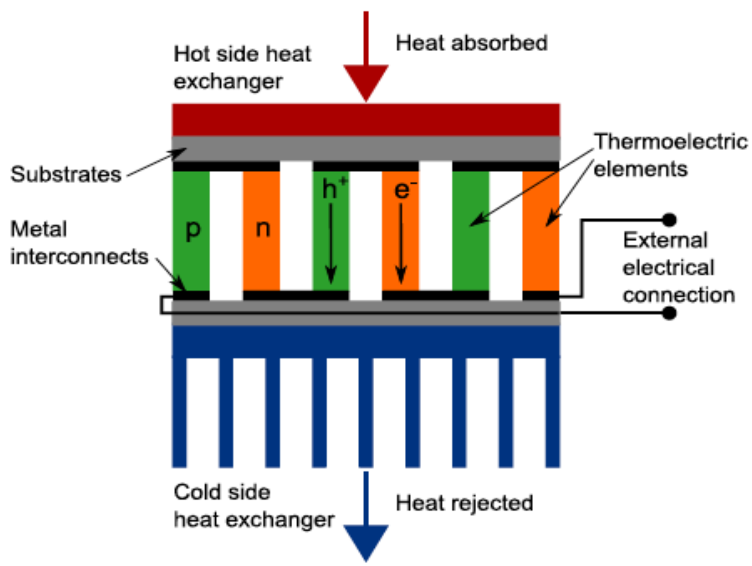
Thermoelectric Device Engineering

G. Jeffrey Snyder

Northwestern University

Evanston IL, USA

http://thermoelectrics.matsci.northwestern.edu



Thermoelectrics Northwestern Materials Science and Engineering

Baranowski, Snyder et al, *J Applied Phys.* 113, 204904, (2013)

Thermoelectric Applications

Solid State Advantage

- No moving parts
- No maintenance
- Long life
- Scalability

Cooling - Thermal Management

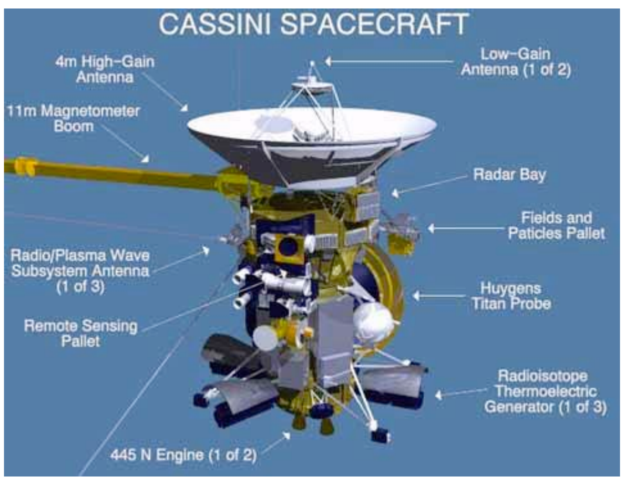
- Small Refrigerators
- Optoelectronics
- Detectors

Power Generation (heat to electricity)

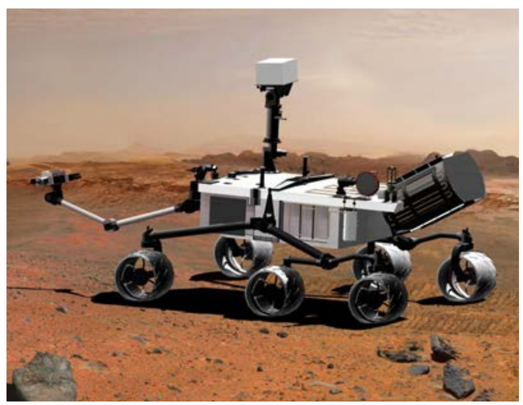
- Spacecrafts
  - Voyager nearly 40 years!
- Remote power sources

Future Possibilities

- Waste Heat Recovery
  - Automobiles
- Distributed Thermal Management



Saturn Orbiter Cassini

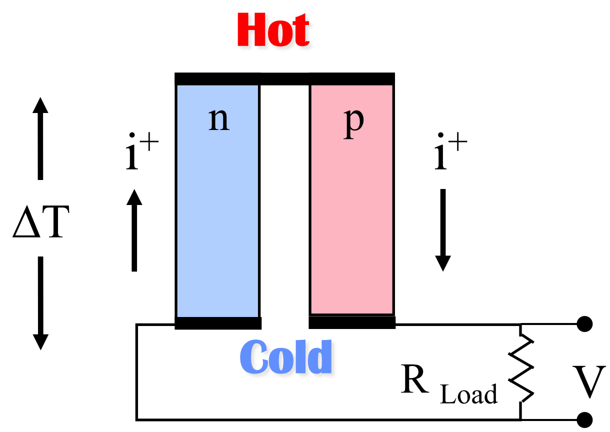


2012 Mars Rover Curiosity

Thermoelectrics
Northwestern Materials Science and Engineering

Power Generation

Draw power through a load



Voltage : $V = \alpha \Delta T$

Power : $IV = V^2/R = \alpha^2 \sigma \times \Delta T^2 \times \frac{A}{l}$

Materials Parameters

Power Factor: $\alpha^2\sigma$

Electrical conductivity $\sigma = 1/\rho$

Maximum Efficiency

$$\frac{\text{Electrical Power}}{\text{Heat Removed}} \approx \frac{\alpha^2 \sigma \cdot \Delta T^2}{\kappa \cdot \Delta T + \alpha T_h I + \frac{I^2 \rho}{2}}$$

κ = Thermal conductivity

Thermal short, reduces efficiency

Complete Result (approximate)

