Research-Based Mathematics Instruction

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Describe your strengths in supporting mathematics.

Describe an opportunity for growth.

Teachers should use systematic and explicit instruction to help students develop a strong foundation for specific mathematics skills.

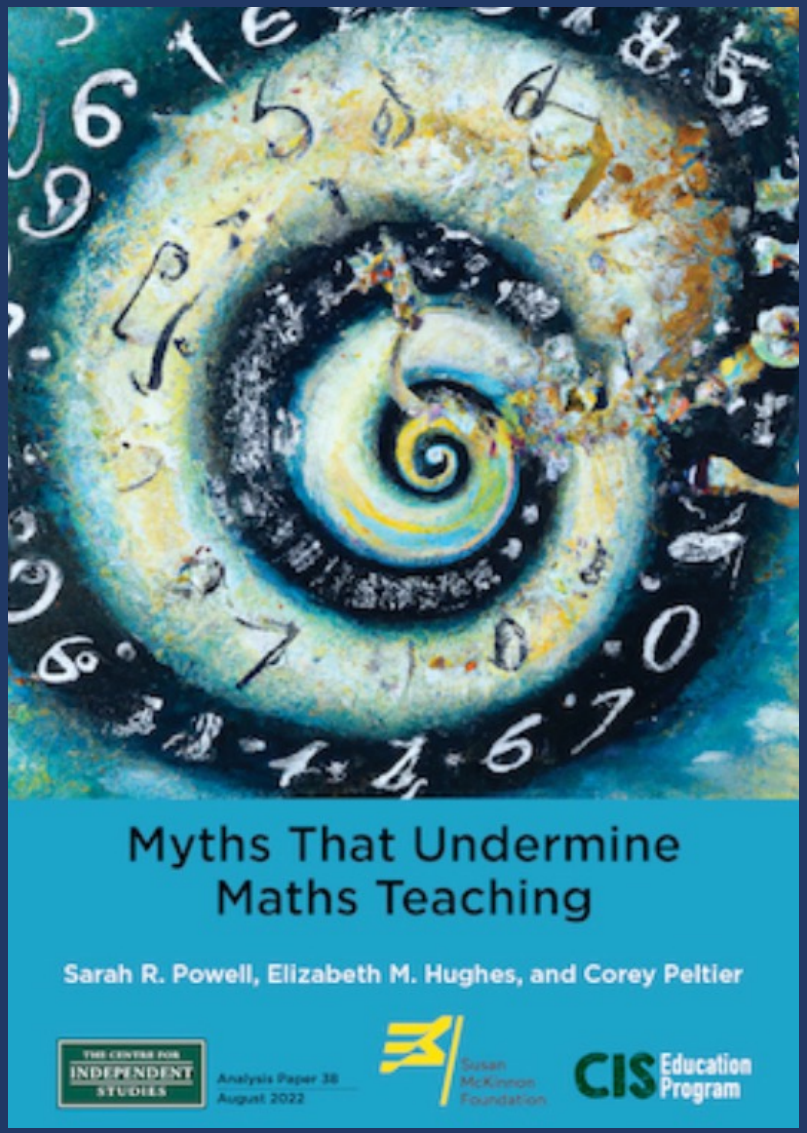
Students require modeling and practice on how to use the language of mathematics.

Students should use hands-on tools, virtual manipulatives, drawings, and other visuals to understand mathematics concepts and procedures.

Teachers should use fluency building activities to build counting fluency and fluency with the operations.

Students should learn how to set up and solve word problems by combining an attack strategy with a focus on word-problem schemas.

Myths



Centre for Independent Studies

THE SCIENCE OF MATH

MISCONCEPTION

Some educators believe students should not be exposed to procedural instruction until they have demonstrated adequate conceptual understanding of a topic.

TRUTH

Conceptual knowledge supports procedural knowledge AND procedural knowledge supports conceptual knowledge. They should be taught together!

There is a two-way relationship between conceptual and procedural knowledge -

conceptual understanding and procedural fluency develop together[a]

Concepts and procedures overlap - they are difficult to measure independently[b]

WHEN TEACHING MATH

Teaching and practicing conceptual understanding can help with the selection and use of problem-solving procedures.[c]

Teaching and practicing procedures helps to develop and deepen understanding of concepts. Procedures include more than algorithms.[d]

Teaching conceptual and procedural knowledge together can help strengthen each over time.[e]

MISCONCEPTION

Many educators believe algorithms promote memorization, and this would contribute to a superficial understanding of steps, conventions, and rules. This belief leads to the idea that students should not be taught algorithms.

TRUTH

An algorithm is a step-by-step procedure for solving a problem. Using an algorithm requires conceptual understanding of what is happening in the porblem and procedural knowledge to accurately solve. Algorithms can serve as a link between conceptual understanding and procedural knowledge.

Examples of algorithms

$$\begin{array}{r} 23 \\ \times\ 6 \\ \hline 120 \\ +\ 18 \\ \hline 138 \end{array}$$

$$\begin{array}{r} 192 \\ +\ 133 \\ \hline \end{array}$$

100 + 100 = 200
90 + 30 = 120
2 + 3 = 5

200 + 120 + 5 = 325

$$\begin{array}{r} {\scriptstyle 7\ 11} \\ \not{8}\not{1}7 \\ -\ 653 \\ \hline 164 \end{array}$$

ALGORITHMS[a]

Promote flexibility

Lead to deeper understanding

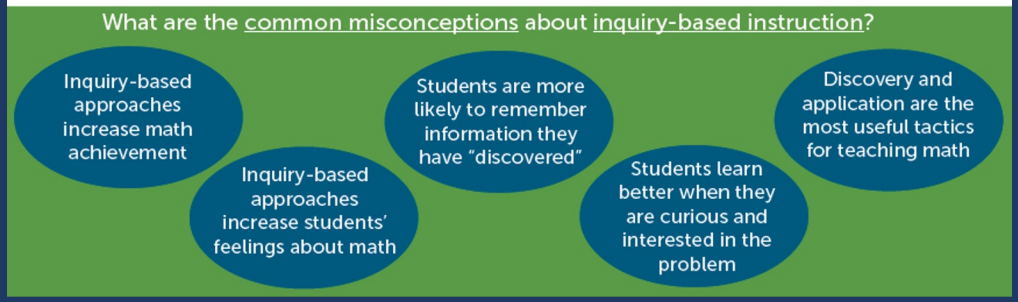
Help know when and how to use strategies

MISCONCEPTION

Inquiry-based instruction should be the primary tactic used to teach math. Explicit instruction only is beneficial for struggling learners. Explicit instruction is an instructional tactic where students are provided with correct answers and and this only promotes rote learning.

TRUTH

Explicit instruction offers value through sequencing of tasks in increments of difficulty, fluency building that promotes effective practice, and scaffolded opportunities for students to combine learned skills with new knowledge. Explicit instruction facilitates creativity and is effective for all learners[a].



HOW DOES EXPLICIT INSTRUCTION PROMOTE CREATIVITY?

Explicit instruction provides sequences of instruction tied to students' needs to promote mastery of the fundamental skill and provide opportunities to expand new understandings.

The process of mastering fundamental skills and demonstrating new knowledge is identical to that followed by athletes, musicians, artists, and experts in all fields.

MISCONCEPTION

Many educators believe that struggling or grappling with challenging math tasks causes students to gain a deeper understanding than would be achieved if they learned the same skill without a struggle.

TRUTH

Productive struggle does not deepen understanding, grit, or creative problem solving. Productive struggle can lead to frustration and cause students to develop misconceptions.[a] In addition, the 'false starts' involved in struggling with challenging tasks without adequate support or guidance lead to lost instructional time and inefficiency.[b]

What is the problem with productive struggle?

- No evidence suggests giving students partial information for making connections leads to learning.[c]
- The idea comes from constructivism, which runs counter to what we understand about math learning.[d]
- Students learning new skills require clear demonstrations and guided practice with immediate feedback.
- New concepts are not learned by struggling. Making connections relies on a foundation of learned knowledge.

REFRAMING PRODUCTIVE STRUGGLE

Using productive struggle for generalization involves providing effective explicit instruction for learning and building proficiency with new math content first.

After verifying students have learned the content, teachers can provide practice opportunities for productive struggle in which students work to generalize their learning to a novel, challenging problem or task (i.e., moving the 'struggle' from the beginning to the end of the instructional sequence).

MISCONCEPTION

Many educators believe interventions targeting a growth mindset will improve academic achievement.

Many educators are concerned with fostering a growth mindset (i.e., "I can work hard to improve my success in math each day") rather than a fixed mindset (i.e., "I'm just not good at math") in students to promote math achievement.

TRUTH

Intervention research on stand-alone growth mindset interventions yield minimal gains on GPA in mathematics courses[a] and replication attempts have failed.[b] The most effective way to improve academic achievement is to deliver skill-building intervention.[c]

What is Growth Mindset Theory (GMT)?

Individuals who believe intelligence is malleable will obtain higher attainment than students who view intelligence as fixed.

In GMT, teachers support students to:
(1) believe they can improve their performance;
(2) identify the effort and persistence required;
(3) seek input or feedback to improve performance;
(4) try new strategies or approaches if old ones fail.

ADVICE FOR USING GROWTH MINDSET IN INTERVENTIONS

- Use praise statements based on students' effort, understanding, and persistence on challenging math work.[d]
- Encourage students to master skills by providing choice of interventions, feedback, and goals on learning, and opportunities to monitor their own progress, reflect on learning goals, and record learning accomplishments.[e]
- Establish individual learning goals rather than promoting work exemplars from high achievers.[d]

MISCONCEPTION

Targeted interventions on increasing executive functioning will increase mathematics performance. Many people believe that improving executive functioning through direct training (e.g., working memory, cognitive training programs) will improve mathematics achievement.

TRUTH

Most evidence suggests there is a small to negligible relationship between cognitive measures and student response to intervention.[a] In the few studies examining the causal link between executive function interventions and academic outcomes, researchers only found improvements on measures of executive function but no improvements on academic achievement.[b]

What Does the Evidence Support?

Students at-risk for math disabilities also may have difficulties with attention, motivation, self-regulation, and working memory.[cd]

Interventions should be tailored and intensified according to students' needs using direct evaluation of students' math skills to make low-inference decisions about intervention tactics.

Effective use of evidence-based instructional approaches negates the potential influence of executive function difficulties.[e]

IMPLICATIONS FOR PRACTICE

Interventions should:

(1) include self-regulation and reinforcement strategies;

(2) minimize cognitive load on working memory through explicit instruction and breaking down problems into smaller, more manageable parts;

(3) minimize language load by using visual representations;

(4) include fluency-building practice.[f]

MISCONCEPTION

Many educators believe math anxiety is caused by instructional activities and timed tests.

In schools, educators may interpret students disengaging in math activities or saying they dislike math as math anxiety. Educators may reduce the difficulty of a math lesson or remove timed tests as a way to reduce math anxiety.

TRUTH

No studies have determined that timed tests cause math anxiety - defined as feelings of apprehension, tension, or fear that may interfere with performance on math-related tasks.[a] In fact, timed tactics improve math performance.[b]

Why are timed tests helpful?

- Timed tests provide critical information for student mastery of key skills and concepts.
- Once a student reaches 100% accuracy, the metric cannot capture additional learning.
- Rate-based metrics are reliable and better indicate a student's instructional level.[c]

Why are timed activities helpful?

- Timed tasks are fluency-building activities.
- Timed activities are necessary to promote math mastery when students have established a high level of accuracy and conceptual understanding.
- Fluency is a necessary dimension of math mastery associated with robust understanding and flexible problem solving.[d]

HOW DO YOU ADDRESS MATH ANXIETY?

Promote skill development through effective instruction.[e]

- Use language focused on working hard and showing growth rather than attaining a benchmark criterion.
- Include fluency-building tactics in core instruction every day.
- Avoid tasks where students have to "figure it out," and save those tasks for students who have mastered the fundamentals.
- Support practices with tasks that increase in difficulty as students master skills.

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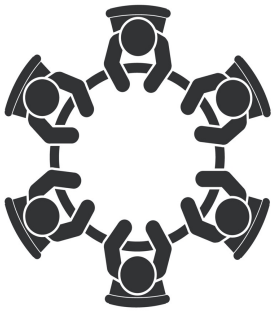

Share misconceptions and truths you have encountered.

Instructional Platform

Research-Based Mathematics Intervention

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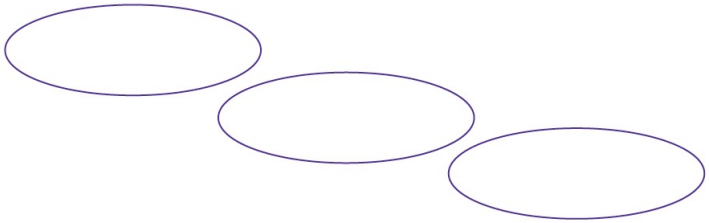
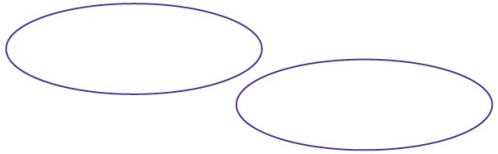
Instructional Platform

Instructional Delivery



Instructional Strategies



Instructional Platform

INSTRUCTIONAL DELIVERY

- Explicit instruction
- Precise language
- Multiple representations

INSTRUCTIONAL STRATEGIES

- Fluency building
- Problem solving instruction

Explicit Instruction

Instructional Platform

INSTRUCTIONAL DELIVERY

Explicit instruction

INSTRUCTIONAL STRATEGIES

Explicit Instruction

MODELING	PRACTICE

SUPPORTS

Over a half century of research supports explicit (i.e., direct, systematic) instruction.
(Stockard et al., 2018)

When compared to discovery approaches, explicit instruction demonstrates higher outcomes.
(Alfieri et al., 2011; Kroesbergen et al., 2004; Poncy et al., 2010)

Numerous meta-analyses and large-scale studies have identified explicit instruction as essential for the teaching and learning of mathematics.
(Chodura et al., 2015; Ennis & Losinski, 2019; Jitendra et al., 2018; Kong et al., 2021; Morgan et al., 2015; Nelson & McMaster, 2019; Powell et al., 2021).

MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

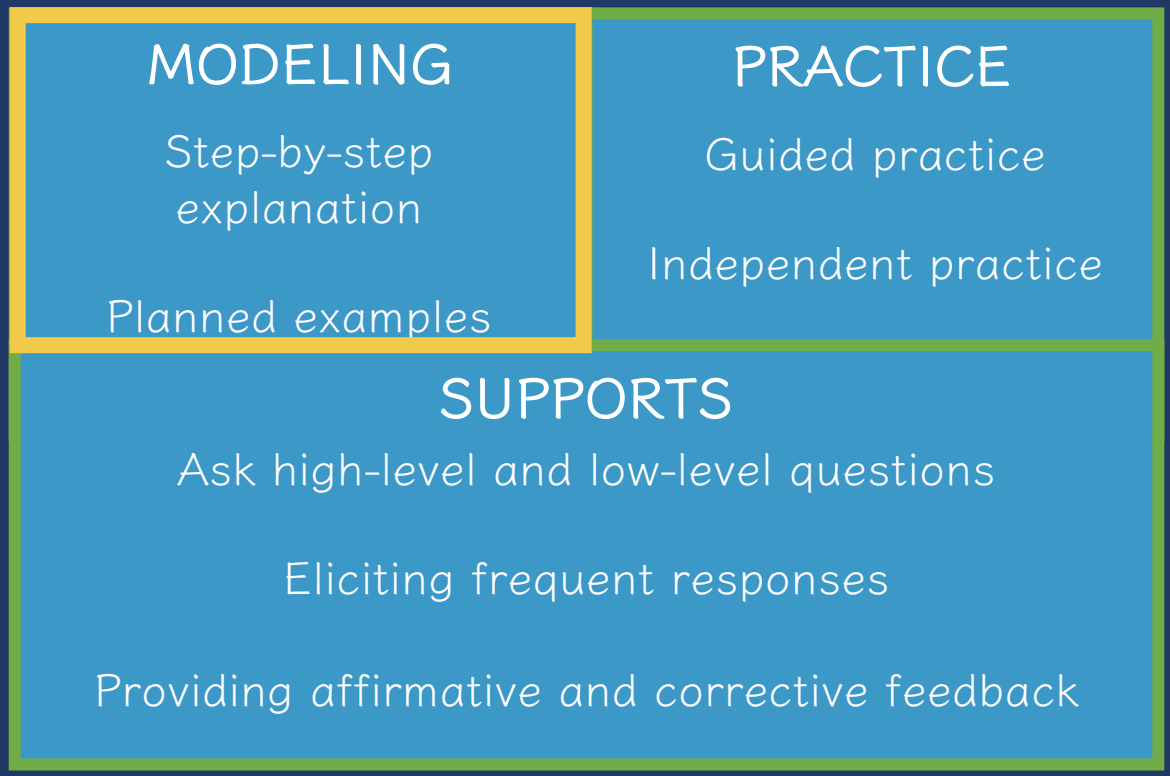
SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

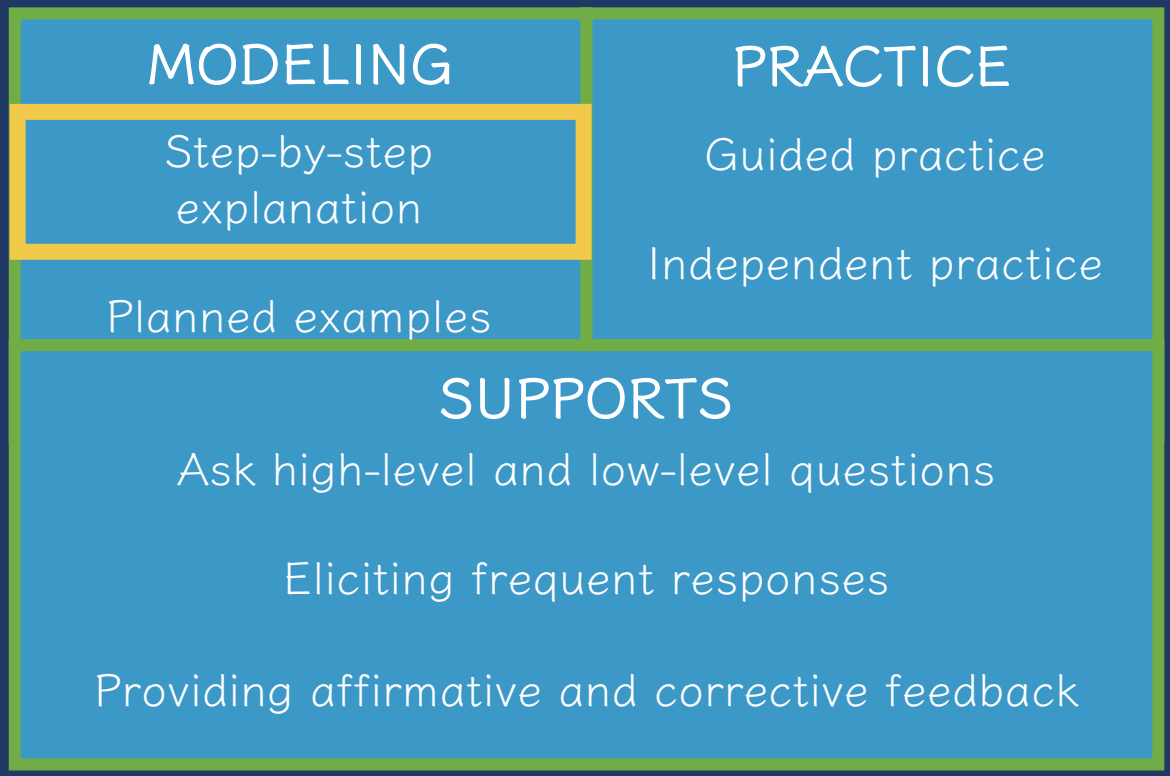
Providing affirmative and corrective feedback

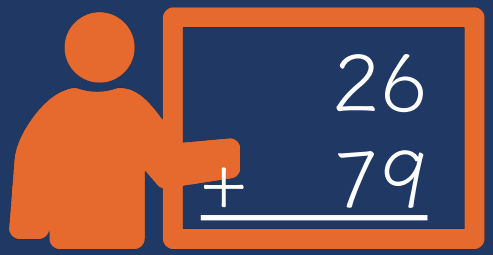
Modeling is a dialogue between the teacher and students.



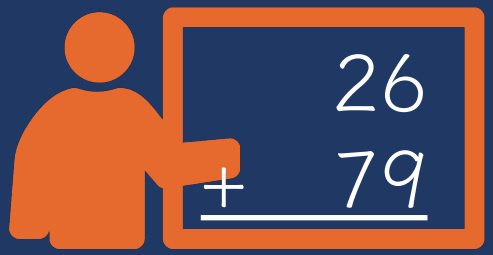
Modeling includes a step-by-step explanation of how to do a math problem.

A teacher may do 1 modeled problem or several.





"Today, we are learning about addition. This is important because sometimes you have different amounts – like money – and you want to know how much money you have altogether."



"Let's solve this problem. What's the problem?"

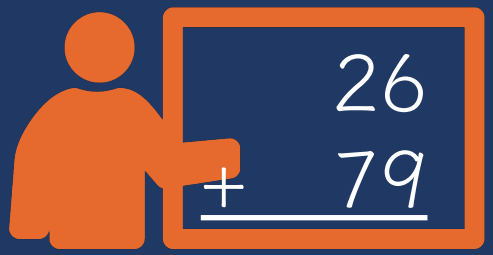
"26 plus 79."

"To solve 26 plus 79, first decide about the operation. Should we add, subtract, multiply, or divide?"

"Add."

"How did you know we want to add?"

"There's a plus sign."



"The plus sign tells us we want to add. To add, let's use the partial sums strategy. What strategy?"

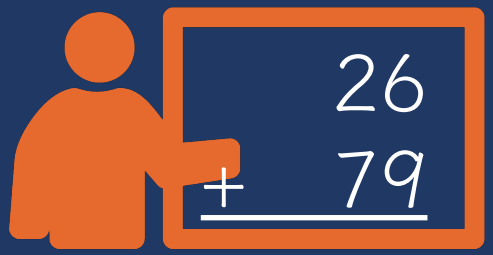
"Partial sums."

"What might partial mean?"

"Part of."

"We'll find parts – or partial sums - then add them together. With the partial sums strategy, we start adding in the greatest place value. What's the greatest place value in this problem?"

"The tens."



"So, let's add the tens. What's 20 plus 70? Use your base-10 blocks or other tools."

"90."

"20 plus 70 equals 90. Let's write 90 right here below the equal line. What will we write?"

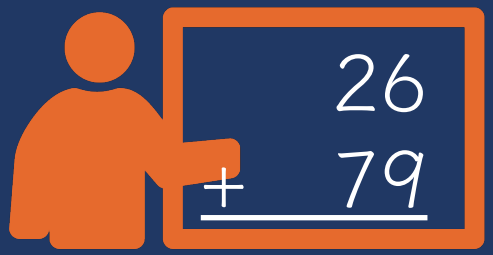
"90."

"90 is the partial sum when you add the tens. What does 90 represent?"

"It's the partial sum of adding 20 plus 70."

"Now, let's add the ones. What should we add?"

"6 plus 9."



"6 plus 9 equals what? Use your base-10 blocks or other tools."

"15."

"How did you get 15?"

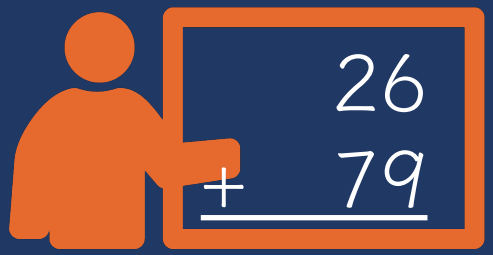
"We knew we had 9, then we added on 6."

"Let's write 15 below the 90. Where do we write the 15?"

"Below the 90."

"15 is the partial sum when you add the ones. Now, let's add the partial sums together. What will we add?"

"90 plus 15."



"What's 90 plus 15? Use your go-to strategy."

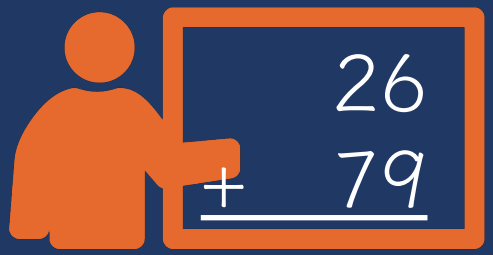
"105."

"How did you add those addends?"

"I added 90 plus 10 then added 5 more."

"So, when you add 26 plus 79, the sum is 105. Who can share how we solved this problem?"

"We used the partial sums strategy. We added the tens then added the ones. Then we added the partial sums."

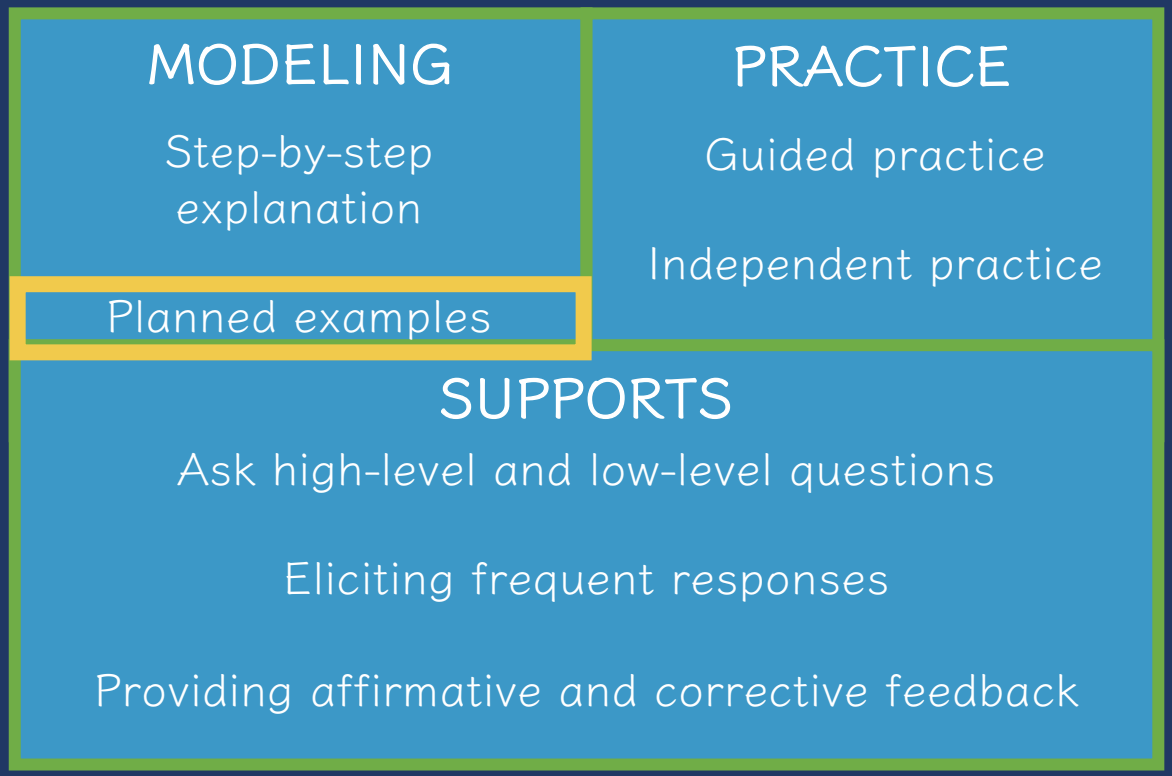


What did you observe?

How would you improve this example?

Modeling needs to include planned examples.

These examples should be sequenced so easier skills lead to more difficult skills.



MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

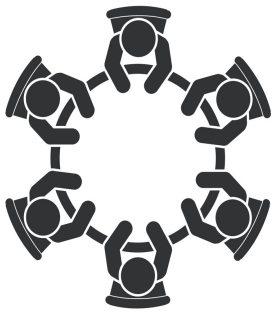
SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

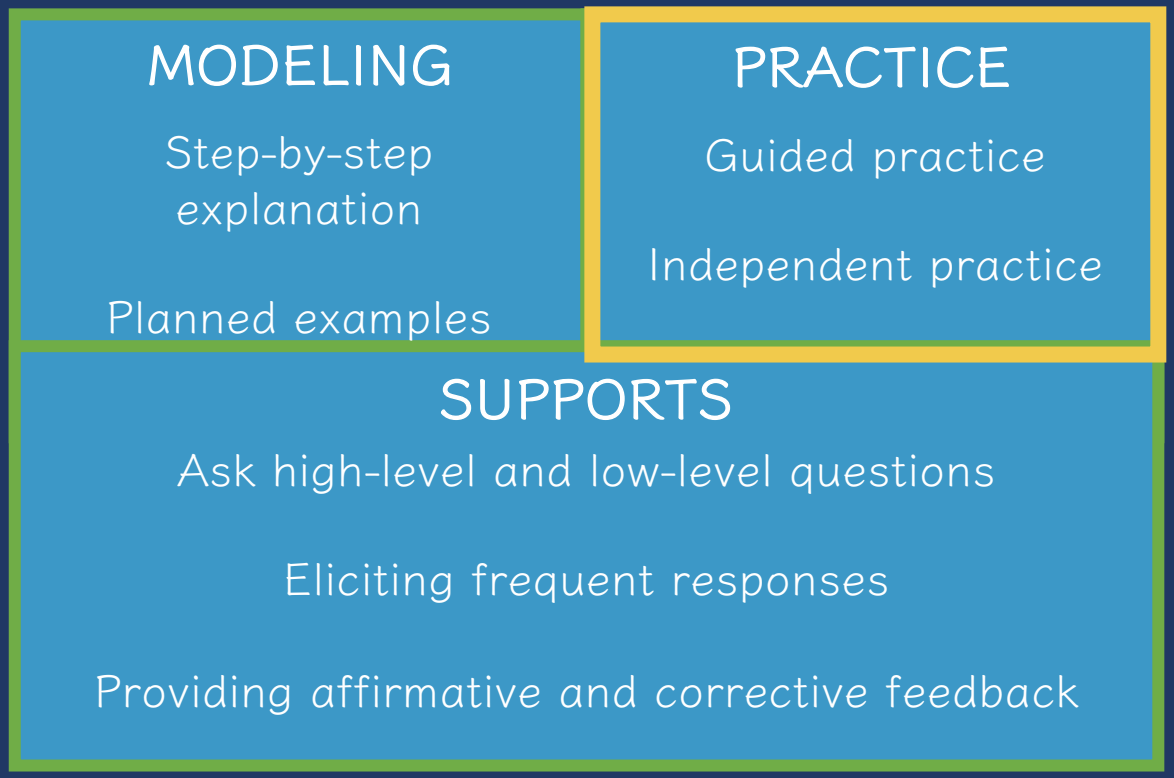
Providing affirmative and corrective feedback

Explicit Instruction



Select a math problem.

