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Darkness Clusters in Cosmology

I. Introduction

In several previous notes, the concept of darkness has been developed and discussed. This note is a small addendum to those contributions, which deals with an overlooked detail.

Darkness is a presumed topological quantity associated with a very large Gauss-Bonnet term generated in the MacDowell-Mansouri extension of the Einstein-Cartan first-order formalism for general relativity. It is a presumably a "number density of topological structures", which for dark-energy-dominated deSitter space has the value, in order of magnitude

$$n \equiv \frac{N(t)}{V(t)} \sim H M_{pl}^2 \equiv \Lambda_Z^3 \sim \left(10^{-20} M_{pl}\right)^3$$

Here, the quantity H is the (constant !) Hubble parameter describing the exponential expansion of dark-energy-dominated deSitter space:

$$\frac{\Lambda}{3} = H^2$$

The natural scale associated with this purported deSitter "darkness" is of order 100 MeV, and is denoted by Λ_Z. The Z stands for Zeldovich, who in 1967 first identified this possible scale present in pure gravity.

For geometries which are static and spherically symmetric, this expression for darkness density generalizes to the expression

$$n \sim \frac{M_{pl}^2}{H^2}\left(\frac{v^2}{r^2}\frac{\partial v}{\partial r}\right)$$

Here the velocity parameter is defined via the Painleve-Gullstrand form of the metric tensor:

$$ds^2 = dt^2 - \left(dr - v(r)\,dt\right)^2 - r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)$$

For Schwarzschild-deSitter space, one has

$$\left|\frac{v}{r}\frac{\partial v}{\partial r}\right| \sim \frac{v^2}{2r^2} = \frac{GM}{r^3} + \frac{H^2}{2}$$

For a source of nuclear matter density ρ_{NM}, this leads to the conclusion that the darkness density near the surface of the source is Planckian

$$n(r) \sim \frac{M_{pl}^2}{H^2}\left(\frac{\rho_{NM}}{M_{pl}^2}\right)^{3/2} \sim \frac{(\Lambda_Z^4)^{3/2}}{H^2 M_{pl}} \sim \frac{(H M_{pl}^2)^2}{H^2 M_{pl}} \sim M_{pl}^3$$

Here $R \sim \Lambda_Z^{-1} A^{1/3}$ is the radius of the nuclear-matter source, and $M \sim \Lambda_Z A$.

The total darkness N which is clustered near such a source scales linearly with atomic number A , with a coefficient of order 10^{60} :

$$N \sim \frac{M_{pl}^2}{H^2}\int_R^\infty r^2 dr \left(\frac{GM}{r^3}\right)^{3/2} \sim \frac{M_{pl}^2}{H^2}\left(\frac{GM}{R}\right)^{3/2} \sim \frac{1}{H^2 M_{pl}}\left(\frac{\Lambda_Z^2 A}{A^{1/3}}\right)^{3/2} \sim \left(\frac{M_{pl}}{H}\right) A \sim 10^{60} A$$

On the other hand, for FRW cosmology it turns out that the darkness density in the past is inferred to be of order

$$n \sim \frac{M_{pl}^2}{H^2} H^3(t), \quad \text{where} \quad H^2(t) \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3M_{pl}^2}\rho(t)$$

When the temperature was of order 10 Mev, this density was Planckian:

$$\rho(t) \sim \Lambda_Z^4 \sim (H M_{pl}^2)^{4/3} \Rightarrow n \sim \frac{M_{pl}^2}{H^2}\cdot\frac{1}{M_{pl}^3}(H M_{pl}^2)^{4/3 \cdot 3/2} \sim M_{pl}^3$$

All this strongly suggests that the MacDowell-Mansouri description is only appropriate for distance scales larger than about 10^{-13} cm, namely the Zeldovich scale:

The point of this note has to do with the observation that the total darkness in the universe nowadays is dominated by the darkness surrounding baryonic matter. Compared to the amount of darkness one would anticipate from a smooth FRW source of matter, radiation (and of course dark energy itself), the baryonic contribution dominates nowadays by about 20 powers of ten. Only after 10 to 15 efoldings of future inflation will the darkness contributed by the ocean of deSitter space overwhelm that residing in the small, widely-scattered, matter-dominated galactic-cluster islands (including our own).

