



CEC
2024
SPECIAL EDUCATION
CONVENTION & EXPO
SAN ANTONIO

What's Important in Mathematics at Tier 2 and 3



Sarah R. Powell, Ph.D.

Professor
College of Education
The University of Texas at Austin



srpowell@utexas.edu



[@sarahpowellphd](https://www.instagram.com/sarahpowellphd)



Introduce yourself.

Describe your strengths in supporting mathematics.

Describe an opportunity for growth.

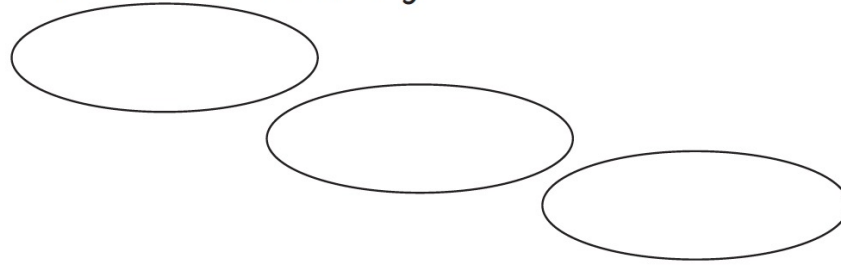
Instructional Platform

What's Important in Mathematics at Tier 2 and 3?

srpowell@utexas.edu @sarahpowellphd
www.sarahpowellphd.com

Instructional Platform

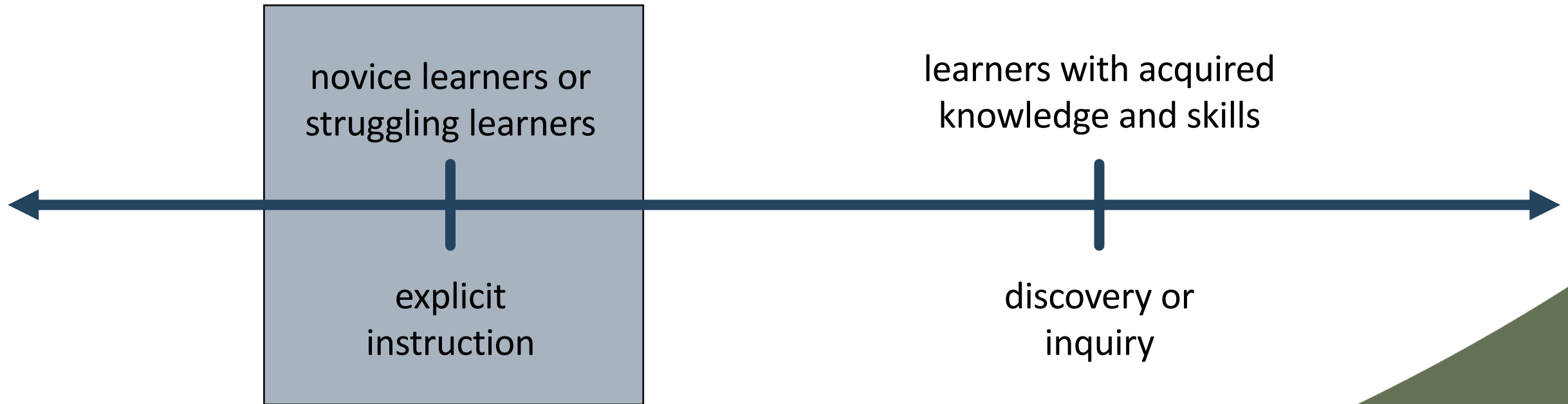
Instructional Delivery



Instructional Strategies



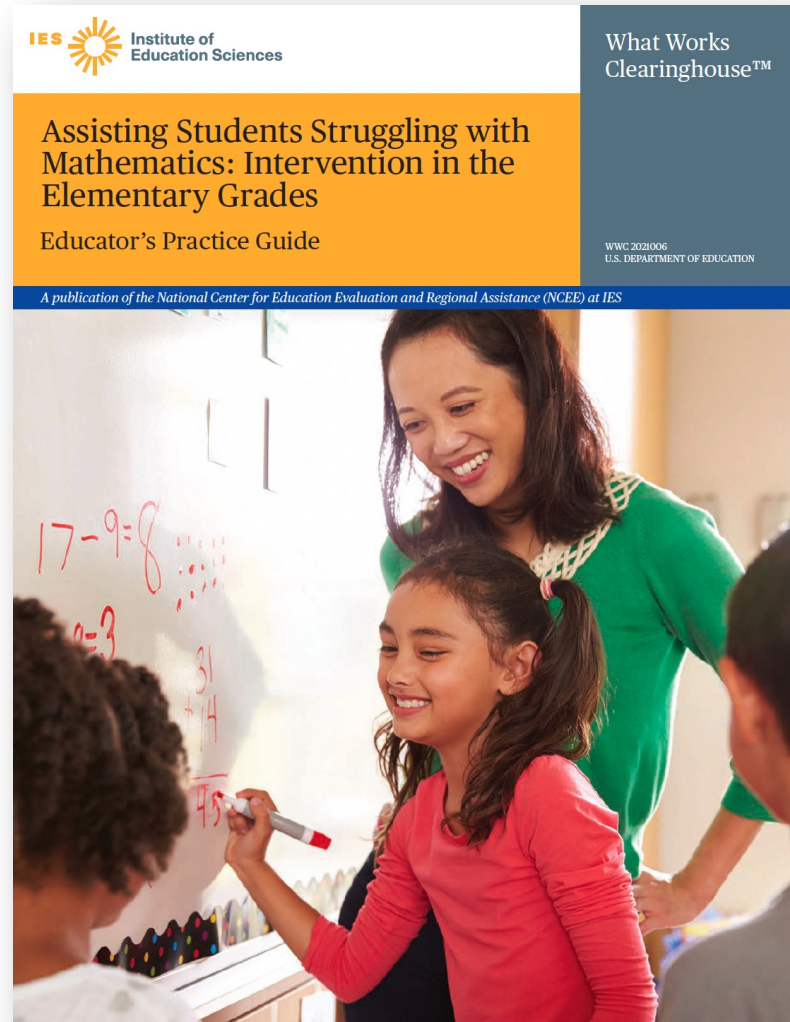
What's the continuum of mathematics support?



Anita Archer (2019)



Instructional Platform



Students require modeling and practice on how to use the language of mathematics.

Students should use hands-on tools, virtual manipulatives, drawings, and other visuals to understand mathematics concepts and procedures.

Teachers should use systematic and explicit instruction to help students develop a strong foundation for specific mathematics skills.

Teachers should use fluency building activities to build counting fluency and fluency with the operations.

Students should learn how to set up and solve word problems by combining an attack strategy with a focus on word-problem schemas.

Instructional Platform

INSTRUCTIONAL DELIVERY

Vocabulary

Representations

Model and Practice

INSTRUCTIONAL STRATEGIES

Fluency

Word Problems

Vocabulary

VOCABULARY

Research and Information

Use Formal Mathematics Language

Instead of that...	Say this...



Instructional Platform

INSTRUCTIONAL DELIVERY

Vocabulary

INSTRUCTIONAL STRATEGIES



Significant correlation ($r = .49$) between mathematics vocabulary and mathematics performance. Mathematics vocabulary appears most important for word-problem performance ($r = .58$).

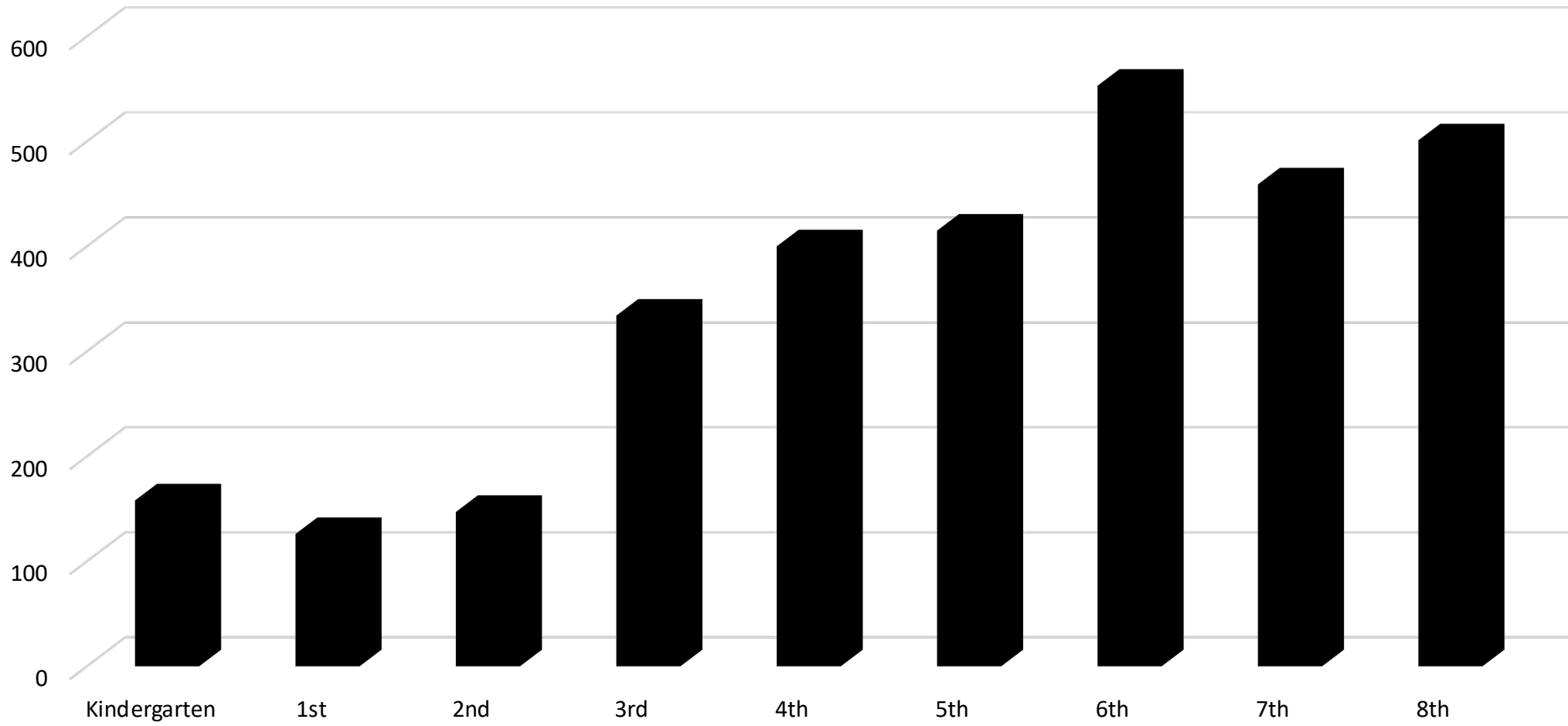
(Lin et al., 2021)