Work with a partner to outline a step-by-step explanation.



Practice continues as a dialogue between the teacher and students.

MODELING

Step-by-step explanation

Planned examples

PRACTICE

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Independent practice

SUPPORTS

Ask high-level and low-level questions

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Providing affirmative and corrective feedback

Guided practice is practice in which the teacher and students practice problems together.

"Let's work on a problem together."

MODELING

Step-by-step explanation

Planned examples

PRACTICE

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Independent practice

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Ask high-level and low-level questions

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Independent practice is practice in which the students practice independently with teacher support.

"Now, you'll practice a problem on your own. Use your attack strategy!"

MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

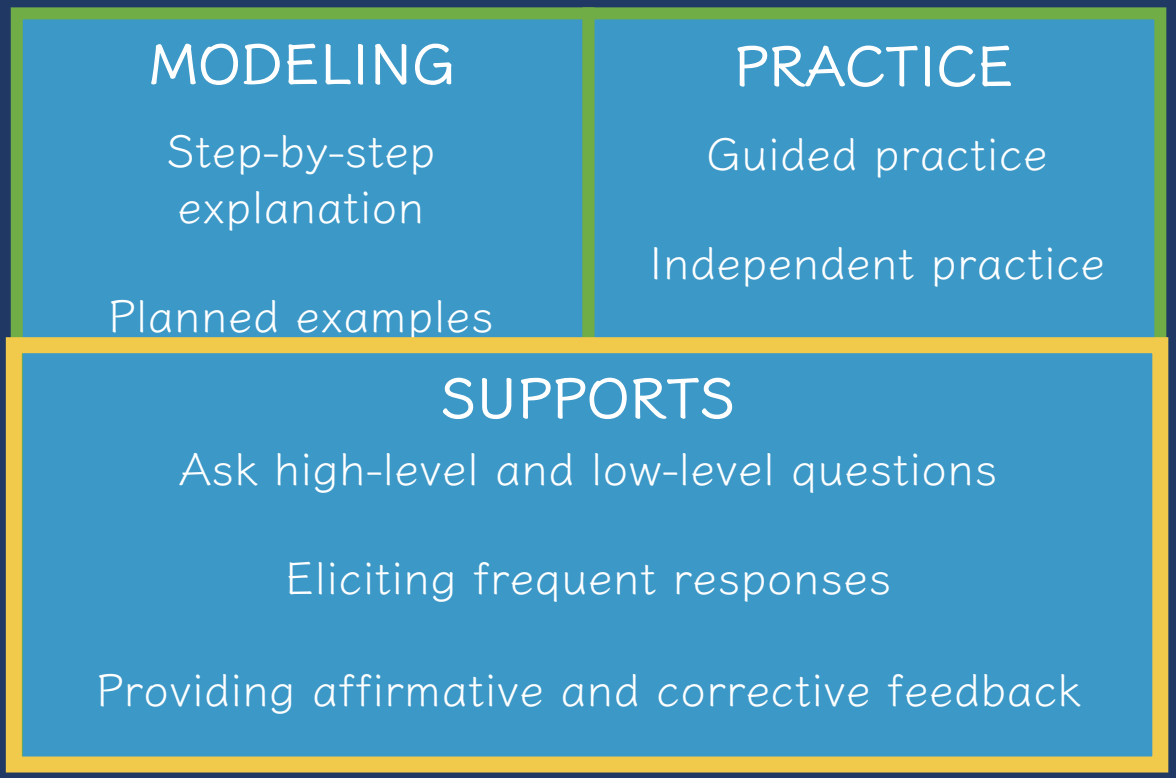
Ask high-level and low-level questions

Eliciting frequent responses

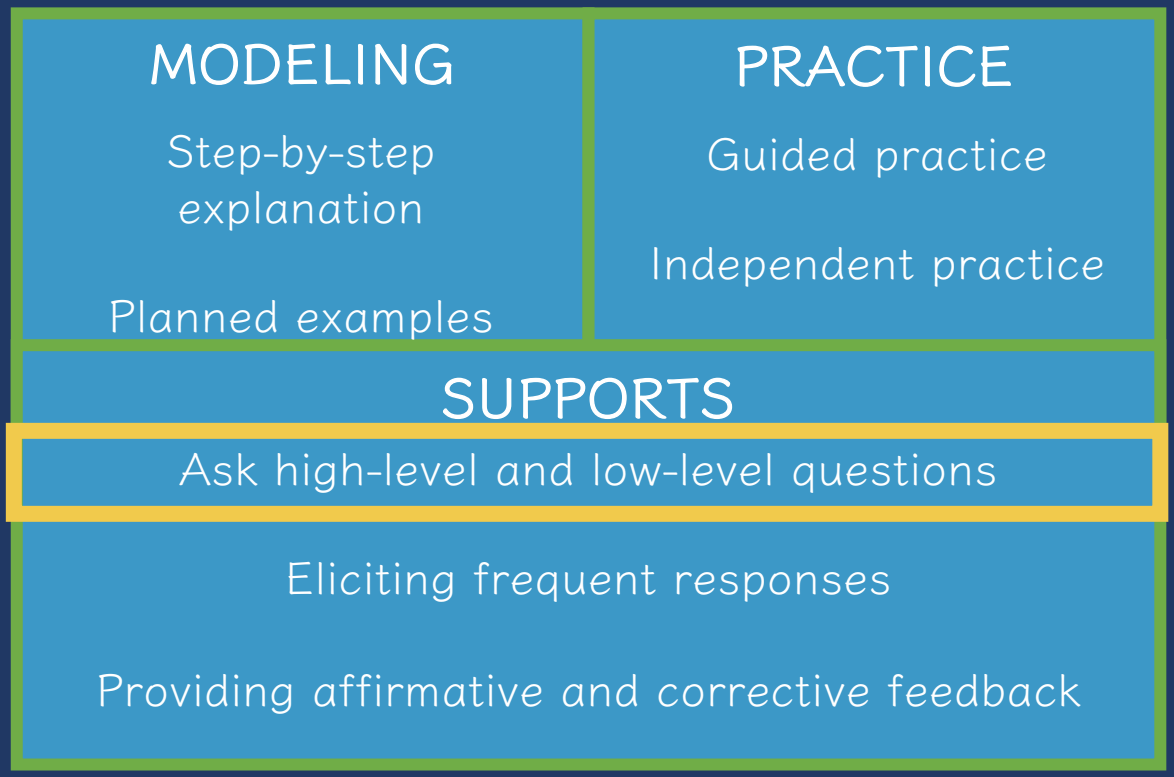
Providing affirmative and corrective feedback

Explicit Instruction

Describe how you would engage students in practice.



These **Supports** should be used in both **Modeling** and **Practice.**



During **Modeling** and **Practice**, it is essential to engage students and check for understanding.

MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

"What is 7 times 9?"

"63."

MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

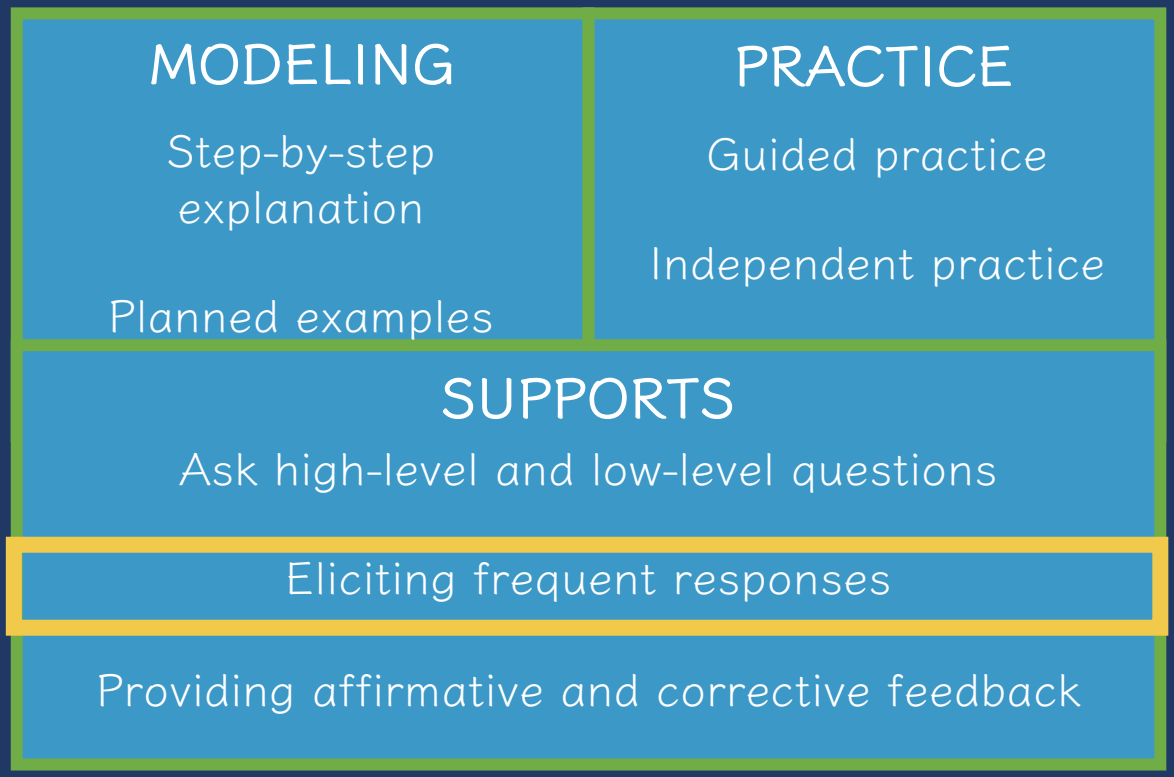
Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

"Why do you use zero pairs?"

"Because a positive 1 and a negative 1 equal 0. I use the zero pair to help me subtract."



During **Modeling** and **Practice**, students should frequently respond. The frequent responses keeps student attention and keeps student learning active.

MODELING

Step-by-step explanation

Planned examples

PRACTICE

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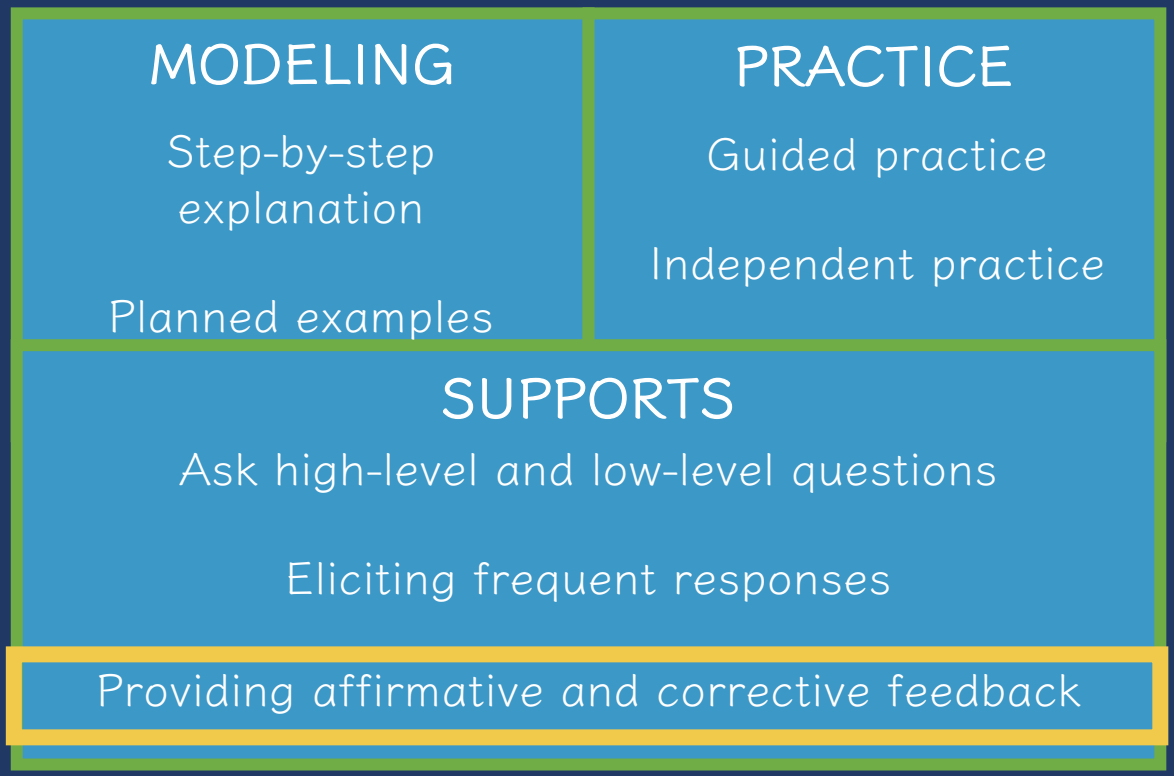
SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback



During **Modeling** and **Practice**, students should receive immediate feedback on their responses.

MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

"Nice work using your word problem attack strategy."

MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

"Let's look at that again. Tell me how you added in the hundreds column."

MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

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Ask high-level and low-level questions

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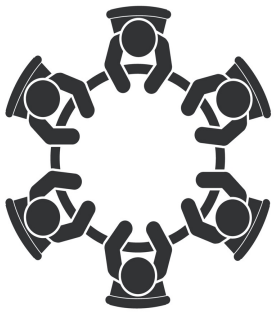
Explicit Instruction



Provide several of your questions.

Provide examples of your feedback.

MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

Teachers should use systematic and explicit instruction to help students develop a strong foundation for specific mathematics skills.

What are your strengths with explicit instruction?

What are the opportunities for growth?

What are your immediate next steps?

Teachers should use systematic and explicit instruction to help students develop a strong foundation for specific mathematics skills.

Students require modeling and practice on how to use the language of mathematics.

Students should use hands-on tools, virtual manipulatives, drawings, and other visuals to understand mathematics concepts and procedures.

Teachers should use fluency building activities to build counting fluency and fluency with the operations.

Students should learn how to set up and solve word problems by combining an attack strategy with a focus on word-problem schemas.

Mathematical Language

Instructional Platform

INSTRUCTIONAL DELIVERY

Explicit instruction

Precise language

INSTRUCTIONAL STRATEGIES

Mathematical Language

Research and Information

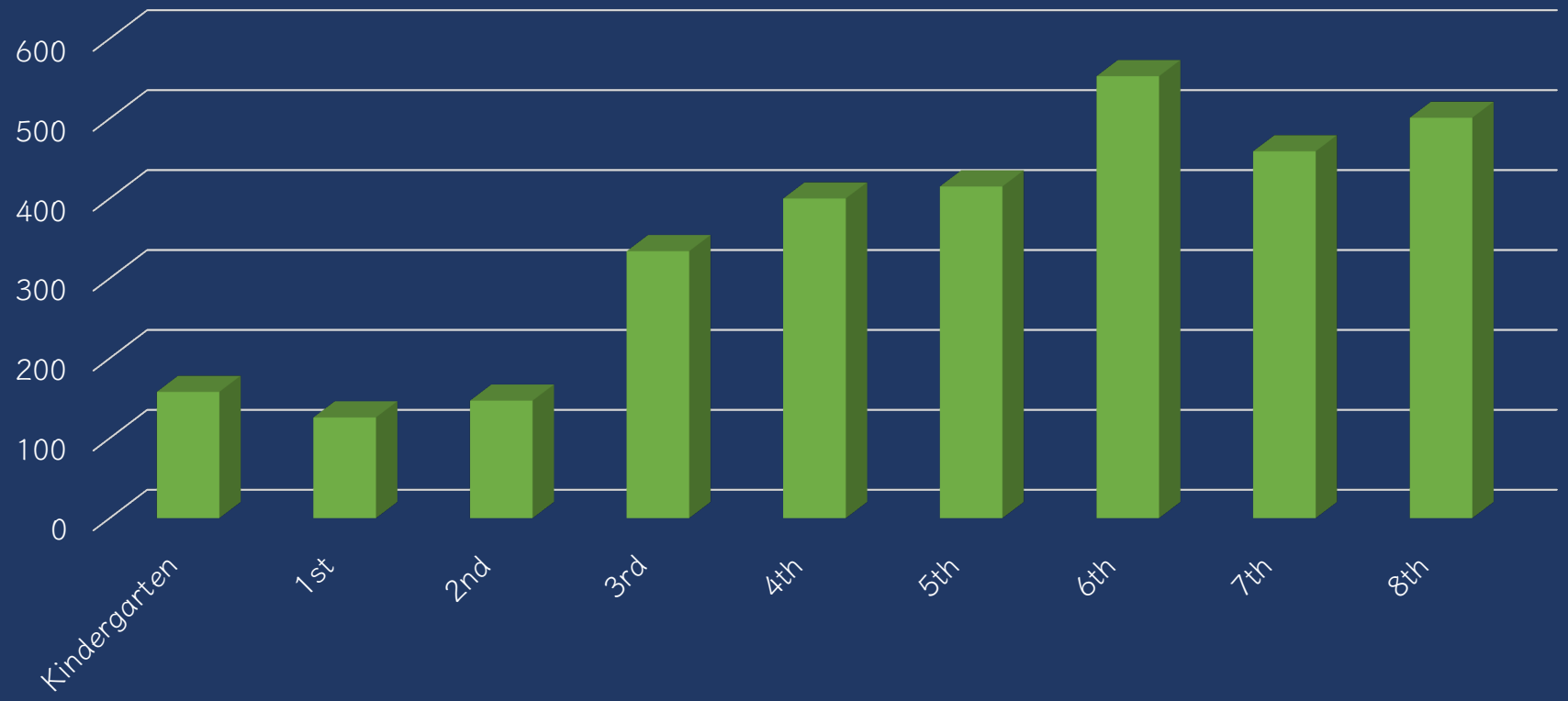
Use Formal Mathematics Language

Instead of that...	Say this...

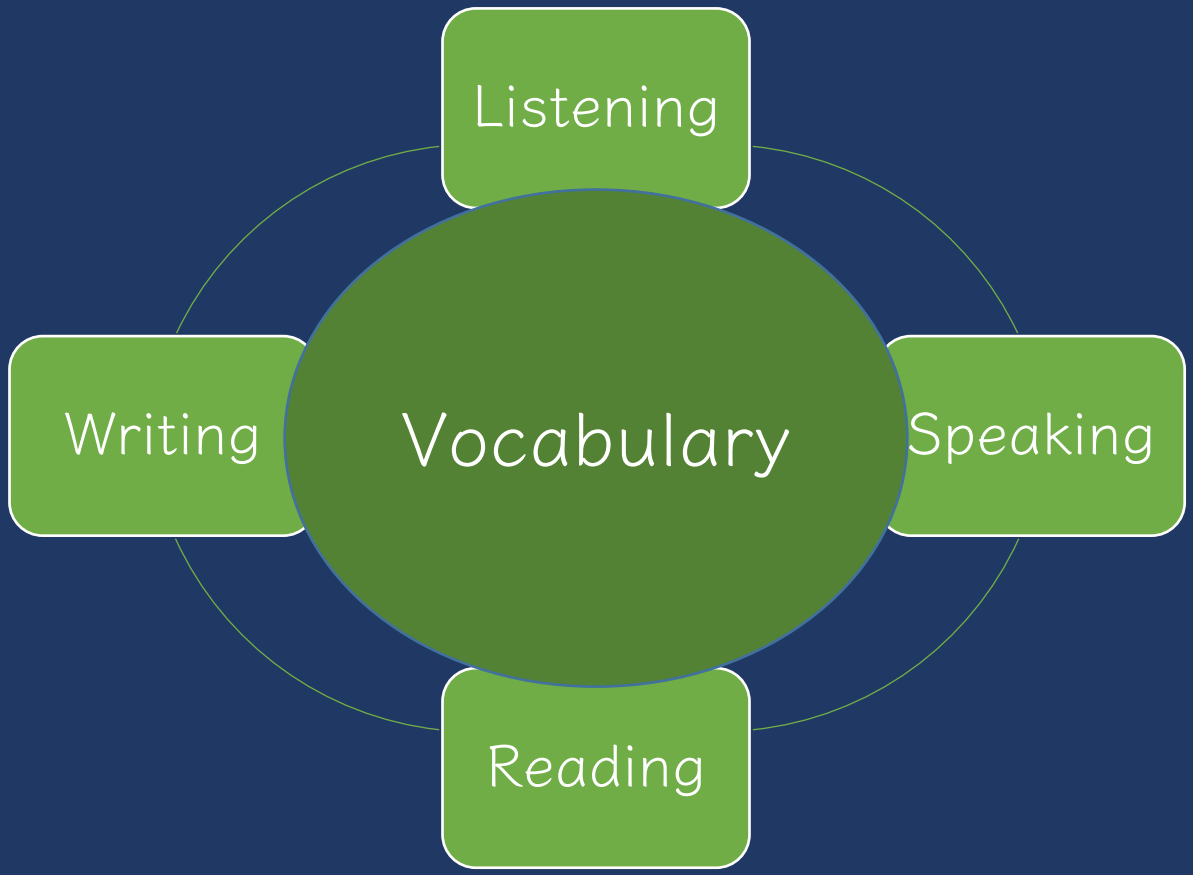
Significant correlation (r = .49) between mathematics vocabulary and mathematics performance. Mathematics vocabulary appears most important for word-problem performance (r = .58).
(Lin et al., 2021)

Early mathematics vocabulary related to mathematics and literacy.
(Hornburg et al., 2018; Purpura et al., 2017)

Students who experience difficulty with mathematics demonstrate lower mathematics vocabulary performance.
(Hughes et al., 2020; Powell & Nelson, 2017; Powell et al., 2017; Unal et al., 2021)



Powell, Bos, & Lin (2019)



1. Some math terms are shared with English but have different meanings

base

right

degree

Rubenstein & Thompson (2002)

1. Some math terms are shared with English but have different meanings

2. Some math words are shared with English with similar meanings
   (but a more precise math meaning)

difference

even

Rubenstein & Thompson (2002)

1. Some math terms are shared with English but have different meanings

2. Some math words are shared with English with similar meanings
   (but a more precise math meaning)

3. Some math terms are only used in math

trapezoid

numerator

parallelogram

Rubenstein & Thompson (2002)

1. Some math terms are shared with English but have different meanings

2. Some math words are shared with English with similar meanings
   (but a more precise math meaning)

3. Some math terms are only used in math

4. Some math terms have more than one meaning

round

square

second

base

Rubenstein & Thompson (2002)

1. Some math terms are shared with English but have different meanings

2. Some math words are shared with English with similar meanings (but a more precise math meaning)

3. Some math terms are only used in math

4. Some math terms have more than one meaning

5. Some math terms are similar to other content-area terms with different meanings

divide vs. Continental Divide

variable vs. variably cloudy

Rubenstein & Thompson (2002)

1. Some math terms are shared with English but have different meanings

2. Some math words are shared with English with similar meanings (but a more precise math meaning)

3. Some math terms are only used in math

4. Some math terms have more than one meaning

5. Some math terms are similar to other content-area terms with different meanings

6. Some math terms are homographs

eight vs. ate

sum vs. some

rows vs. rose

base vs. bass

Rubenstein & Thompson (2002)

1. Some math terms are shared with English but have different meanings

2. Some math words are shared with English with similar meanings (but a more precise math meaning)

3. Some math terms are only used in math

4. Some math terms have more than one meaning

5. Some math terms are similar to other content-area terms with different meanings

6. Some math terms are homographs

7. Some math terms are related but have distinct meanings

factor vs. multiple

hundreds vs. hundredths

numerators vs. denominator

Rubenstein & Thompson (2002)

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8. An English math term may translate into another language with different meanings

mesa vs. tabla

Rubenstein & Thompson (2002)

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9. English spelling and usage may have irregularities

four vs. forty

Rubenstein & Thompson (2002)

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10. Some math concepts are verbalized in more than one way

skip count vs. multiples

one-fourth vs. one quarter

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7. Some math terms are related but have distinct meanings

8. An English math term may translate into another language with different meanings

9. English spelling and usage may have irregularities

rhombus vs. diamond

10. Some math concepts are verbalized in more than one way

vertex vs. corner

11. Informal terms may be used for formal math terms

Use formal math language

Use terms precisely

What number is in the tens place?

What digit is in the tens place?
What is the value of the digit in the tens place?

135

Why this is important...

- A number refers to the entire amount.
- The 3 in the tens place value is not a number, but rather a digit in the number 135.
- Reinforces conceptual understanding of place value.
- Emphasizes that 3 is part of the number 135 with a value of 30.

The alligator eats the bigger number

is less than
OR
is greater than

Why this is important...

- Students must learn how to read and write the inequality symbols.
- Students must learn to read equations correctly from left to right because < and > are two distinct symbols.

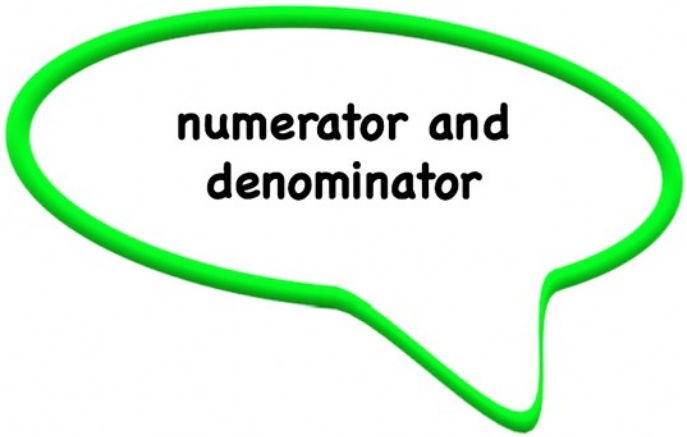
carry OR borrow

regroup OR trade OR exchange

$$\begin{array}{r} 167 \\ +\ 294 \\ \hline \end{array}$$

Why this is important...

- "Carry" or "borrow" is procedural.
- The other terms reinforce the conceptual understanding or regrouping ones into tens, tens into hundreds, and so on (i.e., the total amount does not change) *or* ungrouping hundreds into tens, tens into ones, and so on.



Why this is important...

- Identifying that there are two separate (whole) numbers suggests that whole number properties can be applied to fractions.
- Emphasizing that a fraction is ONE number with ONE magnitude on a number line that is communicated with a numerator and denominator is important.

reduce the fraction

rename OR
find equivalent OR
simplify

Why this is important...

- Reducing suggests that the quantity or magnitude of the new number will be less than the original number.