$$\text{Efficiency} \approx \frac{\Delta T}{T_h} f(zT)$$

Carnot efficiency

$$f \approx \frac{\sqrt{1+z\bar{T}} - 1}{\sqrt{1+z\bar{T}} + \frac{T_c}{T_h}}$$

Materials figure of merit

$$zT = \frac{\alpha^2 \sigma}{\kappa} T = \frac{\alpha^2}{\rho \kappa} T$$

Thermoelectrics
Northwestern Materials Science and Engineering

zT vs. *ZT*

Materials figure of merit *zT*

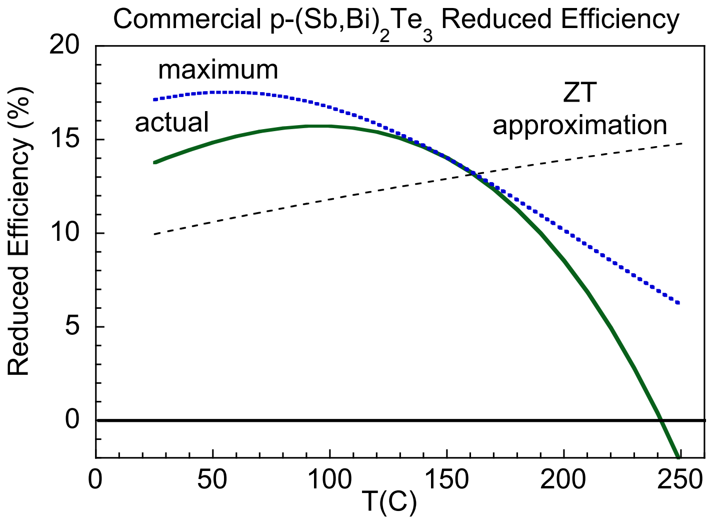
$$zT = \frac{\alpha^2}{\rho\kappa} T$$

Determines maximum reduced efficiency at any given point

$$\max \eta_r = \frac{\sqrt{1 + zT} - 1}{\sqrt{1 + zT} + 1}$$

Actual efficiency is less

- series current not optimal everywhere

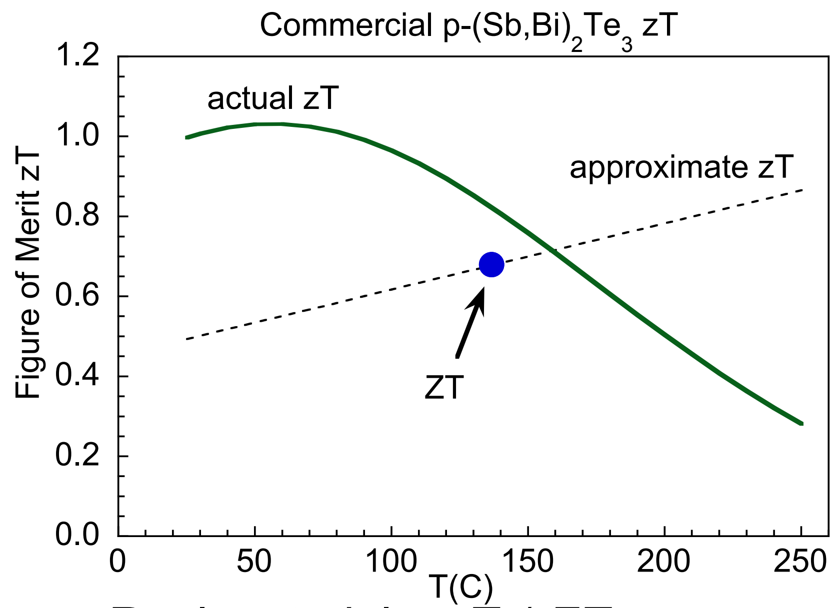


Device Figure of Merit *ZT*

$$\eta = \frac{\Delta T}{T_h} \cdot \frac{\sqrt{1 + Z\overline{T}} - 1}{\sqrt{1 + Z\overline{T}} + T_c/T_h}$$

$ZT = zT$ is approximation for

$\alpha(T)$, $\rho(T)$, $\kappa(T)$, are constant



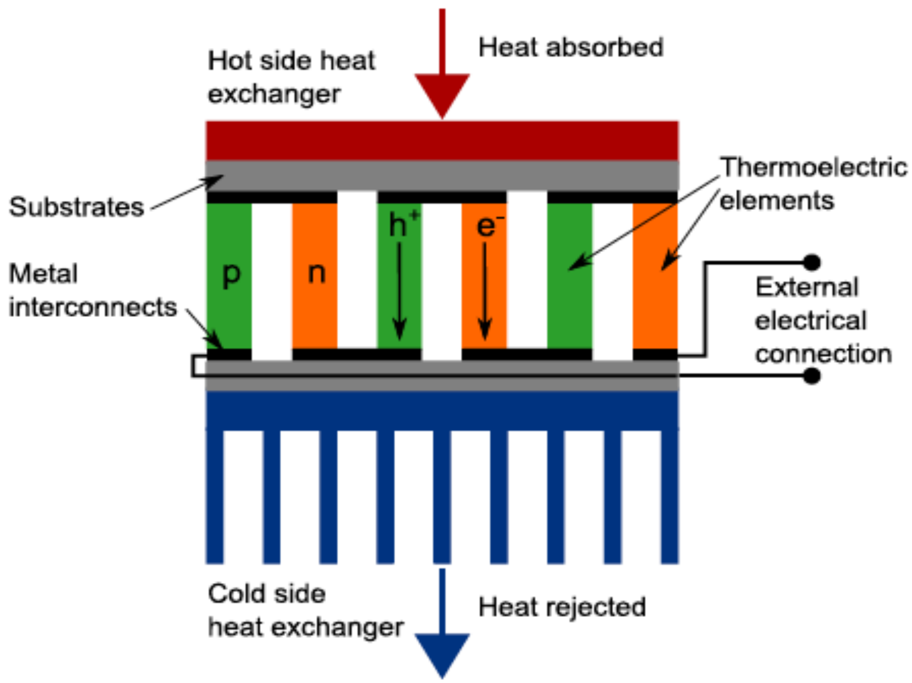
Real materials: $zT \neq ZT$

especially max $zT \neq ZT$

Beware conclusions about changing ΔT

Goldsmid, H. J. *Applications of Thermoelectricity* (Methuen, London, 1960).

Thermal Impedance Match



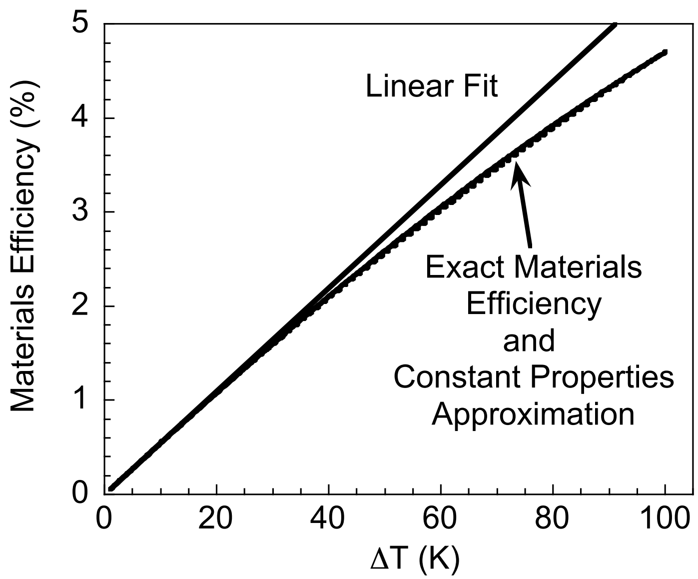
Thermoelectrics
Northwestern Materials Science and Engineering

Small ΔT Thermoelectric Generator

Efficiency proportional to ΔT

$$\eta = \eta_1 \cdot \Delta T$$

$$\eta_1 \approx 0.05\% / K$$



$$\eta = \frac{\Delta T}{T_h} \cdot \frac{\sqrt{1+Z\overline{T}} - 1}{\sqrt{1+Z\overline{T}} + T_c / T_h}$$

Power requires Heat flow

$$P = \eta Q$$

Heat Flow related to thermal conductance of TE

$$Q_h = K\Delta T + SIT_h - \tfrac{1}{2} I^2 R$$

$$Q \approx K_{eff} \Delta T$$

$$K = \kappa_{eff} \frac{A}{l}$$

$$\kappa_{eff} \approx \kappa \cdot \sqrt{1+Z\overline{T}}$$

P - electrical power out

Q - Heat input

η - Efficiency

K - Thermal Conductance of TE

κ - Thermal Conductivity

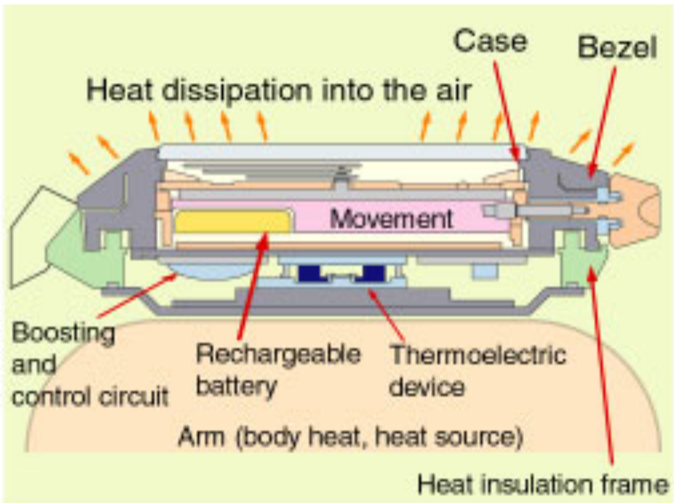
A - Area of TE

l - length of TE

Thermal Impedance Matching

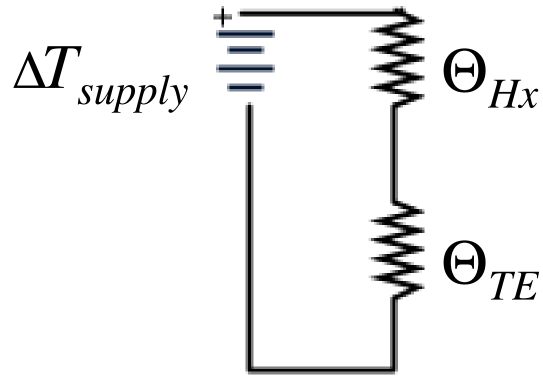
Claim: 'All the heat you need'