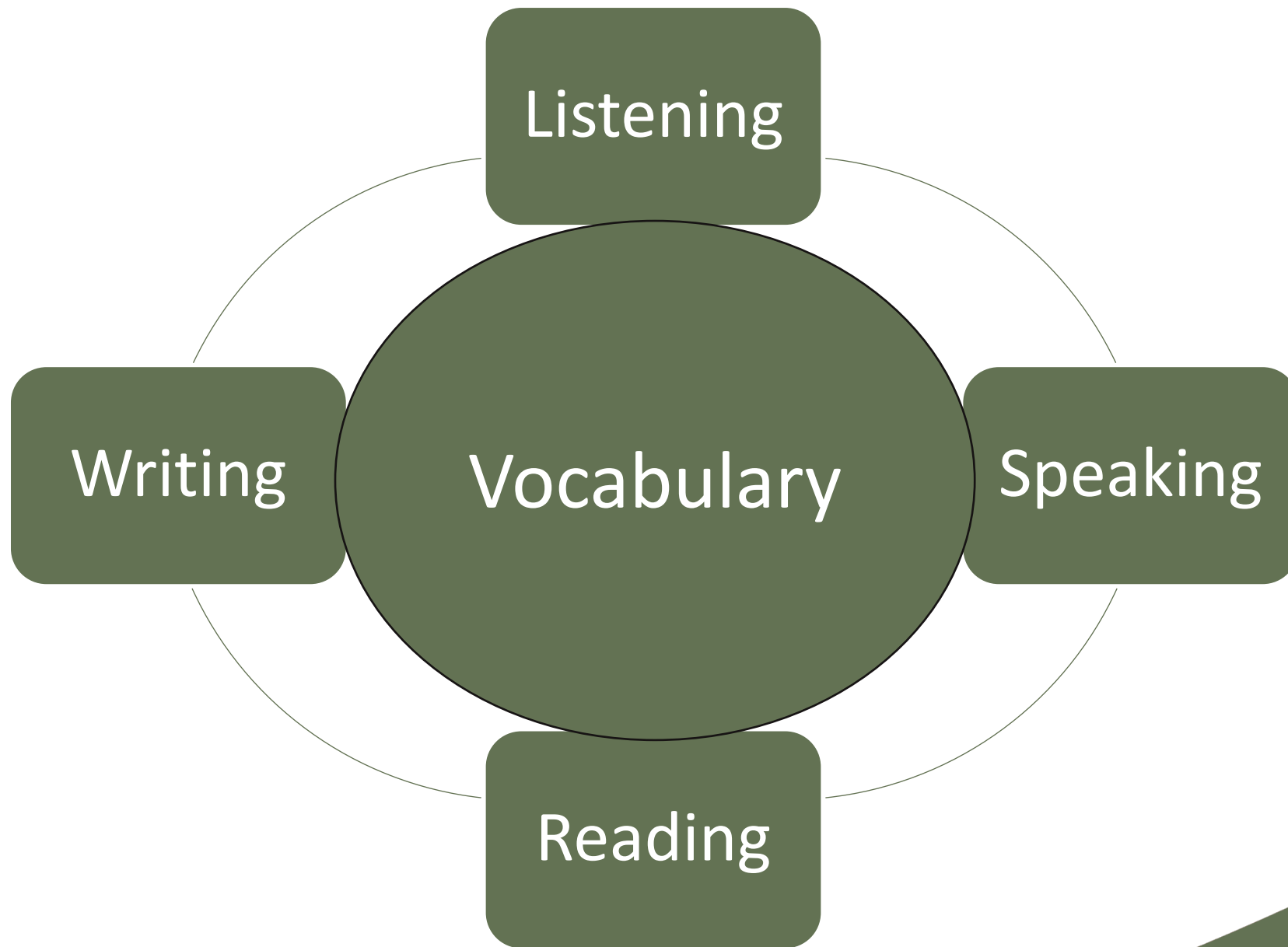
Early mathematics vocabulary related to mathematics and literacy.

(Hornburg et al., 2018; Purpura et al., 2017)

Students who experience difficulty with mathematics demonstrate lower mathematics vocabulary performance.

(Hughes et al., 2020; Powell & Nelson, 2017; Powell et al., 2017; Unal et al., 2021)





1. Some math terms are shared with English but have different meanings

base

right

degree

Rubenstein & Thompson (2002)



1. Some math terms are shared with English but have different meanings
2. Some math words are shared with English with similar meanings (but a more precise math meaning)

difference

even

1. Some math terms are shared with English but have different meanings
2. Some math words are shared with English with similar meanings (but a more precise math meaning)
3. Some math terms have more than one meaning

round

square

second

base

1. Some math terms are shared with English but have different meanings
2. Some math words are shared with English with similar meanings (but a more precise math meaning)
3. Some math terms have more than one meaning
4. Some math terms are only used in math

trapezoid

numerator

parallelogram

1. Some math terms are shared with English but have different meanings
2. Some math words are shared with English with similar meanings (but a more precise math meaning)
3. Some math terms have more than one meaning
4. Some math terms are only used in math
5. Some math terms are homophones

eight vs. ate

sum vs. some

rows vs. rose

base vs. bass

1. Some math terms are shared with English but have different meanings
2. Some math words are shared with English with similar meanings (but a more precise math meaning)
3. Some math terms have more than one meaning
4. Some math terms are only used in math
5. Some math terms are homophones
6. Some math terms are related but have distinct meanings

**factor vs.
multiple**

**hundreds vs.
hundredths**

**numerators vs.
denominator**

1. Some math terms are shared with English but have different meanings
2. Some math words are shared with English with similar meanings (but a more precise math meaning)
3. Some math terms have more than one meaning
4. Some math terms are only used in math
5. Some math terms are homophones
6. Some math terms are related but have distinct meanings
7. Some math concepts are verbalized in more than one way

one-fourth vs.
one quarter

skip count vs.
multiples



1. Some math terms are shared with English but have different meanings
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4. Some math terms are only used in math
5. Some math terms are homophones
6. Some math terms are related but have distinct meanings
7. Some math concepts are verbalized in more than one way
8. Some informal terms may be used for formal math terms

vertex vs.
corner

rhombus vs.
diamond







Why might your students have difficulty with math vocabulary?

Use formal math language

Use terms precisely




The alligator eats the
bigger number




is less than
OR
is greater than

Why this is important...

- Students must learn how to read and write the inequality symbols.
- Students must learn to read equations correctly from left to right because $<$ and $>$ are two distinct symbols.



What number is in
the tens place?



What digit is in the tens place?
What is the value of the digit in
the tens place?

135

Why this is important...

- A number refers to the entire amount.
- The 3 in the tens place value is not a number, but rather a digit in the number 135.
- Reinforces conceptual understanding of place value.
- Emphasizes that 3 is part of the number 135 with a value of 30.



carry OR borrow



regroup OR
trade OR
exchange

$$\begin{array}{r} 167 \\ + 294 \\ \hline \end{array}$$

Why this is important...

- “Carry” or “borrow” is procedural.
- The other terms reinforce the conceptual understanding or regrouping ones into tens, tens into hundreds, and so on (i.e., the total amount does not change) *or* ungrouping hundreds into tens, tens into ones, and so on.





top number and
bottom number



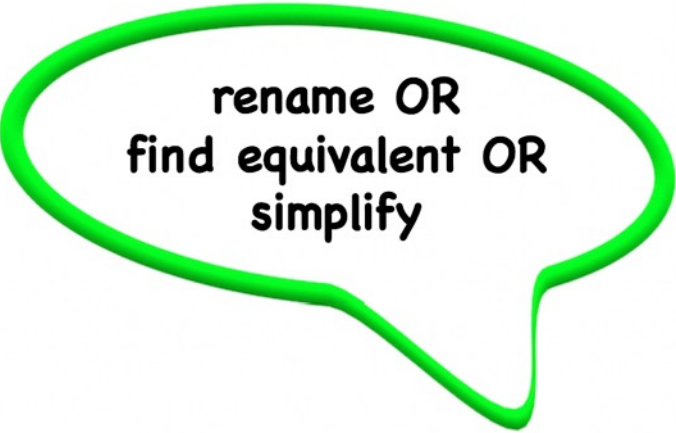

numerator and
denominator

Why this is important...

- Identifying that there are two separate (whole) numbers suggests that whole number properties can be applied to fractions.
- Emphasizing that a fraction is ONE number with ONE magnitude on a number line that is communicated with a numerator and denominator is important.




reduce the fraction




rename OR
find equivalent OR
simplify

Why this is important...

- Reducing suggests that the quantity or magnitude of the new number will be less than the original number.



Four point seven
Four point oh seven



Four and seven tenths
Four and seven hundredths

4.7
4.07

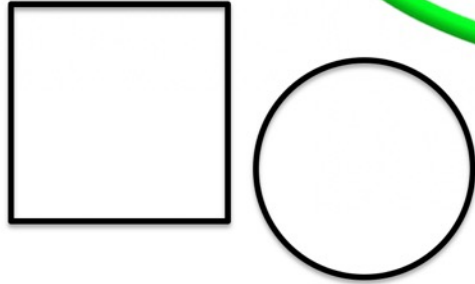
Why this is important...

- Accurately shares the magnitude of the decimal.
- Emphasizes place value.



box OR ball

square OR
circle



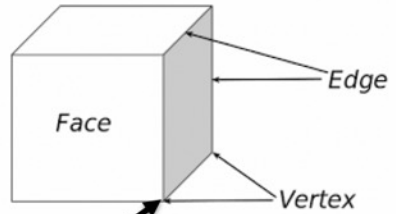
Why this is important...

- Use the formal language of shapes to confirm informal language.



point

vertex



Why this is important...

- This is the endpoint where two or more line segments or rays meet.





flips, slides, turns



reflections,
translations,
rotations

Why this is important...

- The informal language helps children remember the actions, but this vocabulary is not used on assessments.
- Use the formal mathematical terms.



long hand and
short hand

minute hand and
hour hand



Why this is important...

- The informal language describes the length of clock hands but not the meaning.
- Help students understand the hours and minutes.



Identify examples of “Instead of _____, say _____.”

Use formal math language

Use terms precisely

VOCABULARY

Use Terms With Precision

Strategies for Teaching Mathematics Language



What are your strengths?



What are your opportunities for growth?



What are your plans for next Monday?
Next month?
Next year?