Four point seven
Four point oh seven

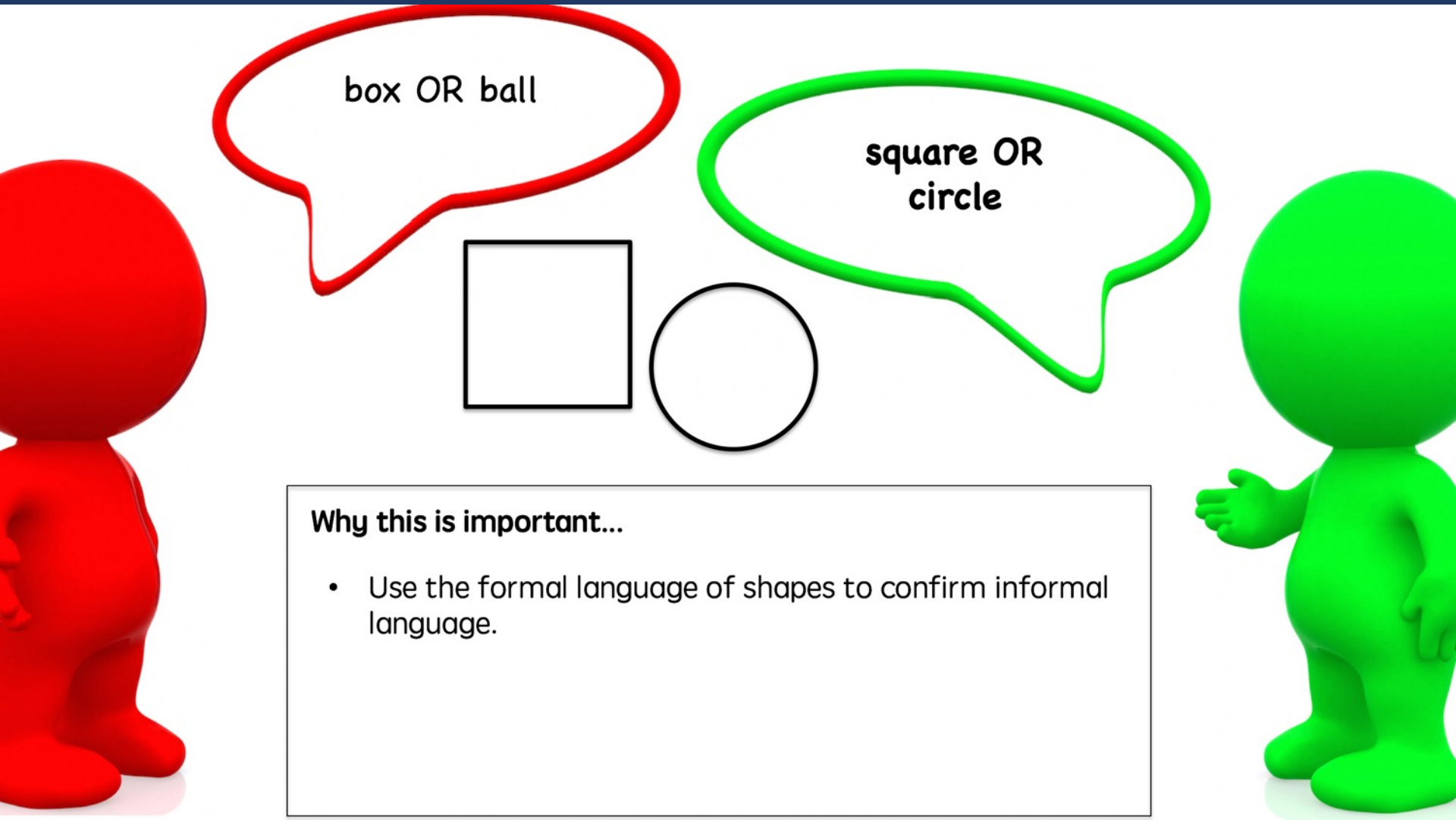
Four and seven tenths
Four and seven hundredths

4.7

4.07

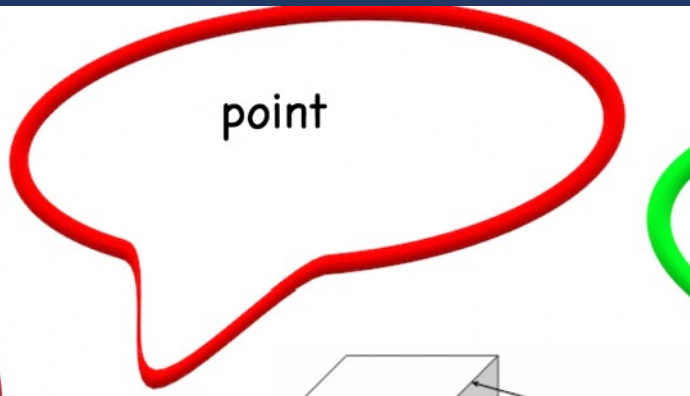
Why this is important...

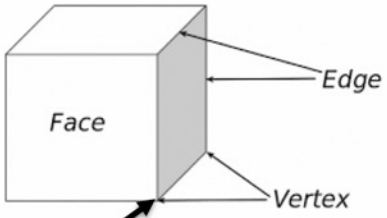
- Accurately shares the magnitude of the decimal.
- Emphasizes place value.



Why this is important...

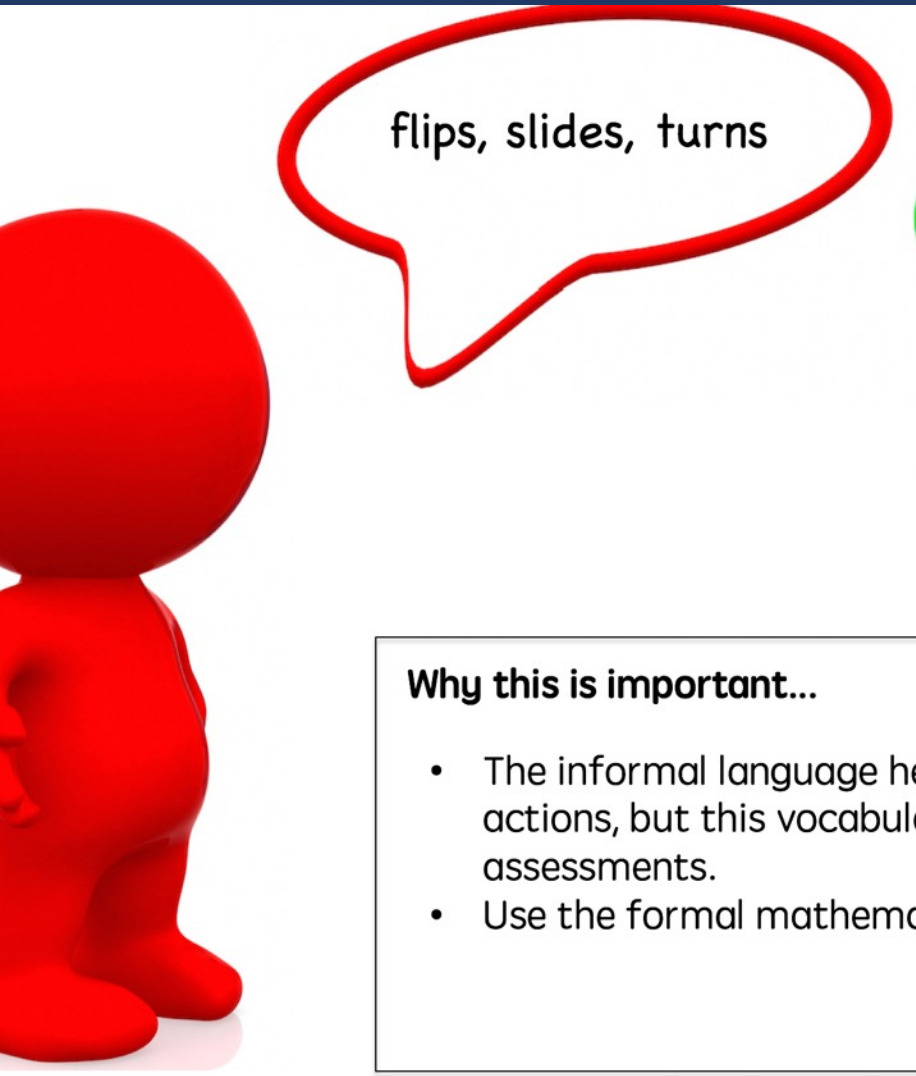
- Use the formal language of shapes to confirm informal language.

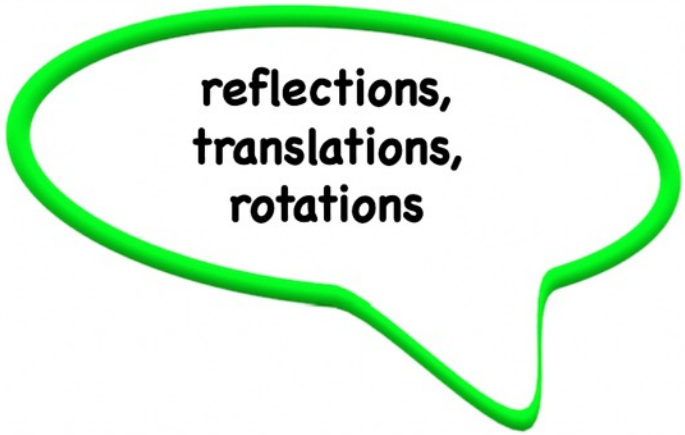




Why this is important...

- This is the endpoint where two or more line segments or rays meet.





Why this is important...

- The informal language helps children remember the actions, but this vocabulary is not used on assessments.
- Use the formal mathematical terms.

Why this is important...

- The informal language describes the length of clock hands but not the meaning.
- Help students understand the hours and minutes.

Mathematical Language

Research and Information

Use Formal Mathematics Language

Instead of that...	Say this...

Identify examples of "Instead of ____, say ____."

Use formal math language

Use terms precisely

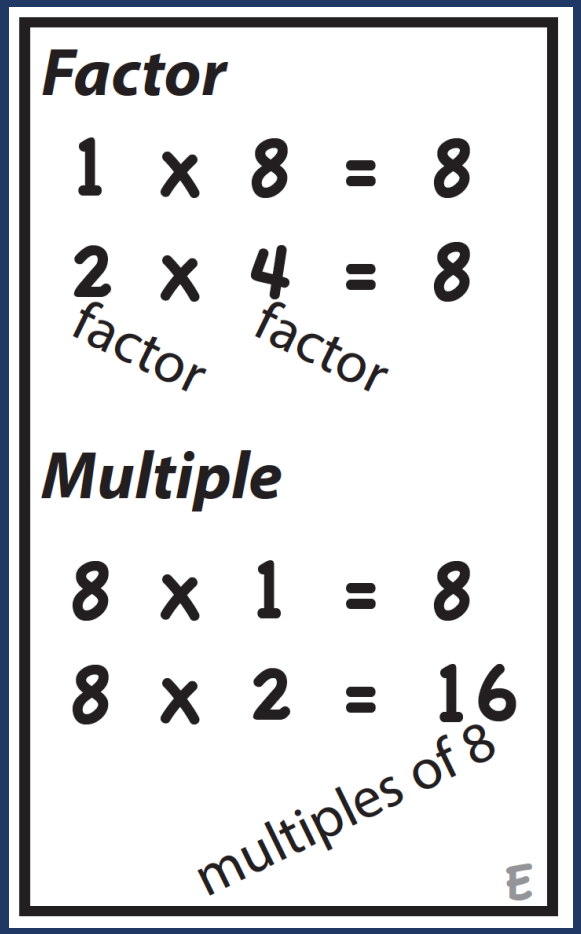
Use Terms With Precision

Strategies for Teaching Mathematics Language

What are your strengths?

What are your opportunities for growth?

What are your immediate next steps?



Improper fraction

$$\frac{8}{5}$$

Proportion

$$\frac{2}{5} = \frac{8}{20}$$

Mixed number

$$1\frac{3}{5}$$

Ratio

4:3

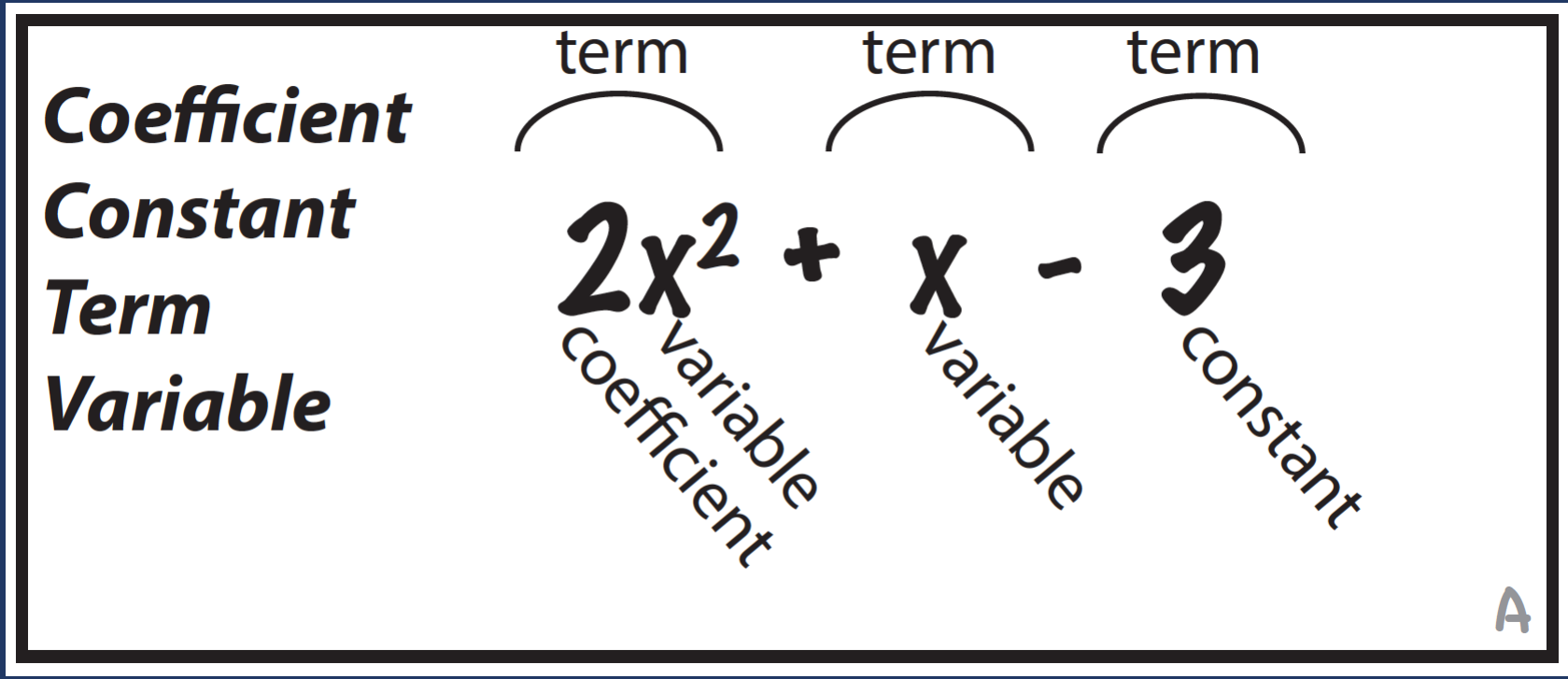
Proper fraction

$$\frac{2}{9}$$

Unit fraction

$$\frac{1}{6}$$

D

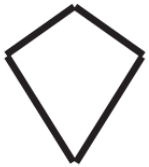


Equation	$9x - 4 = 7x$
Expression	$9x - 4$
Formula	$a^2 + b^2 = c^2$
Function	$f(x)$
Inequality	$9x - 4 > 6x$

C

Quadrilaterals

Kite



Rhombus

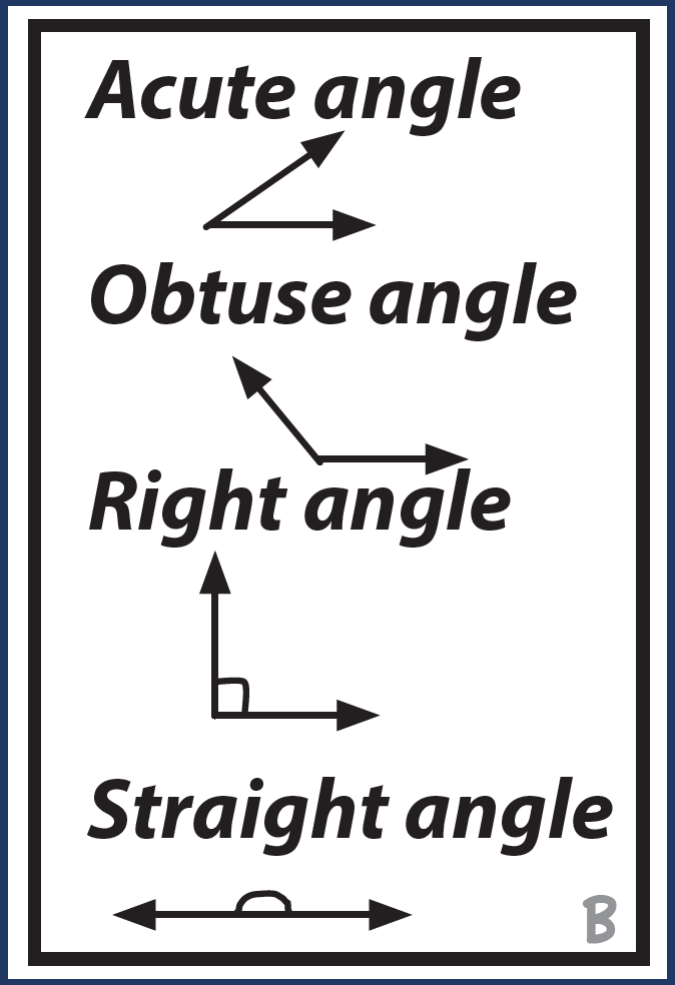
Parallelogram

Square

Rectangle

Trapezoid

A



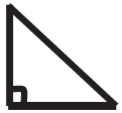
Acute triangle

Equilateral triangle

Obtuse triangle

Isosceles triangle

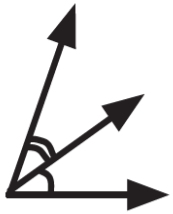
Right triangle



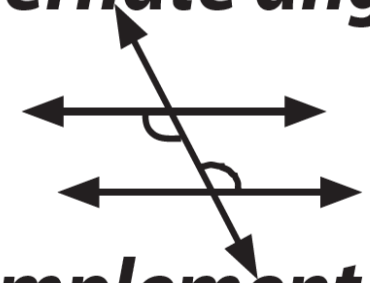
Scalene triangle

C

Adjacent angles

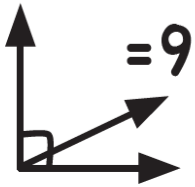


Alternate angles

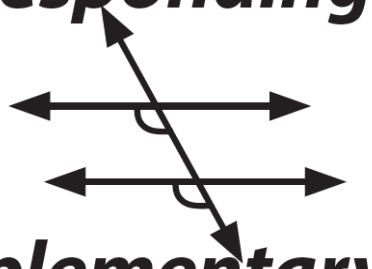


Complementary angles

=90°



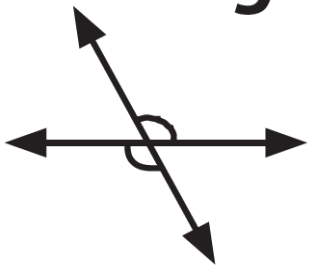
Corresponding angles



Supplementary angles

=180°

Vertical angles

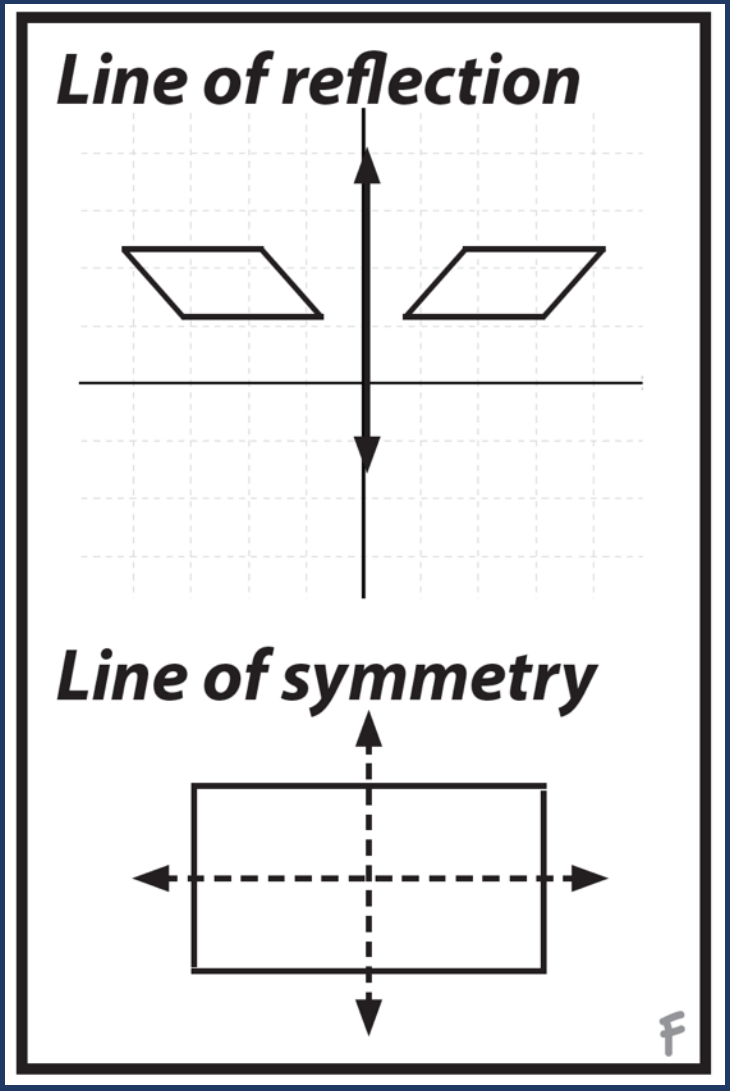


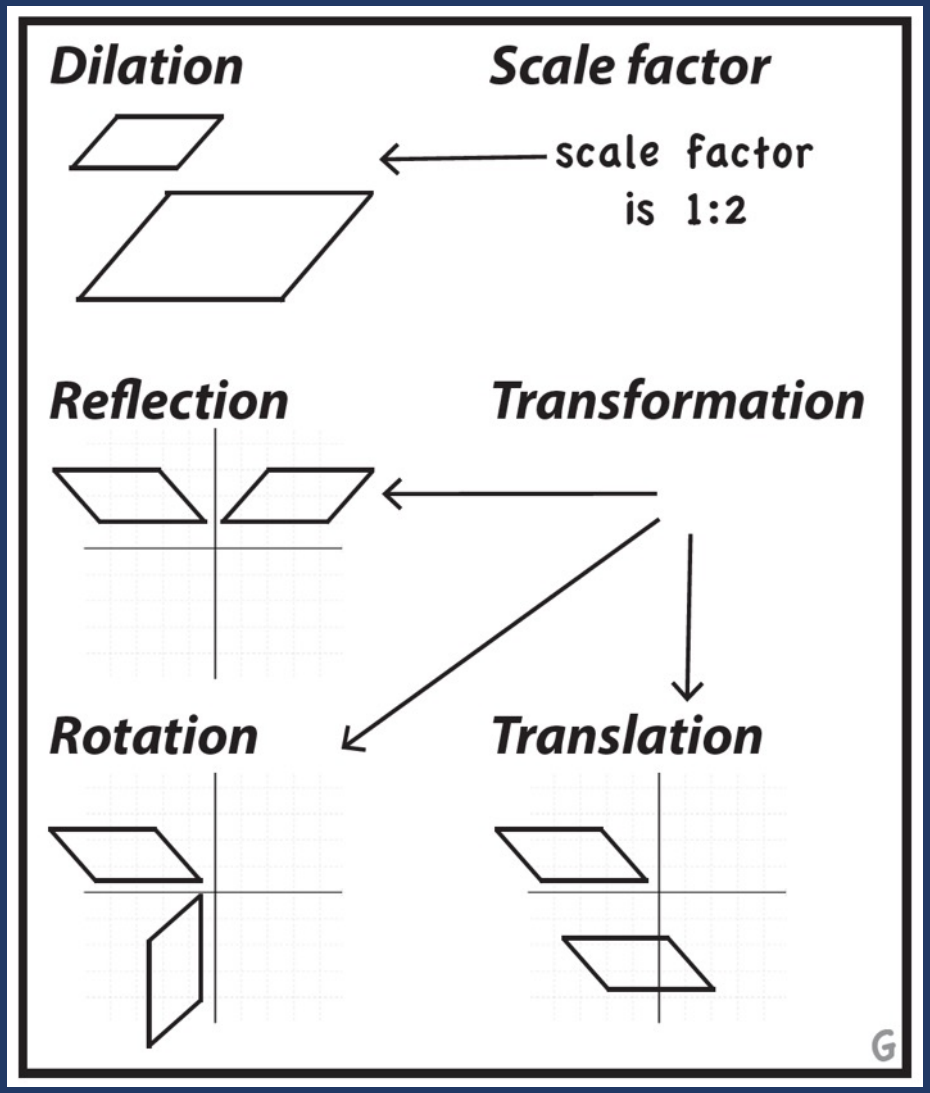
D

Congruent figures

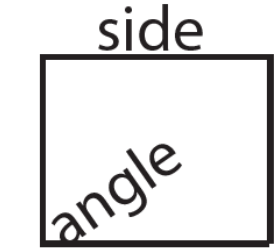
Similar figures

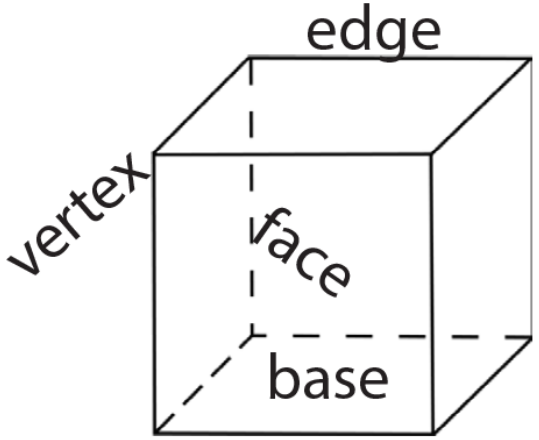
E

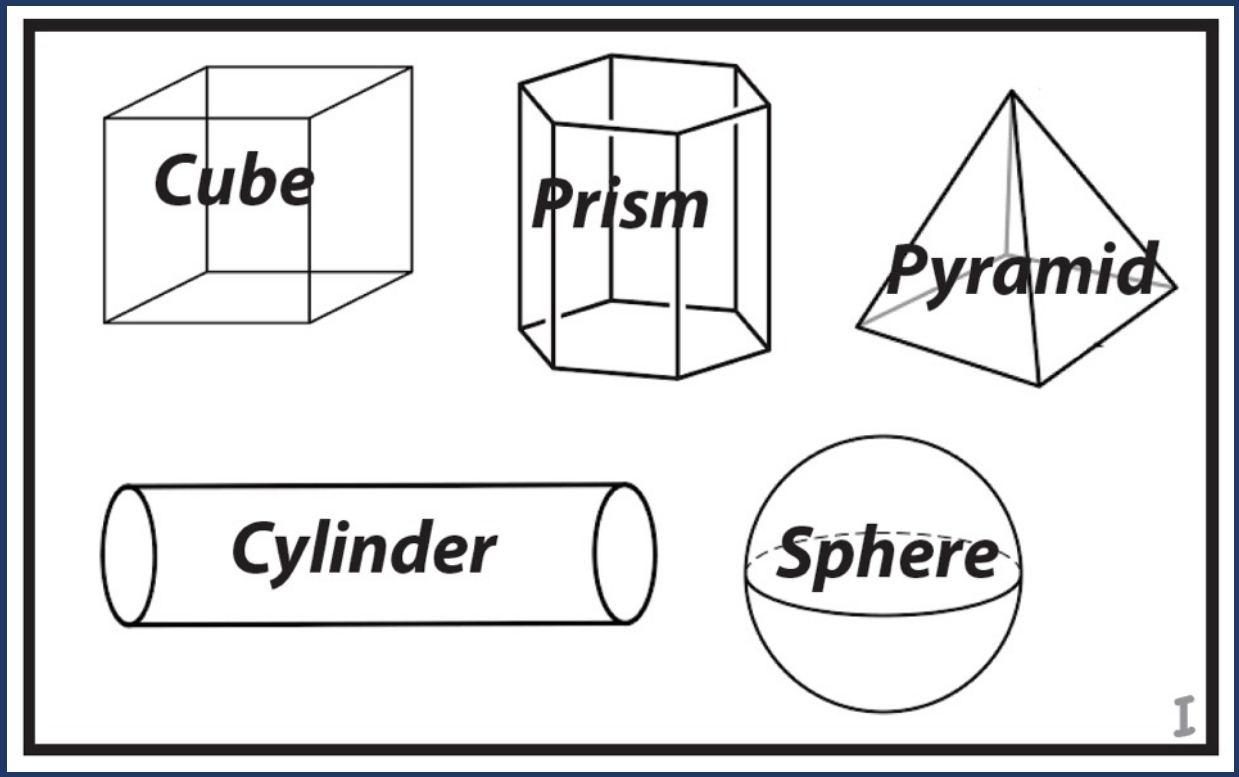




Angle
Base
Edge
Face
Side
Vertex







Coordinate plane

Ordered pair

Quadrants

x

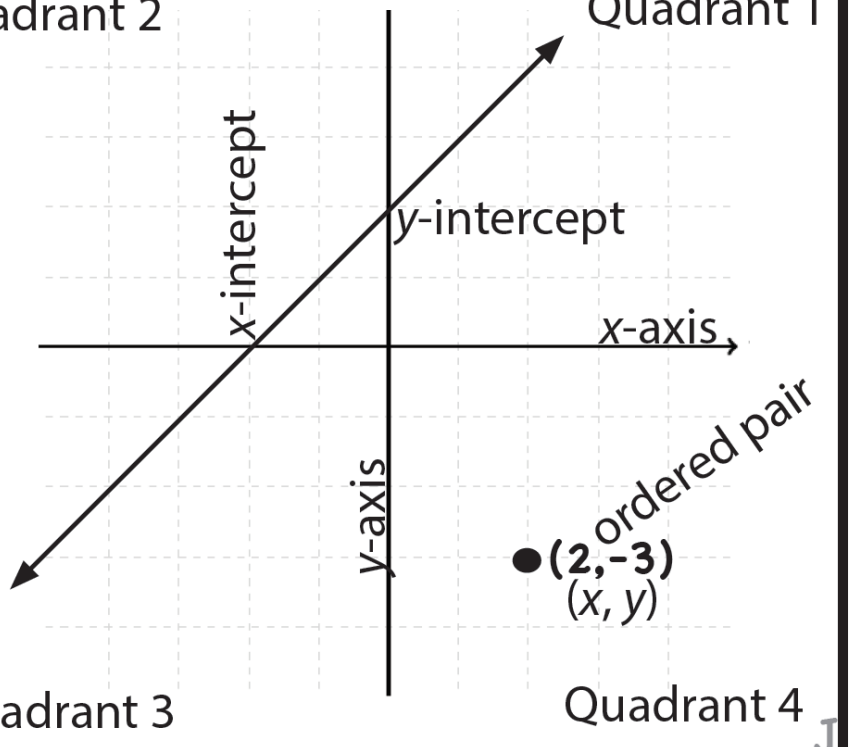
x-axis

x-intercept

y

y-axis

y-intercept



Use Terms With Precision

Strategies for Teaching Mathematics Language

What are your strengths?