Reality: Space limits thermal impedance of source + sink



Example: air cooled TEG

- Typical air Heat Exchanger
  - <~ 0.2 W/K cm^2
- Maximum power when thermal impedance is matched
  - $\Theta_{TE} = \Theta_{Hx}$



$$Q = \frac{\Delta T_{\text{supply}}}{\Theta_{\text{Hx}} + \Theta_{TE}} = \frac{\Delta T_{TE}}{\Theta_{TE}}$$

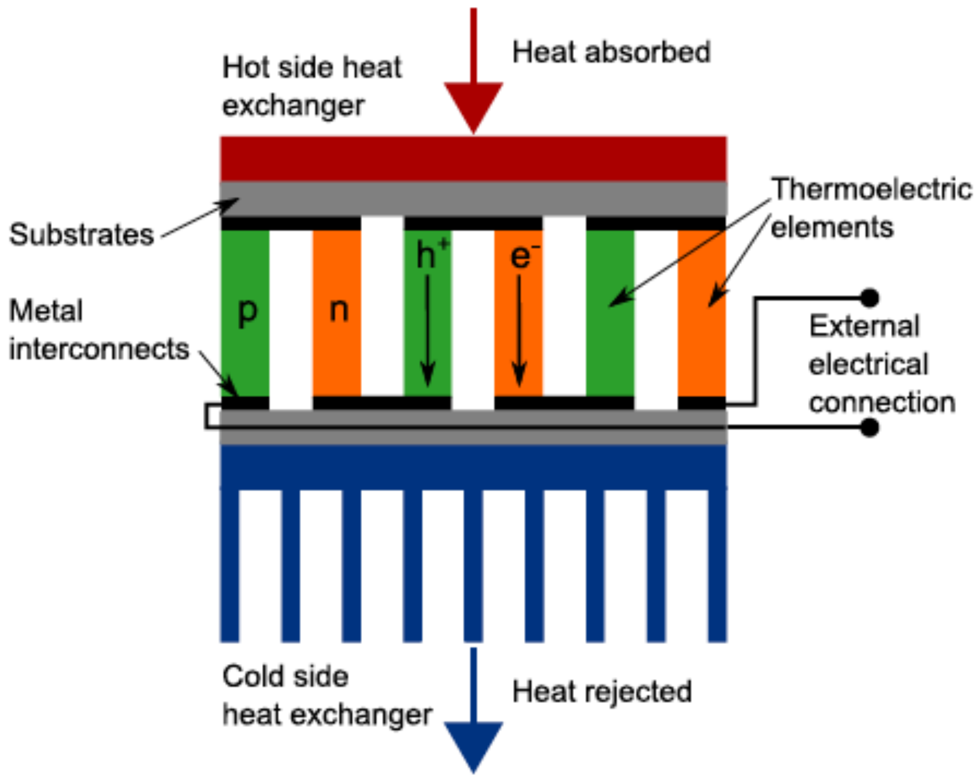
$$P = \eta_1 \Delta T_{\text{supply}}^2 \frac{\Theta_{TE}}{\left(\Theta_{Hx} + \Theta_{TE}\right)^2}$$

$$\frac{\partial P}{\partial \Theta_{TE}} = 0 \Rightarrow \Theta_{TE} = \Theta_{\text{Hx}}$$

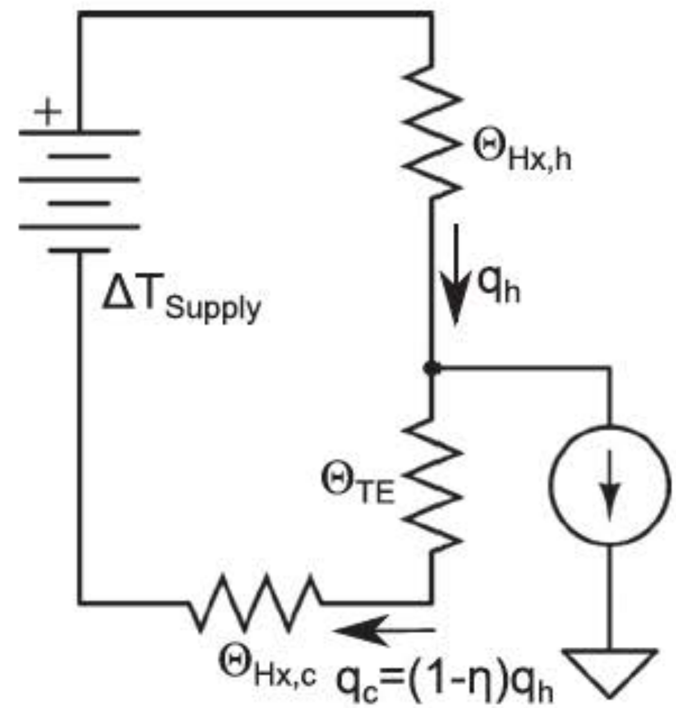
Thermoelectrics
Northwestern Materials Science and Engineering

Effective Thermal Conductivity

Thermal model



Thermal Circuit

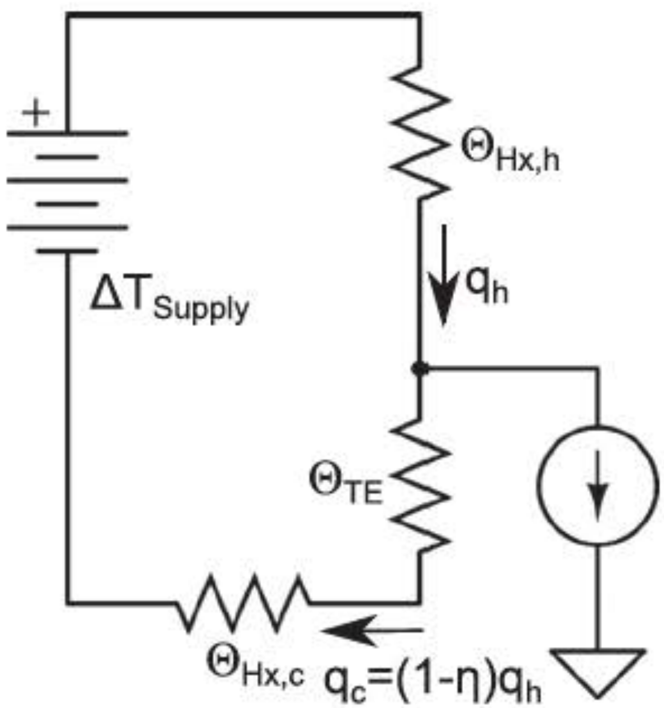


Thermoelectrics
Northwestern Materials Science and Engineering

Baranowski, Snyder et al, *J Applied Phys.* 113, 204904, (2013)

Thermal Circuit Analysis

Thermal Circuit



Mathematical Model

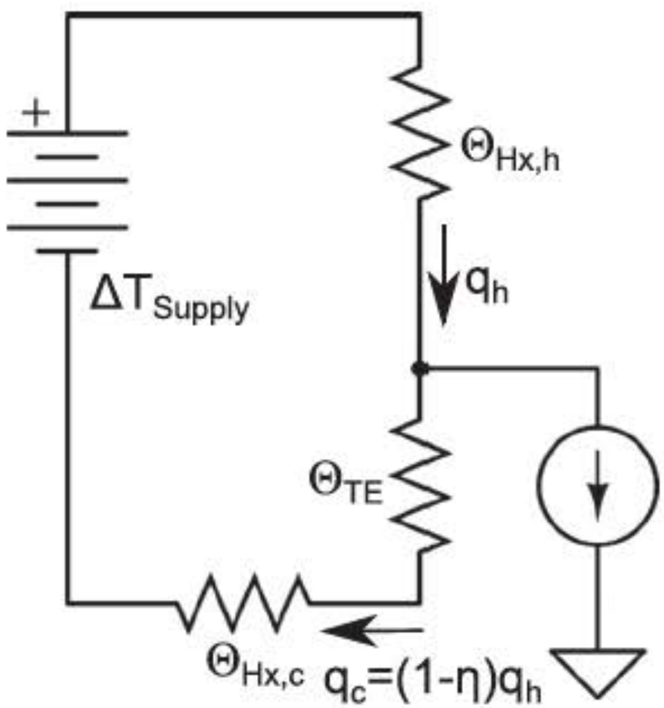
$$\Delta T_{Hx} = q_h(\Theta_{Hx,h} + (1-\eta)\Theta_{Hx,c}) = q_h\Theta_{Hx}$$

$$\Delta T_{TE} = \Delta T_{supply}\frac{\Theta_{TE}}{\Theta_{Hx} + \Theta_{TE}}$$