Factor

$$1 \times 8 = 8$$

$$2 \times 4 = 8$$

factor factor

Multiple

$$8 \times 1 = 8$$

$$8 \times 2 = 16$$

multiples of 8

E

Improper fraction

$$\frac{8}{5}$$

Mixed number

$$1\frac{3}{5}$$

Proper fraction

$$\frac{2}{9}$$

Proportion

$$\frac{2}{5} = \frac{8}{20}$$

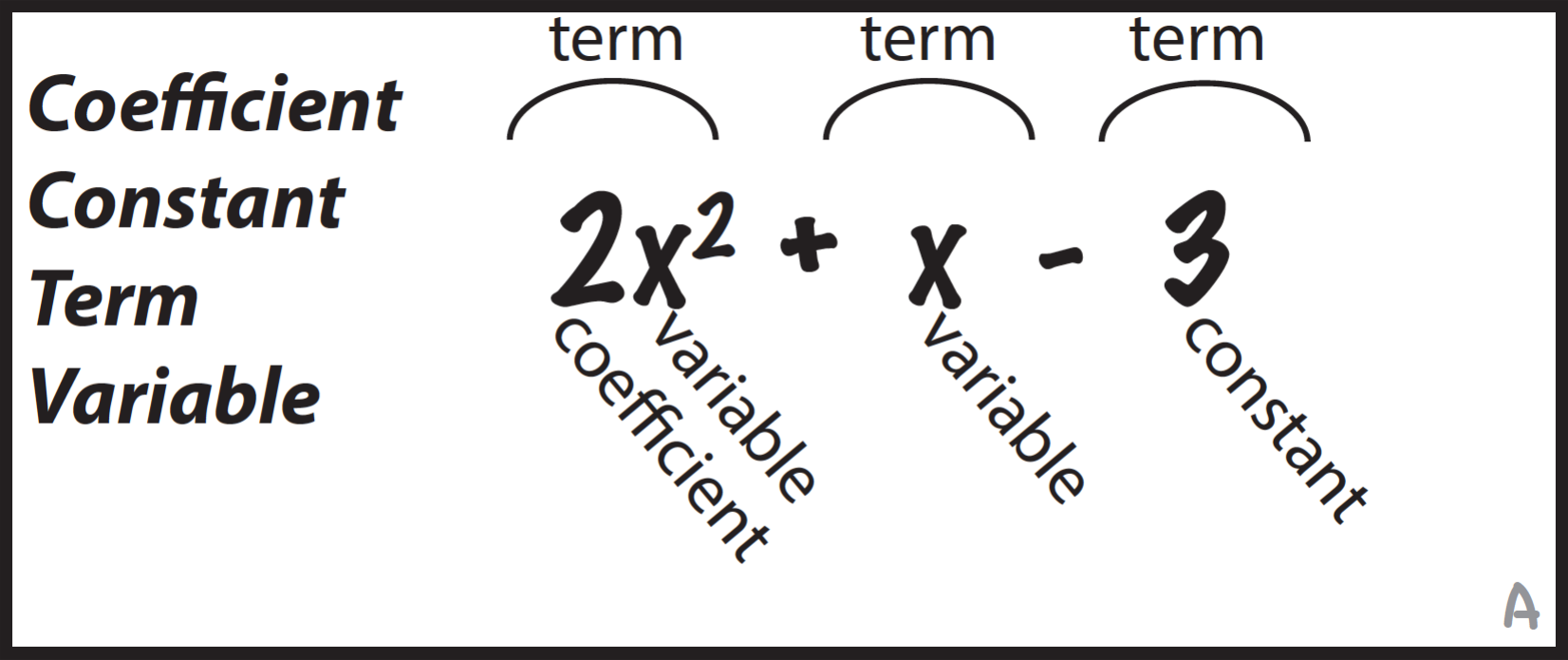
Ratio

$$4:3$$

Unit fraction

$$\frac{1}{6}$$

D



A

Equation $9x - 4 = 7x$

Expression $9x - 4$

Formula $a^2 + b^2 = c^2$

Function $f(x)$

Inequality $9x - 4 > 6x$

c

Quadrilaterals

Kite



Rhombus



Parallelogram



Square



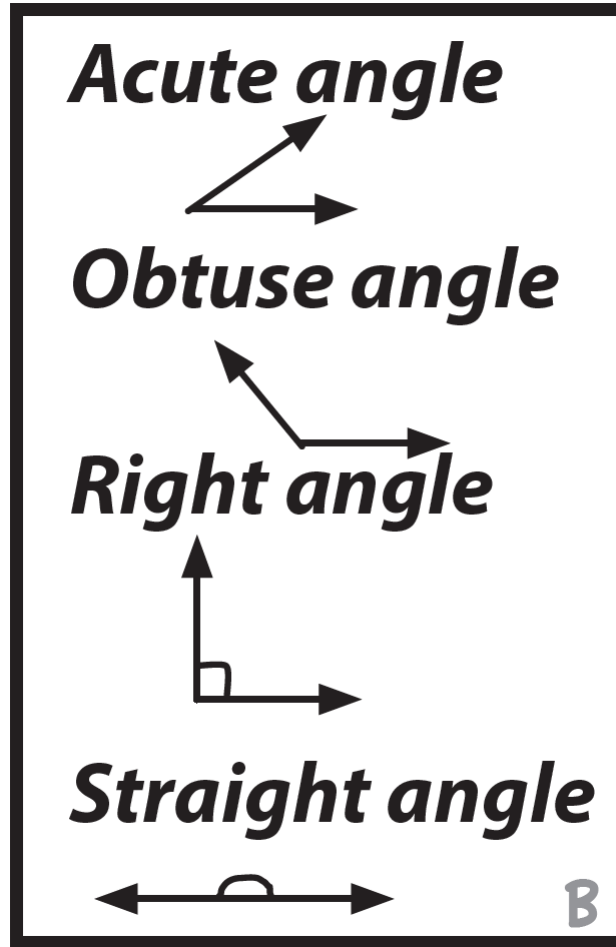
Rectangle



Trapezoid



A



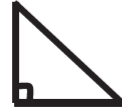
Acute triangle



Obtuse triangle



Right triangle



Equilateral triangle



Isosceles triangle

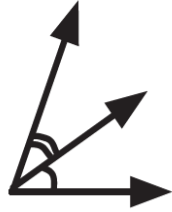


Scalene triangle

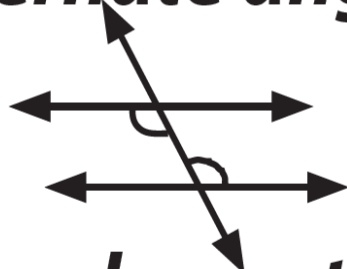