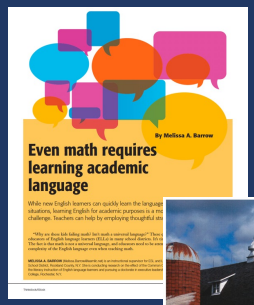
What are your opportunities for growth?

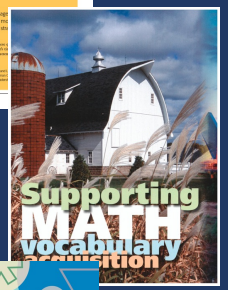
What are your immediate next steps?

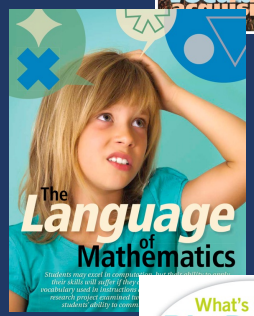
Discuss terms you want your students to use with precision.

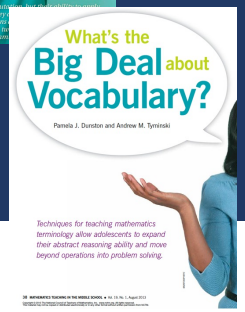
Use formal math language

Use terms precisely









Use explicit instruction.
(Powell & Driver, 2015; Stevens et al., 2022)

Use semantic maps.
(Stevens et al., 2022)

Use visuals.
(Powell & Driver, 2015)

Use flashcards with spaced practice.
(Petersen-Brown et al., 2019)

Use read-alouds.
(Purpura et al., 2017)

Use explicit instruction.
Use multiple representations.
Create opportunities for discussion and feedback.
Monitor student progress.
Coordinate vocabulary instruction across settings.
Create additional practice opportunities.
(Nelson et al., 2020)

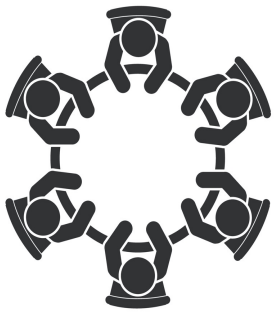
Use Terms With Precision

Strategies for Teaching Mathematics Language

What are your strengths?

What are your opportunities for growth?

What are your immediate next steps?



Discuss your strategy for focusing on mathematical language in your teaching.

Students require modeling and practice on how to use the language of mathematics.

What are your strengths with mathematical language?

What are the opportunities for growth?

What are your immediate next steps?

Teachers should use systematic and explicit instruction to help students develop a strong foundation for specific mathematics skills.

Students require modeling and practice on how to use the language of mathematics.

Students should use hands-on tools, virtual manipulatives, drawings, and other visuals to understand mathematics concepts and procedures.

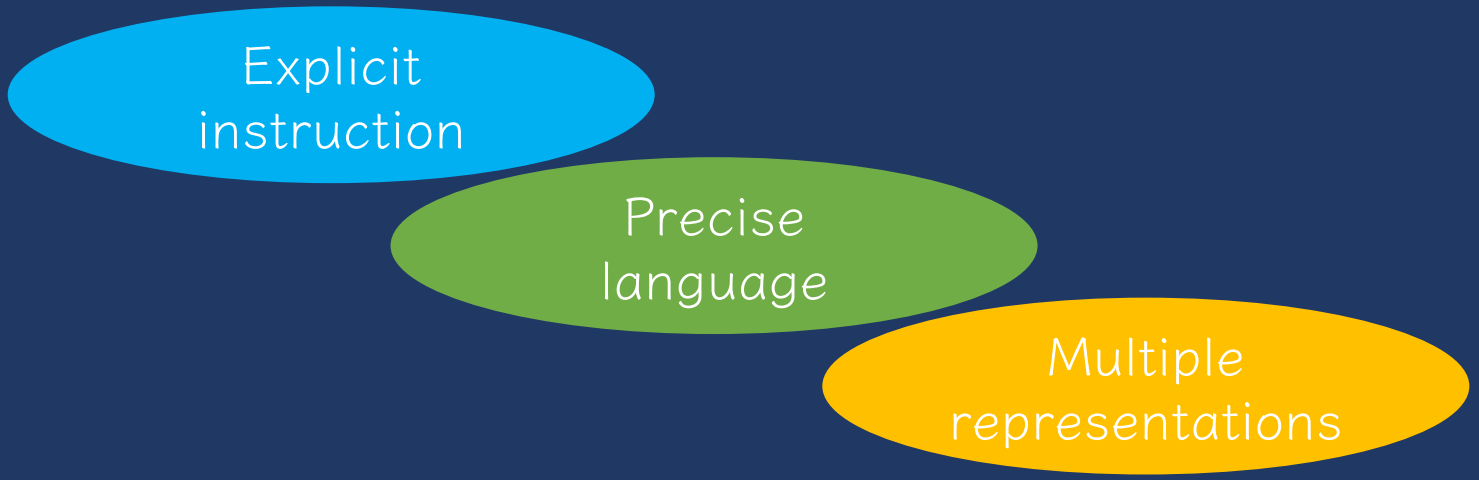
Teachers should use fluency building activities to build counting fluency and fluency with the operations.

Students should learn how to set up and solve word problems by combining an attack strategy with a focus on word-problem schemas.

Multiple Representations

Instructional Platform

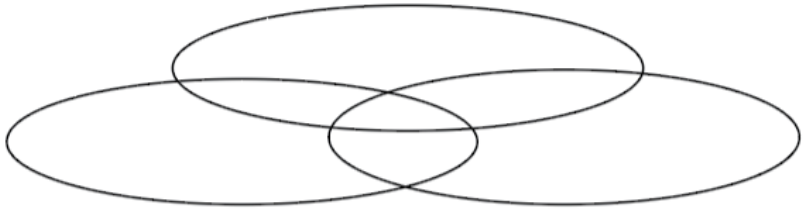
INSTRUCTIONAL DELIVERY



INSTRUCTIONAL STRATEGIES

Multiple Representations

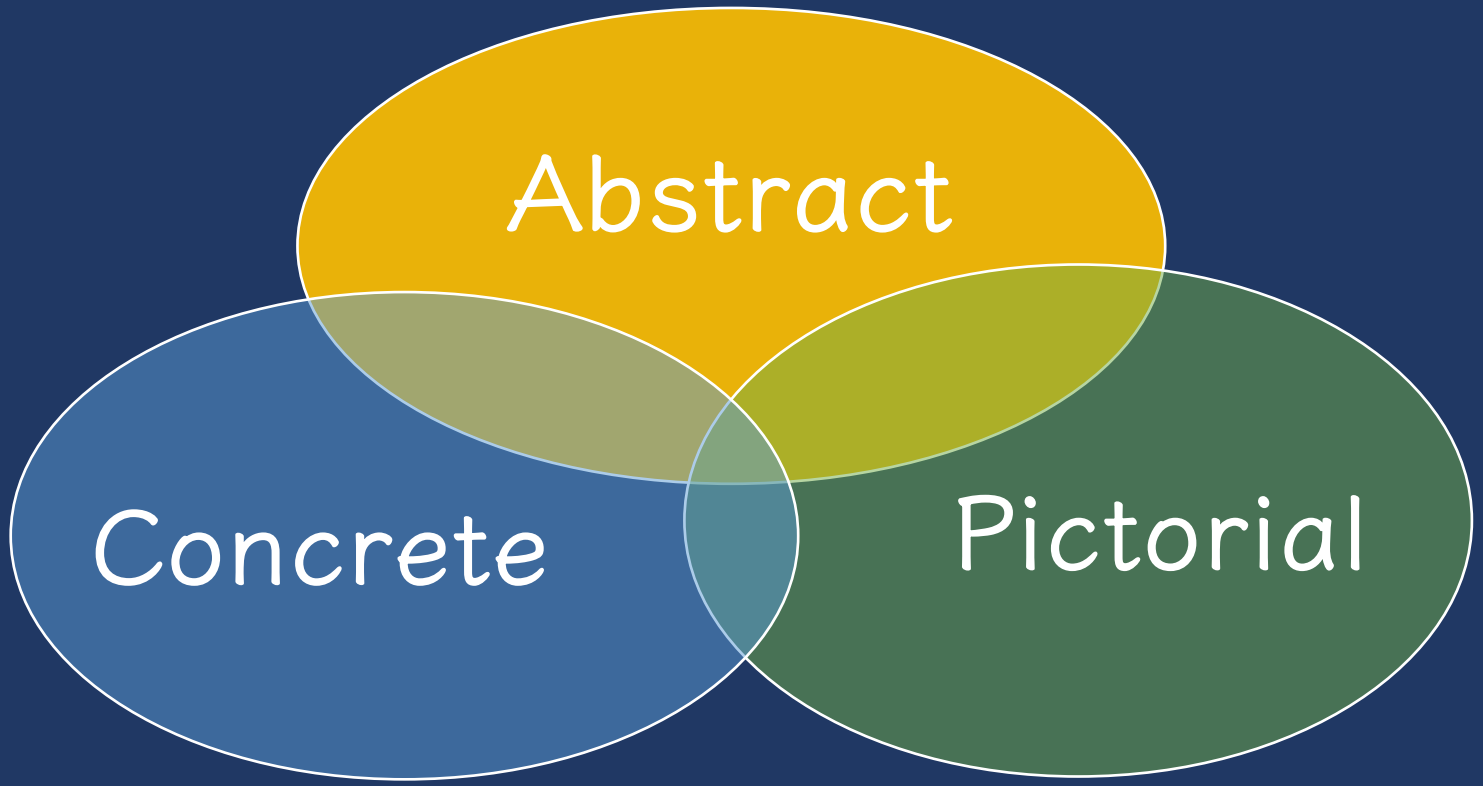
Research and Information

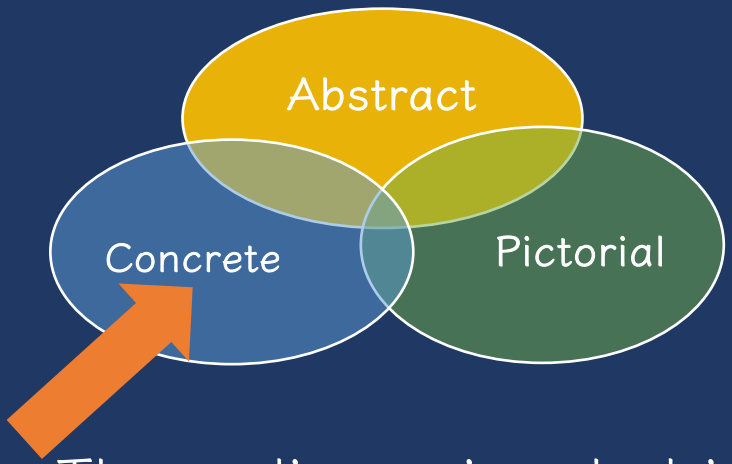


Hands-on manipulatives contribute to increases in mathematics performance.
(Bouck & Park, 2018; Carbonneau et al., 2013; Namkung & Bricko, 2021; Sherman & Bisanz, 2009; Strickland & Maccini, 2012)

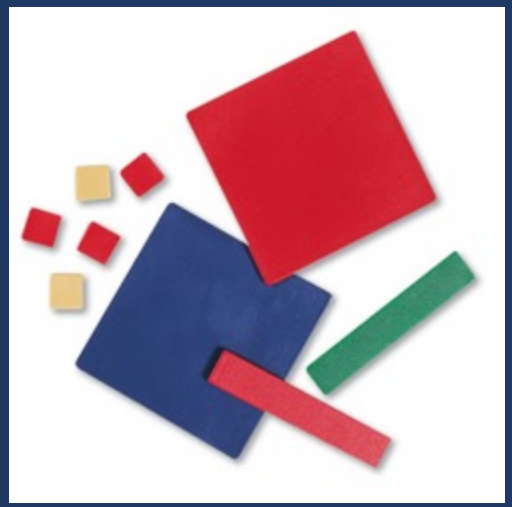
Virtual manipulatives contribute to increases in mathematics performance.
(Bouck et al., 2020; Satsangi et al., 2016)

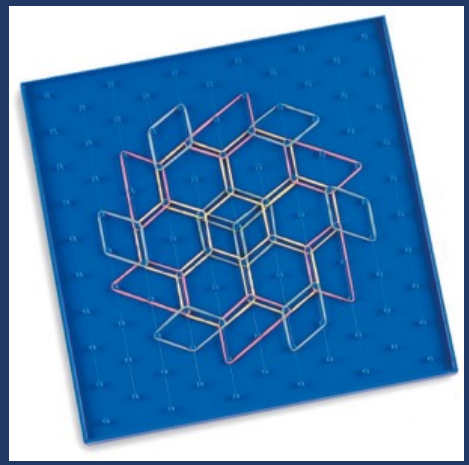
Other visuals (e.g., graphic organizers) contribute to increases in mathematics performance.
(Jitendra et al., 2009; Sharp & Dennis, 2017; van Garderen, 2007; Xin, 2008)

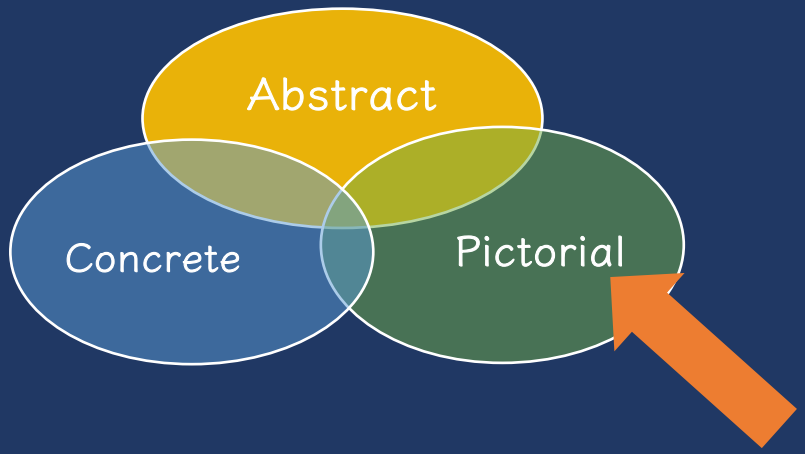




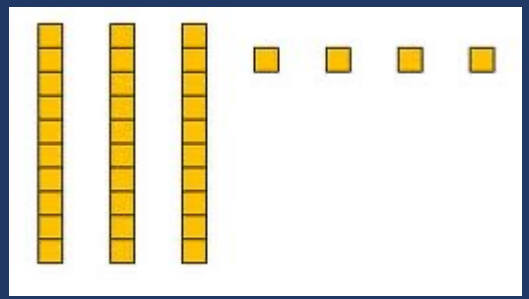
Three-dimensional objects

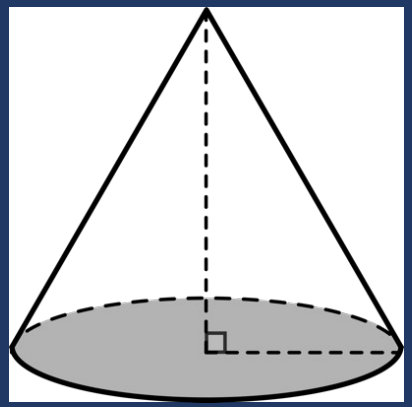


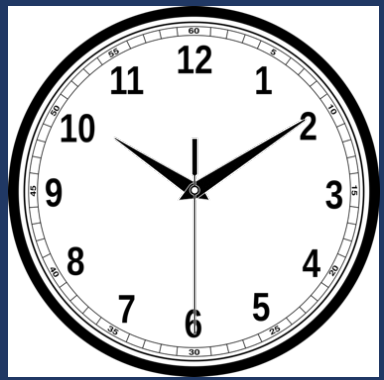


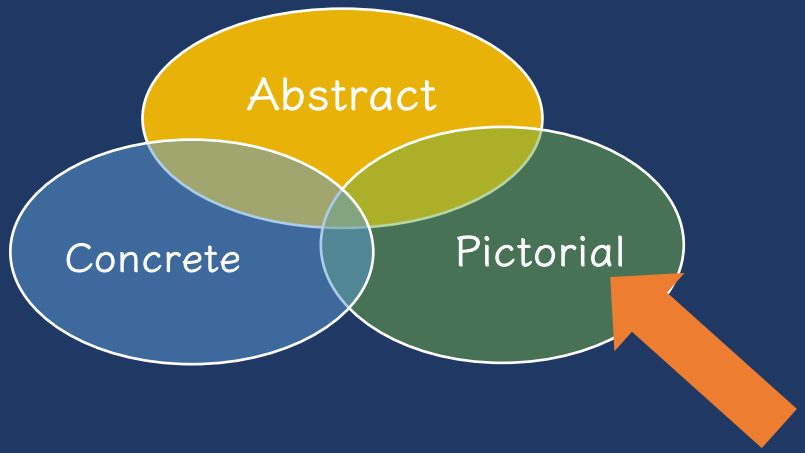


Two-dimensional images

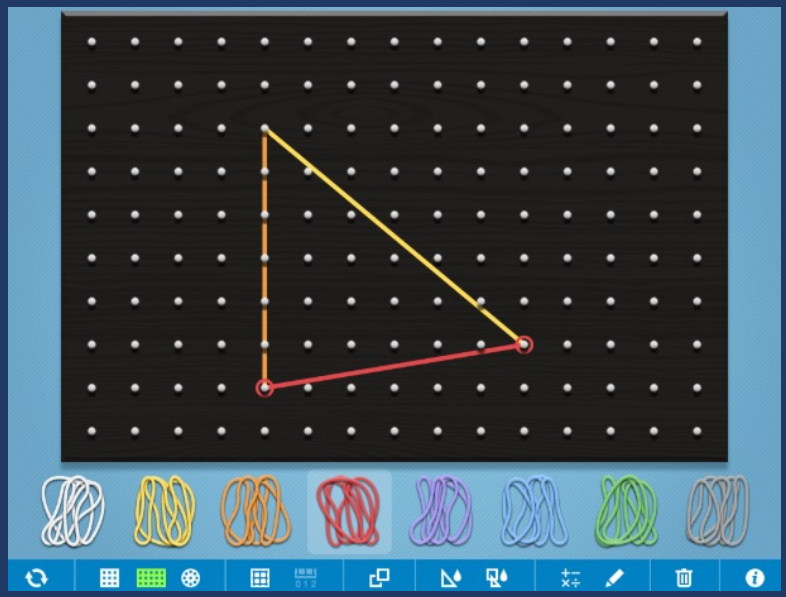


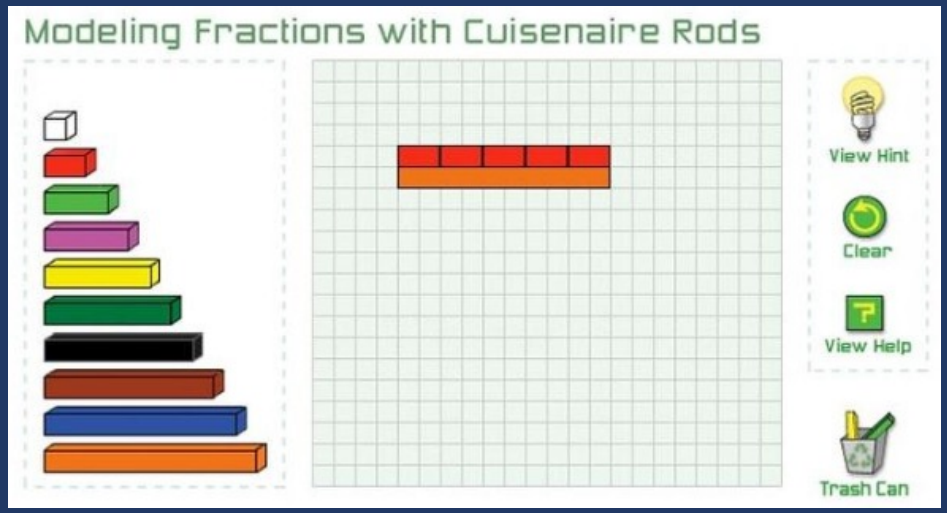


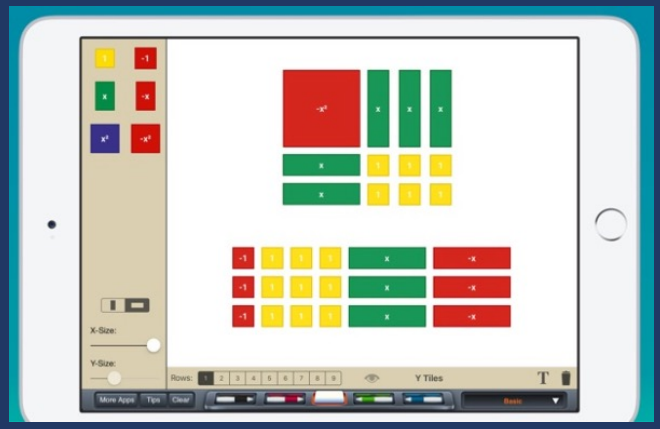


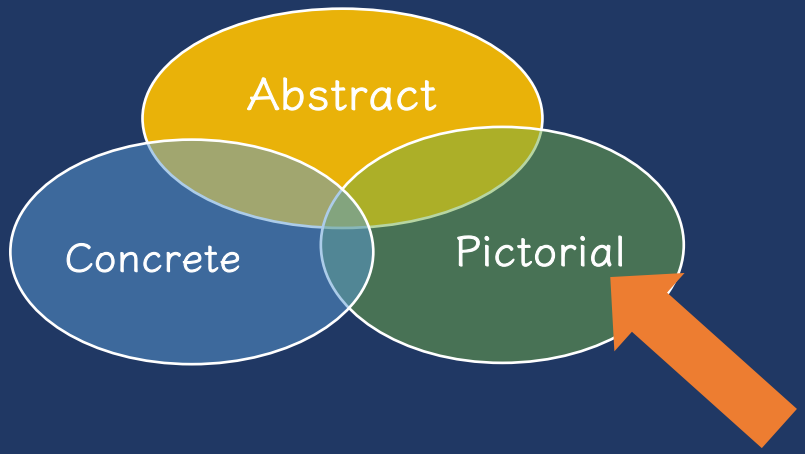


Two-dimensional images

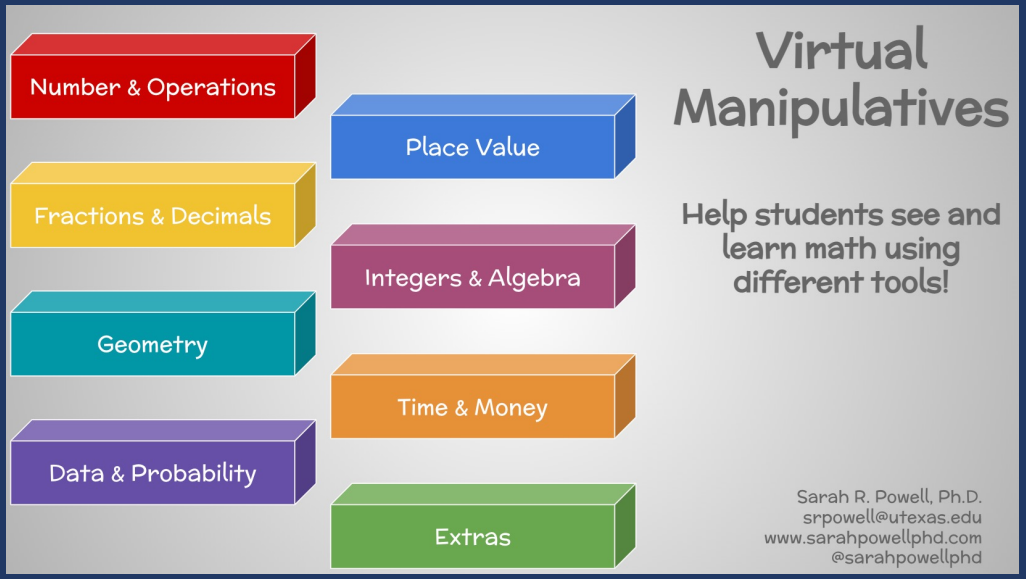


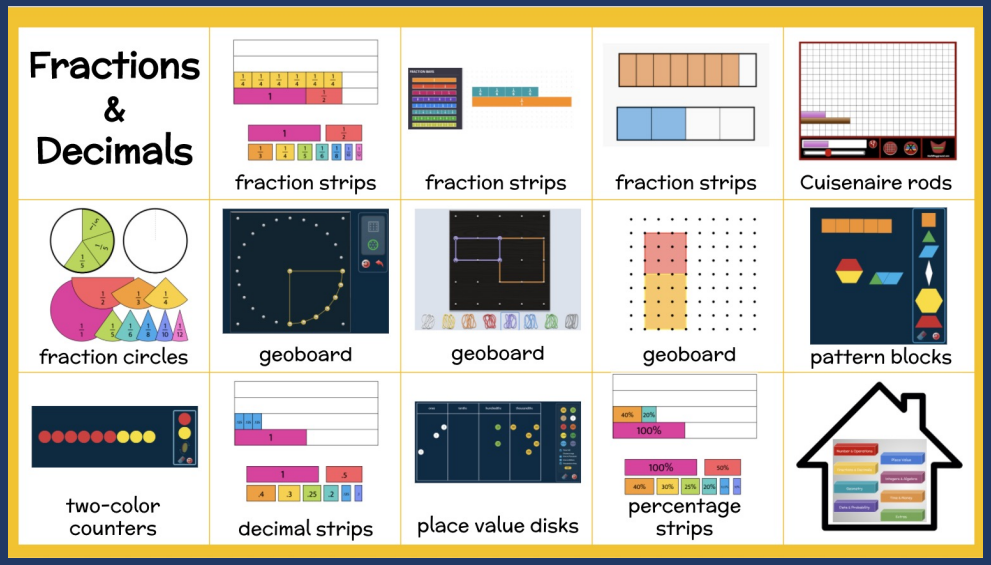






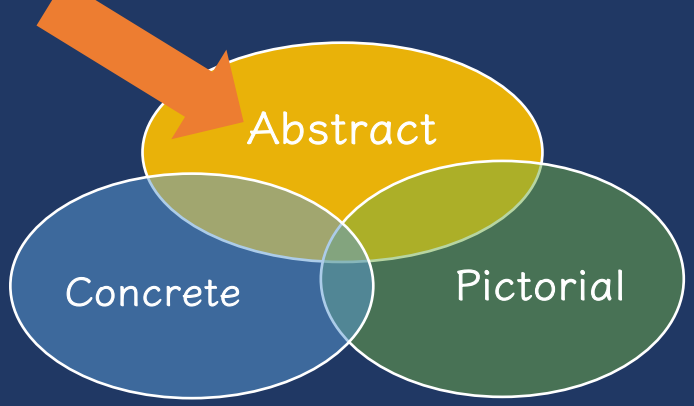
Two-dimensional images





Explore 3 virtual manipulatives.

Share with a partner.



Numerals and symbols and words

$2 + 8 = 10$

34 = 3 tens and 4 ones

$x - 6 = 8$

$$\begin{array}{r} 4{,}179 \\ +\quad 569 \\ \hline \end{array}$$

Multiple Representations

Fraction Models

What are your strengths?

What are your opportunities for growth?

What are your immediate next steps?