Baranowski, Snyder et al, *J Applied Phys.* 113, 204904, (2013)

Power and Efficiency

Thermal Circuit



Thermoelectric Definitions

$$P = \eta q_h$$

$$\eta = \frac{\Delta T_{TE}}{T_h}\eta_{r,d}$$

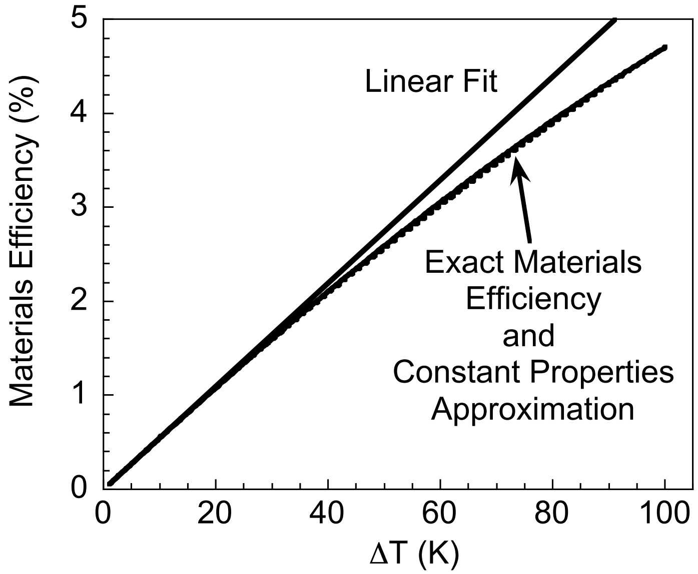
$$\eta = \frac{\Delta T_{TE}}{T_h}\frac{\sqrt{1+ZT}-1}{\sqrt{1+ZT}+T_c/T_h}$$

Thermoelectrics
Northwestern Materials Science and Engineering

Baranowski, Snyder et al, *J Applied Phys.* 113, 204904, (2013)

Efficiency dependence on ΔT

Mathematical Model



$$\eta = \frac{\Delta T_{TE}}{T_h} \frac{\sqrt{1+ZT}-1}{\sqrt{1+ZT}+T_c/T_h}$$

$$\frac{P_{max}}{A_{Hx}} = \frac{\Delta T_{supply}^2 h_{Hx}}{4T_h} \frac{\sqrt{1+ZT}-1}{\sqrt{1+ZT}+T_c/T_h}$$

$$\eta_{r,d} = \frac{2T_h\left(1-\left(\frac{T_c}{T_h}\right)^{\eta_{r,l\max}}\right)}{\Delta T_{supply}}$$

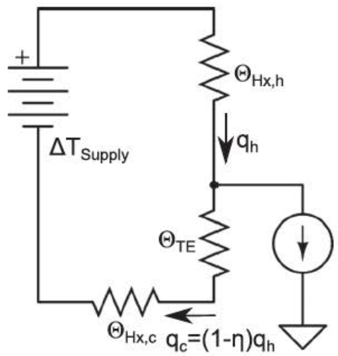
$$\eta_{r,l\max} = \frac{\sqrt{1+zT}-1}{\sqrt{1+zT}+1}$$

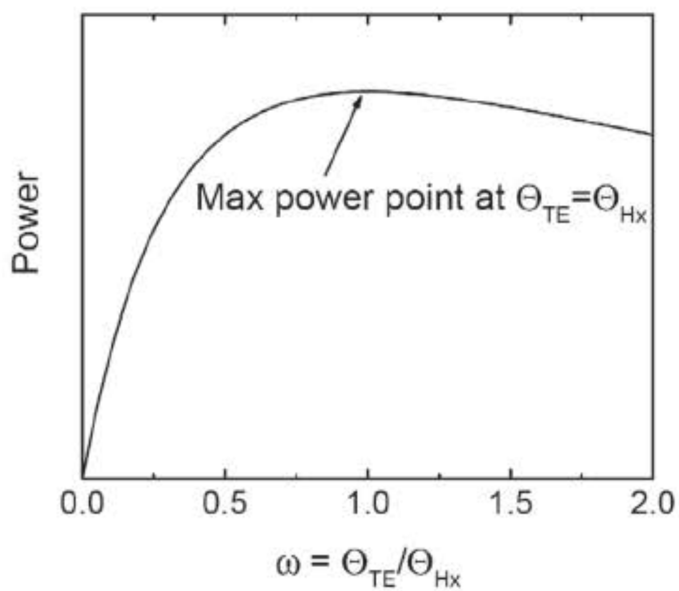
$$\frac{P_{max}}{A_{Hx}} = \frac{\Delta T_{supply} h_{Hx}}{2}\left(1-\left(\frac{T_c}{T_h}\right)^{\eta_{r,l\max}}\right)$$

Baranowski, Snyder et al, *J Applied Phys.* 113, 204904, (2013)

Thermal Impedance Match

Thermal Circuit





Thermal Impedance match derivation

$$\Delta T_{TE} = \Delta T_{supply} \frac{\Theta_{TE}}{\Theta_{Hx} + \Theta_{TE}}$$

$$\Delta T_{supply} = q_h (\Theta_{Hx} + \Theta_{TE})$$

$$P = \frac{\Delta T_{supply}^2 \eta_{r,d}}{T_h} \frac{\Theta_{TE}}{(\Theta_{Hx} + \Theta_{TE})^2}$$

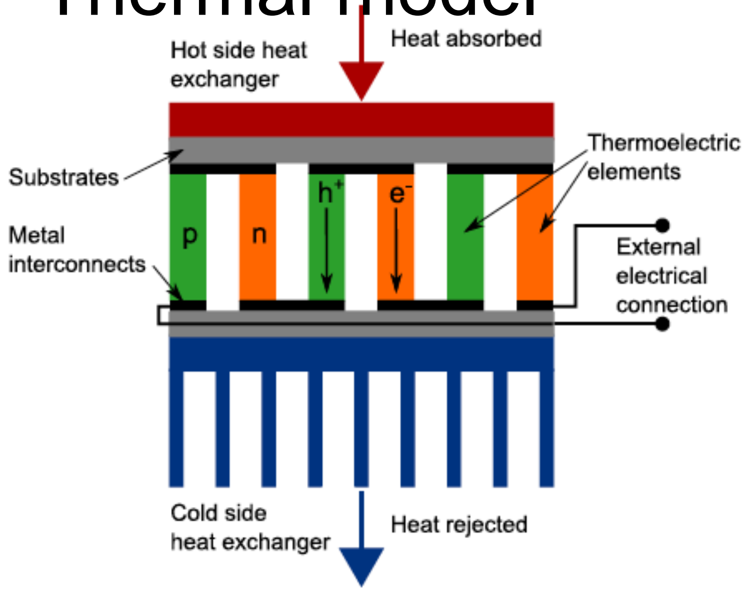
$$P_{max} = \frac{\Delta T_{supply}^2 \eta_{r,d}}{4 T_h \Theta_{Hx}}$$

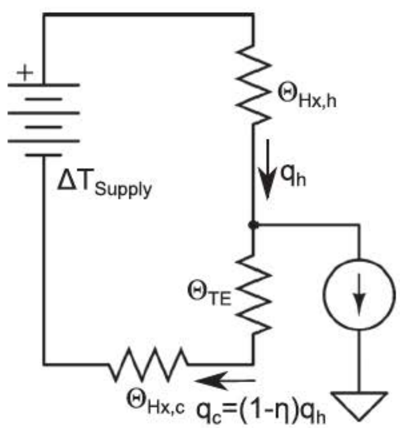
Thermoelectrics
Northwestern Materials Science and Engineering