C

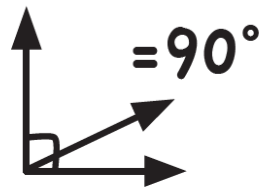
Adjacent angles



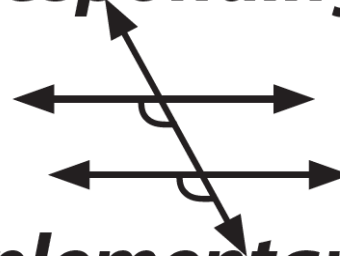
Alternate angles



Complementary angles



Corresponding angles

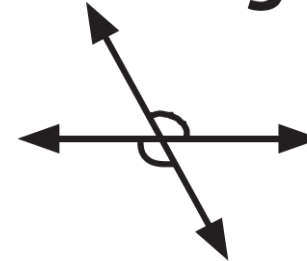


Supplementary angles

= 180°



Vertical angles



D

Congruent figures

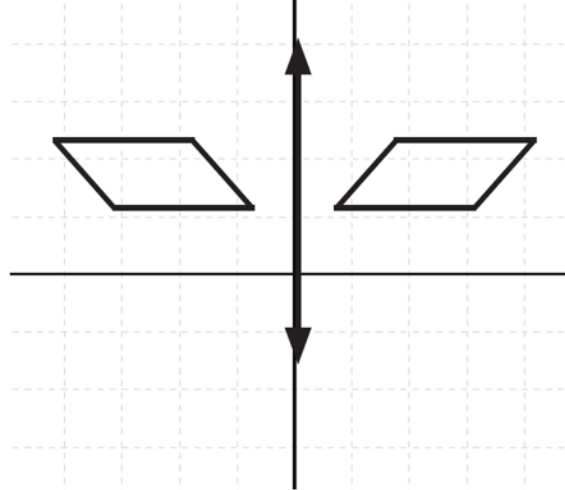


Similar figures

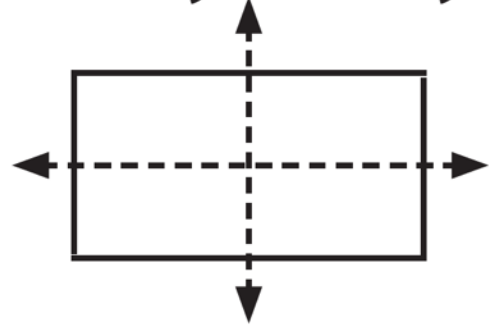


E

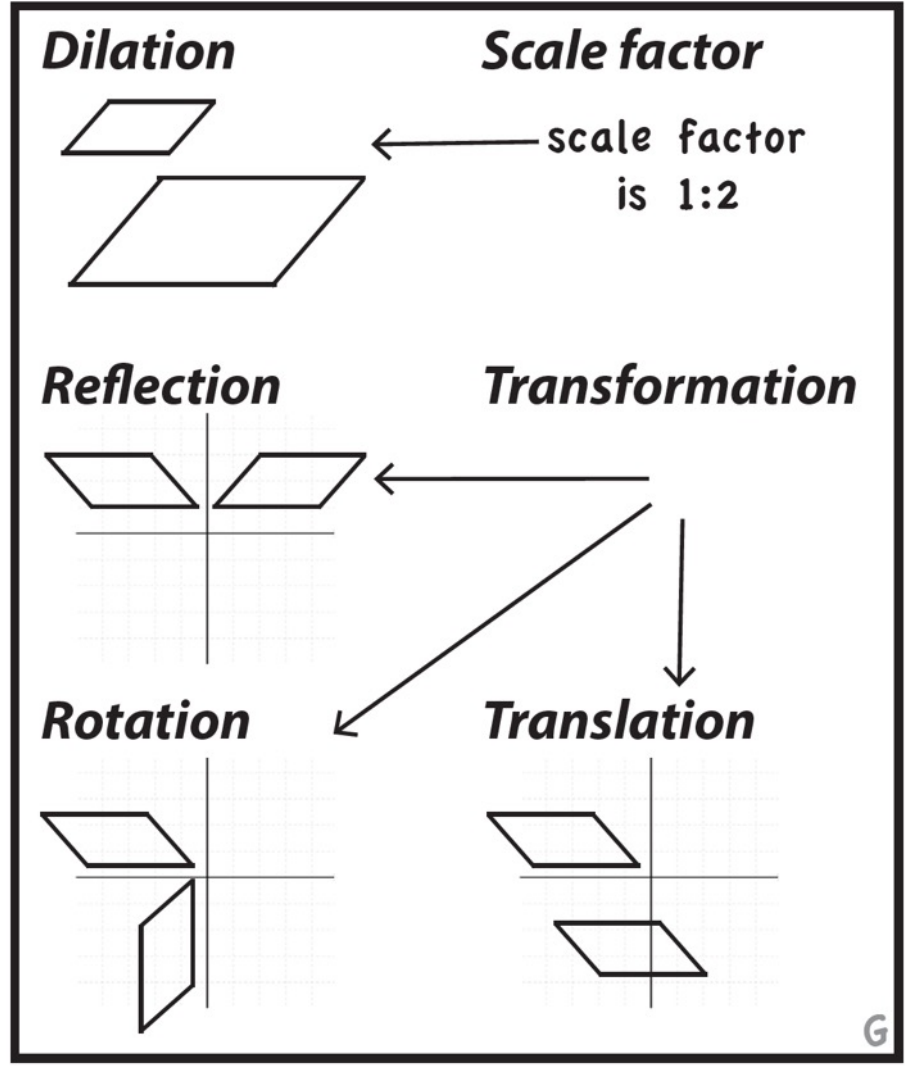
Line of reflection

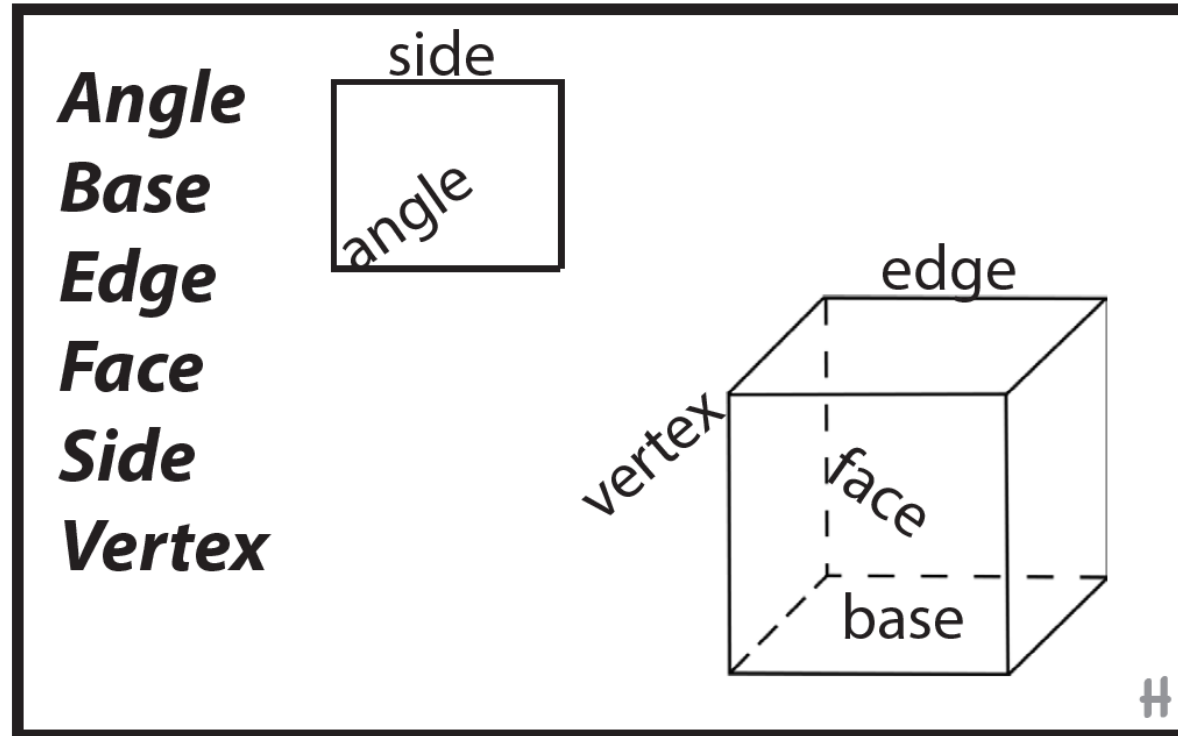


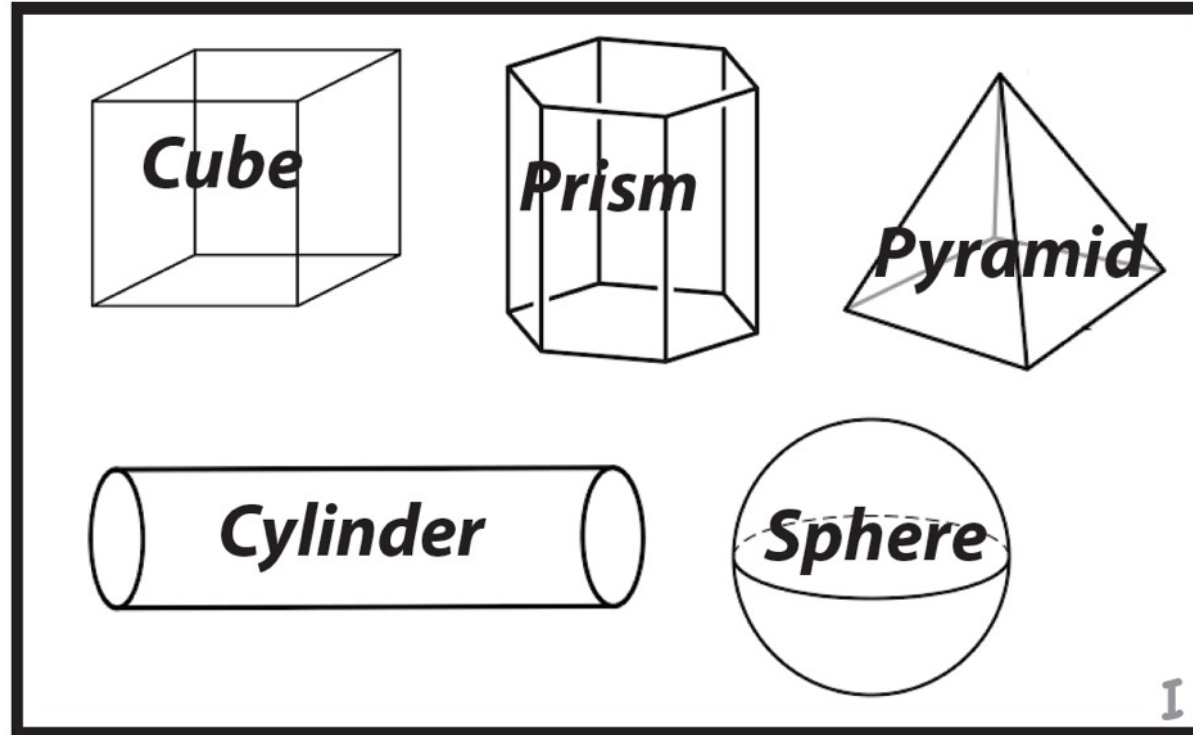
Line of symmetry



F







Coordinate plane

Ordered pair Quadrant 2

Quadrants

x

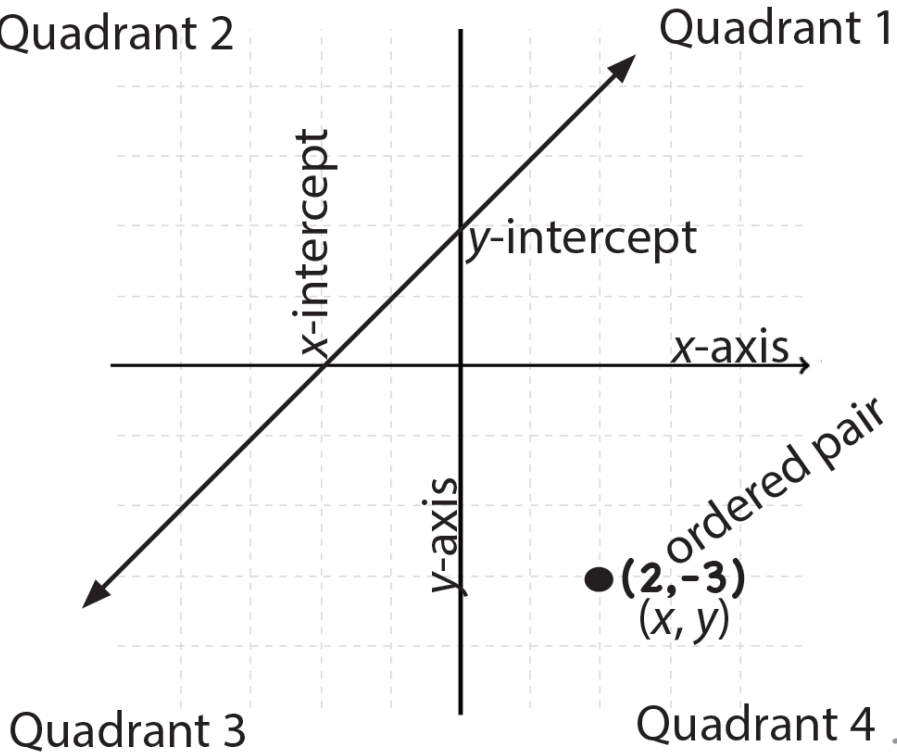
x-axis

x-intercept

y

y-axis

y-intercept



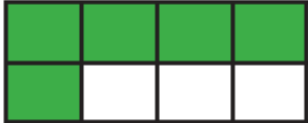


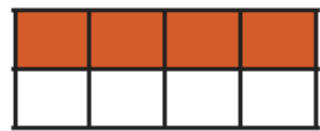


Discuss terms you want your students to use with precision.