LENGTH

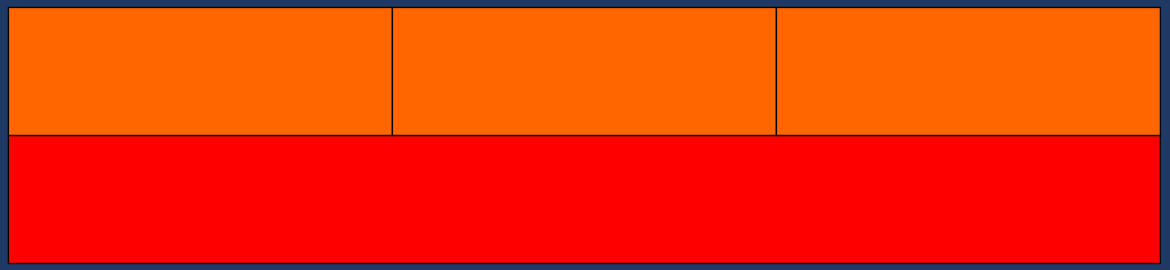
AREA

SET

LENGTH

Fractions are appropriated by length

$\frac{2}{3}$

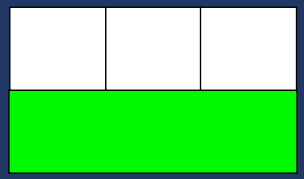


Fraction tiles/bars

LENGTH

Fractions are appropriated by length

$\frac{2}{3}$

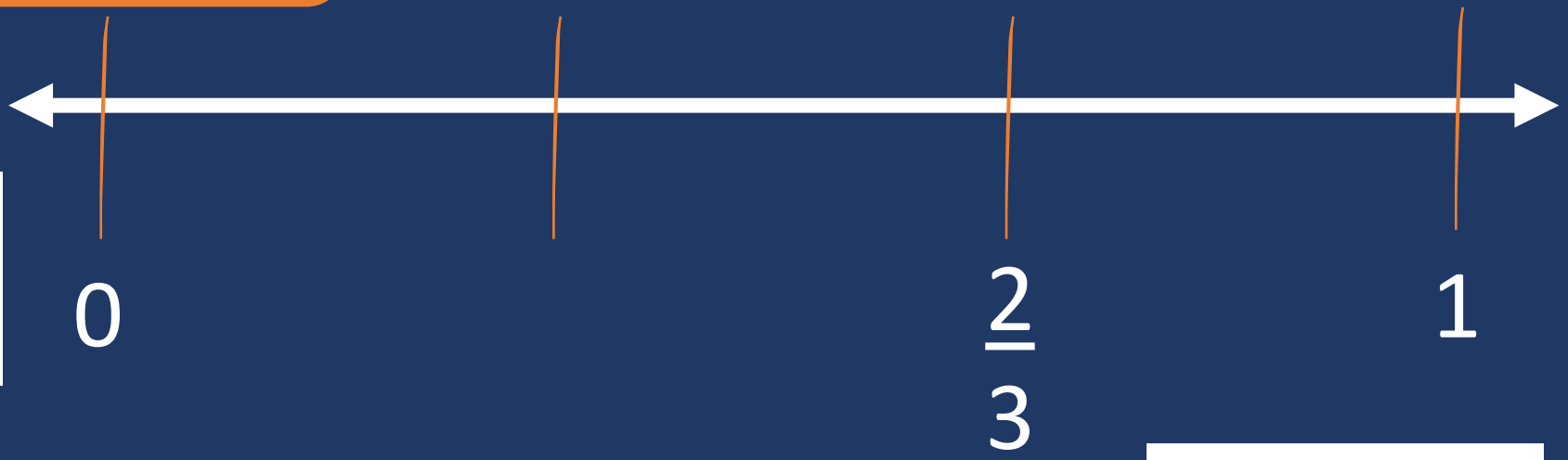


Cuisenaire rods

LENGTH

Fractions are appropriated by length

$\frac{2}{3}$

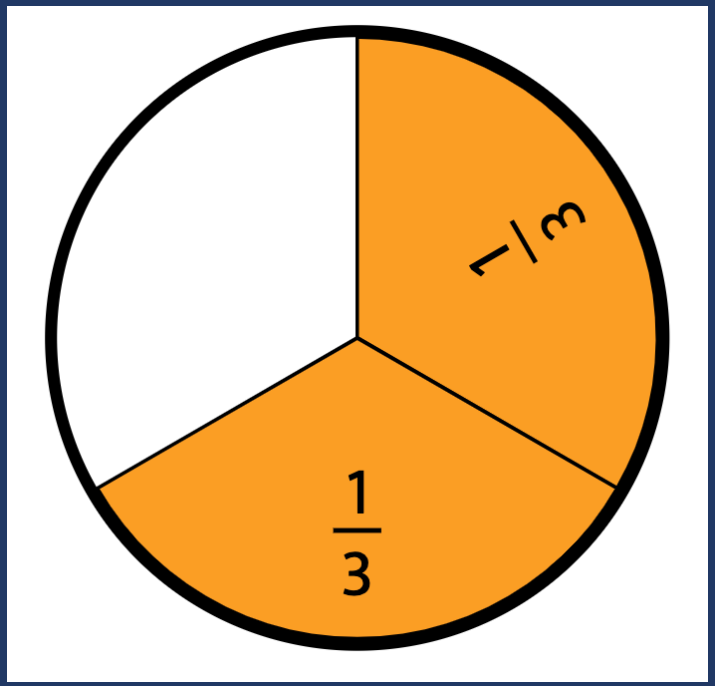


Number line

AREA

Shapes divided into equal sections

$\frac{2}{3}$

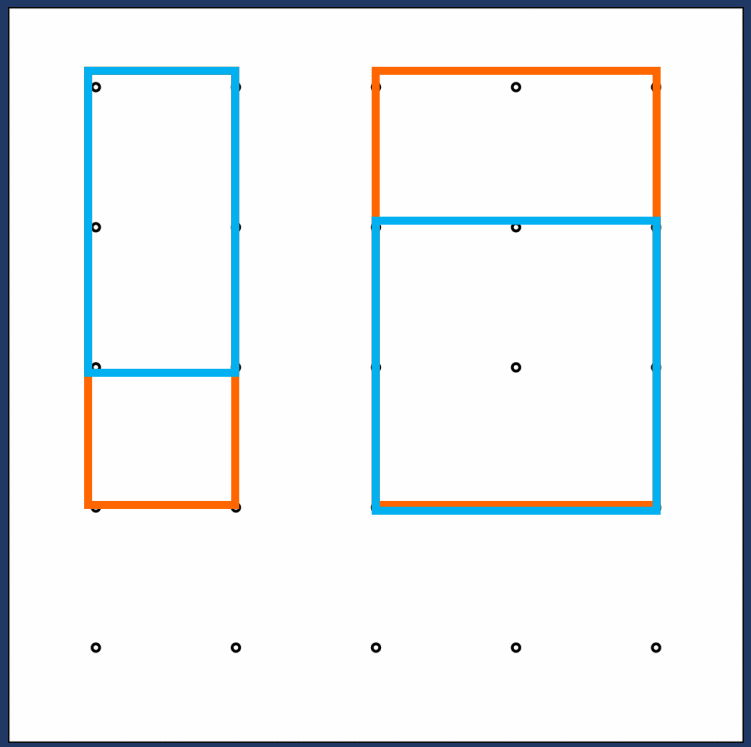


Fraction circles

AREA

Shapes divided into equal sections

$\frac{2}{3}$



Geoboards

AREA

Shapes divided into equal sections

$\frac{2}{3}$

Pattern blocks

AREA

Shapes divided into equal sections

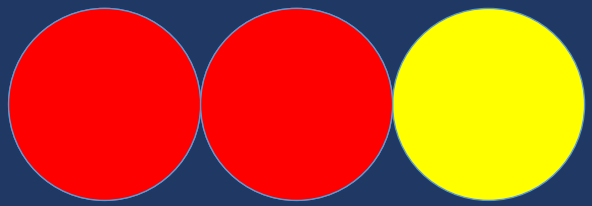
$\frac{2}{3}$

Legos

SET

Individual shapes match the fraction

$\frac{2}{3}$

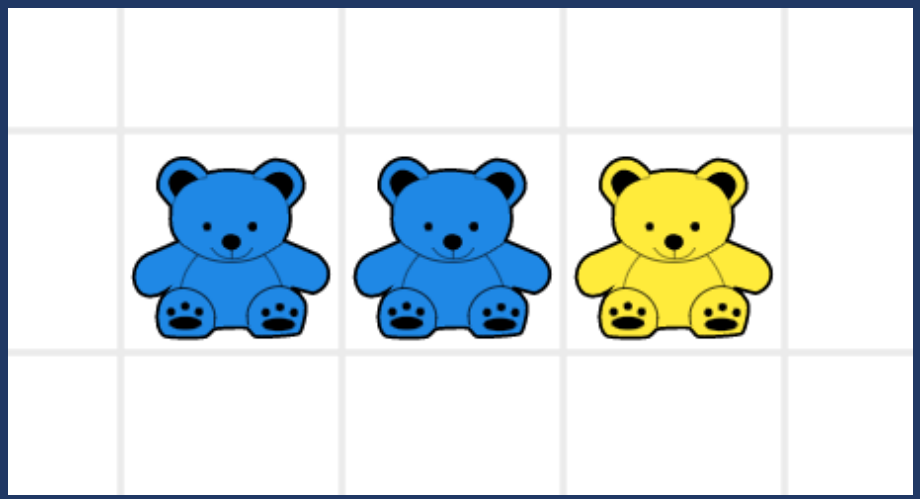


Two-color counters

SET

Individual shapes match the fraction

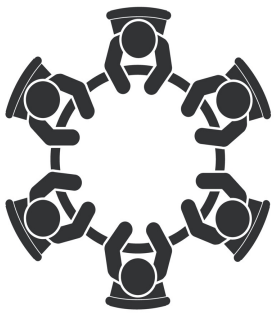
$\frac{2}{3}$



LENGTH

AREA

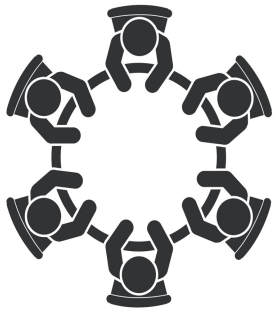
SET



Show a fraction using all three models.

Reflect upon explicit instruction and using mathematical language.

Students should use hands-on tools, virtual manipulatives, drawings, and other visuals to understand mathematics concepts and procedures.



What are your strengths with multiple representations?

What are the opportunities for growth?

What materials do you need?

Teachers should use systematic and explicit instruction to help students develop a strong foundation for specific mathematics skills.

Students require modeling and practice on how to use the language of mathematics.

Students should use hands-on tools, virtual manipulatives, drawings, and other visuals to understand mathematics concepts and procedures.

Teachers should use fluency building activities to build counting fluency and fluency with the operations.

Students should learn how to set up and solve word problems by combining an attack strategy with a focus on word-problem schemas.

Building Fluency

Instructional Platform

INSTRUCTIONAL DELIVERY

Explicit instruction

Precise language

Multiple representations

INSTRUCTIONAL STRATEGIES

Fluency building

Fluency

Research and Information

Fact Fluency

Mathematics fluency, particularly fluency with facts and computation, is related to overall mathematics performance.
(Bailey *et al.*, 2012; Cirino *et al.*, 2019; Koponen *et al.*, 2007, 2017; Vukovic *et al.*, 2014

Fact fluency practice improves mathematics fact performance.
(Burns *et al.*, 2010; Codding *et al.*, 2011; McCallum *et al.*, 2004; Nelson *et al.*, 2013; Poncy *et al.*, 2010; Schutte *et al.*, 2015; Stocker & Kubina, 2017)

Fluency is doing mathematics *easily and accurately.*

Fluency in mathematics makes mathematics *easier.*

Fluency provides less stress on working memory.

Fluency helps students build confidence with mathematics.

With fluency, it is important to emphasize both *conceptual learning and procedural learning.*

Addition	Subtraction
Multiplication	Division

Counting

Comparing numbers

Counting coins

Telling time

Identifying equivalent fractions

Knowing multiples

Identifying shapes

Knowing formulas

Fluency

Research and Information

Fact Fluency

Addition

100 addition facts

Single-digit addends sum to a single- or double-digit number

$$\begin{array}{rl} 5 & \text{(addend)} \\ +\ 4 & \text{(addend)} \\ \hline 9 & \text{(sum)} \end{array}$$

Total

Addition

Count one set, count another set, put sets together, count sum

$$2 + 3 = 5$$

Change

Addition

Start with a set, add the other set, count sum

$$2 + 3 = 5$$

Total

Addition

Parts put together into a total

Karly saw 4 cardinals and 5 blue jays. How many birds did Karly see?

Change

Addition

An amount that **increases** or decreases

Premila had $4. Then they earned $5 for cleaning their room. How much money does Premila have now?

$3 + 9 = \_\_$

Addition

If you have brown *eyes*:

What's a Total story to show addition?

If you don't have brown *eyes*:

What's a Change/Join story to show addition?

Subtraction

100 subtraction facts

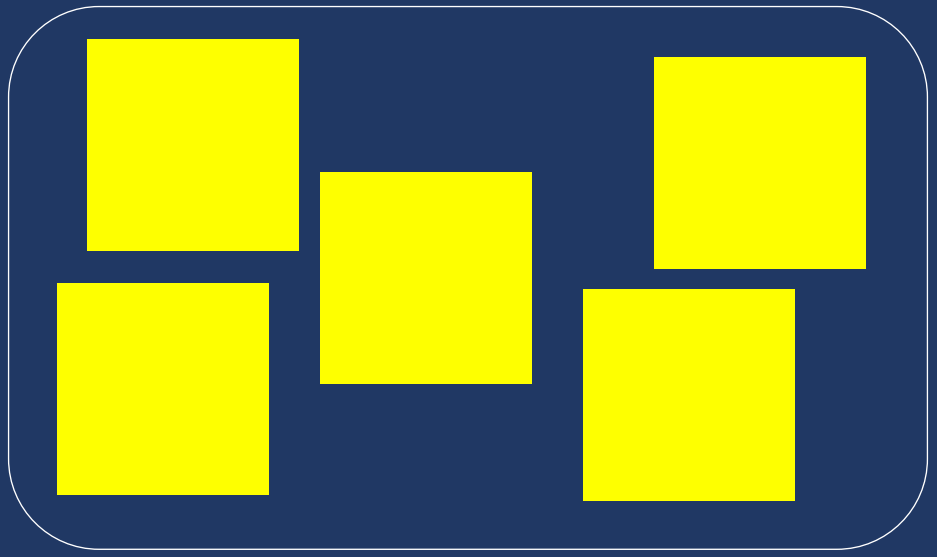
Subtrahend and difference are single-digit numbers and minuend is single- or double-digit number

$$\begin{array}{rl} 16 & \text{(minuend)} \\ -\ \ 8 & \text{(subtrahend)} \\ \hline 8 & \text{(difference)} \end{array}$$

Change

Subtraction

Start with a set, take away from that set, count difference

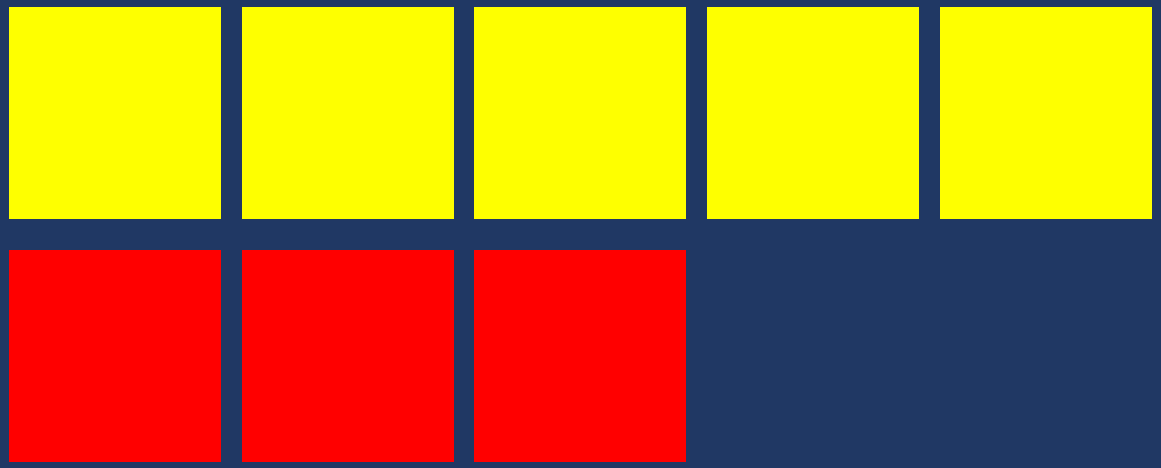


$$5 - 3 = 2$$

Difference

Subtraction

Compare two sets, count difference



$$5 - 3 = 2$$

Change

Subtraction

An amount that increases or **decreases**

Bronwyn had 9 cookies. Then they ate 2 of the cookies. How many cookies does Bronwyn have now?

Difference

Subtraction

Greater and lesser amounts compared for a difference

Rachel has 9 apples. Jodie has 2 apples. How many more apples does Rachel have? (How many fewer does Jodie have?)

$9 - 5 = \_\_$

Subtraction

If you would chose beaches:

What's a Change/Separate story to show subtraction?

If you would chose mountains:

What's a Difference story to show subtraction?

Multiplication

100 multiplication facts

Multiplication of single-digit factors results in a single- or double-digit product

$$\begin{array}{rl} 2 & \text{(factor)} \\ \times\ 3 & \text{(factor)} \\ \hline 6 & \text{(product)} \end{array}$$

Equal Groups

Multiplication

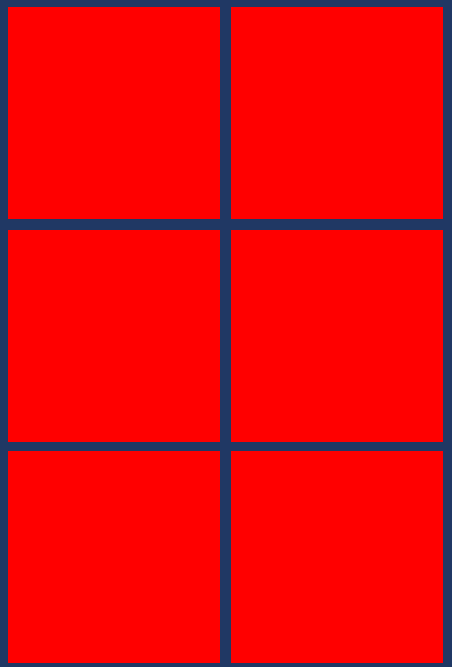
Show the groups, show the amount for each group, count product

$$3 \times 2 = 6$$

Equal Groups

Multiplication

Show the groups, show the amount for each group, count product



$$3 \times 2 = 6$$

Comparison

Multiplication

Show a set, then multiply the set

$$3 \times 2 = 6$$

Equal Groups

Multiplication

Groups multiplied by number in each group for a product

Rhiannon has 2 boxes of crayons. There are 12 crayons in each box. How many crayons does Rhiannon have altogether?

Comparison

Multiplication

Set multiplied by a number of times for a product

Vivienne had 12 stickers. Jessica had 2 times as many stickers as Vivienne. How many stickers did Jessica have?

$2 \times 5 = \_\_$

Multiplication

If you wear glasses:

What's an Equal Groups story to show multiplication?

If you don't wear glasses:

What's a Comparison story to show multiplication?

Division

90 division facts

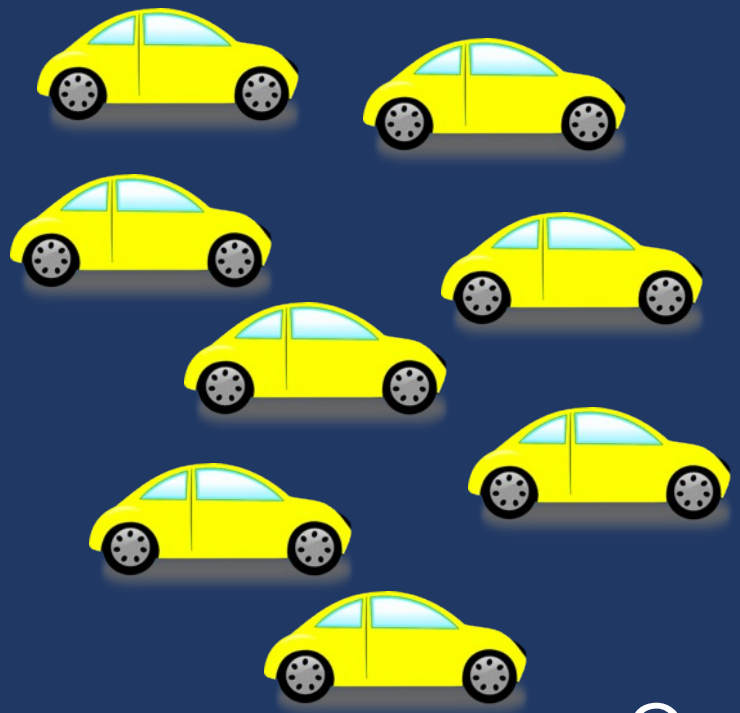
Divisor and quotient are single-digit numbers and dividend is single- or double-digit number

8	÷	4	=	2
(dividend)		(divisor)		(quotient)

Equal Groups (Partitive Division)

Division

Show the dividend, divide equally among divisor, count quotient

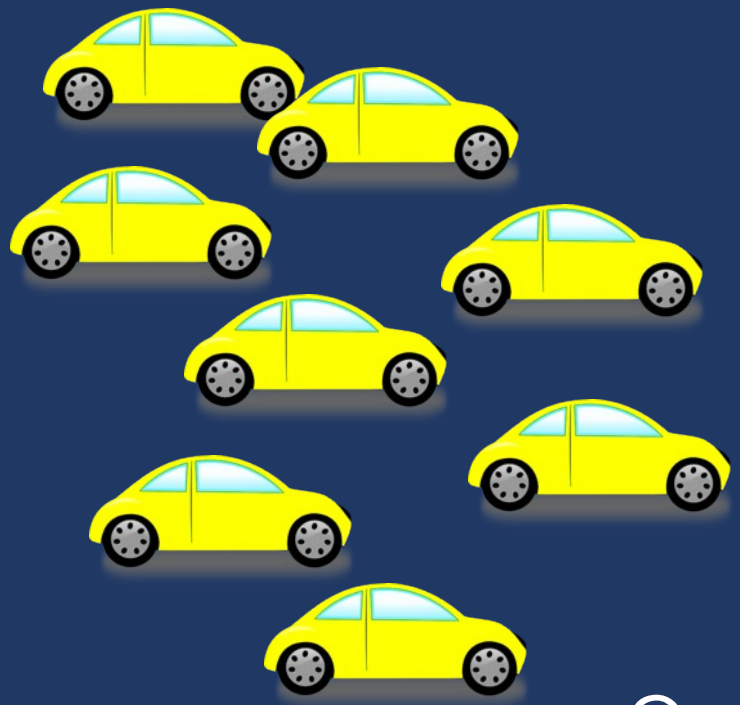


$$8 \div 2 = 4$$

Equal Groups (Quotative Division)

Division

Show the dividend, make groups of the divisor, count groups



$$8 \div 2 = 4$$

Equal Groups

Groups multiplied by number in each group for a product

Stefanie has 12 pencils. She wants to share them equally among her 2 friends. How many pencils will each friend receive?

Nicole has 12 pencils. She put them into pencil pockets with 6 pencils each. How many pencil pockets did Nicole use?

$12 \div 4 = \_\_$

Division

If you watch *Stranger Things*:

What's a Partitive story to show division?

If you watch *Ted Lasso*:

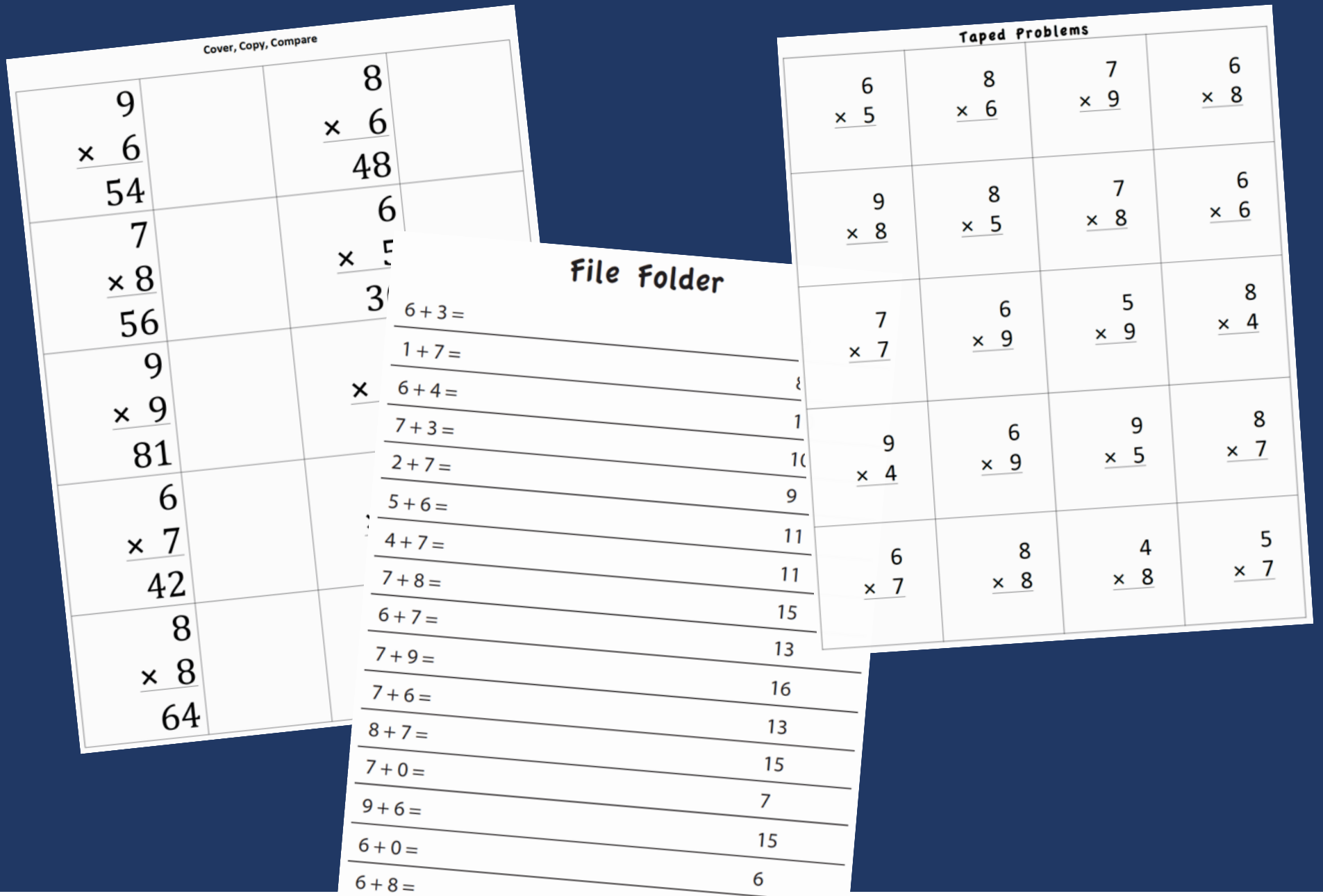
What's a Quotative story to show division?

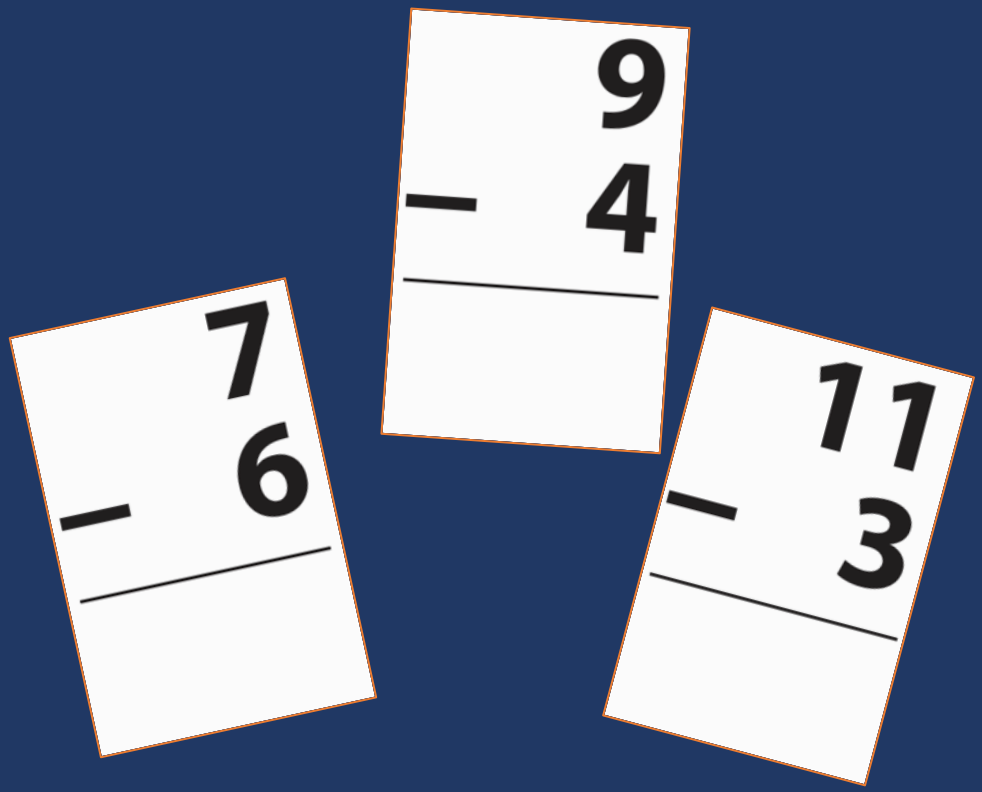
Addition	Subtraction
Multiplication	Division

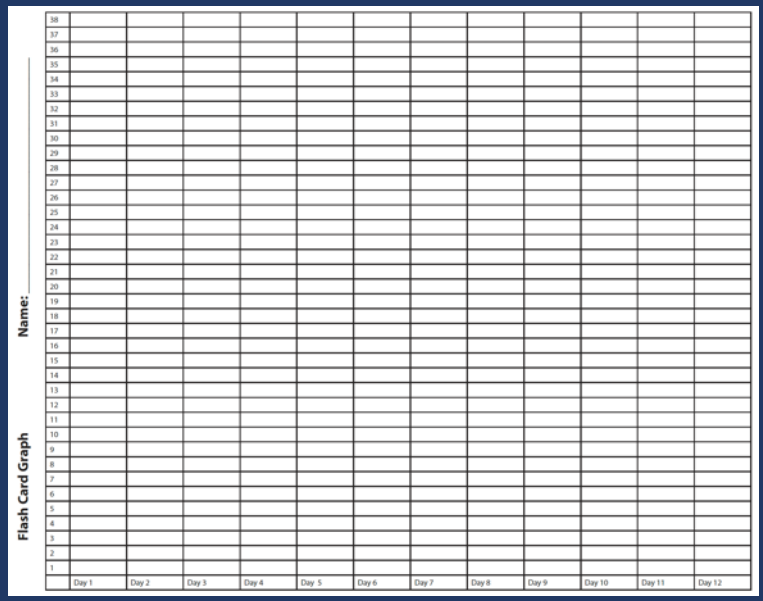
Build fluency with math facts.