Baranowski, Snyder et al, *J Applied Phys.* 113, 204904, (2013)

Effective Thermal Conductivity

Thermal model





Area Specific Formulation

Cross sectional Area A

$$h_{Hx} = 1/\Theta_{Hx}A_{Hx}$$

forced air

$$h_{Hx} \approx 0.004\,\mathrm{W/cm^2K}$$

forced water

$$h_{Hx} \approx 0.6\,\mathrm{W/cm^2K}$$

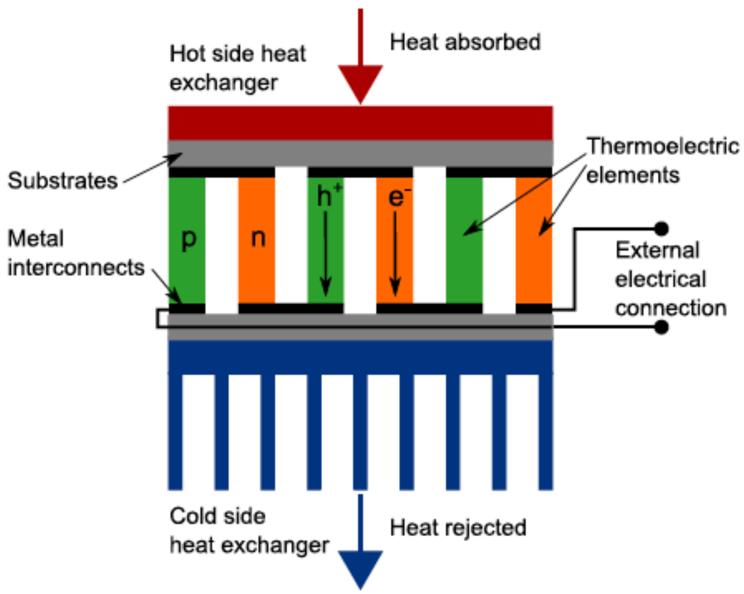
$$\frac{P_{max}}{A_{Hx}} = \frac{\Delta T_{supply}^2\, h_{Hx}\, \eta_{r,d}}{4T_h}$$

Thermoelectrics
Northwestern Materials Science and Engineering

Baranowski, Snyder et al, *J Applied Phys.* 113, 204904, (2013)

Effective Thermal Conductivity

Thermal model



Area Specific Formulation

Cross sectional Area A

$$\Theta_{TE} = \frac{l}{\kappa_{\text{eff}} A_{TE}}$$

filling factor: $f = A_{TE}/A_{Hx}$

$$\frac{1}{h_{Hx}} = \Theta_{Hx} A_{Hx} = \Theta_{TE} A_{Hx} = \Theta_{TE} \frac{A_{TE}}{f}$$

$$l = \frac{f \, \kappa_{\text{eff}}}{h_{Hx}}$$

Thermoelectrics Northwestern Materials Science and Engineering

Baranowski, Snyder et al, *J Applied Phys.* 113, 204904, (2013)

Effective Thermal Conductivity

Effective κ at point of optimum operation

Heat flux q''

$$q_h'' = \frac{\kappa_{\text{eff}}}{l}(T_h - T_c)$$

Compatibility formalism

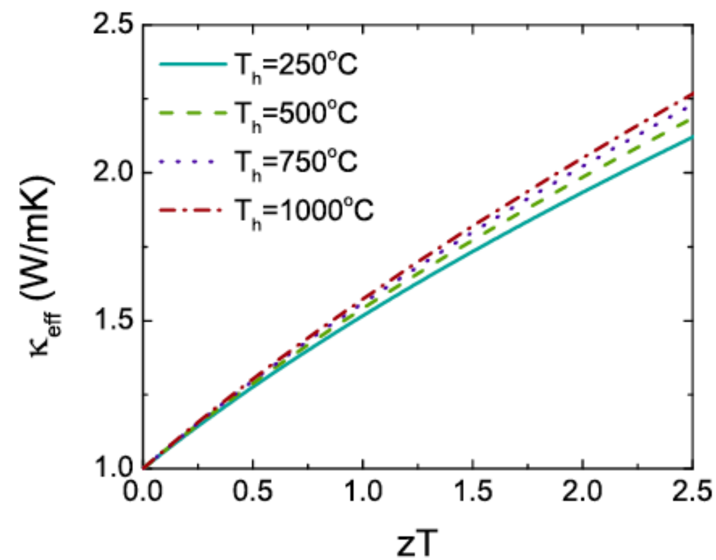
$$q_h'' = \frac{\Phi_h \int_{T_c}^{T_h} \kappa u \, dT}{l}$$

gives

$$\kappa_{\text{eff}} = \frac{\Phi_h}{(T_h - T_c)} \int_{T_c}^{T_h} \kappa u \, dT$$

κ_{eff} Approximate Forms

$$\kappa_{eff} \approx \kappa \cdot \sqrt{1 + Z\bar{T}}$$



$$\kappa_{\text{eff}} = \frac{\kappa T_h (1 + zT + \sqrt{1 + zT})}{2(T_h - T_c)} \left(1 - \left(\frac{T_h}{T_c}\right)^{k_g}\right)$$

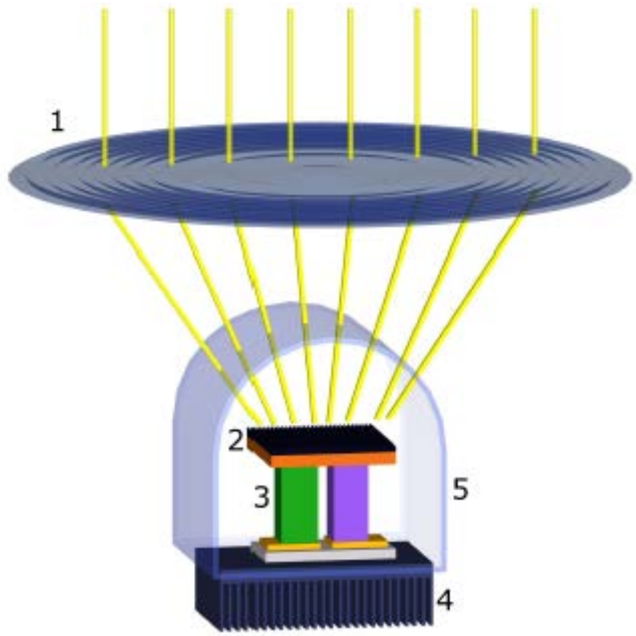
$$k_g = \frac{2 - 2\sqrt{1 + zT}}{zT}$$

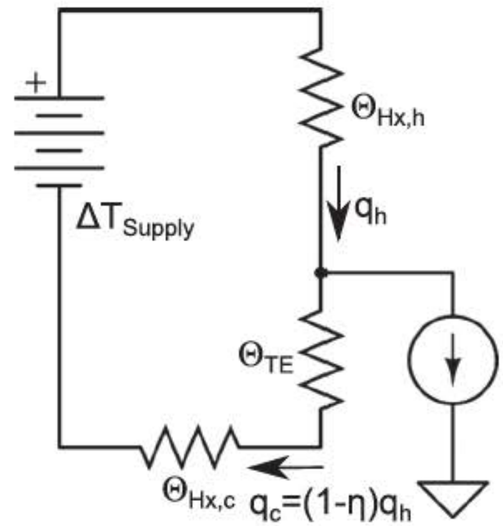
Thermoelectrics
Northwestern Materials Science and Engineering

Baranowski, Snyder et al, *J Applied Phys.* 113, 204904, (2013)

