-4701

jreichert@teachspin.com
www.teachspin.com

VOL. V No. 1 ***NEWSLETTER OF TEACHSPIN, INC.*** ***April 2016***

How Low Can You Go?

By now, every physicist alive has probably seen this published waveform: It comes from the LIGO consortium, and it represents the detection of a gravitational-wave burst, ascribed to the merger of a very distant pair of black holes. The vertical axis gives 10^{+21} times the 'dimensionless strain' detected at Hanford, WA, as the wave passed. That means the detector's 4-km-long baselines deformed by only ±4 x10^{-18} m during the event. How does one detect such minuscule displacements? Hint: the acronym stands for Laser Interferometer Gravitational-wave Observatory. We thought this might be the perfect time to remind your students of what they can learn, hands-on in the lab, about the detection of tiny displacements using the techniques of *laser interferometry*.



We bring this up because our '**Modern Interferometry**' set-up includes *everything* that is needed to understand the fundamentals of LIGO's technique, at a length scale (and cost!) under 10^{-4} times LIGO's. This instrument puts into students' hands all the tools needed to create, and detect, displacements of sub-microscopic scale.

In a Michelson interferometer, a displacement of one end mirror by a quarter-wavelength (λ/4) of the light used will add a distance λ/2 to the round-trip distance in one arm, and change the output from constructive to destructive interference. For red HeNe-laser light, we can thus detect displacements of λ/4 = 158 nm. That's 0.158 μm (or, if you will, about 6 *micro*-inches). But is that the best one can do? How low *can* you go, in detecting small displacements?

Here's a model for such an interferometer's output signal. If one end mirror's position is denoted by coordinate s, then the intensity output can be written as

$$I(s) = \frac{I_0}{2}\left(1 + \cos 2\pi \frac{s - s_0}{\lambda/2}\right)$$

Here I_0 is the maximum intensity that is achieved at every 'bright fringe', $s - s_0 = 0$, λ/2, 2·(λ/2), and so on. But counting successive full fringes is just the *beginning* of achieving the best sensitivity. A plot of this $I(s)$ function shows that operating neither at the 'bright fringe' ●, nor at the 'dark fringe'◆, but instead on the *slopes* ★ of the $I(s)$-curve, gives the greatest sensitivity $\partial I/\partial s$.

Operating at $s - s_0 = \lambda/8 + \delta s$, for example, we get the prediction

$$I(s) = \frac{I_0}{2} - \frac{I_0}{2}\sin\frac{4\pi\delta s}{\lambda} \simeq \frac{I_0}{2} - I_0\frac{\delta s}{\lambda/2\pi}.$$

Notice that the output value is near neither I_0 nor 0, but lies halfway between; notice also that $I(s)$ now shows *linear*, ie. first-order, sensitivity to displacements δs. Finally, the scale factor of this sensitivity can be fully modelled, to give a calibrated response.

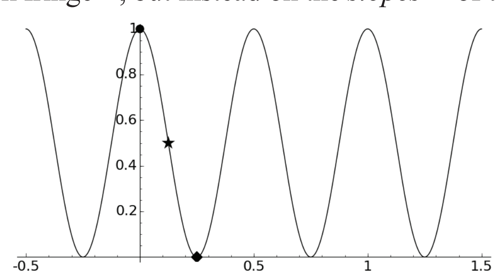


Fig. 1: The expected form of the intensity output of a Michelson interferometer, $I(s)$, as one end mirror is translated through coordinate s. Vertical axis, in units of I_0; horizontal axis, one-way mirror motion in units of wavelength λ.

But how small a displacement you can detect? We now see that depends on how small a change in output intensity you can detect. If you could detect a δI that is 1% of I_0, then $\delta I/I_0 = 0.01 = -\,\delta s\,/\,(\lambda/2\pi)$. That easily gives $\delta s = 0.01(\lambda/2\pi)$, which for our choice of λ gives a detectable δs of just 1.0 nm, or 10^{-9} m, about 10^{-8} of the length of its interferometer arms. (Note LIGO has to do *vastly* better than this!)