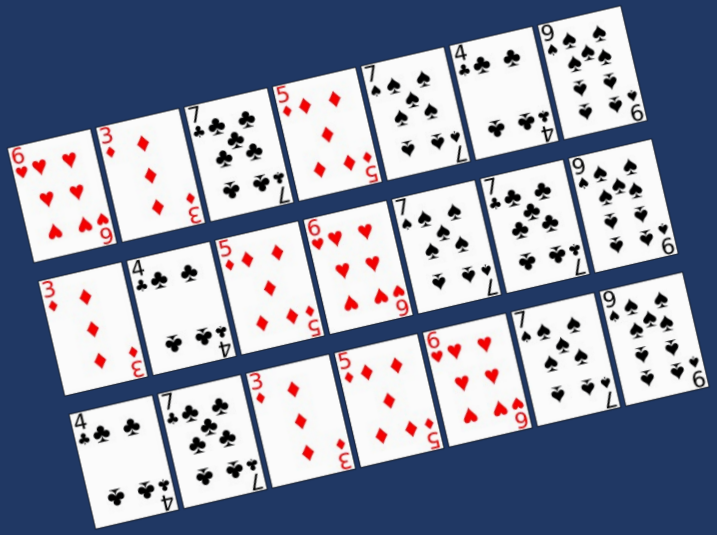
- Addition: single-digit addends
- Subtraction: single-digit subtrahend
- Multiplication: single-digit factors
- Division: single-digit divisor

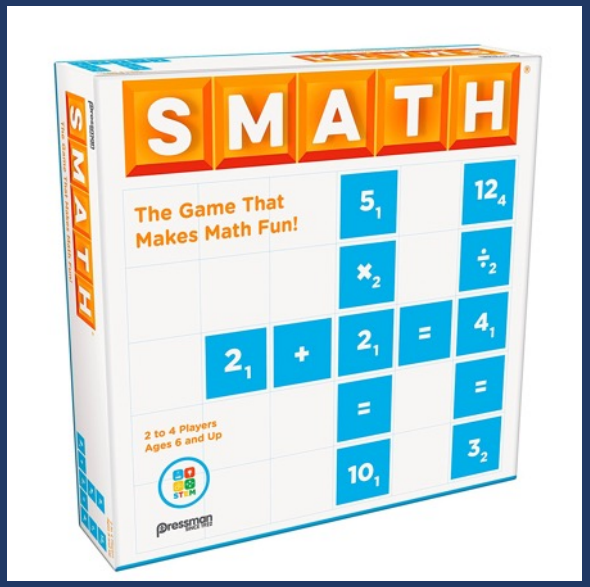
$$\begin{array}{r} 5 \\ +\ 8 \\ \hline \end{array} \qquad \begin{array}{r} 9 \\ -\ 4 \\ \hline \end{array} \qquad \begin{array}{r} 6 \\ \times\ 7 \\ \hline \end{array} \qquad \begin{array}{r} 56 \\ \div\ 8 \\ \hline \end{array}$$

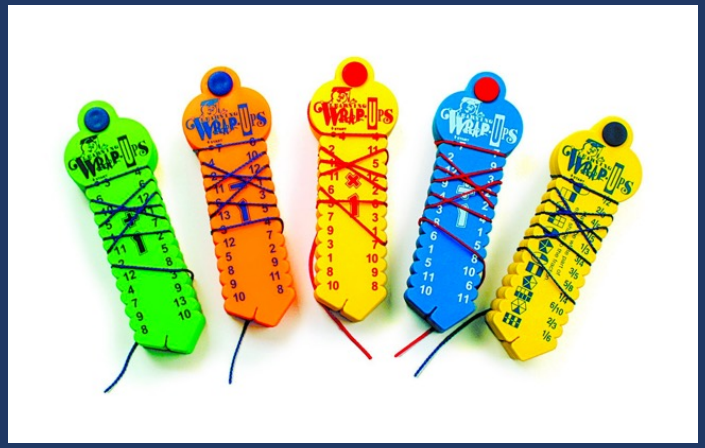


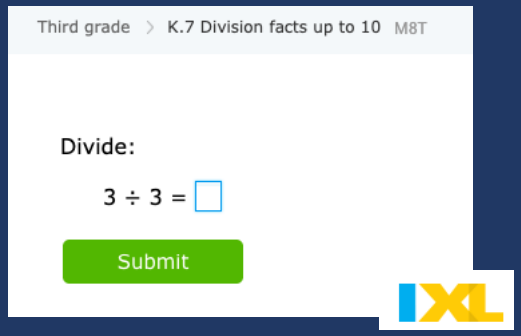


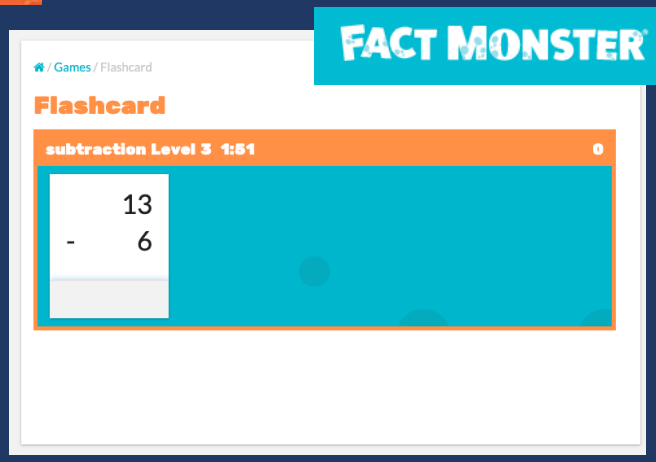












DAILY and BRIEF

Addition	Subtraction
Multiplication	Division

What are five ways you help students build fact fluency?

Fluency

Computational Fluency

What are your strengths?

What are your opportunities for growth?

What are your immediate next steps?

Addition	Subtraction
Multiplication	Division

Build fluency with whole-number computation

$$\begin{array}{r} 15 \\ +\ 28 \\ \hline \end{array}$$

$$\begin{array}{r} 23 \\ \times\ 9 \\ \hline \end{array}$$

$$\begin{array}{r} 1009 \\ -\ 724 \\ \hline \end{array}$$

$$\begin{array}{r} 7250 \\ \div\ 15 \\ \hline \end{array}$$

Addition	Subtraction
Multiplication	Division

Build fluency with rational-number computation

$$\begin{array}{r} 1.4 \\ +\ 3.9 \\ \hline \end{array}$$

$$\frac{2}{3} \times \frac{3}{4}$$

$$\frac{9}{4} - \frac{3}{8}$$

$$\begin{array}{r} 7.892 \\ \div\ 0.14 \\ \hline \end{array}$$

Addition	Subtraction
Multiplication	Division

Build fluency with integer computation

$$\begin{array}{r} 1.4 \\ +\ -3.9 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \\ \times\ -12 \\ \hline \end{array}$$

$-14 - (-7) =$

$-135 \div 2 =$

Partial Sums

A.

$$\begin{array}{r} 74 \\ +\ 18 \\ \hline 80 \\ +12 \\ \hline 92 \end{array}$$

B.

$$\begin{array}{r} 725 \\ +\ 365 \\ \hline 1{,}000 \\ 80 \\ +\ 10 \\ \hline 1{,}090 \end{array}$$

$$\begin{array}{r} 227 \\ +\ 185 \\ \hline \end{array}$$

Opposite Change

A.

$$\begin{array}{r} 74 \xrightarrow{-4} 70 \\ +\ 18 \xrightarrow{+4} +22 \\ \hline 92 \end{array}$$

B.

$$\begin{array}{r} 725 \xrightarrow{+5} 730 \\ +\ 365 \xrightarrow{-5} +360 \\ \hline 1{,}090 \end{array}$$

$$\begin{array}{r} 227 \\ +\ 185 \\ \hline \end{array}$$

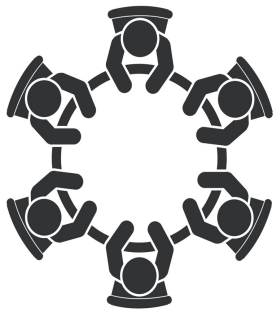
Partial Differences

A.

$$\begin{array}{r} 62 \\ -\ 17 \\ \hline +50 \\ -\ 5 \\ \hline 45 \end{array}$$

B.

$$\begin{array}{r} 305 \\ -\ 96 \\ \hline +300 \\ -90 \\ -1 \\ \hline 209 \end{array}$$



$$\begin{array}{r} 232 \\ -\ 164 \\ \hline \end{array}$$

Same Change

A.

$$\begin{array}{r} 62 \xrightarrow{+3} 65 \\ -\ 17 \xrightarrow{+3} -20 \\ \hline 45 \end{array}$$

B.

$$\begin{array}{r} 305 \xrightarrow{+4} 309 \\ -\ 96 \xrightarrow{+4} -100 \\ \hline 209 \end{array}$$

$$\begin{array}{r} 232 \\ -\ 164 \\ \hline \end{array}$$

Add Up

A.

$$\begin{array}{r} 62 \\ -\ 17 \\ \hline \end{array}$$

$$\begin{array}{r} 17 \\ 20 \quad 3 \\ 60 \quad 40 \\ 62 \quad +\ 2 \\ \hline 45 \end{array}$$

B.

$$\begin{array}{r} 305 \\ -\ 96 \\ \hline \end{array}$$

$$\begin{array}{r} 96 \\ 100 \quad 4 \\ 300 \quad 200 \\ 305 \quad +\ 5 \\ \hline 209 \end{array}$$

$$\begin{array}{r} 232 \\ -\ 164 \\ \hline \end{array}$$

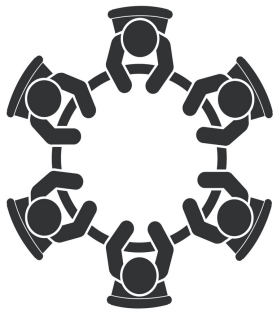
Partial Products

A.

$$\begin{array}{r} 24 \\ \times\ 43 \\ \hline 800 \\ 160 \\ 60 \\ +\ 12 \\ \hline 1{,}032 \end{array}$$

B.

$$\begin{array}{r} 132 \\ \times\ 53 \\ \hline 5000 \\ 1500 \\ 100 \\ 300 \\ 90 \\ +\ 6 \\ \hline 6996 \end{array}$$



$$\begin{array}{r} 13 \\ \times\ 47 \\ \hline \end{array}$$

Area (Array)

A.

$$\begin{array}{r} 24 \\ \times\ 43 \\ \hline \end{array}$$

	20	4	
	800	160	40
	60	12	3

$$\begin{array}{r} {\scriptstyle 1\ 1} \\ 800 \\ 160 \\ 60 \\ +\ 12 \\ \hline 1{,}032 \end{array}$$

B.

$$\begin{array}{r} 132 \\ \times\ 53 \\ \hline \end{array}$$

	100	30	2	
	5000	1500	100	50
	300	90	6	3

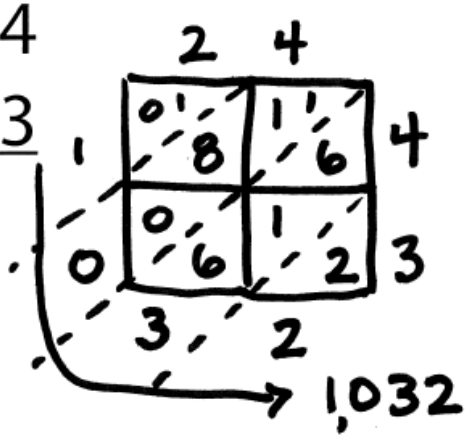
$$\begin{array}{r} 5000 \\ 1500 \\ 100 \\ 300 \\ 90 \\ +\ 6 \\ \hline 6996 \end{array}$$

$$\begin{array}{r} 13 \\ \times\ 47 \\ \hline \end{array}$$

Lattice

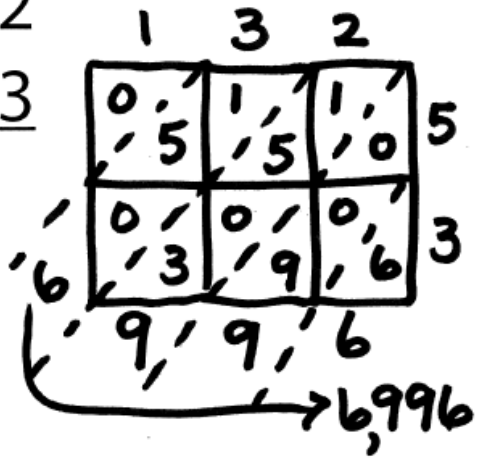
A.

$$\begin{array}{r} 24 \\ \times\ 43 \\ \hline \end{array}$$



B.

$$\begin{array}{r} 132 \\ \times\ 53 \\ \hline \end{array}$$



$$\begin{array}{r} 13 \\ \times\ 47 \\ \hline \end{array}$$

Partial Quotients

A.

$$
\begin{array}{r|l}
12\overline{)158} & \\
-120 & 10 \\
\hline
38 & \\
-36 & +3 \\
\hline
2 & 13\ \text{R}2
\end{array}
$$

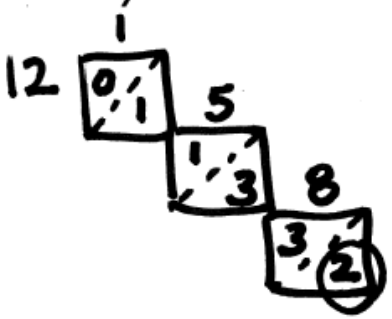
B.

$$
\begin{array}{r|l}
34\overline{)\cancel{89}170} & \\
-680 & 20 \\
\hline
290 & \\
-170 & 5 \\
\hline
\cancel{120}\ 10 & \\
102 & +3 \\
\hline
18 & 28\ \text{R}\ 18
\end{array}
$$

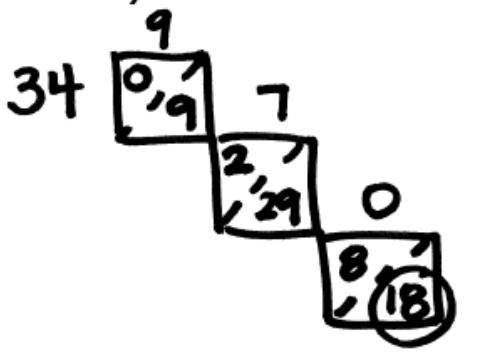
$$
\begin{array}{r}
804 \\
\div\ \ 12 \\
\hline
\end{array}
$$

Lattice

A. $12\overline{)158}$ 13 R2



B. $34\overline{)970}$ 28 R 18



$$804 \div 12$$

Teachers should use fluency building activities to build counting fluency and fluency with the operations.

What are your strengths with building fluency?

What are the opportunities for growth?

What are your immediate next steps?

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Students require modeling and practice on how to use the language of mathematics.

Students should use hands-on tools, virtual manipulatives, drawings, and other visuals to understand mathematics concepts and procedures.

Teachers should use fluency building activities to build counting fluency and fluency with the operations.

Students should learn how to set up and solve word problems by combining an attack strategy with a focus on word-problem schemas.

Word-Problem Solving

Instructional Platform

INSTRUCTIONAL DELIVERY

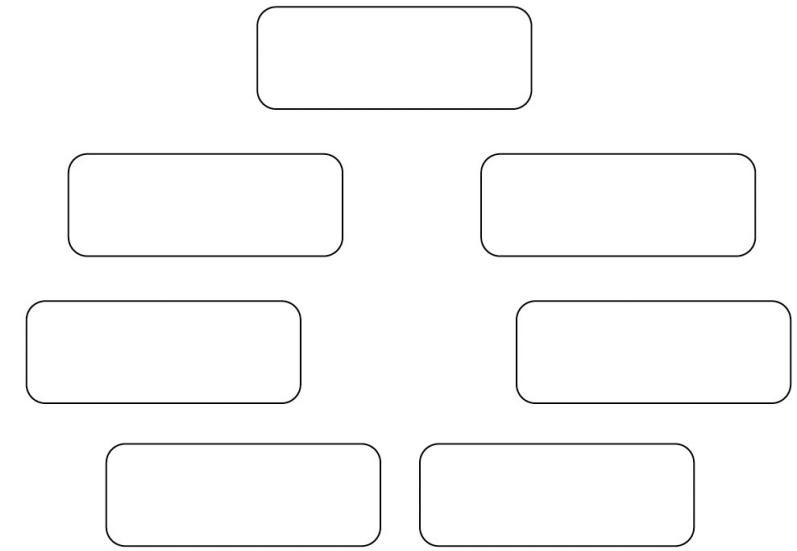
- Explicit instruction
- Precise language
- Multiple representations

INSTRUCTIONAL STRATEGIES

- Fluency building
- Problem solving instruction

Word-Problem Solving

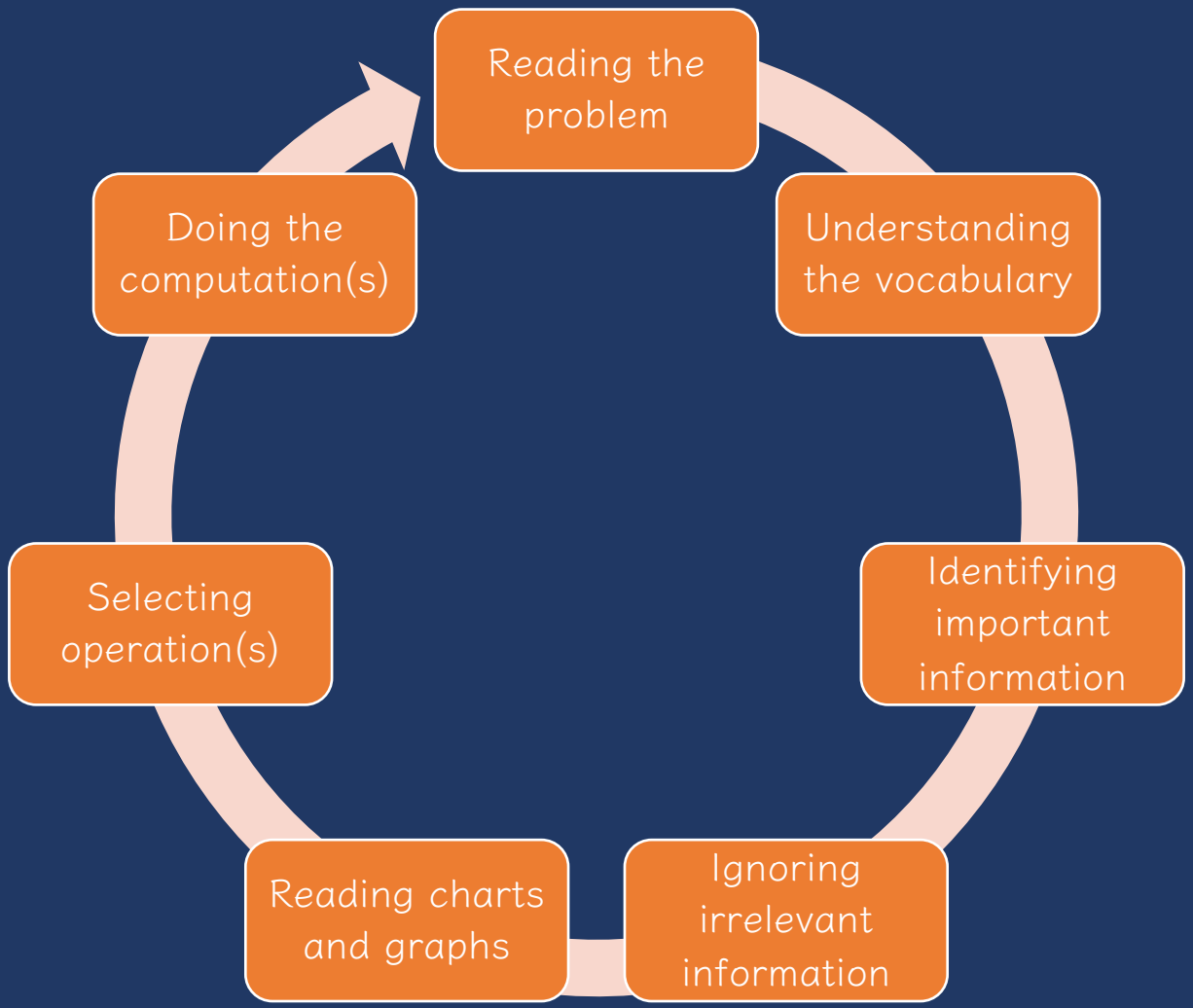
Research and Information



Key words tied to operations is an ineffective word-problem strategy.
(Karp et al., 2019; Powell et al., 2022)

Using a meta-cognitive strategy improves word-problem performance.
(Freeman-Green et al., 2015; Krawec et al., 2012; Montague et al., 2011; Swanson et al., 2014)

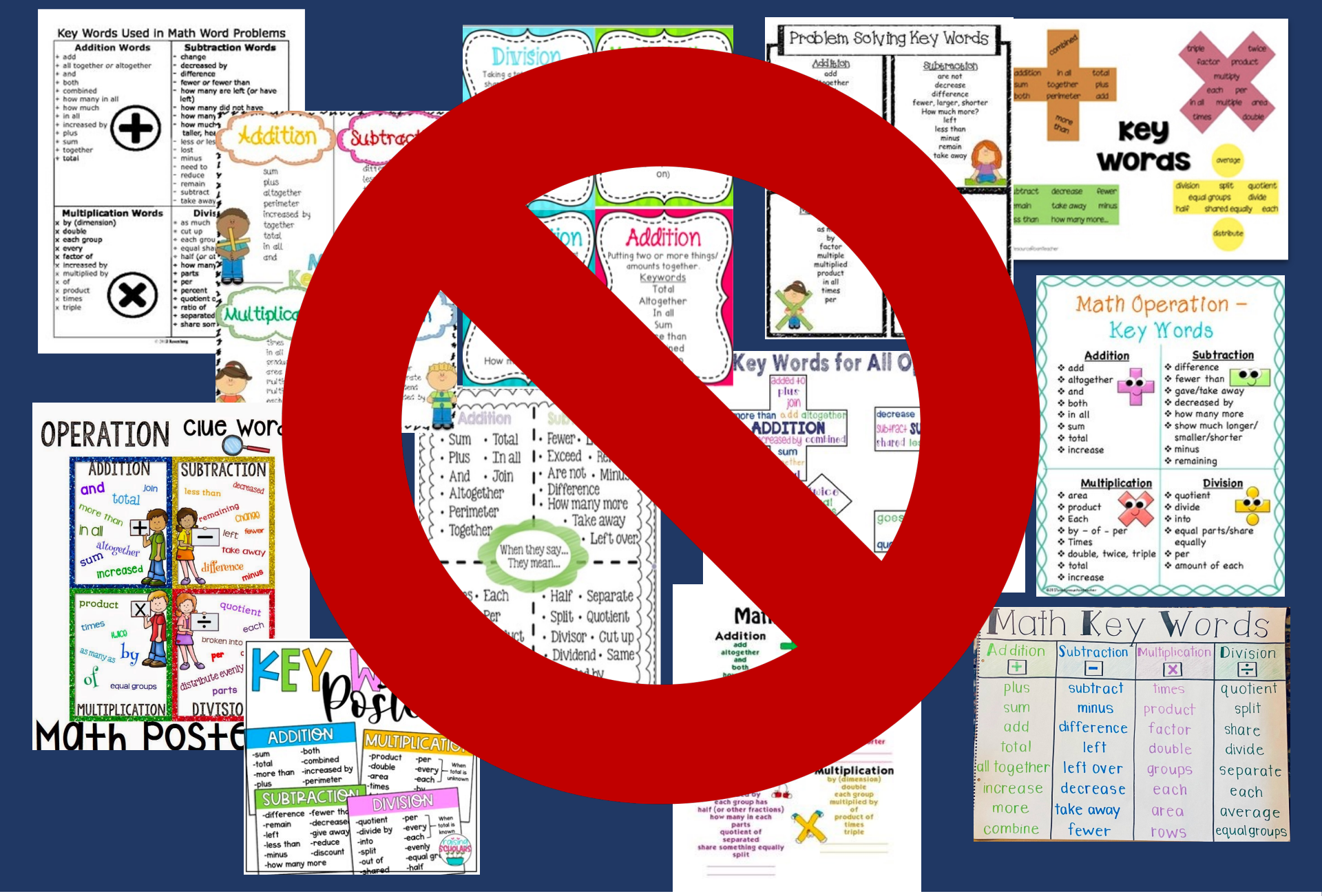
A focus on schemas improves word-problem performance.
(Alghamdi et al., 2020; Cook et al., 2020; Flores et al., 2016; Fuchs et al., 2021; Griffin et al., 2019; Jitendra et al., 2013; Lein et al., 2020; Peltier et al., 2020; Powell et al., 2022; Xin & Xhang, 2009; Zheng et al., 2013)



1. Keywords tied to operations

Lincoln had 8 pencils **fewer** than Roscoe. If Roscoe had 18 pencils, how many pencils did Lincoln have?

Lincoln had 8 pencils **fewer** than Roscoe. If Lincoln had 18 pencils, how many pencils did Roscoe have?



Description of Single-Step Word Problems (n = 132)

Schema	Occurrence of schema		Any keyword		Schema-specific keywords[a]		Multiple keywords[a]		Keyword(s) led to correct solution[a]	
	n	%	*n*	%	*n*	%	*n*	%	*n*	%
Total	27	20.5	26	96.3	23	88.5	5	19.2	21	80.8
Difference	17	12.9	17	100.0	14	82.4	2	11.8	12	70.6
Change	11	8.3	7	63.6	5	71.4	5	71.4	2	28.6
Equal groups	29	22.0	26	89.7	22	84.6	18	69.2	8	30.8
Comparison	10	7.6	9	90.0	9	100.0	4	44.4	5	55.6
Ratios or proportions	29	22.0	23	79.3	9	39.1	9	39.1	6	26.1
Product of measures	9	6.8	9	100.0	8	88.9	1	11.1	5	55.6

[a]When a problem featured a keyword.

Description of Multi-Step Word Problems (n = 84)

	Occurrence of schema[a]		Any keyword		Keyword(s) led to correct solution[b]	
Schema	*n*	%	*n*	%	*n*	%
Total	40	47.6	39	97.5	3	7.7
Difference	11	13.1	11	100.0	1	9.1
Change	21	23.8	19	95.0	1	5.3
Equal groups	49	58.3	48	98.0	1	2.1
Comparison	7	8.3	7	100.0	0	0.0
Ratios or proportions	22	25.0	16	76.2	1	6.3
Product of measures	7	8.3	7	100.0	2	28.6

[a]Sum across schemas does not equal 100 because each word problem featured more than one schema.

[b]When a problem featured a keyword.

Mr. Rivera's taxable income is $20 each hour before taxes are taken out. Mr. Rivera worked a total of 40 hours each week for 50 weeks.

What is the dollar amount, to the nearest dollar, taken out for taxes based on Mr. Rivera's taxable income?

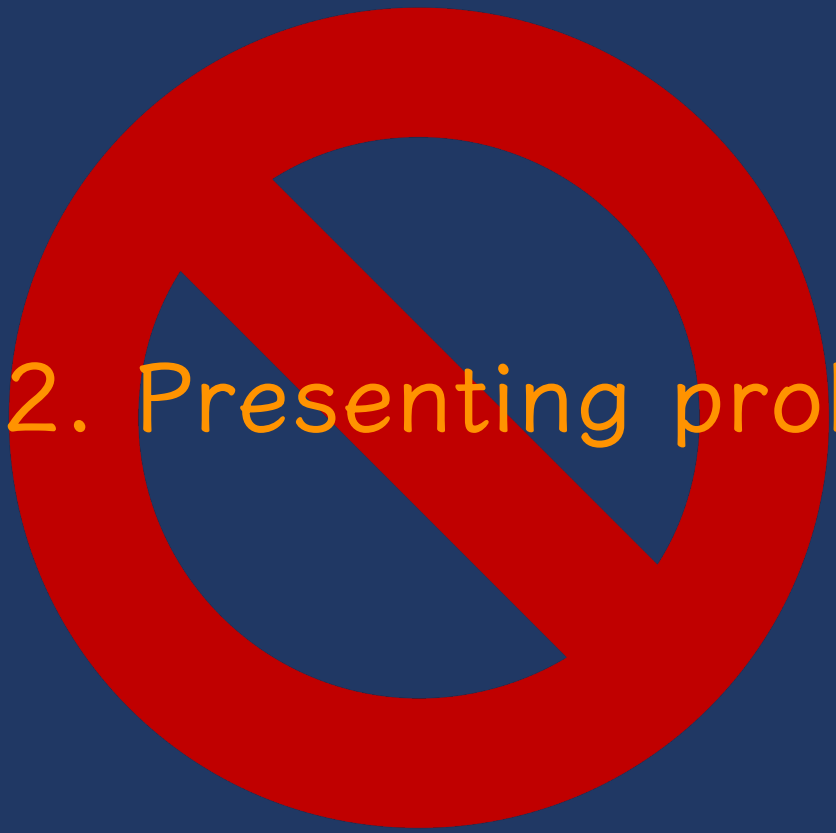
The temperature of a substance decreased by 24°C per minute for 3 minutes. What was the overall change of the temperature of the substance?