Solar Thermoelectric Example

Thermal model

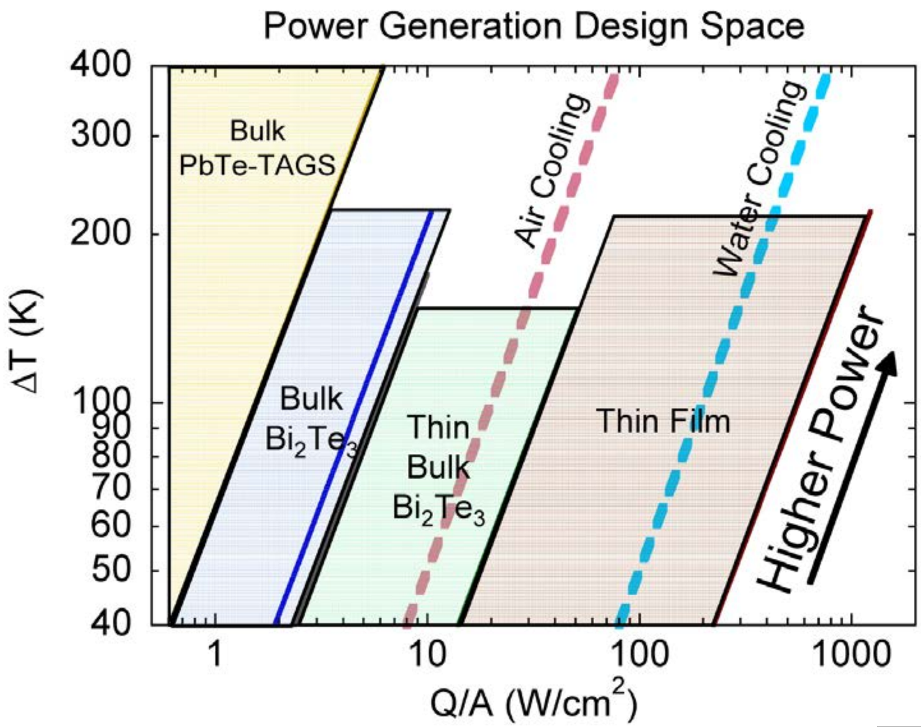


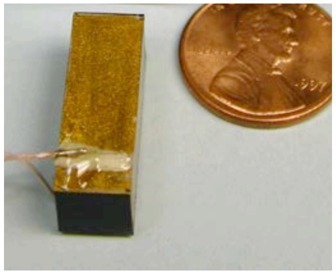


$$\eta_{STEG} = \frac{(T_h - T_c)\kappa_{eff}}{q''_{inc} L_{th}} \left(1 - \left(\frac{T_c}{T_h}\right)^{\eta_r}\right)$$

Baranowski, Snyder et al, *Energy Environmental Sci.* 5, 9055, (2012)

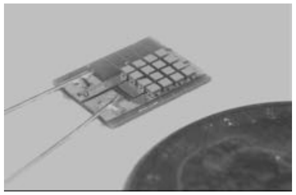
TE phase space





Thermopile
l ~ 20 mm

Commercial Module
l ~ 2 mm



Thin Film Device
l ~ 0.02 mm

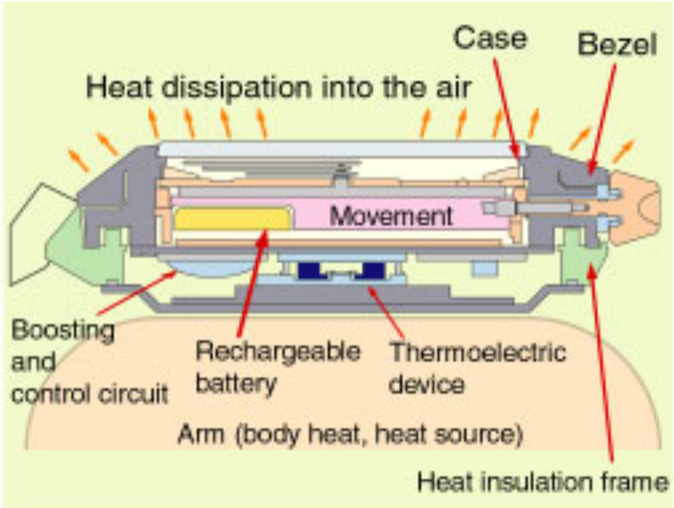
Thermoelectrics
Northwestern Materials Science and Engineering

Example - Wristwatch

How much power could you harvest from body Heat?

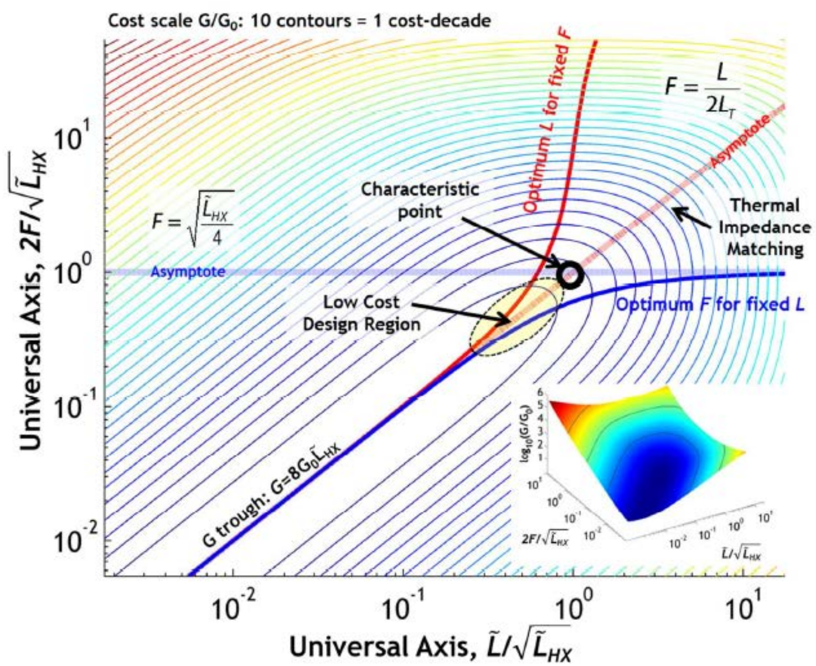
- Body ($2m^2$) releases 100W heat
  - 20 mW/cm^2
- Skin temperature 33C, RT = 21C
  - ΔT = 12K
- Thermal Impedance match TE
  - ΔT_{TE} = 6K
  - Carnot Efficiency = 2%
  - Reduced Efficiency (ZT~1) = 20%
  - Total Efficiency = 0.4%
  - 80 uW/cm^2
- Seiko Wristwatch
  - 22 uW output
  - 20mV/K * 10 modules
  - 300mV OC = 1.5K across TE
  - 600K/W / 10 modules
    = 25mW heat
    = 0.1% efficiency

Seiko Thermic, Thermoelectric Powered Wristwatch



Thermoelectrics
Northwestern Materials Science and Engineering

Power and Cost



TEG Design Optimization

1. Decide type and size of heat exchangers = determine h_{Hx}

   roughly $$\frac{P_{max}}{A_{Hx}} = \frac{\Delta T^2_{supply} h_{Hx} \eta_{r,d}}{4T_h}$$ with $\eta_{r,d}$ ~ 0.15

2. Decide target T_h and T_c, roughly ΔT_{TE} = $\Delta T_{supply}/2$

   Determine TE materials used

3. Determine TEG target length l and fill factor f from $$l = \frac{f\,\kappa_{\text{eff}}}{h_{Hx}}$$

   Determine TE materials used

4. Optimization of module based on optimum efficiency

   Number of couples from Target Voltage (Current from V and P)

   Area of elements from optimum area ratio $$\frac{A_p}{A_n} \approx \sqrt{\frac{\rho_p \kappa_n}{\rho_n \kappa_p}}$$

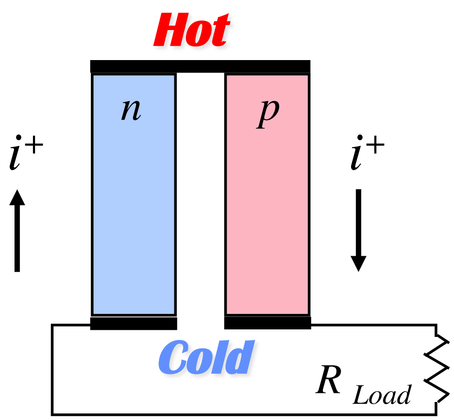
   Load Resistance approx $$R_{Load} = R\sqrt{1+ZT}$$