Use formal math language

Use terms precisely

Use semantic maps

Term	Definition	Example	Nonexample
integer	(... -3, -2, -1, 0, 1, 2, 3...)	15	$\frac{1}{3}$
denominator	The equal parts of a whole or set.	$\frac{5}{8}$ <p>8 is the denominator</p> 	$\frac{5}{8}$ 
numerator	The equal parts of a given fraction.	$\frac{5}{8}$ <p>5 is the numerator</p> 	$\frac{5}{8}$ 

Use word walls

difference

The result of subtracting or the result when comparing two numbers.

$$5 - 4 = 1$$

1 is the difference

horizontal line

A straight line that goes from left to right or right to left.



equivalent

Two numbers that have the same value.

$$\frac{2}{4} \text{ is equivalent to } \frac{1}{2}$$

total

The result or sum when adding numbers.

$$5 + 7 = 12$$

Use flash cards

addend

quotient

divisor

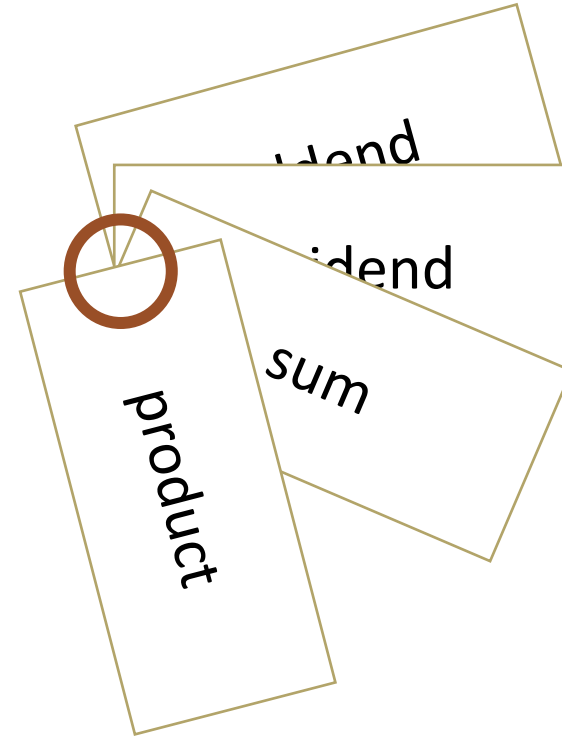
dividend

factor

sum

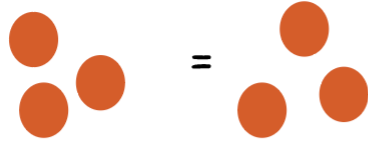
product

difference



Use glossaries

equal - with the same value

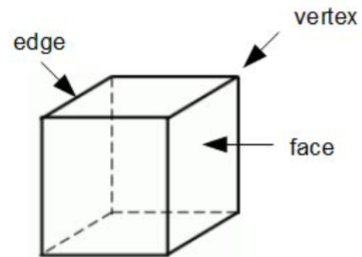


E

equation - expressions with an equal sign

$$5 = 2 + 3$$

edge - line segment between faces



Glosario de Matemáticas

Término	Definición
adición (<i>add</i>)	Juntar o agregar.
algoritmo (<i>algorithm</i>)	Es una serie de pasos organizados que describe el proceso que se debe seguir, para dar solución a un problema específico.
alineal (<i>array</i>)	Un conjunto de objetos, imágenes o números alineados en columnas y filas.
ancho (<i>width</i>)	La medida de un lado de un objeto, generalmente el lado más corto.
ángulo (<i>angle</i>)	Dos semirrectas o segmentos de línea recta que comparten un punto final.
ángulo agudo (<i>acute angle</i>)	Un ángulo que mide menos de 90°.
ángulo obtuso (<i>obtuse angle</i>)	Un ángulo que mide entre 90° y 180°.
ángulo recto (<i>right angle</i>)	Un ángulo que mide exactamente 90°.
área (<i>area</i>)	La cantidad de unidades cuadradas que cubre una figura geométrica cerrada.
balance presupuestario (<i>balance the budget</i>)	Un presupuesto es cuando la cantidad total de dinero gastado, ahorrado y compartido es igual al ingreso total.
báscula (<i>balance scale</i>)	Instrumento de medición que se utiliza para medir el peso o la carga.
base (computación) (<i>base (computation)</i>)	Un número que se multiplica por un exponente.
base (geometría) (<i>base (geometry)</i>)	La forma inferior de una figura tridimensional.



Use anchor charts

Addition Computation

$$\begin{array}{r} 1 \\ 17 \\ + 59 \\ \hline 76 \end{array}$$

← addend
← addend
← sum

Quadrilaterals

Kite



Rhombus



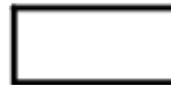
Parallelogram



Square



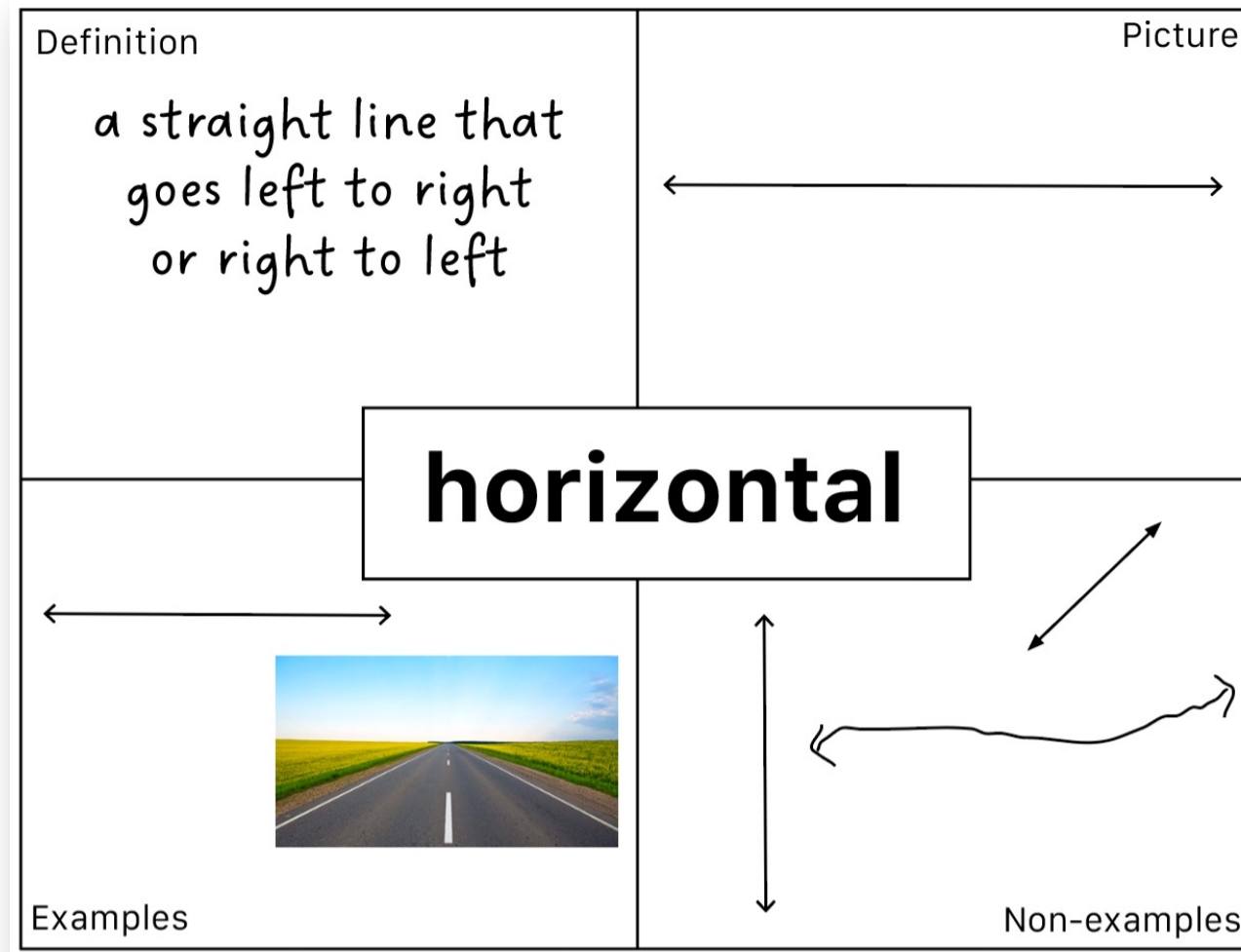
Rectangle



Trapezoid



Use graphic organizers



Use games

Name: _____ Date: _____

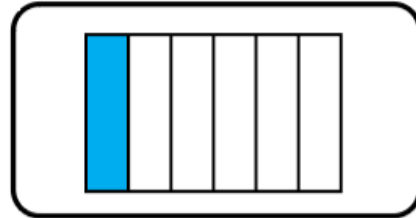
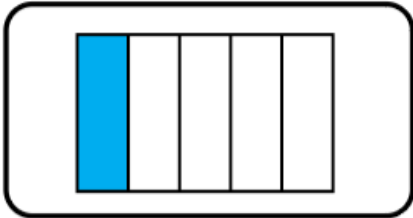
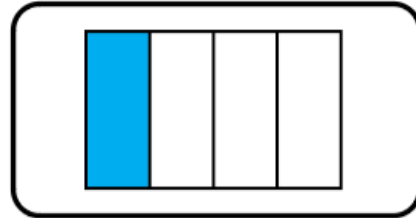
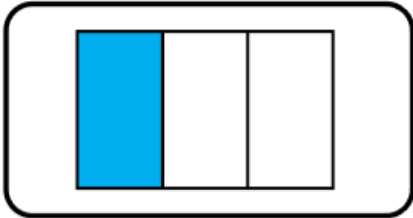
3RD GRADE MATH

T W D H P T R I Y T S F T O Y T S P X G U C I C
M S G C E N T I M E T E R S Y S C Y Z D R G D J
I M I E U J R L H F O A W Z X U A L F J O Q F T
U P F N Z L R V C N N P Z D D H L P H F A V J Z
Y R U P M I A C O R D P I V J T E I H Y M F M S
E K X H R T T Y U J E S V X S V A T Z W M S E T
W Z Z W G E Q G M P C V C Z E H E L W L T T A A
N U V V B R J F H Y N E O Z H T K U K F A Z S W
Q Y J P S H A C D X A M Y G C U K M Z L J E U P
T Z W B P N R Z Y F L I Y D N A J J Z M W D R S
O W W U W T G I T I A T F F I S Q G X K K I E Z
E W R Q C S U U C U B J L E Z P B H G B C V M Q
F R A C T I O N S K Z O W M D X B M M C O I E C
N Z C A T L I J B J M U F O X X A D O Y L D N K
D E V P T U C M Y S A H K Z S U O W R S C V T G
M X N L N S O L V E R R I Y I B Z N Q O X C P W
M V W Z C D Q Z Z U G X L U K P Y O B S W I M N
K F L J U R J G J R O Q E M X C U O H N M S F D
S U B T R A C T G U L Y Y A K W U N U S O L H H
E G F D D A I O I A I Y R R J I C A P A C I T Y
I L W B A T R D O X K C J G H T D X E G C S M Z
X J Q Y A H L T B J R A I W J B C K I U S S I U
N U O D N R L V T I T L C U P C L W C B F S Z Q
J F N P R R K P Y M R E L U R Q H M A E Y Y Z H

