Math 1497 - Sample Test 3

1. Sketch the following parametric curve and find the equation of the tangent at the point of self intersection

$$x = \frac{1+t+t^2-t^3}{1+t^2}, \quad y = \frac{2}{1+t^2}.$$

2. Graph the following polar equations

$$r = 2 + 2\sin\theta, \quad r = 2\sin 2\theta, \quad r^2 = 2\sin 2\theta.$$

3. Find the area inside one leaf of the rose described by

$$r = 2\sin 3\theta.$$

4. Find the area of the following:

(i) inside $r = 2 + 2\sin\theta$,
(ii) inside the outer loop and outside the inner loop of $r = 1 - 2\sin\theta$,
(iii) outside $r = \cos 2\theta$ and inside $r = \sin 2\theta$ on $\left[0, \frac{\pi}{2}\right]$.

5. Find the projection of the vector $\overrightarrow{u}$ onto $\overrightarrow{v}$ where $\overrightarrow{u} =< 2, 3 >$, and $\overrightarrow{v} =< 4, 2 >$. Sketch both vectors, the projected vector and the orthogonal complement.

6. Find the area of the triangle whose vertices are located at the points $(1,1,1)$, $(2,4,6)$ and $(-2,3,7)$.

7. (i) Find the equation of the plane that contains the vector $< 1,2,4 >$ and the points $(1,1,1)$ and $(-2,3,7)$.
   (ii) Find the equation of the plane that contains the points $(1,3,5)$, $(2,-1,2)$ and $(0,4,6)$.

8. (i) Find the equation of the line that passes through the points $(1,2,4)$ and $(-2,3,7)$.
   (ii) Find the equation of the line perpendicular to the plane $x + 2y - 3z = 6$ passing through the point $(1,-1,3)$.

9. Sketch and name the following surfaces

$$(i)\ \ y - z^2 = 1, \qquad (ii)\ \ -x^2 + y^2 + z^2 = 1, \qquad (iii)\ \ x^2 - y + z^2 = 0.$$