Thermoelectrics
Northwestern Materials Science and Engineering

Snyder, *CRC Handbook of Thermoelectrics.* Chapter 9, (2005)
Baranowski, Snyder et al, *J Applied Phys.* 113, 204904, (2013)

Power from TEG

Power derivation



$$P = I^2 R_L$$

$$I = \frac{\alpha \Delta T}{R_{TE} + R_L}$$

$$P = \frac{\alpha^2 \Delta T^2}{(R_L + R_{TE})^2} R_L$$

$$m = R_L / R_{TE}$$

$$P = \frac{\alpha^2 \Delta T^2 A}{\rho l} \frac{m}{(1+m)^2}$$

Thermoelectrics
Northwestern Materials Science and Engineering

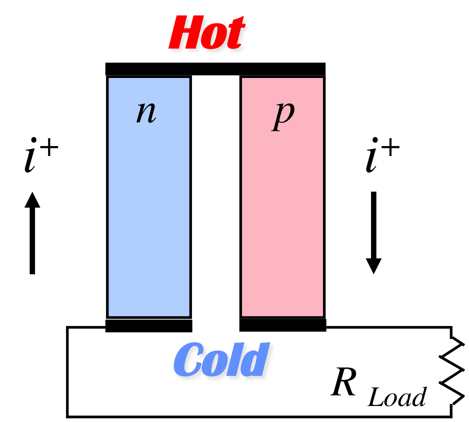
Baranowski, Snyder et al, *J Applied Phys.* 115, 126102, (2014)
Baranowski, Snyder et al, *J Applied Phys.* 113, 204904, (2013)

Power from TEG

$$P = \frac{\alpha^2 \Delta T^2 A}{\rho l} \frac{m}{(1+m)^2}$$

Maximum power ?

mathematically this function has no maximum!



A = infinity

- •consider power density P/A

ΔT = maximum?

- •actually one should thermal impedance match $\Delta T \sim \Delta T_{supply}/2$

l = zero

- •**set l to a constant** ?!?
  - –off the shelf component? = how to get most power from given TEG
  - are we optimizing the use of a given TEG or design a TEG for application?

$$\frac{\partial}{m}\left(\frac{P}{A}\right)_{l,\alpha,\rho,\Delta T} = 0 \Rightarrow m = 1$$

used to derive electrical load match condition $m = 1$

Not mathematically rigorous when l is design parameter !

used to derive power factor α^2/ρ more important than zT for power generating

NO! Faulty derivation – this function has no maximum! beware of limits to infinity

Thermoelectrics
Northwestern Materials Science and Engineering

Baranowski, Snyder et al, *J Applied Phys.* 115, 126102, (2014)
Baranowski, Snyder et al, *J Applied Phys.* 113, 204904, (2013)

TEG Power Competition

Max power TE

T_h and T_c, with q_h heat through it

$$R_{Load} = R$$

has higher current

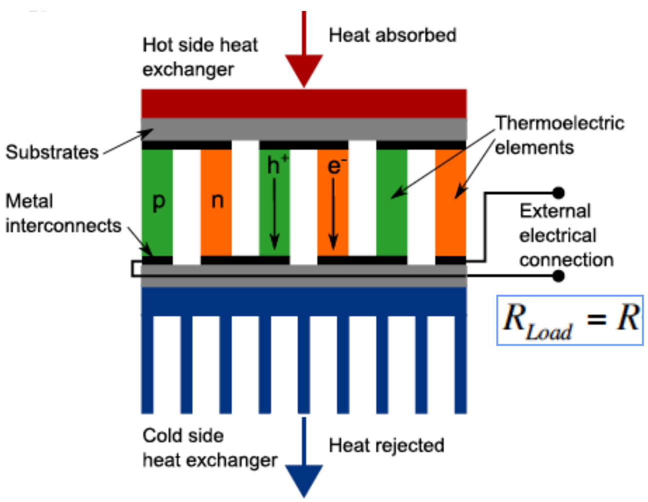
Max efficiency TE with same

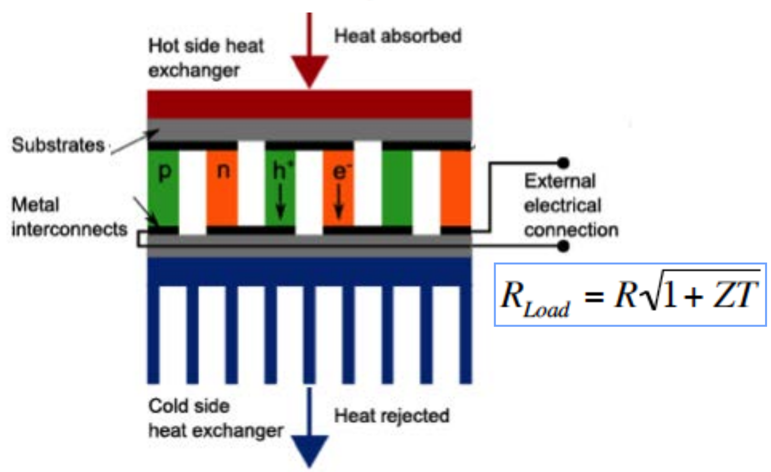
T_h and T_c, with q_h heat through it

$$R_{Load} = R\sqrt{1+ZT}$$

has lower current so reduce l to keep q_h as Max power TE

$$q''_h = \frac{\kappa_{\text{eff}}}{l}(T_h - T_c)$$





Thermoelectrics
Northwestern Materials Science and Engineering

Baranowski, Snyder et al, *J Applied Phys.* 115, 126102, (2014)
Baranowski, Snyder et al, *J Applied Phys.* 113, 204904, (2013)

TEG Power Competition

Max power TE

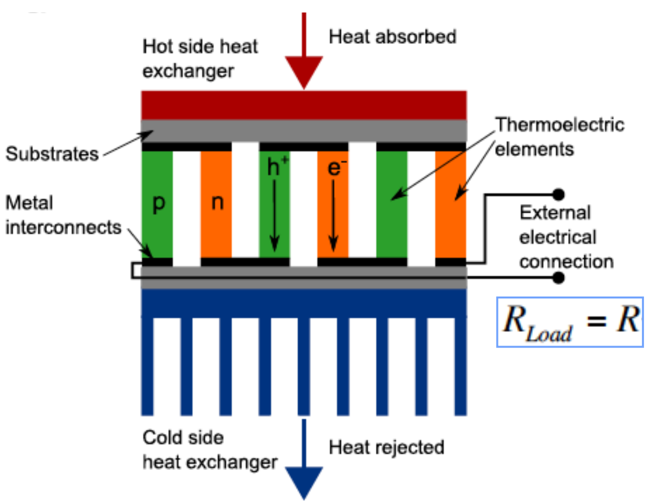
T_h and T_c, with q_h heat through it

$$R_{Load} = R$$

has lower efficiency

$$P = \eta q_h$$

therefore ***lower*** power



Max efficiency TE with same

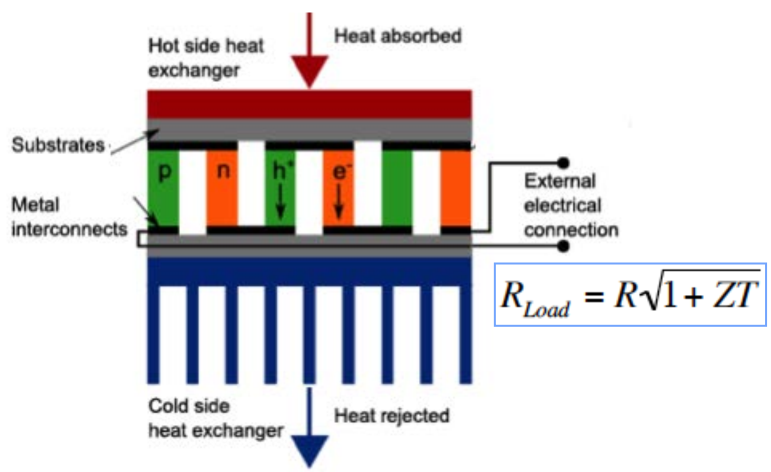
T_h and T_c, with q_h heat through it

$$R_{Load} = R\sqrt{1+ZT}$$

has higher efficiency

$$P = \eta q_h$$

therefore ***higher*** power



Thermoelectrics
Northwestern Materials Science and Engineering

Baranowski, Snyder et al, *J Applied Phys.* 115, 126102, (2014)
Baranowski, Snyder et al, *J Applied Phys.* 113, 204904, (2013)