ADD	BALANCED	CAPACITY	CENTIMETERS
CLOCK	DIVIDE	FRACTIONS	GRAM
INCHES	KILO GRAM	LITER	MEASUREMENT
MULTIPLY	NOON	RULER	SCALE
SOLVE	SUBTRACT	TIME	

square	circle	decagon	triangular prism	cone
cylinder	cube	sphere	line	pyramid
parallelogram	octagon	FREE	trapezoid	oval
kite	pentagon	cylinder	rectangle	line segment
hexagon	rhombus	triangle	quadrilateral	rectangular prism

Use games



one

one-half

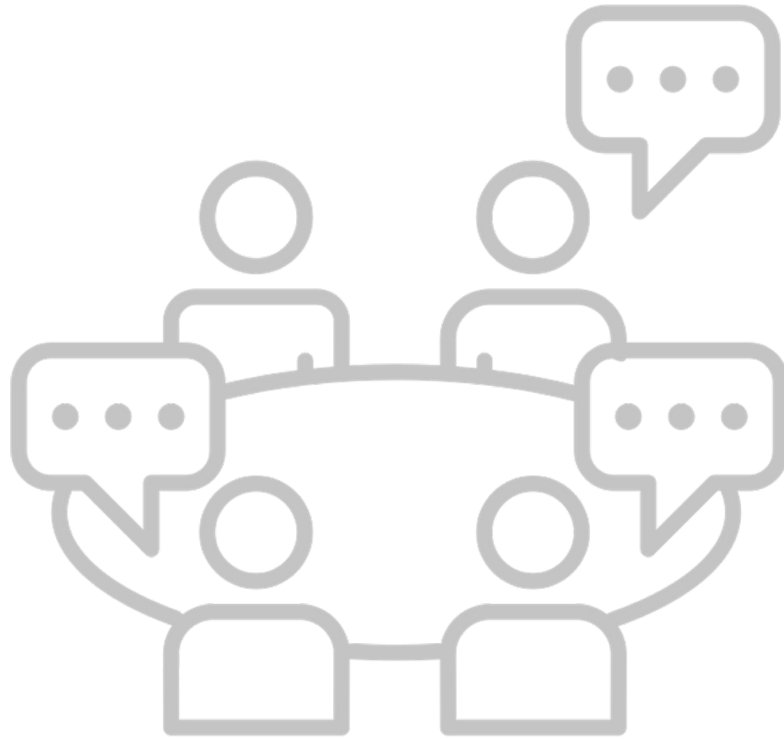
one-third

one-fourth

one-fifth

one-sixth

Use discussion



add
change
compare
decrease
difference
increase
part
put together
subtract
total

Use technology

The screenshot shows the Flocabulary website interface. At the top left is the Flocabulary logo with the tagline 'BY nearpod'. To the right of the logo are navigation links: 'Lessons', 'Mixes', and 'About', followed by a search icon. Further right are buttons for 'Join a Class', 'Log In', and 'Free Trial'. Below the navigation is a teal header with the word 'Math' in white. The main content area features seven topic cards, each with an image and a title:

- Addition & Subtraction**: Image showing a purple plus sign and an orange minus sign.
- Multiplication & Division**: Image showing a pink multiplication sign and a green division sign.
- Numbers & Operations**: Image showing a collage of various numbers.
- Expressions & Equations**: Image showing a yellow pencil on a notebook with mathematical equations.
- Geometry & Measurement**: Image showing various colorful geometric shapes like pyramids, cylinders, and spheres.
- Statistics & Probability**: Image showing three dice with numbers 1, 2, and 3.
- Ratios & Proportional Relationships**: Image showing a pie chart with a 25% orange slice and a 75% green slice.

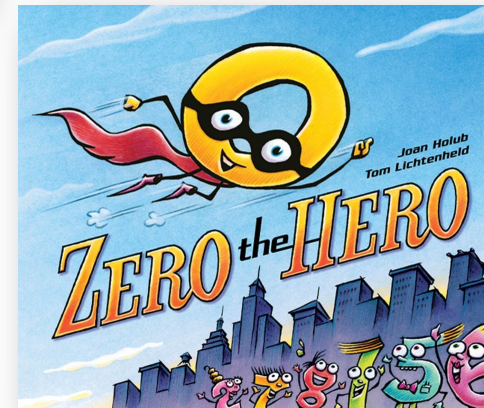
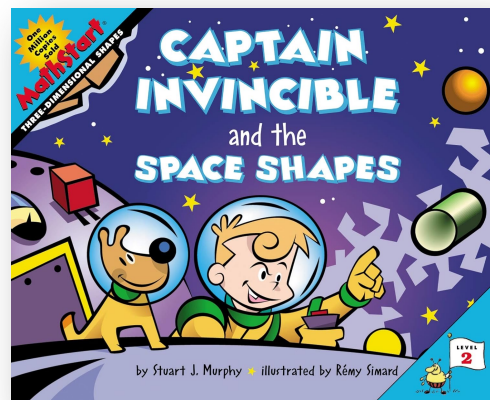
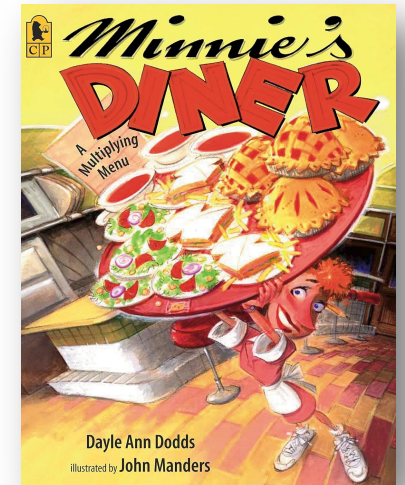
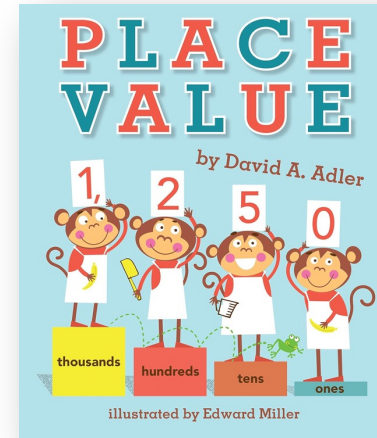
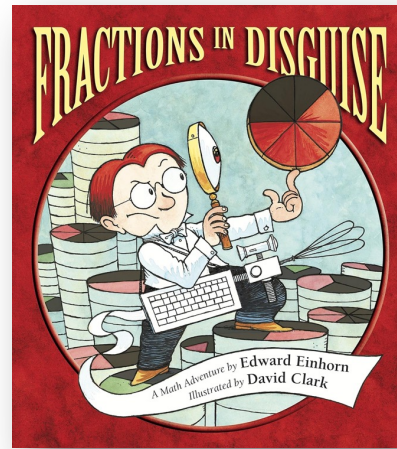
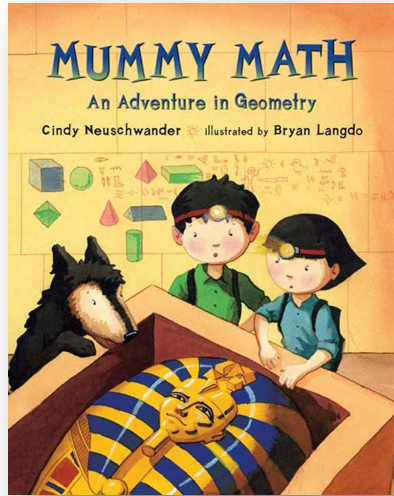
Use math writing

I would help Cole out by not putting them into thirds because he put 1 and $\frac{2}{3}$ but the answer should be $\frac{3}{5}$. So next I would draw the lines to make them into 5 pieces but put next to each other. Then you shade in 3 of the squares and keep the others alone. Then that would be the correct way to do $\frac{3}{5}$.

What Alex did wrong was that it was that he has five rectangles but they are different sizes so first what I would do is draw 5 rectangles side by side then what you would do is shade 3 of them and that would equal $\frac{3}{5}$ so that would be the correct way to solve it.

I would help Bo out by he shaded three rectangles which is correct but he left 5 shaded and there should only be 2 dark rectangles so if you erase 3 rectangles then then answer would be $\frac{2}{5}$ and that is the correct way to solve it.

Use read-alouds



Model and practice

Teacher Let's work on addition. Today, let's think about addition as combining. What does it mean to combine?

Students Put together.

Teacher When we combine, we put things together. When you cook, you put ingredients together. For example, to make macaroni and cheese, you combine what?

Students Macaroni noodles and cheese!

Teacher That's right. You combine macaroni and cheese! Now, let's think about combining numbers. Look at this problem.
(Show problem.)

Make sure students use
mathematics vocabulary
as much as adults do!



What are your strengths with vocabulary?

What are your opportunities for growth?

What are your plans for next Monday?

Next month?

Next year?

Students require modeling and practice on how to use the language of mathematics.

Students should use hands-on tools, virtual manipulatives, drawings, and other visuals to understand mathematics concepts and procedures.

Teachers should use systematic and explicit instruction to help students develop a strong foundation for specific mathematics skills.

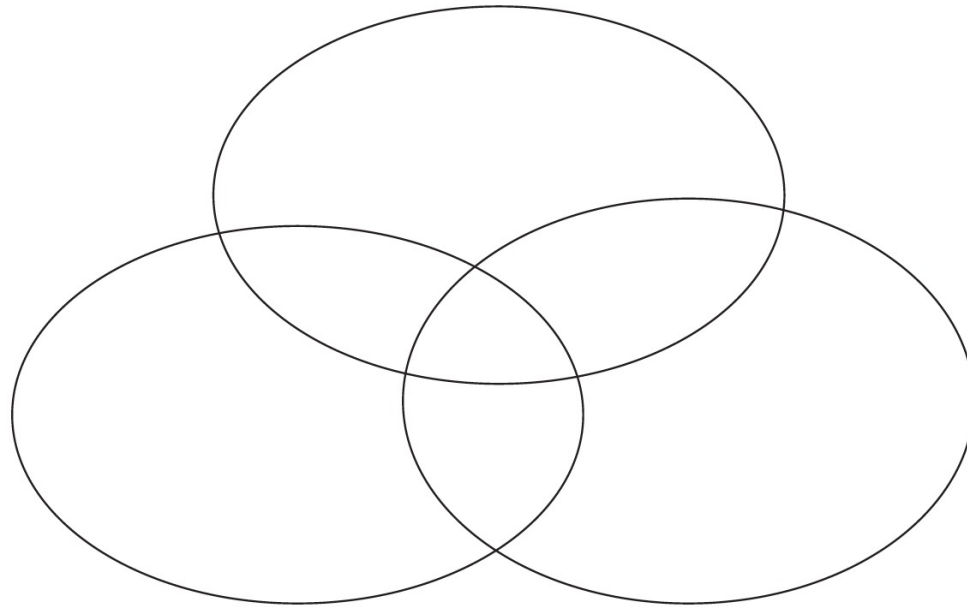
Teachers should use fluency building activities to build counting fluency and fluency with the operations.