Keywords are important to identify and understand

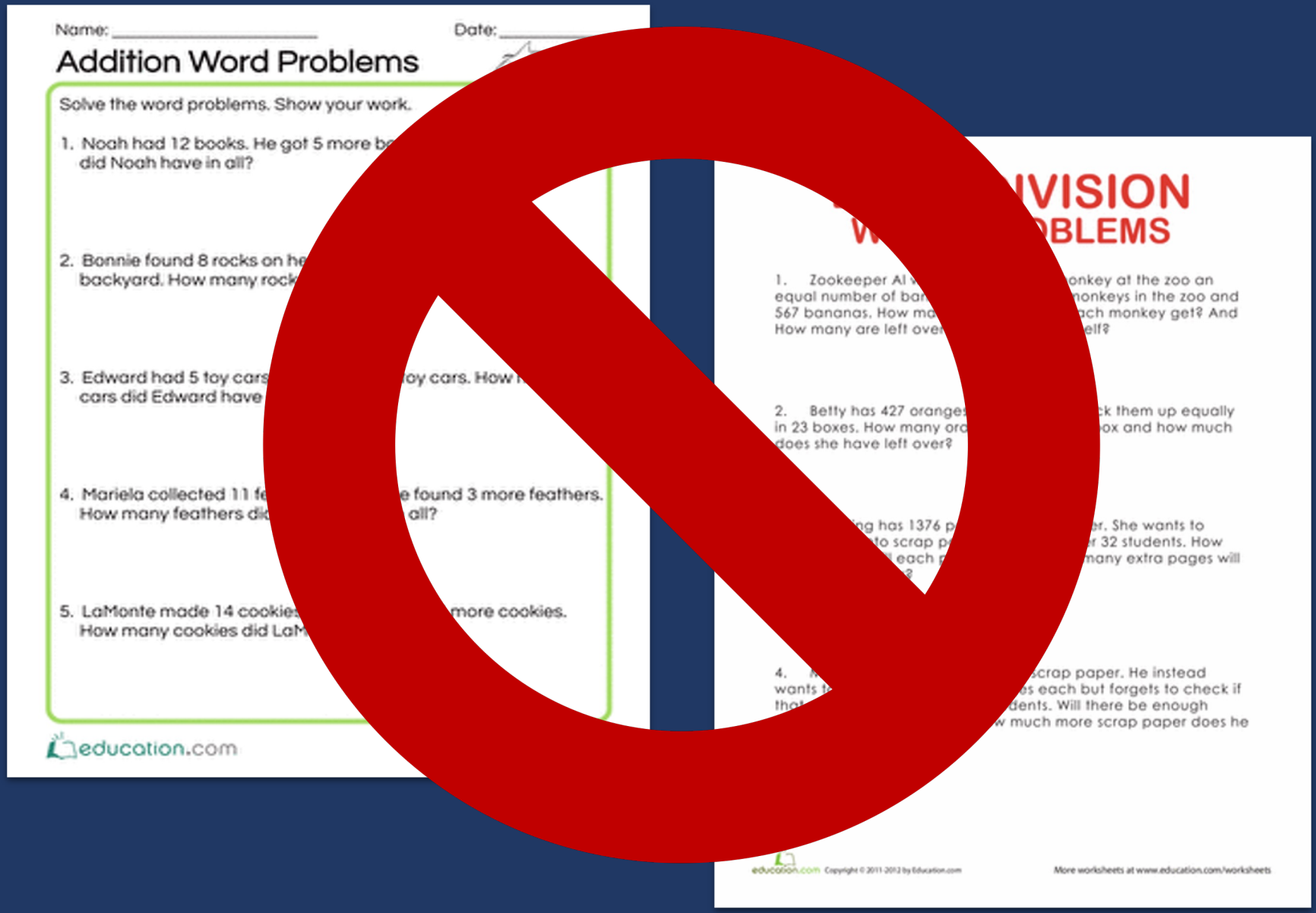
Keywords are the mathematical vocabulary that help an students understand what the story is about and what they need to do

Talk about keywords ("What does *more than* tell you about?")

But, *do not* tie a keyword to a specific operation!



2. Presenting problems by operation



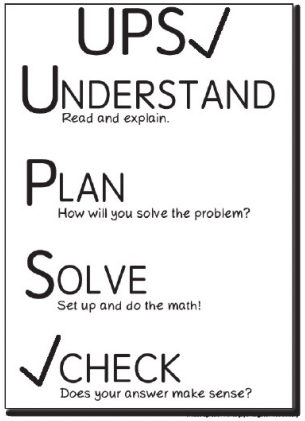
Word-Problem Solving

SOLVE

Study the problem.
Organize the facts.
Line up the plan.
Verify the plan with computation.
Examine the answer.

R-CUBES

Read the problem.
Circle key numbers.
Underline the question.
Box action words.
Evaluate steps.
Solve and check.



Teach an attack strategy

Teach about schemas

RIDE

Read the problem.

Identify the relevant information.

Determine the operation and unit for the answer.

Enter the correct numbers and calculate, then check the answer.

RIDGES

Read the problem.
I know statement.
Draw a picture.
Goal statement.
Equation development.
Solve the equation.

RICE

Read and record the problem.
Illustrate your thinking.
Compute.
Explain your thinking.

STAR

Stop and read the problem carefully.
Think about your plan and the strategy you will use.
Act. Follow your plan and solve the problem.
Review your answer.

SUPER

Slowly read the story problem twice.
Underline the question and circle the numbers you need.
Picture it. Draw the scenario to show what is happening.
Explain the problem with a number sentence.
Rewrite the answer in a sentence.

SHINES

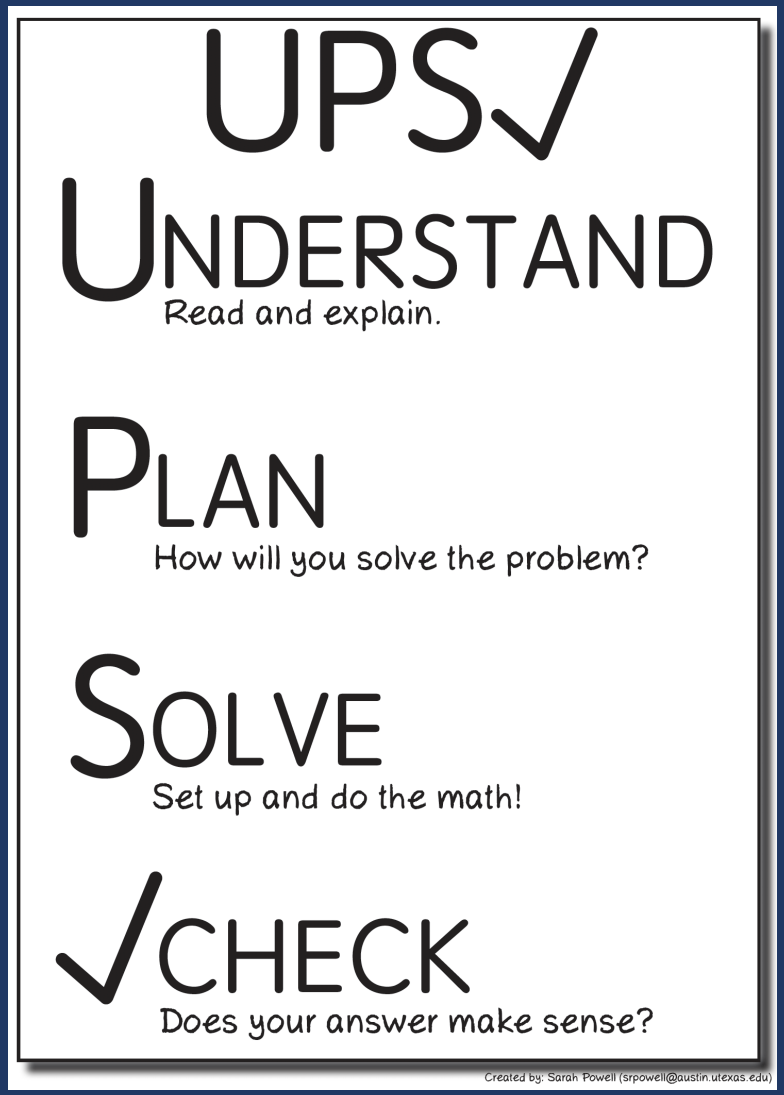
Slowly and carefully read the problem.
Highlight or underline key information.
Identify the question by drawing a circle around it.
Now solve the problem. Show your work.
Examine your work for precision, accuracy, and clarity.
Share your answer by writing a sentence.

R-CUBES

Read the problem.
Circle key numbers.
Underline the question.
Box action words.
Evaluate steps.
Solve and check.

SOLVE

Study the problem.
Organize the facts.
Line up the plan.
Verify the plan with computation.
Examine the answer.



Share your favorite attack strategy.

Teach an attack strategy

Teach about schemas

Total

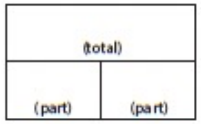
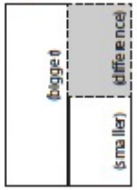
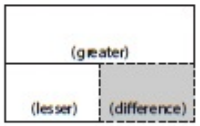
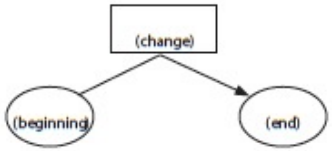
Difference

Change

Equal Groups

Comparison

Ratios/Proportions

Schema and Definition	Equations and Graphic Organizers	Examples			Variations
Total (Combine; Part-part-whole) Parts combined for a sum	P1 + P2 = T (part + part = total)  	Sum unknown: Lyle has 11 red apples and 18 green apples. How many apples does Lyle have altogether?	Part unknown: Lyle has 29 red and green apples. If 11 of the apples are red, how many green apples does Lyle have?		More than two parts: Lyle has 34 apples. Of the apples, 11 are red, 18 are green, and the rest are yellow. How many yellow apples does Lyle have?
Difference (Compare) Sets compared for a difference	B – s = D (bigger – smaller = difference)  G – L = D (greater – less = difference)  	Difference unknown: Sasha wrote 85 words in her essay, and Tabitha wrote 110 words. How many fewer words did Sasha write than Tabitha?	Bigger/greater unknown: Tabitha wrote 25 more words than Sasha. If Sasha wrote 85 words, how many words did Tabitha write?	Smaller/lesser unknown: Tabitha wrote 110 words in her essay. Sasha wrote 25 words fewer than Tabitha. How many words did Sasha write?	(None)
Change (Join; Separate) An amount that increases or decreases	ST +/– C = E (start +/– change = end)   	End (increase) unknown: Jorge had $52. Then, he earned $16 babysitting. How much money does Jorge have now?	Change (increase) unknown: Jorge had $52. Then, he earned some money babysitting. Now, Jorge has $68. How much did Jorge earn babysitting?	Start (increase) unknown: Jorge has some money, and then he earned $16 for babysitting. Now, Jorge has $68. How much money did he have to start with?	Multiple changes: Jorge had $78. He stopped and bought a pair of shoes for $42 and then he spent $12 at the grocery. How much money does Jorge have now?
		End (decrease) unknown: Jorge had $52. Then, he spent $29 at the ballpark. How much money does Jorge have now?	Change (decrease) unknown: Jorge had $52 but spent some money when he went to the ballpark. Now, Jorge has $23. How much did Jorge spend at the ballpark?	Start (decrease) unknown: Jorge had some money. Then, he spent $29 at the ballpark and has $23 left. How much money did Jorge have before going to the ballpark?	

Word-Problem Solving

What are your strengths?

What are your opportunities for growth?

What are your immediate next steps?

Total

Part-part-whole
Combine

Parts put together into a total

Daniela saw 3 canoes and 8 kayaks. How many boats did Daniela see?

Total

Daniela saw 11 boats. If 3 of the boats were canoes, how many were kayaks?

Part

Daniela saw 11 boats. 8 of the boats were kayaks, how many were canoes?

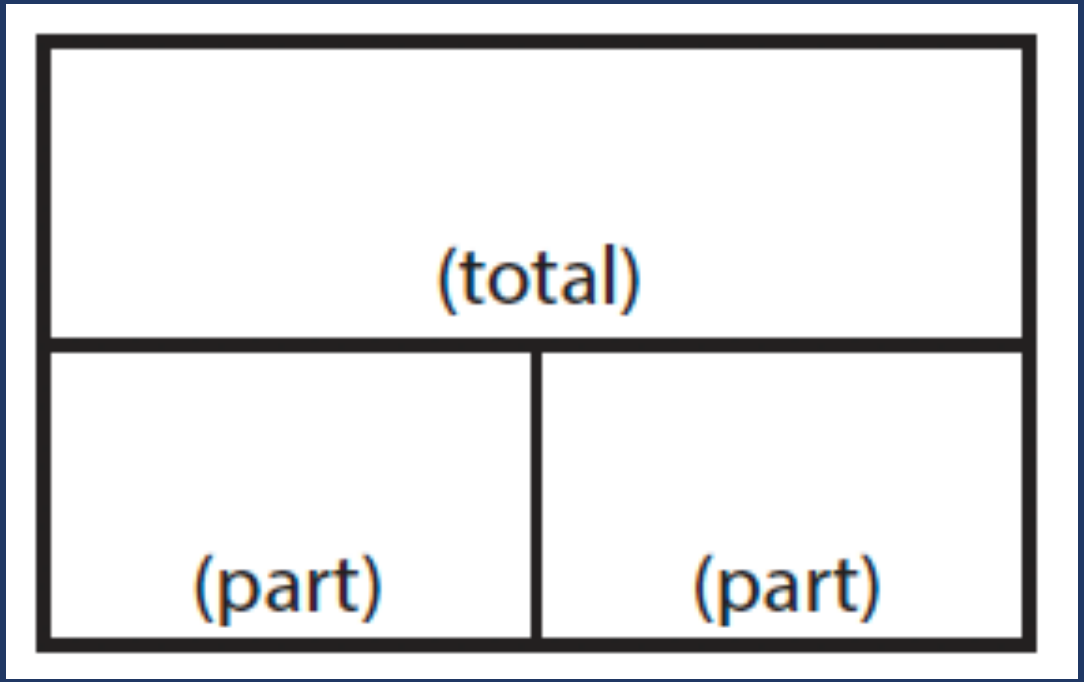
Part

Total

"Are parts put together for a total?"

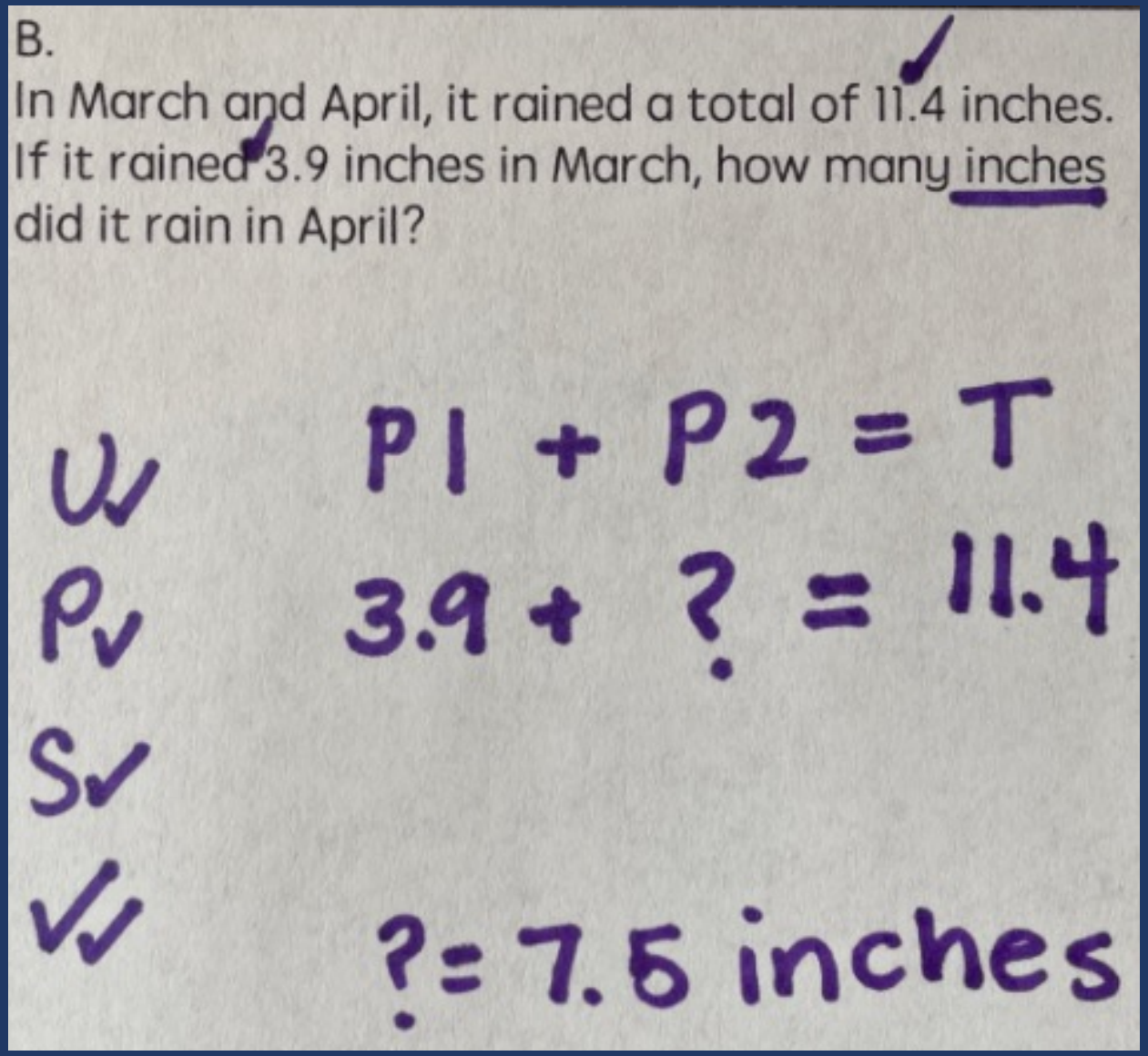
Total

P1 + P2 = T



Fuchs et al. (2008); Griffin & Jitendra (2009)

Total



Total

Share a Total problem.

Difference

Compare

Greater and lesser amounts compared for a difference

Adrianna has 10 pencils. Tracy has 4 pencils. How many more pencils does Adrianna have?

Difference

Adrianna has 6 more pencils than Tracy. If Tracy has 4 pencils, how many does Adrianna have?

Greater amount

Tracy has 6 fewer pencils than Adrianna. Adrianna has 10 pencils. How many pencils does Tracy have?

Lesser amount

Total

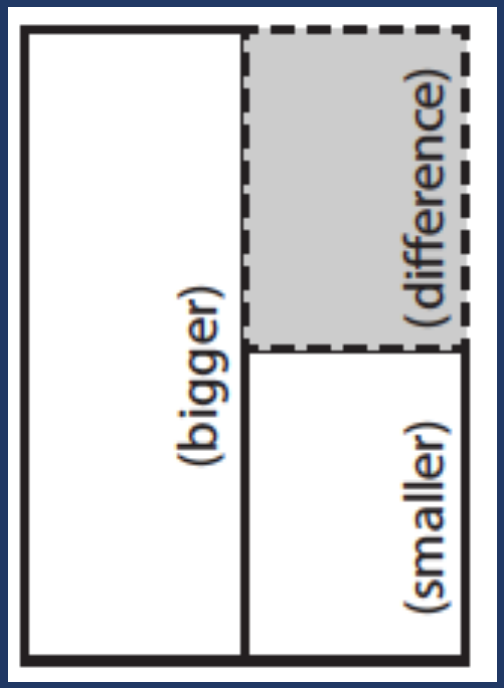
"Are parts put together for a total?"

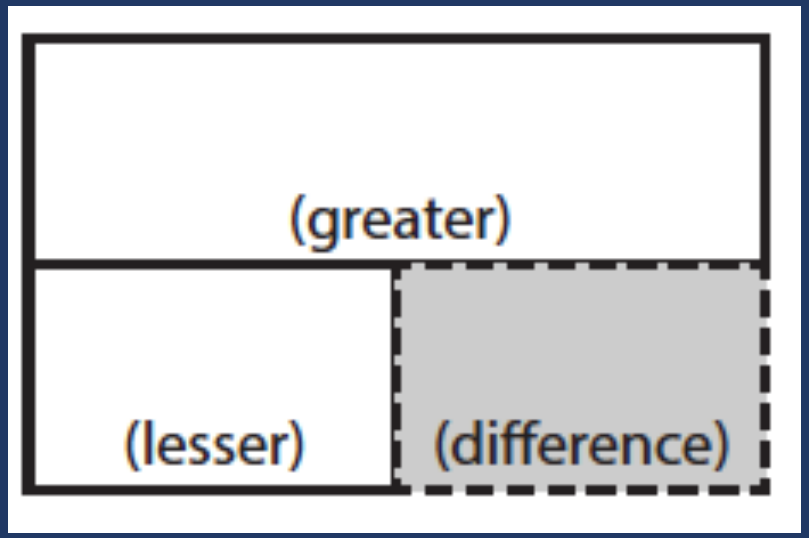
Difference

"Are amounts compared for a difference?"

Difference

G – L = D



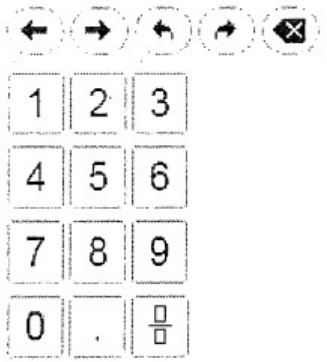


Fuchs et al. (2008); Griffin & Jitendra (2009)

Difference

Jana has 107 wooden beads and 68 glass beads. How many more wooden beads than glass beads does Jana have?

Enter your answer in the response box.



U G - L = D

P 107 - 68 = ?

S ?=39 more wooden beads

✓

Fuchs et al. (2008); Griffin & Jitendra (2009)

Difference

Share a Difference problem.

Change

Join

An amount that **increases** or decreases

Nickole had 6 notebooks. Then, she bought 3 notebooks. How many notebooks does Nickole have now?

Nickole had 6 notebooks. Then, she bought a few more notebooks. Now, Nickole has 9 notebooks. How many notebooks did she buy?

Nickole had some notebooks. Then, she bought 3 notebooks. Now, Nickole has 9 notebooks. How many notebooks did she have to start with?

End amount

Change amount

Start amount

Change

Separate

An amount that increases or **decreases**

Samantha baked 20 cookies. Then, she ate 3 of the cookies. How many cookies does Samantha have now?

End amount

Samantha baked 20 cookies. Then, she ate some of the cookies. Now, she has 17 cookies. How many cookies did Samantha eat?

Change amount

Samantha baked some cookies. She ate 3 of the cookies and has 17 cookies left. How many cookies did Samantha bake?

Start amount

Total

"Are parts put together for a total?"

Difference

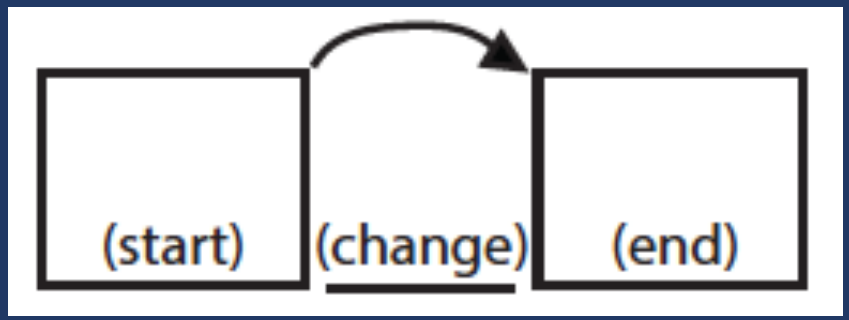
"Are amounts compared for a difference?"

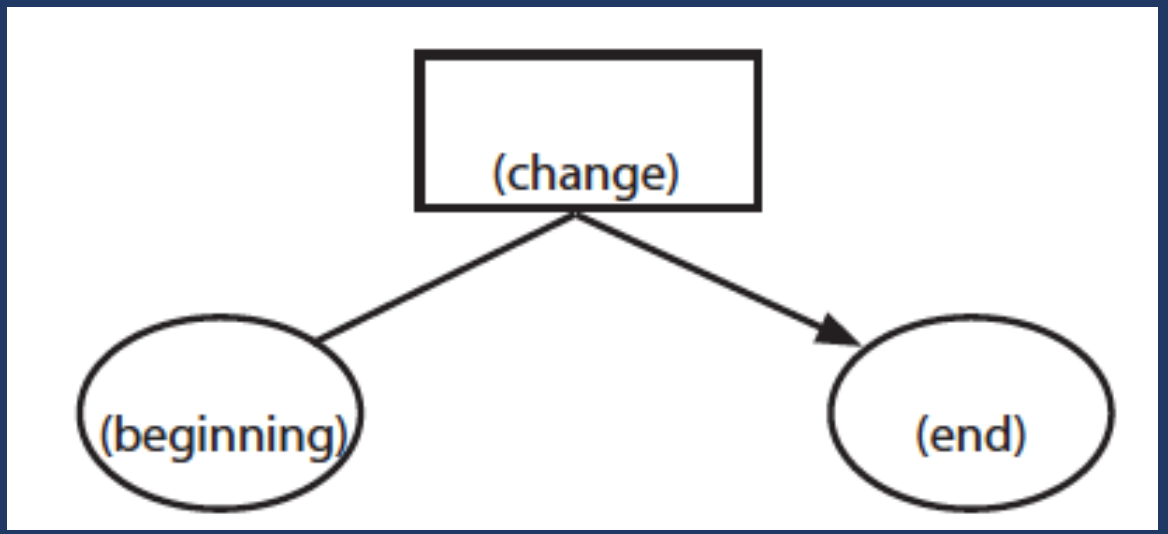
Change

"Does an amount increase or decrease?"

Change

ST +/− C = E

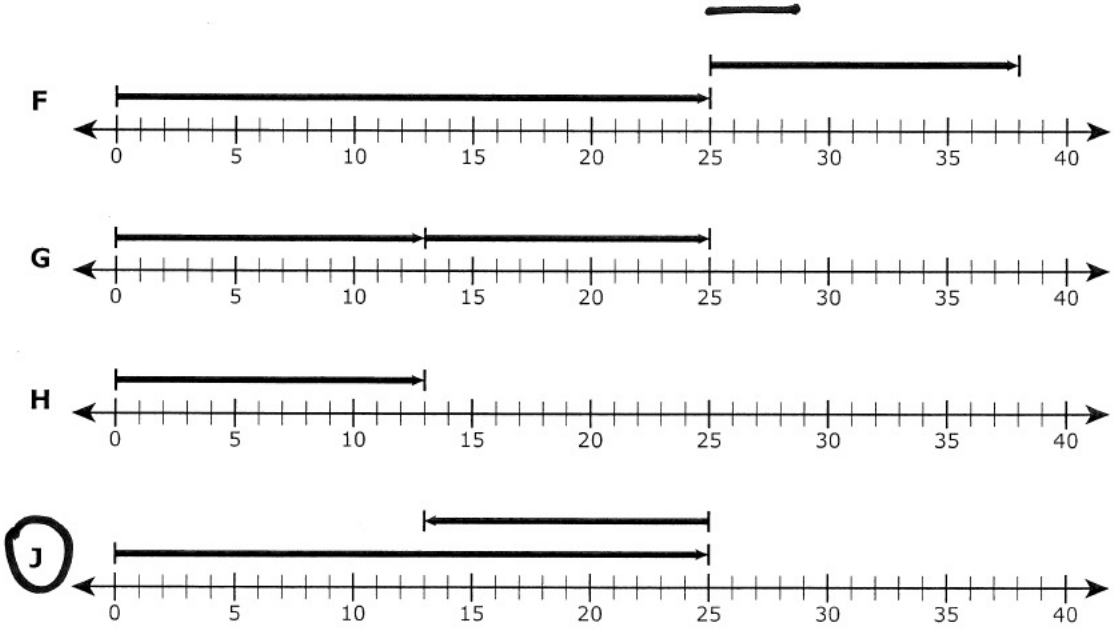




Fuchs et al. (2008); Griffin & Jitendra (2009)

Change

28 There were 25 people in a library. Some people left the library and went home. Then there were 13 people remaining in the library. Which number line represents one way to determine the number of people who left the library?



U

P

S

✓

25 - ? 13

? = 12 people left

Fuchs et al. (2008); Griffin & Jitendra (2009)

Change

Share a Change problem.

Schema Check!

Change

Pablo goes to a stamp show where he can share, buy, and sell stamps.