TEG Power Competition

Proof by contradiction

where TE height l is design variable

1) Suppose: $R_{Load} = R$ always provides more power than $R_{Load} = R\sqrt{1+ZT}$

2) give example of $R_{Load} = R$

3) show counter example where $R_{Load} = R\sqrt{1+ZT}$ provides more power

4) supposition 1) must be false

Thermoelectrics
Northwestern Materials Science and Engineering

Baranowski, Snyder et al, *J Applied Phys.* 115, 126102, (2014)
Baranowski, Snyder et al, *J Applied Phys.* 113, 204904, (2013)

TEG Power Competition

Can be used to show maximum power design

where TE height l is design variable

$$R_{Load} = R\sqrt{1+ZT}$$

$$\frac{A_p}{A_n} \approx \sqrt{\frac{\rho_p \kappa_n}{\rho_n \kappa_p}}$$

zT is figure of merit for power

not power factor $\alpha^2\sigma$!

high κ matters !

Thermoelectrics
Northwestern Materials Science and Engineering

Baranowski, Snyder et al, *J Applied Phys.* 115, 126102, (2014)
Baranowski, Snyder et al, *J Applied Phys.* 113, 204904, (2013)

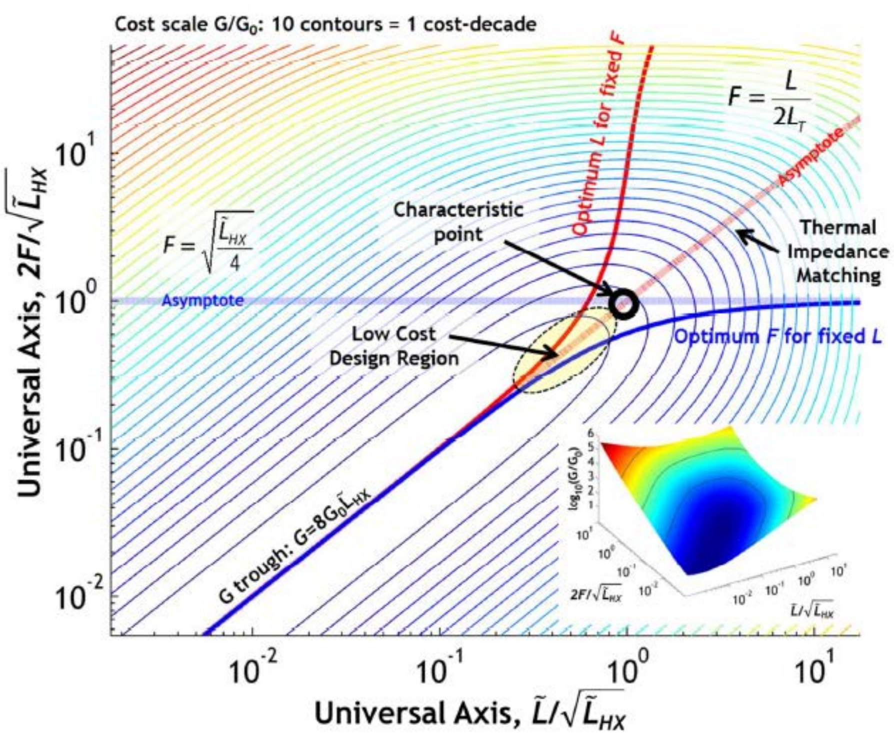
ZT at any cost?

Linear Thermal model using κ_{eff} facilitates cost modeling

- lowest system cost/W is usually dominated by heat exchanger cost rather than TE material cost ($200/kg material)
- Power increases with ZT so cost directly depends on ZT but not TE cost

$$\frac{Cost}{W} \approx \left(\frac{Cost}{Area}\right)_{HX} \cdot \frac{1}{Z\overline{T}}$$

ZT is TE cost metric



Thermoelectrics
Northwestern Materials Science and Engineering

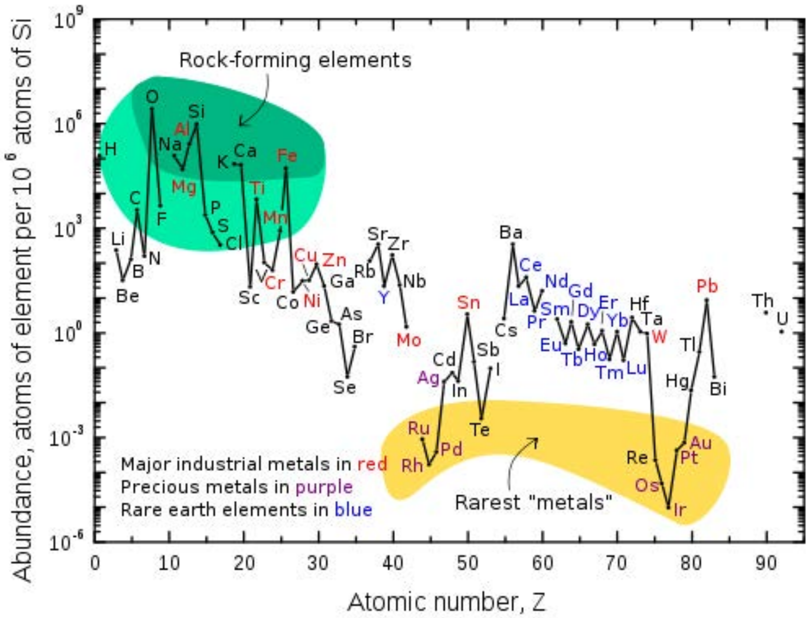
N.R Kristiansen, Snyder, et al. *J. Electronic Materials* 6, 1024 (2012)
Dames et al, *Energy Environmental Sci.* 6, 2561, (2013)

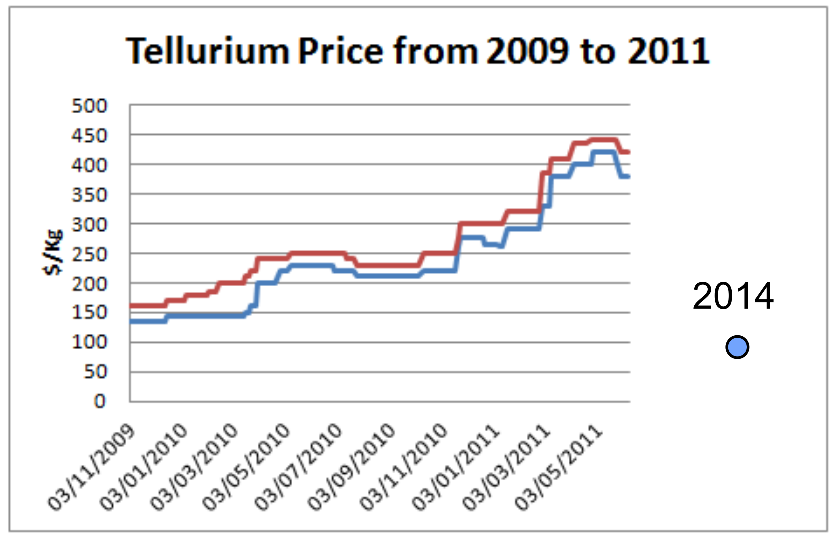
ZT at any cost?

What about High Cost and abundance of Te ?!?

Te has low abundance on Earth crust

Cost of Te above $400/kg in 2011 (back to $100/kg in 2014)





Te is a by product of Cu refining, capacity not at full production

Bi is $20/kg, Pd is $25,000/kg

Thermoelectrics
Northwestern Materials Science and Engineering