Students should learn how to set up and solve word problems by combining an attack strategy with a focus on word-problem schemas.

Representations

REPRESENTATIONS

Research and Information



bit.ly/srpowell



Instructional Platform

INSTRUCTIONAL DELIVERY

Vocabulary

Representations

Model and Practice

INSTRUCTIONAL STRATEGIES



Hands-on manipulatives contribute to increases in mathematics performance.

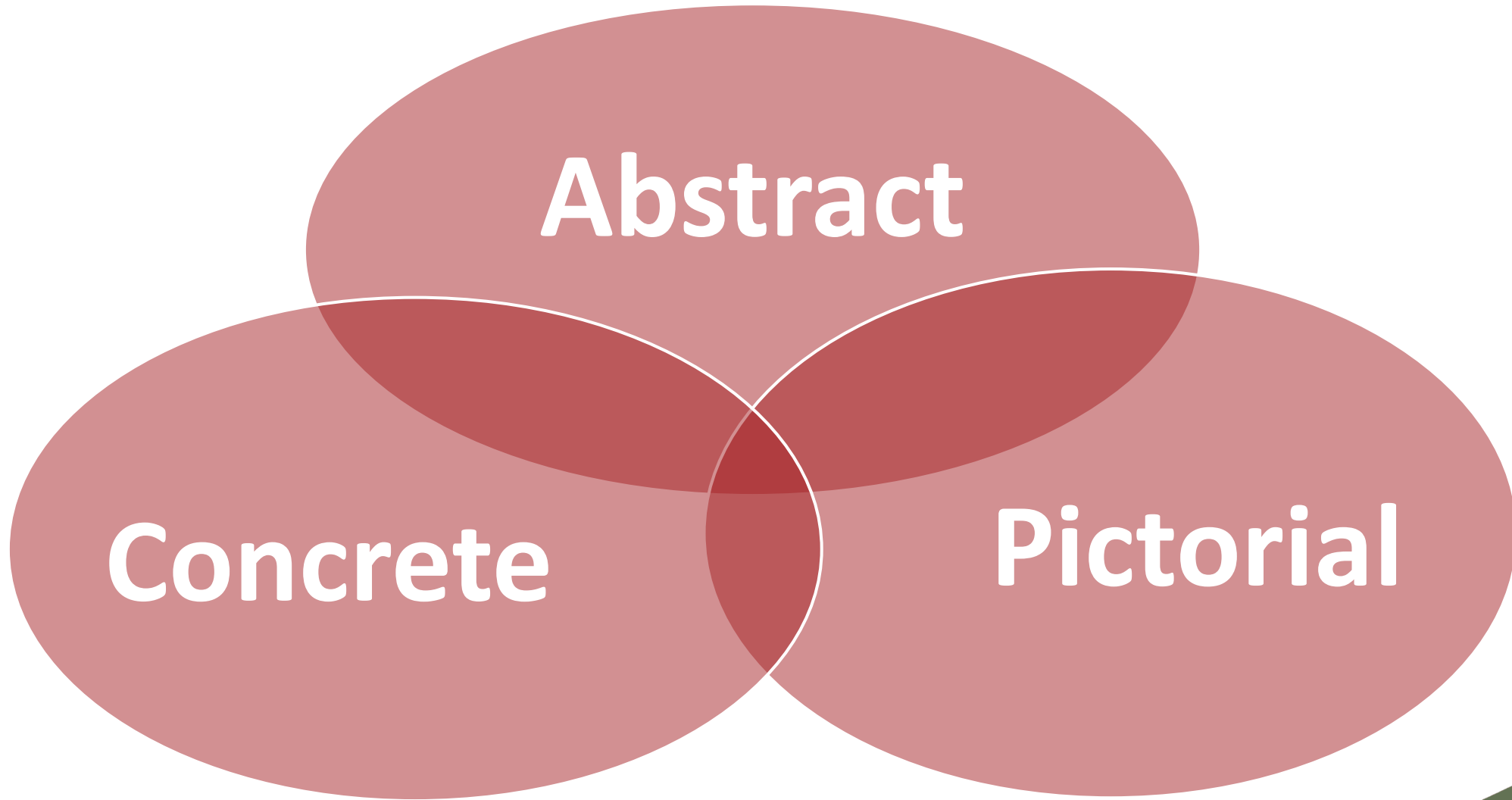
(Bouck & Park, 2018; Carbonneau et al., 2013; Namkung & Bricko, 2021; Sherman & Bisanz, 2009; Strickland & Maccini, 2012)

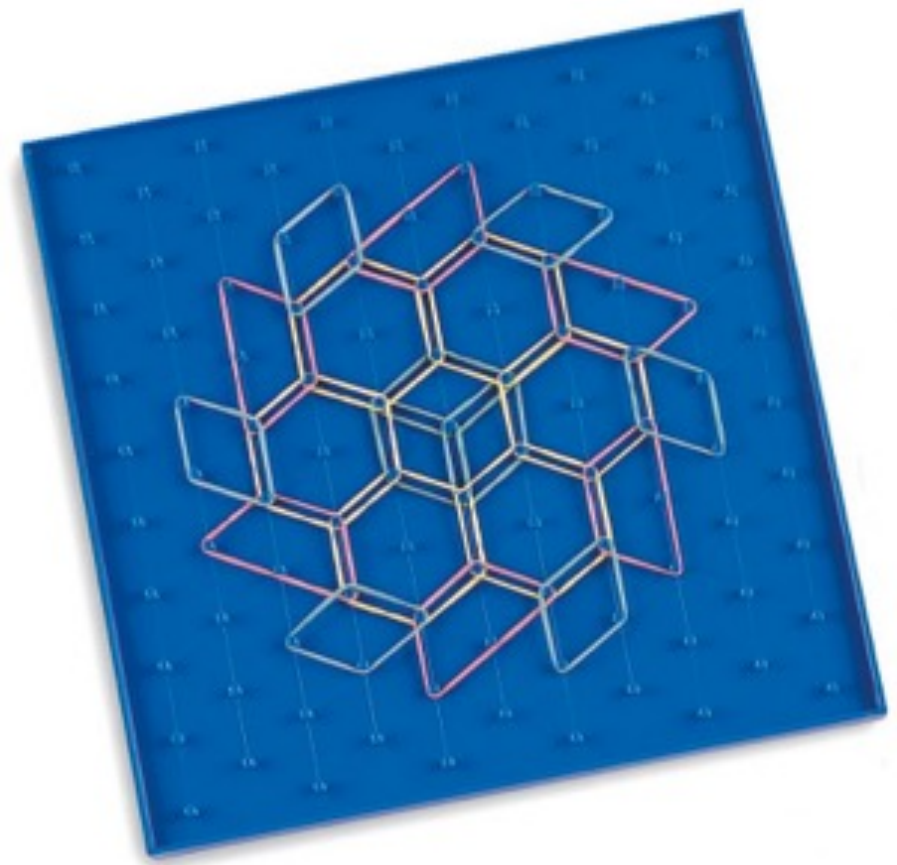
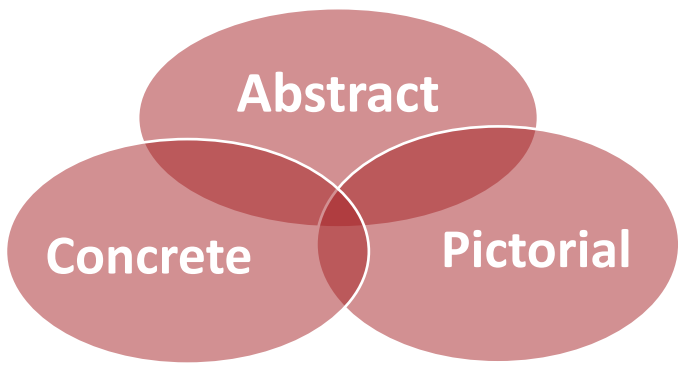
Virtual manipulatives contribute to increases in mathematics performance.

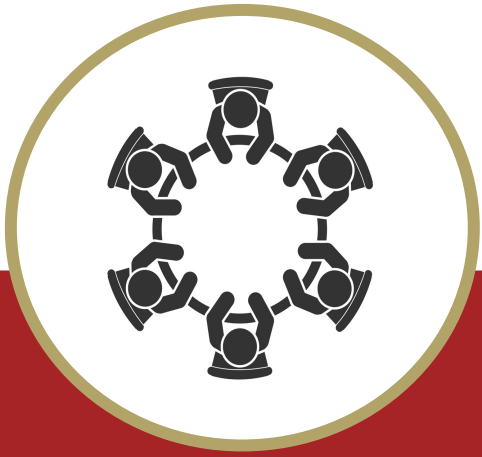
(Bouck et al., 2020; Satsangi et al., 2016)

Other visuals (e.g., graphic organizers) contribute to increases in mathematics performance.