Here's how your students can generate, detect, and calibrate this sort of displacement using the tools included in **Modern Interferometry**. They can set up a Michelson interferometer, using our special translation stage for mounting one end mirror, and they can drive it using a differential micrometer and a piezoelectric transducer to achieve sub-μm control of its motion. Motor drive of that micrometer gives a picture of $I(s)$ fringes, where here optical intensities I get mapped into (negative) voltage signals via a photodetector. A plot such as Fig. 2 tells us that (-)7.8 Volts correspond to the intensity scale factor I_0 in our model.

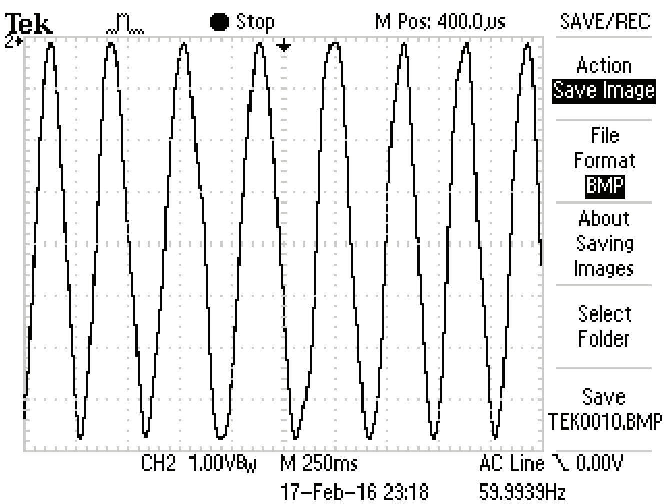


Fig. 2: The output voltage from a photodetector, showing the successive bright-fringe (more negative) and dark-fringe (near 0) voltage outputs, as the end-mirror position is monotonically translated.

Now if we park the micrometer so that the signal I is at any half-of-I_0 location, we are operating at a maximum-slope point noted in Fig. 1. Then if we drive our piezoelectric actuator with a triangle-wave signal, we can find empirically what amplitude of drive is needed to give one-half-of-a-fringe in the output:

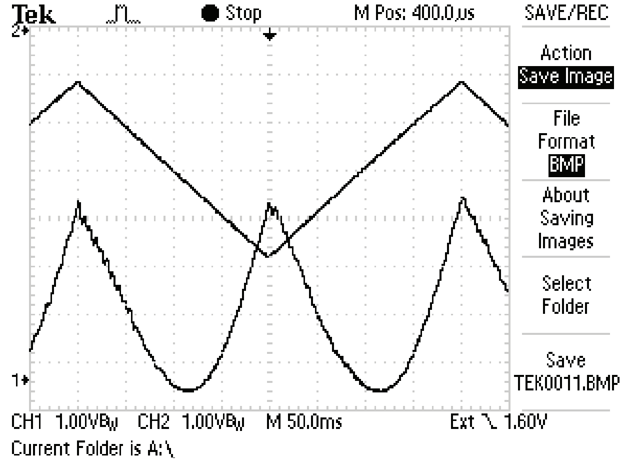


Fig. 3: Upper trace: The waveform used to drive a piezoelectric actuator, 10-fold attenuated; Lower trace: the one-half-of-a-fringe displacement revealed in the interferometric output signal $I(s)$.

The result shows that we've achieved a peak-to-peak excursion of $\lambda/4$ in the end mirror ($\lambda/2$ in the round-trip distance), and so we can tell what peak-to-peak voltage excursion on the piezo is needed to achieve this.

Turning down that triangle-wave amplitude by 10-fold ought to give a mirror motion of $\lambda/40$, peak-to-peak. The predicted range of detector voltages comes from

$$\delta I = -I_0 \frac{\lambda/40}{\lambda/2\pi} = -I_0(\pi/20) \; .$$

Using $I_0 = -7.8$ V, this gives $\delta I = 1.2$ V, to be compared with the results shown in Fig. 4:

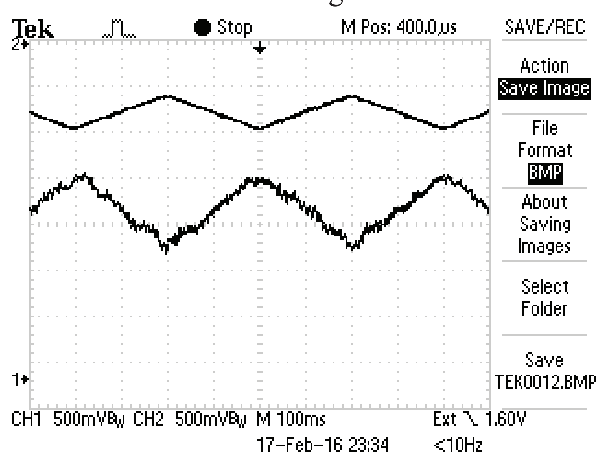


Fig. 4: Upper and lower traces just as in Fig 3, but now with a 10-fold smaller drive to the piezo.

Thus far, we've seen 'cause' and 'effect' waveforms using single real-time sweeps on an oscilloscope, but now that we're in pursuit of ever-smaller displacements, we can start to exploit signal-averaging techniques. (LIGO does *not* have this luxury!) A first method is just to average multiple traces on a 'scope: below we use a further 10-fold reduction in the drive, to $\lambda/400$ peak-to-peak, and show the detection of this motion on a 'scope, using the average of 4 waveforms.

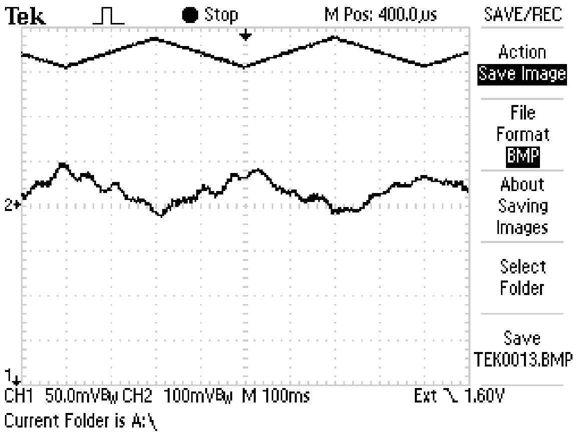


Fig. 5: Again, the piezo drive signal above, and the interferometer output signal below, for displacement of λ/400 peak-to-peak, detected using the averaging of 4 sweeps.

But that's not the limit, either! We can change the drive from triangle-wave to sinusoidal, for convenience, and then we expect the resulting signal to be also sinusoidal in time. But a sinusoid-in-time means that the response will be 'all at one point' in frequency space. And hence the signal is best detected, in the presence of noise, via a Fourier transform that ought to show a signal peak above a noise background.

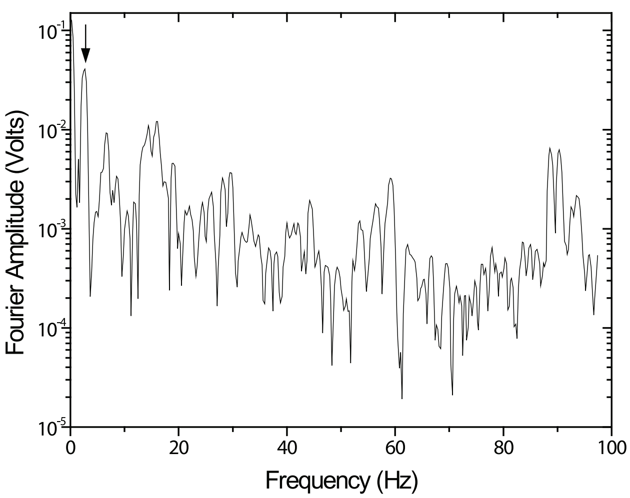


Fig. 6: The Fourier amplitude spectrum (log scale) of the interferometer response (in Volts), as a function of frequency (in Hz), again for displacement of λ/400 peak-to-peak, showing the signal peak (arrowed) at 2.5 Hz, lying atop a noise background of various origins.