26. Part A

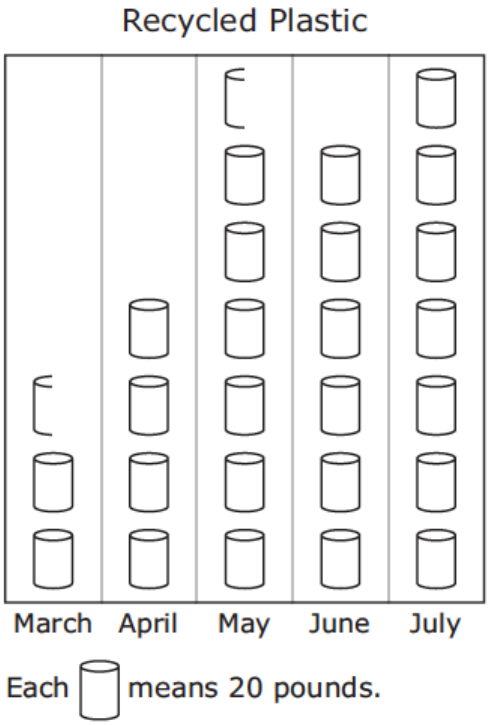
The first day, Pablo starts with 744 stamps. He buys 27 stamps from his friend. He then sells 139 stamps.

What is the total number of stamps that Pablo has after the first day of the stamp show?

Grade 3 PARCC

Difference

The graph below shows the number of pounds of plastic the Keller family recycled for five months.



Based on the graph, how many more pounds of plastic did the family recycle in July than in April?

Grade 3 STAAR

Total

Mr. Conley delivers packages. The bar graph shows the total number of packages he delivered on five days last week.

10. Part A

What is the total number of packages Mr. Conley delivered on Monday and Tuesday?

Ⓐ 300

Ⓑ 340

Ⓒ 350

Ⓓ 360

Grade 3 PARCC

Total

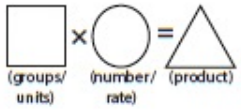
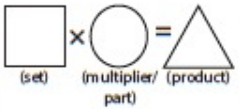
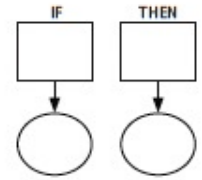
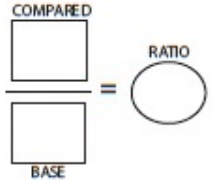
Difference

Change

Equal Groups

Comparison

Ratios/Proportions

Schema and Definition	Graphic Organizers	Examples			Variations
Equal Groups (Vary) A number of equal sets or units		Product unknown: Maria bought 5 cartons of eggs with 12 eggs in each carton. How many eggs did Maria buy?	Groups unknown: Maria bought 60 eggs. The eggs were sold in cartons with 12 eggs each. How many cartons of eggs did Maria buy?	Number unknown: Maria bought 5 cartons of eggs for a total of 60 eggs. How many eggs were in each carton?	With rate: Maria bought 5 cartons of eggs. Each carton cost $2.95. How much did Maria spend on eggs?
Comparison One set as a multiple or part of another set		Product unknown: Malik picked 7 flowers. Danica picked 3 times as many flowers. How many flowers did Danica pick?	Set unknown: Danica picked 3 times as many flowers as Malik. If Danica picked 21 flowers, how many flowers did Malik pick?	Times unknown: Malik picked 7 flowers. Danica picked 21 flowers. How many times more flowers did Danica pick?	With fraction: Malik picked 25 red and yellow flowers. If 1/5 of the flowers were yellow, how many were red?
Proportions		Subject unknown: Sally typed 56 words in 2 minutes. How many words could Sally type in 7 minutes?	Object unknown: Sally typed 56 words in 2 minutes. How many minutes would it take Sally to type 192 words?		With percentage: Watson received an 80% on his science quiz. If the test had 40 questions, how many questions did Watson answer correctly?
		Base unknown: Justin baked cookies and brownies. The ratio of cookies to brownies was 3:5. If he baked 15 cookies, how many brownies did he bake?	Compared unknown: Justin baked cookies and brownies. The ratio of cookies to brownies was 3:5. If he baked 25 brownies, how many cookies did he bake?	Ratio unknown: Justin baked 15 cookies and 25 brownies. What's the ratio of cookies to brownies?	With unit rate: Paula bought 5 boxes of markers. She spent $9.75. What is the price of one box of markers?

Equal Groups

Array
Vary

Groups multiplied by number in each group for a product

Toni has 2 boxes of crayons. There are 12 crayons in each box. How many crayons does Toni have altogether?

Product

Toni has 24 crayons. They want to place them equally into 2 boxes. How many crayons will Toni place in each box?

Number in each group

Toni has 24 crayons. They put them into boxes with 12 crayons each. How many boxes did Toni use?

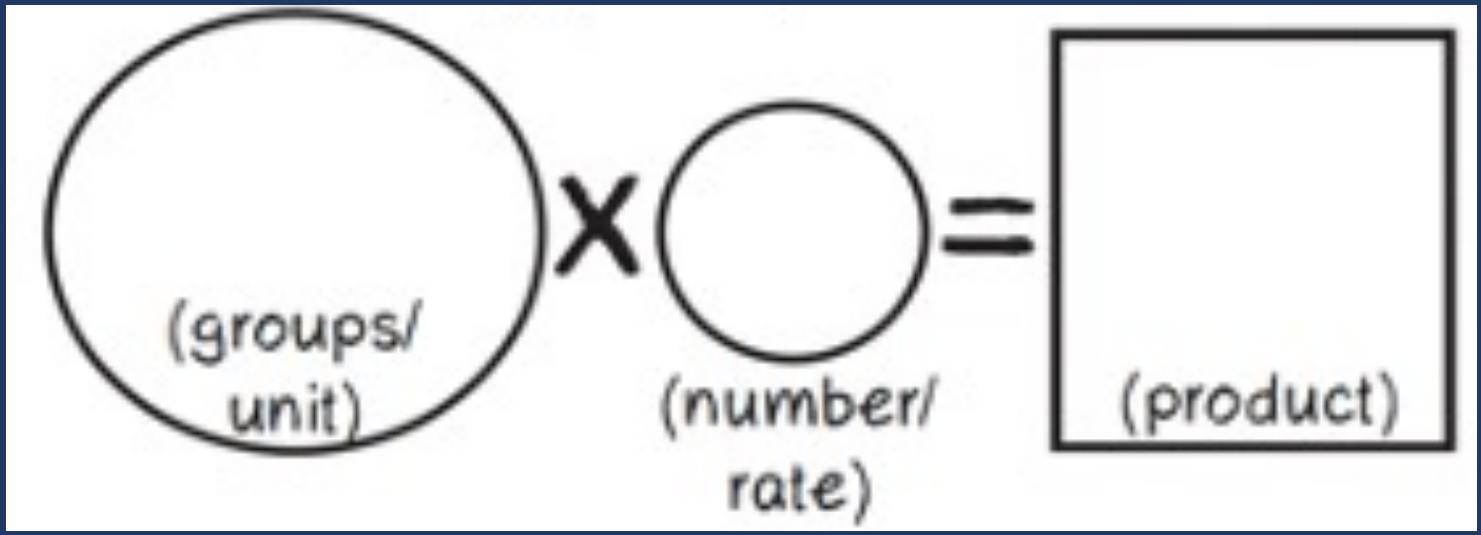
Groups

Equal Groups

"Are there groups with an equal number in each group?"

Equal Groups

GR × N(E) = P



Equal Groups

Jack has 24 fish. He puts them into 4 bowls. Each bowl has an equal number of fish.

How many fish are in each bowl?

U

P

S

✓

4 × ? = 24

$\times \begin{matrix} 4 \\ ? \end{matrix}$

$\overline{24}$

$4\overline{)24}$

? = 6 fish

Equal Groups

Share an Equal Groups problem.

Comparison

Set multiplied by a number of times for a product

Brooke ran 6 minutes. Shaleeni ran 4 times longer than Brooke. How many minutes did Shaleeni run?

Set

Number of times

Product

Equal Groups

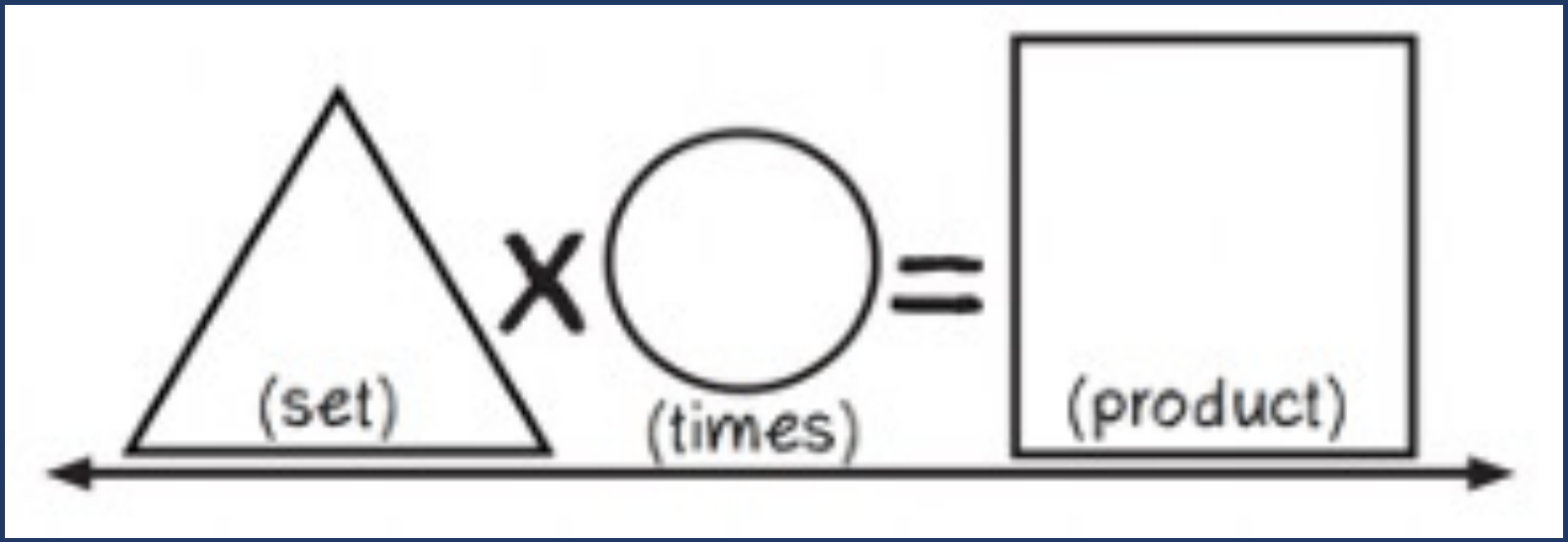
"Are there groups with an equal number in each group?"

Comparison

"Is a set compared a number of times?"

Comparison

S × T = P



Comparison

Susan has 3 times as many books as Mary. Mary has 18 books. Which equation can be solved to figure out how many books Susan has?

(A) $\square - 3 = 18$

(B) $3 + 18 = \square$

(C) $18 \div \square = 3$

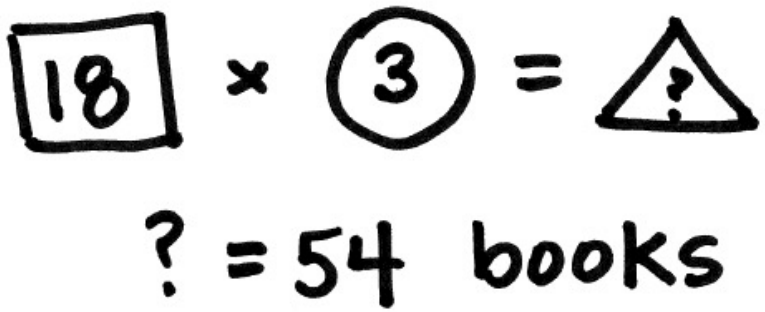
(D) $3 \times 18 = \square$

U

P

S

✓



Comparison

Share a Comparison problem.

Ratios/Proportions

Description of relationships among quantities

Melissa baked cookies and brownies. The ratio of cookies to brownies was 3:5. If she baked 25 brownies, how many cookies did she bake?

Emma typed 56 words in 2 minutes. At this rate, how many words could Emma type in 7 minutes?

Xin et al. (2005)

Equal Groups

"Are there groups with an equal number in each group?"

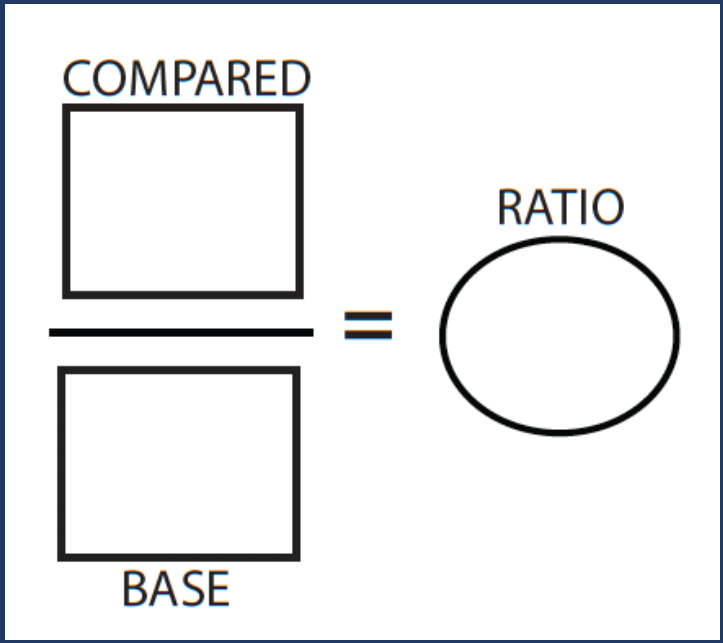
Comparison

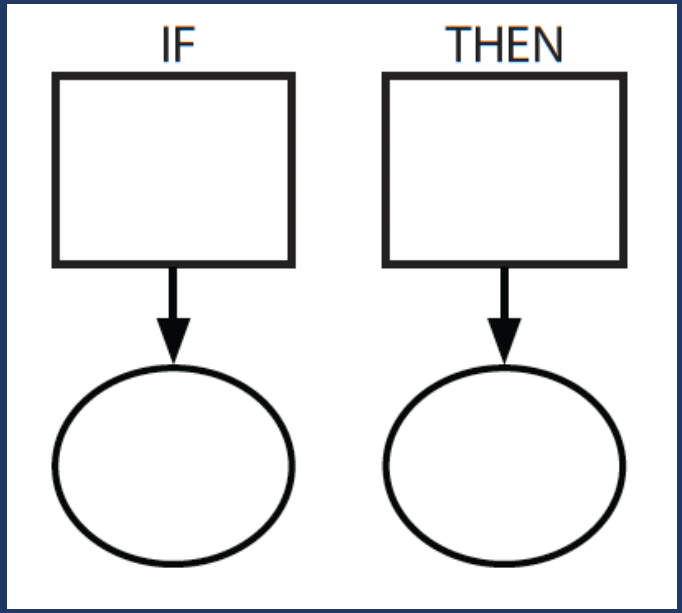
"Is a set compared a number of times?"

Ratios/Proportions

"Are there relationships among quantities - if this, then this?"

Ratios/Proportions





Xin et al. (2005)

Schema Check!

Equal Groups

Mr. Kowolski ordered 35 boxes of granola bars. Each box contained 24 granola bars.

What is the total number of granola bars Mr. Kowolski ordered?

Grade 4 PARCC

Ratios/Proportions

A company makes 625 cell phone cases each day. How many cell phone cases does the company make in 31 days?

Grade 5 STAAR

Comparison

Danielle's full-grown dog weighs 10 times as much as her puppy. The puppy weighs 9 pounds.

Enter the number of pounds the full-grown dog weighs.

Grade 4 Smarter Balanced

Total

Difference

Change

Equal Groups

Comparison

Ratios/Proportions

Teach an attack strategy

Teach about schemas

Students should learn how to set up and solve word problems by combining an attack strategy with a focus on word-problem schemas.

What are your strengths with word-problem solving?

What are the opportunities for growth?

What's one thing you can start doing next week?

Teachers should use systematic and explicit instruction to help students develop a strong foundation for specific mathematics skills.

Students require modeling and practice on how to use the language of mathematics.

Students should use hands-on tools, virtual manipulatives, drawings, and other visuals to understand mathematics concepts and procedures.

Teachers should use fluency building activities to build counting fluency and fluency with the operations.

Students should learn how to set up and solve word problems by combining an attack strategy with a focus on word-problem schemas.

Pirate Math Equation Quest

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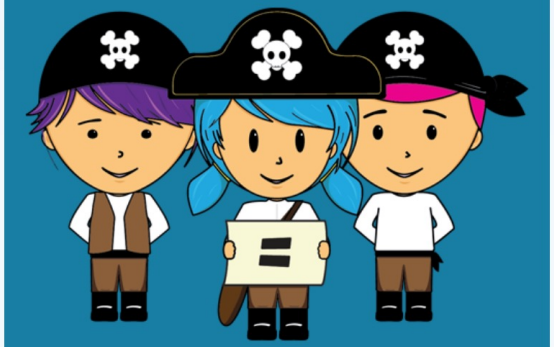
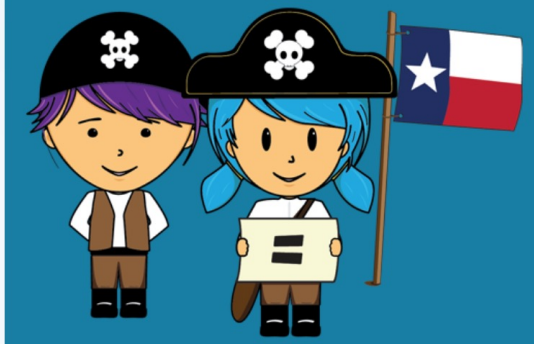
Welcome to Pirate Math Equation Quest!

Individual Word-Problem Intervention

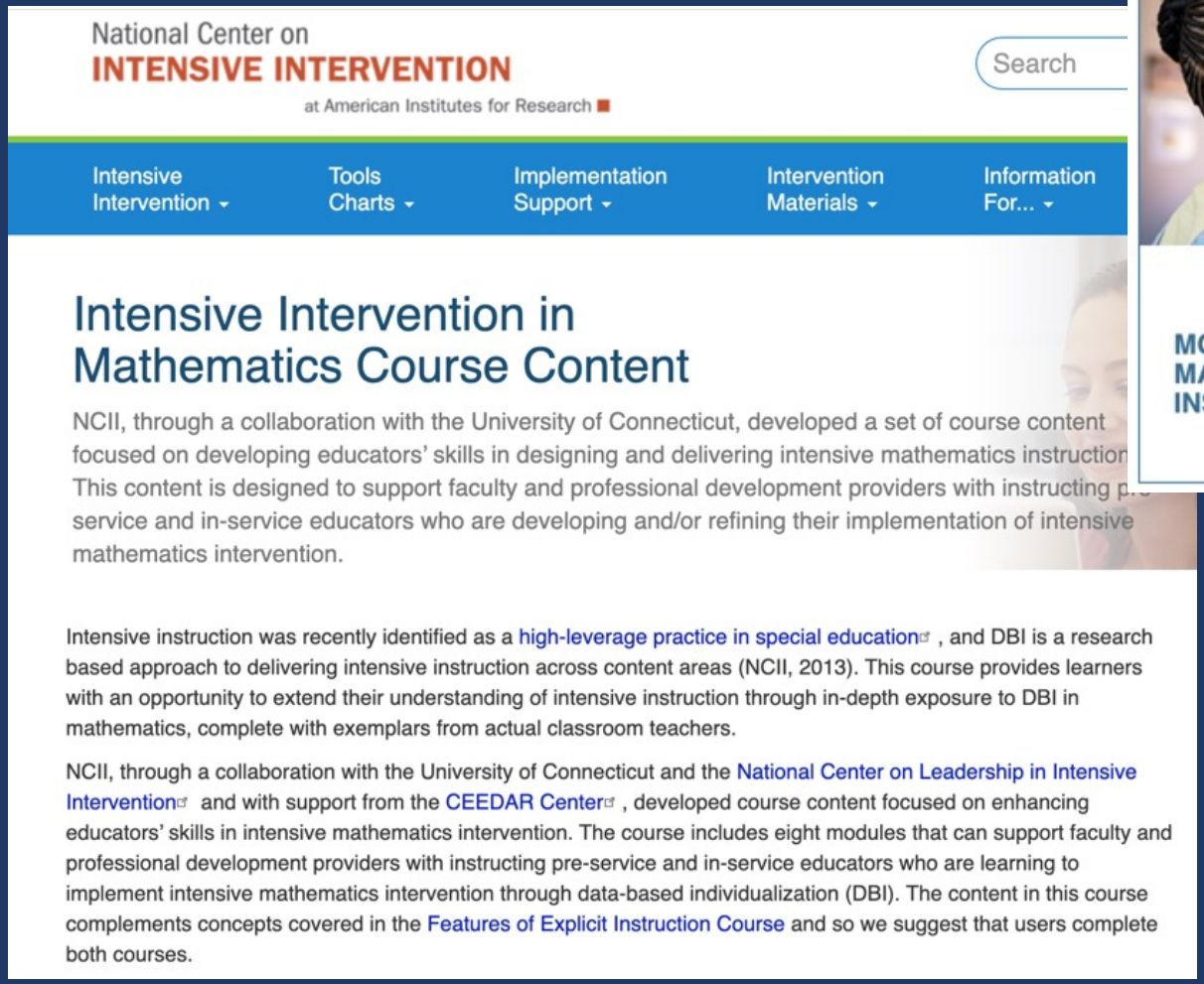
Small-Group Word-Problem Intervention

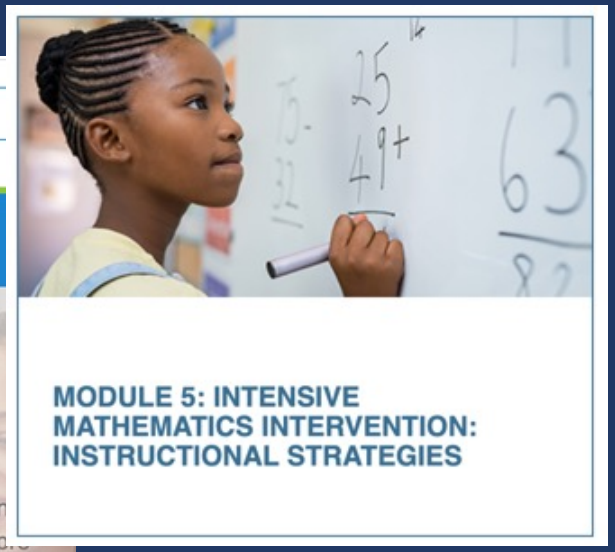


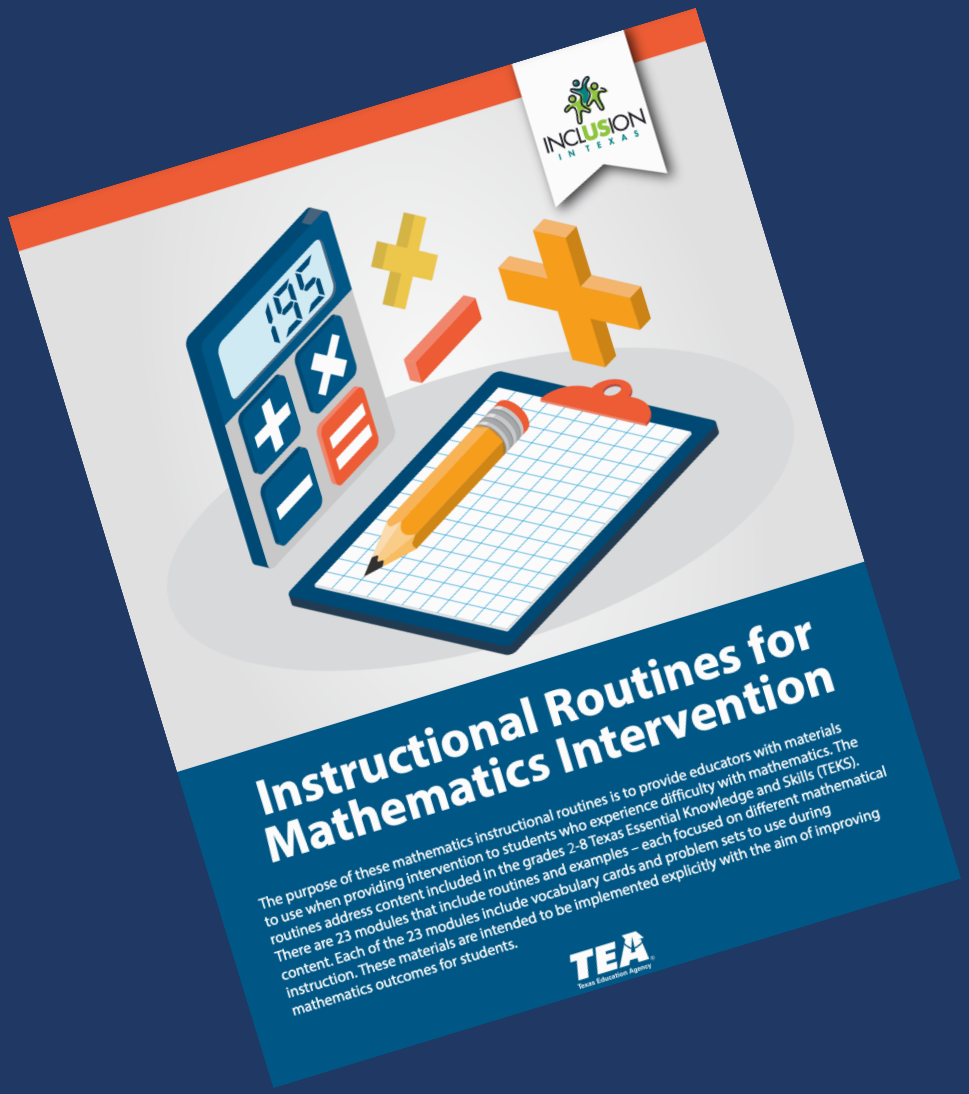
Small-Group Word-Problem Intervention for STAAR



https://intensiveintervention.org/intensive-intervention-math-course

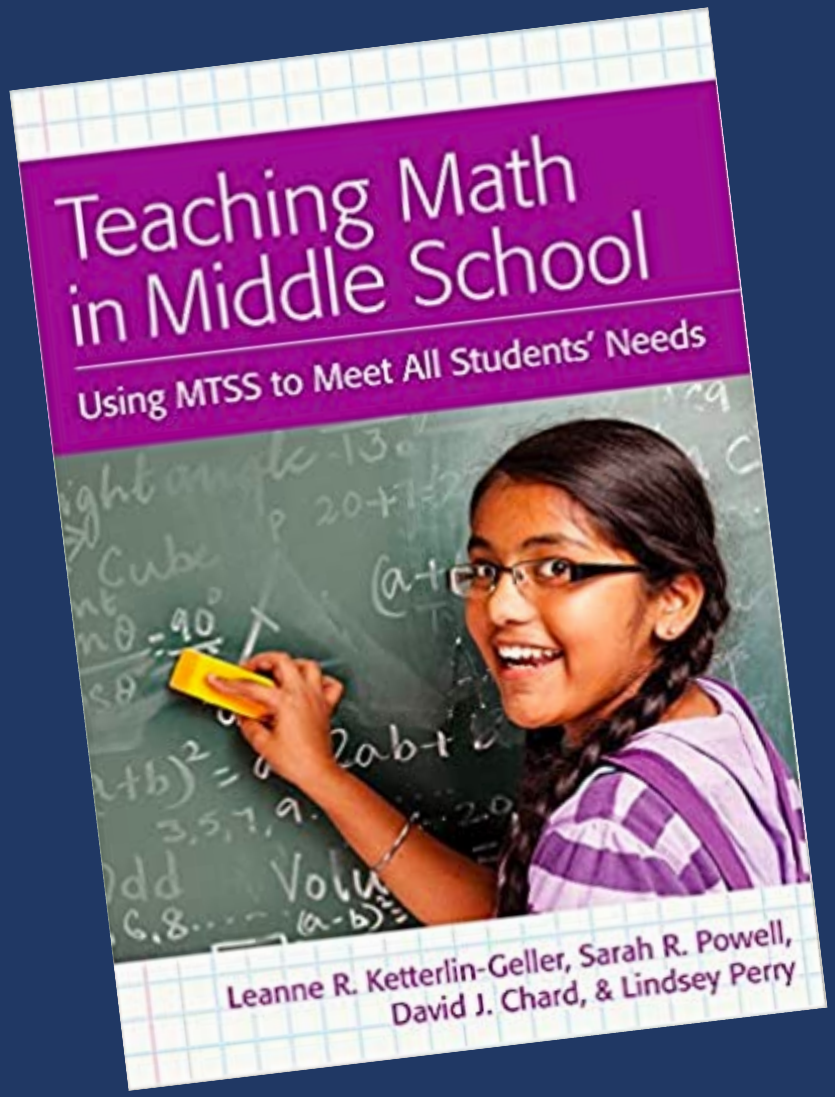






https://www.inclusionintexas.org/apps/pages/index.jsp?uREC_ID=2155039&type=d&pREC_ID=2169859

https://www.amazon.com/Teaching-Math-Middle-School-Students/dp/1598572741



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