Math 2471 Calc 3

min / max

Critical pt

$$f_x(a,b) = 0 \quad f_y(a,b) = 0$$

2nd Derivative test

$$\Delta = f_{xx} f_{yy} - f_{xy}^2 \text{ at } P(a,b)$$

(1) if $\Delta > 0$ $f_{xx}|_p > 0$ min

(2) if $\Delta > 0$ $f_{xx}|_p < 0$ max

(3) if $\Delta < 0$ saddle

(4) if $\Delta = 0$ no conclusion

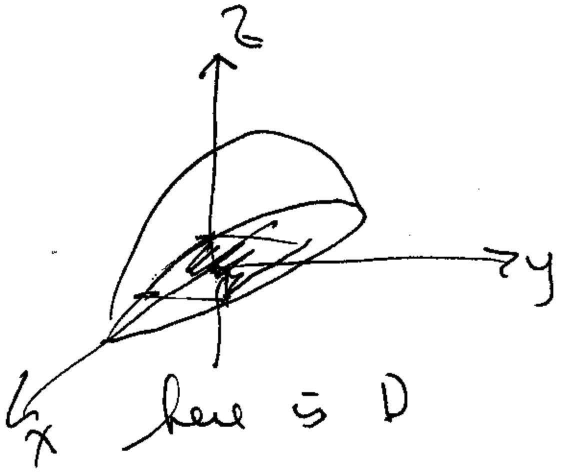
If we restrict ourselves to a closed domain D, we will have a min / max

It will be located either inside D

or on the boundary of D.

Consider

$$z = 6 - x^2 - 4y^2 \text{ on } [-1,1] \times [-1,1]$$



top view when $z = 0$

$$x^2 + 4y^2 = 6$$

$$\frac{x^2}{6} + \frac{2}{3}y^2 = 1$$

Now $z_x = -2x, \quad z_y = -8y$

CP $x = 0, y = 0$

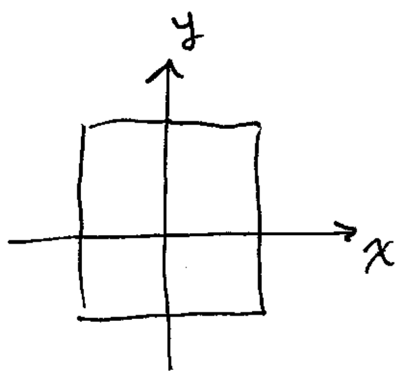
$$z_{xx} = -2 \quad z_{xy} = 0 \quad z_{yy} = -8$$

$$\Delta = z_{xx} z_{yy} - z_{xy}^2 = 16 > 0$$

$z_{xx} < 0$ so a max at $(0,0)$ and is 6

$z = 6 - x^2 - 4y^2$ is cont's so on a closed domain there ~~has~~ to be a m m

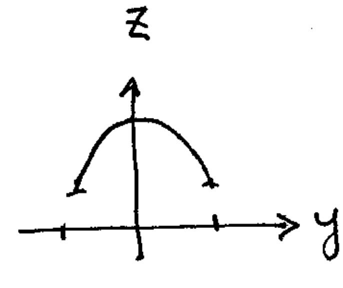
Top view



if $x = \pm 1$

$z = 6 - 1 - 4y^2$

$= 5 - 4y^2$



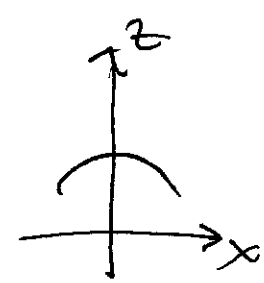
~~zy~~ The max is at $y = 0$ when $x = \pm 1$ which is 5 be we know 6 is absolute max

but min occurs at end pts when $y = \pm 1$

so min = 1

when $y = \pm 1$ $z = 6 - x^2 - 4 = 2 - x^2$

min ~~here~~ is at $x = \pm 1$ $z = 1$



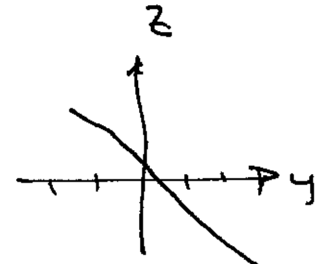
Ex 2 $\quad z = xy$ for $-1 \le x \le 2, \; -2 \le y \le 3$

Now $\quad z_x = y \quad z_y = x \quad$ so CP $(0,0)$

$z_{xx} = 0, \; z_{xy} = 1, \; z_{yy} = 0$

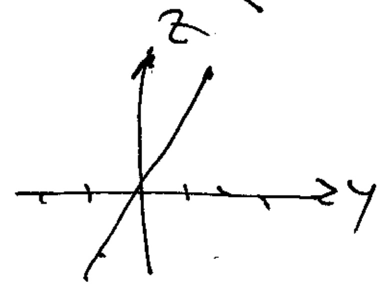
$\Delta = z_{xx} z_{yy} - z_{xy}^2 = 1 < 0$ so saddle

then we go along boundary



$x = -1 \qquad z = -y \qquad -2 \le y \le 3$

$x = 2 \qquad z = 2y \qquad$ " $\quad$ "



$y = -2 \qquad z = -2x \qquad -1 \le x \le 2$

$y = 3 \qquad z = 3x \qquad -1 \le x \le 3$

Plug in $\quad (-1,-2) \quad (-1,3) \quad (2,-2) \quad (2,3)$

$\qquad z = 2 \quad z = -3 \quad z = -4 \quad z = 6$

min $\qquad$ max