This view of the signal-amid-noise also teaches us that the output is less noisy in some frequency bands than in others (just as is LIGO!). From Fig. 6 we see that it would be advantageous to arrange for our signal to appear (for example) at 71 Hz, rather than the 2.5 Hz previously used. Now using a good spectrum analyzer (such as the SR770 that's in our 'Fourier Methods' package), we can 'zoom in' in frequency space to isolate such a signal peak. Using 16 seconds of acquisition time, we see the following response to a sinusoidal displacement of λ/400, peak-to-peak, of one end mirror.

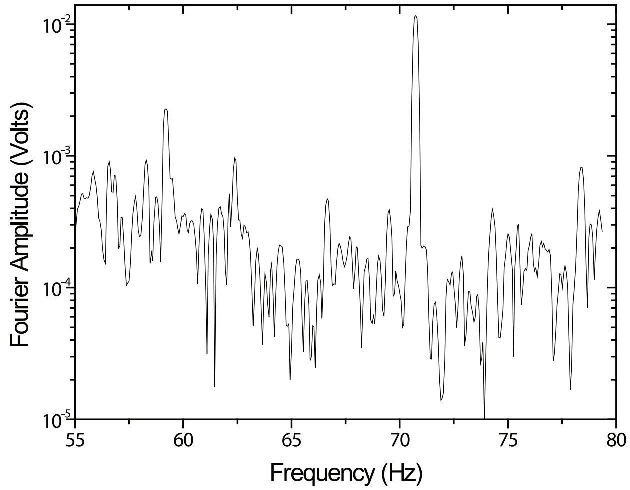


Fig. 7: The Fourier amplitude spectrum of the interferometer response, still for displacement of λ/400 peak-to-peak, showing the signal peak, now at 71 Hz, lying atop the local noise background.

With that signal peak standing so high above the noise background, it's clear that the piezo's displacement could be reduced by yet another factor of 10, to λ/4000 peak-to-peak, and still leave the signal standing visibly above the noise.

Now a displacement of λ/4000 peak-to-peak represents a mirror motion of ≈0.16 nm, which is just about three times the Bohr radius a_0. So we're moving our aluminum end-mirror support back and forth on our optical table by about the diameter of one aluminum atom! Yet we're *still* not at the limits of detectability: since we're in an arena in which we cause the signal, and hence we can be sure of its frequency and phase, we could alternatively use *lock-in detection* to tease the signal out of the noise.

How small a displacement can your students create, calibrate, and detect? How can they be sure it's a genuine displacement signal they're detecting? What's their best value for the 'dimensionless strain' detected, $\delta s/L$, where L is the length of their interferometer arms? How far do they have yet to go to compete with LIGO's strain sensitivity? Think of all that they will come to understand about signals (and about noise!) on their way to a 'hero experiment'. Which students of yours will work at the gravitational-wave observatories of the future?

TEACH SPIN

Tri-Main Center, Suite 409
2495 Main Street
Buffalo, NY 14214-2153

How Low Can You Go?
Learning how LIGO works

SUPPORTING ADVANCED LABORATORY DEVELOPMENT

There's an emerging combination of 'modules', which together can form a *closed loop with feedback* with the potential of improving the advanced-lab experience for everybody:

- There's the umbrella organization ALPhA which supervises the Immersions program (http://www.advlab.org/immersions.html), designed to help advanced-lab instructors learn new experiments.
- ALPhA is also already sponsoring the 'ALPhA Miners' program (www.advlab.org/alpha-mining.html); this is designed to spot great ideas emerging in the research-conference world, and bring them to the attention of advanced-lab instructors.
- Now there's a new program, the 'ALPhA Mining Actualization' or AMA (http://jfreichertfoundation.org/alpha-mining-actualization-ama/), supported by the Reichert Foundation, which has some $money$ available to assist those instructors who, with student co-workers, would like to develop such a 'mined' idea into a working advanced-lab experiment.
- Note that successful projects of this nature would also be eligible for competition for the AAPT-ALPhA Award (https://www.aapt.org/Programs/awards/aapt_alpha_award.cfm), which recognizes student achievement in the advanced lab.

How's that for a full circle? And who will be the first student winner of the AAPT-ALPhA Award, whose future research discoveries might someday be mined for yet more advanced-lab ideas?