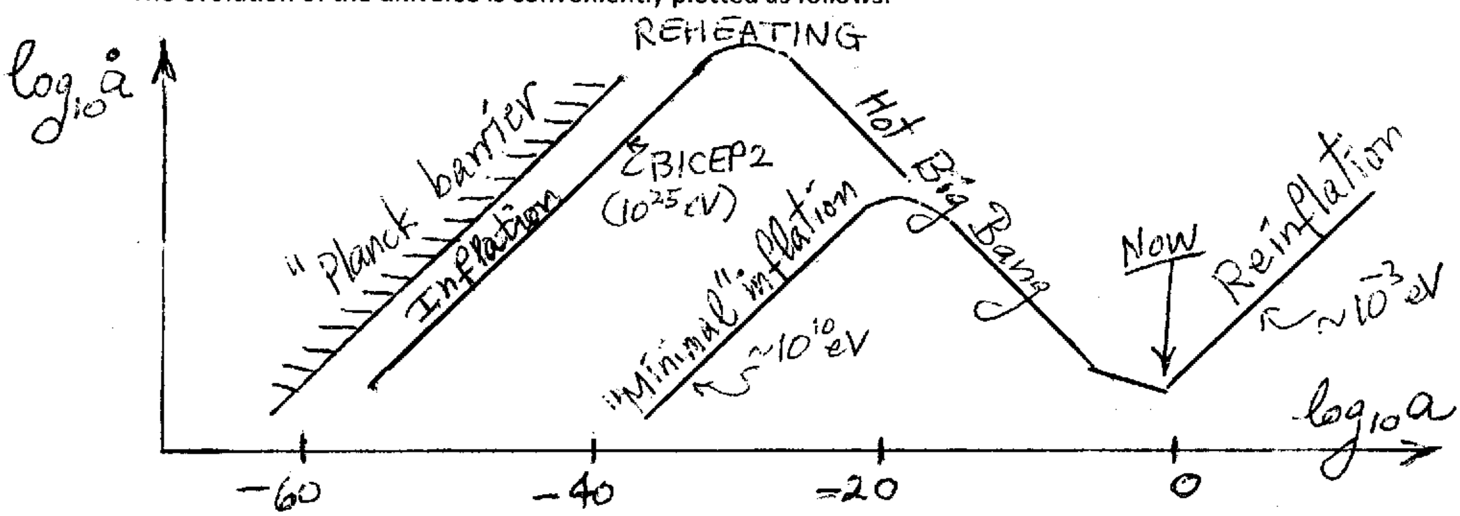
In the next section we review the cosmological evolution of darkness, with the above observation included. The final section is devoted to a few comments regarding possible implications of this observation.

II. Cosmological Evolution

The story begins, for us, during the radiation-dominated era, when the temperature was tens of MeV, and the darkness density was Planckian. At this time, the QCD phase transition was just completed, and the components of the hot plasma consisted of baryons, electrons, photons, and neutrinos. What is important is that the baryons were present, and were dilute---the antibaryons were annihilated out of existence before that time. But each proton or neutron contained in its neighborhood 10^{60} units of darkness already. And, as the universe expanded---all the way to the present time---the amount of darkness per comoving volume did not change. This occurred because the total darkness in the comoving volume scaled with baryon number, which did not change. Up to numerical factors of order unity, this situation persisted right through the epoch of nucleosynthesis.

The consequence of this is that the amount of darkness present nowadays is much larger than what would be anticipated from a smooth FRW scenario without the density fluctuations given by the baryonic component. In particular, the total baryon number in the universe, out to distances of order the deSitter horizon ($\sim 10^{28}$ cm) is $\sim 10^{80}$, which leads to a total amount of cluster-darkness of $\sim 10^{140}$. The deSitter darkness inside the horizon is of order $H^{-3}(H M_{pl}^2) \sim 10^{120}$, 20 powers of ten smaller. Therefore it will take of order 7 powers of ten of inflation for the deSitter darkness to overwhelm the clustered darkness.

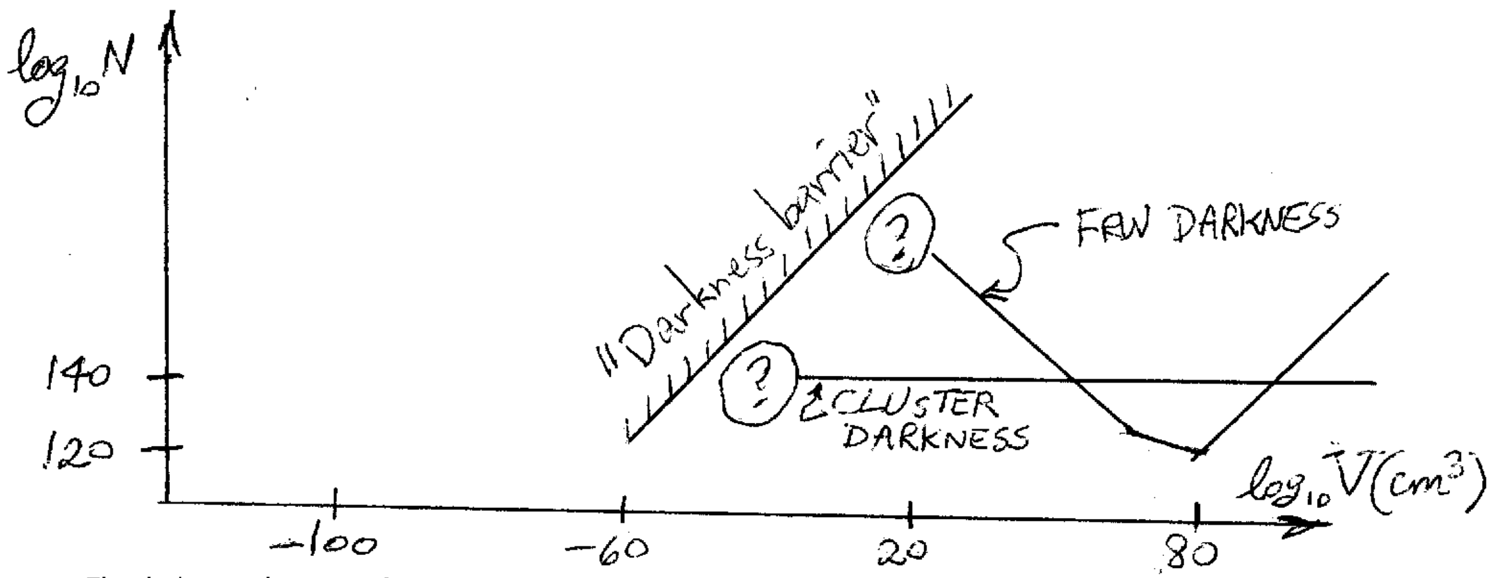
The evolution of the universe is conveniently plotted as follows:



The components with positive slope are inflationary, characterized by a constant value of the dark energy scale, as indicated. We have described two choices of the inflationary dark energy scale. The BICEP scale of 10^{16} GeV is maximal, and comparable to the default option of most theorists. However, there is very little if any empirical evidence that precludes a much lower scale. The curve labeled "minimal" is about as low as is thinkable, with a dark energy scale of order 10 TeV.

In the plot is also the "Planck barrier", to the left of which transPlanckian effects occur.

In describing the cosmological evolution of darkness, It is convenient to plot the log of the darkness N within a comoving volume against the logarithm of the comoving volume $V \propto a^3$:



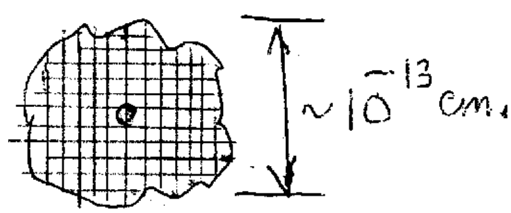
The darkness-cluster evolution is evidently described by a horizontal line, as shown. The FRW darkness evolution follows the same trajectory as in the first figure, because the FRW darkness within a comoving volume is proportional to the cube of $\dot{a}$, while the comoving volume itself is proportional to the cube of a .

In the above figure there is depicted a "darkness barrier" , analogous to the "Planck barrier" in the first figure. To the left of it, we can expect transplanckian darkness densities. This is again an indicator of the limitations of the MacDowell-Mansouri description. We return to this issue in the following section.

III. Possible Implications for the Inflationary Epoch

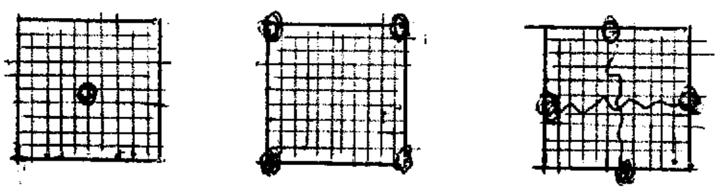
In previous notes we have speculated that the way that the MacDowell description gets generalized involves six compactified extra dimensions, each at the Zeldovich scale of order 10^{-12} cm. We will assume this is the case in what follows.

For each of the three ordinary space dimensions, there are two extra dimensions. Assume that in the present universe, each darkness element is of Planckian size and lies at the origin of coordinates of the extra dimensions, as shown below:



This means that there are of order 10^{40} unoccupied cells in the two extra dimensions associated with, say, the x-coordinate. Then, as we go back in time and cross the "darkness barrier", these unoccupied cells get filled from the origin outward. Evidently this allows the darkness density to increase by a factor of 10^{40} before all the cells become filled. Since, in the radiation-dominated era, the darkness density increases as the sixth power of the scale factor a, we can go 20 powers of ten in the scale factor a(t) beyond the darkness barrier before getting into trouble. But, happily, this takes us all the way to the Planck barrier.

In this visualization, we should recognize that the details may be quite different depending on what topology is chosen for the compactified extra dimensions, and whether one appeals to an orbifold construction. Some of the choices are illustrated below



However, we expect that a common feature is, as we proceed backward in time, that the darkness grows outward from the seed structures present at late times.

What happens to the darkness clusters as we go backward in time and cross the darkness barrier? The baryon number is now carried by quarks, which can be regarded as massless. And the cosmological history of the origin of nonvanishing baryon number becomes entangled with CP violation, as well as lepton number violation. So it is tempting to simply extend the horizontal line to the left of the darkness barrier, but associate it somehow with these phenomena. Again extra dimensions can be expected to be involved. But in this case it might be that only half the extra dimensions need be activated, since the large number to be dealt with is 10^{60} instead of 10^{120}. But the devil will be in the details, which are far beyond the scope of this little note.

IV. Concluding Comments

The bottom line is that, until the universe expands by about seven powers of ten, the darkness will be dominated by that surrounding baryons in matter-dominated islands, and not by the darkness in the voids, which has been the primary emphasis I have made in previous notes. Maybe this will turn out to be relevant, maybe not. I simply do not know.