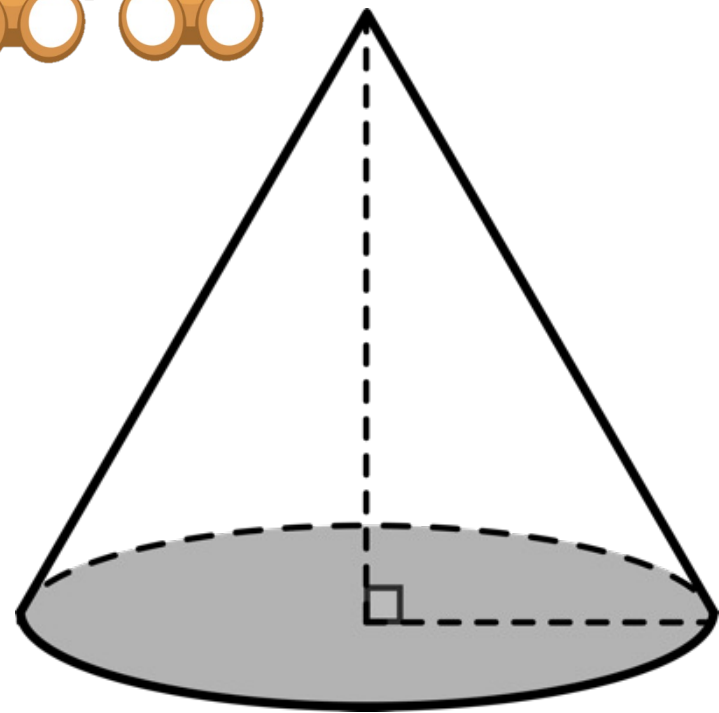
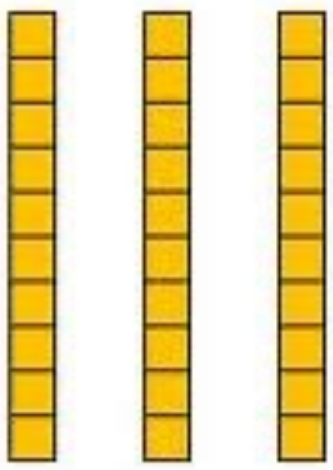
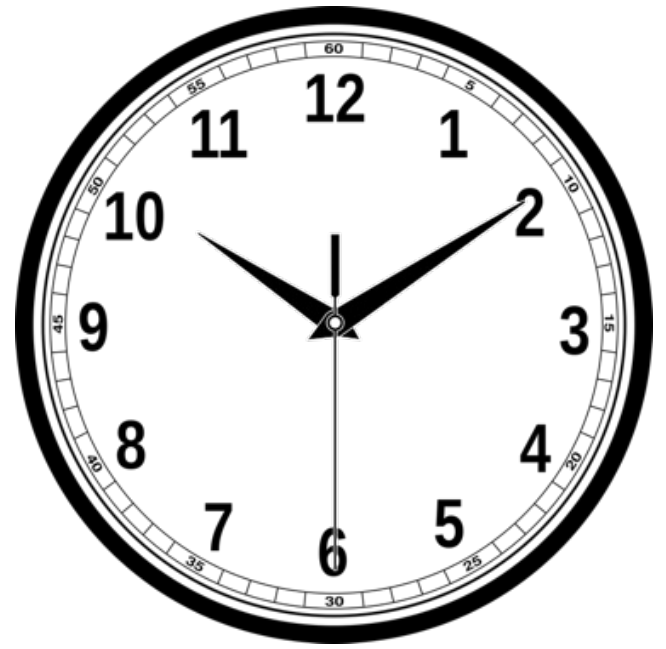
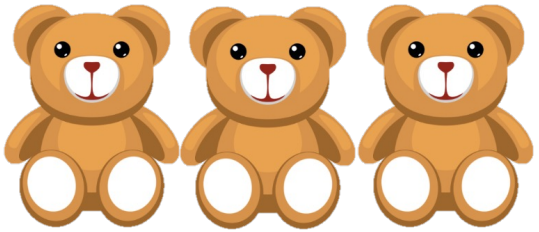
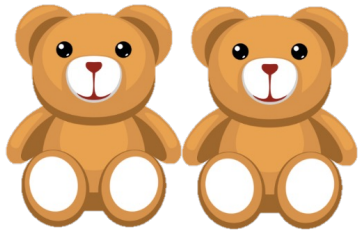
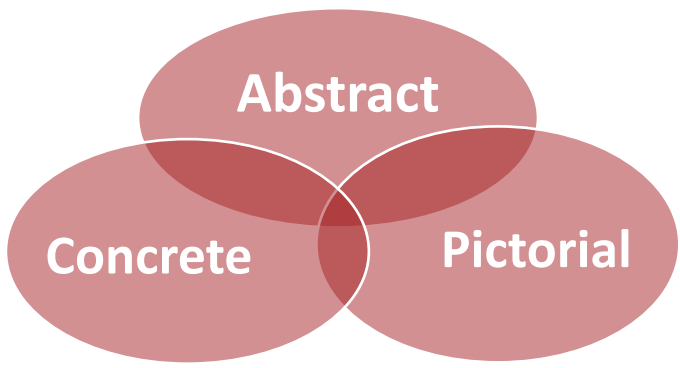
(Jitendra et al., 2009; Sharp & Dennis, 2017; van Garderen, 2007; Xin, 2008)

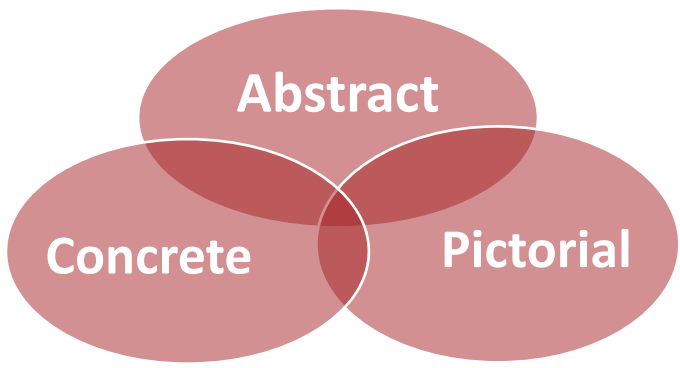






Describe your favorite hands-on materials.

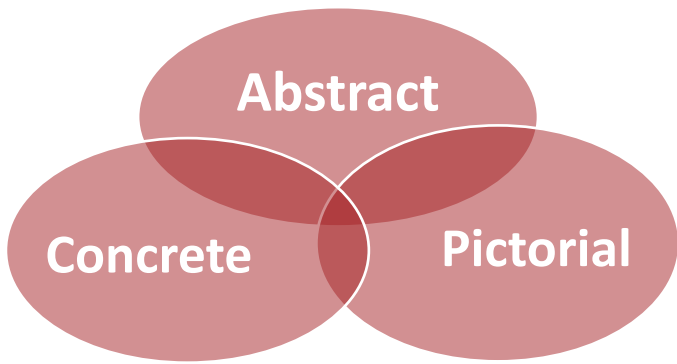




Modeling Fractions with Cuisenaire Rods

A screenshot of a digital interface for modeling fractions with Cuisenaire rods. On the left, a vertical list of rods is shown: a white cube, a red rod (1 unit), a green rod (2 units), a purple rod (3 units), a yellow rod (4 units), a dark green rod (5 units), a black rod (6 units), a brown rod (7 units), a blue rod (8 units), and an orange rod (9 units). In the center, a grid shows a horizontal bar composed of four red rods and one orange rod, representing the fraction 4/9. On the right, a control panel includes icons for 'View Hint' (lightbulb), 'Clear' (circular arrow), 'View Help' (question mark), and 'Trash Can' (trash bin).

A screenshot of a digital interface for geometry on a dot grid. The grid is black with white dots. A triangle is drawn with vertices at grid points. The left vertical side is orange, the bottom horizontal side is red, and the hypotenuse is yellow. Below the grid is a toolbar with icons for various geometric shapes: a white circle, a yellow circle, an orange circle, a red circle, a purple circle, a blue circle, a green circle, and a grey circle. At the bottom, there is a navigation bar with icons for undo, redo, grid, zoom, and other functions.

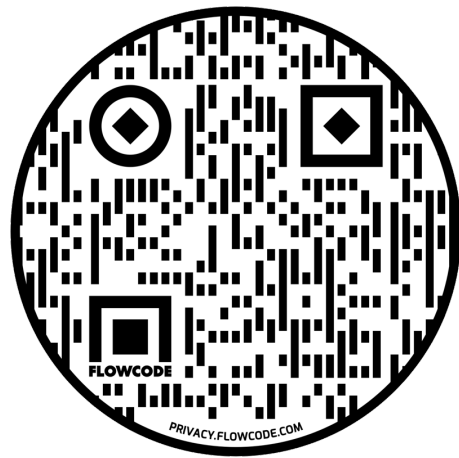


Virtual Manipulatives

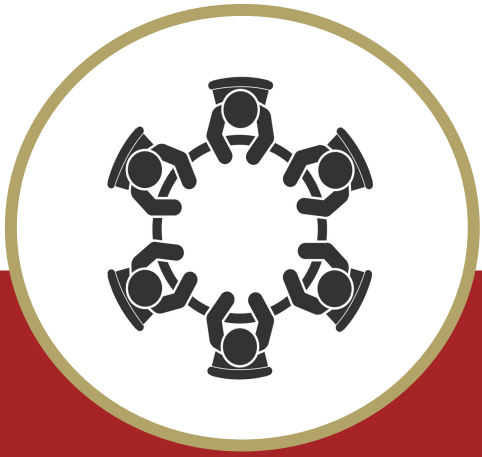
Help students see and learn math using different tools!

Number & Operations	Place Value
Fractions & Decimals	Integers & Algebra
Geometry	Time & Money
Data & Probability	Extras

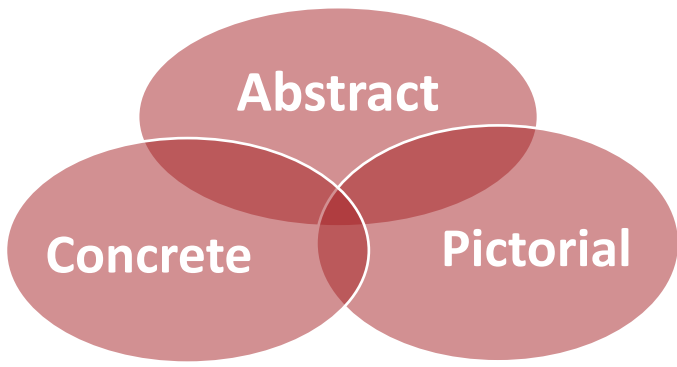
Sarah R. Powell, Ph.D.
srpowell@utexas.edu
www.sarahpowellphd.com
@sarahpowellphd



Fractions & Decimals	fraction strips	fraction strips	fraction strips	Cuisenaire rods
	fraction circles	geoboard	geoboard	geoboard
	two-color counters	decimal strips	place value disks	percentage strips
				pattern blocks



Describe your favorite virtual manipulatives.



$$2 + 8 = 10$$

$$34 = 3 \text{ tens and } 4 \text{ ones}$$


0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10


$$\begin{array}{r} 4,179 \\ + \underline{569} \end{array}$$


REPRESENTATIONS

Representations

Item	Representations

 What are your strengths?

 What are your opportunities for growth?

 What are your plans for next Monday?
Next month?
Next year?





What are your strengths with representations?

What are your opportunities for growth?

What are your plans for next Monday?

Next month?

Next year?

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Model and Practice

MODEL AND PRACTICE

Research and Information

MODELING

PRACTICE

SUPPORTS



Instructional Platform

INSTRUCTIONAL DELIVERY

Vocabulary

Representations

Model and Practice

INSTRUCTIONAL STRATEGIES



Over a half century of research supports explicit (i.e., direct, systematic) instruction.
(Stockard et al., 2018)

When compared to discovery approaches, explicit instruction demonstrates higher outcomes.
(Alfieri et al., 2011; Kroesbergen et al., 2004; Poncy et al., 2010)

Numerous meta-analyses and large-scale studies have identified explicit instruction as essential for the teaching and learning of mathematics.
(Chodura et al., 2015; Ennis & Losinski, 2019; Jitendra et al., 2018; Kong et al., 2021; Morgan et al., 2015; Nelson & McMaster, 2019; Powell et al., 2021).

MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

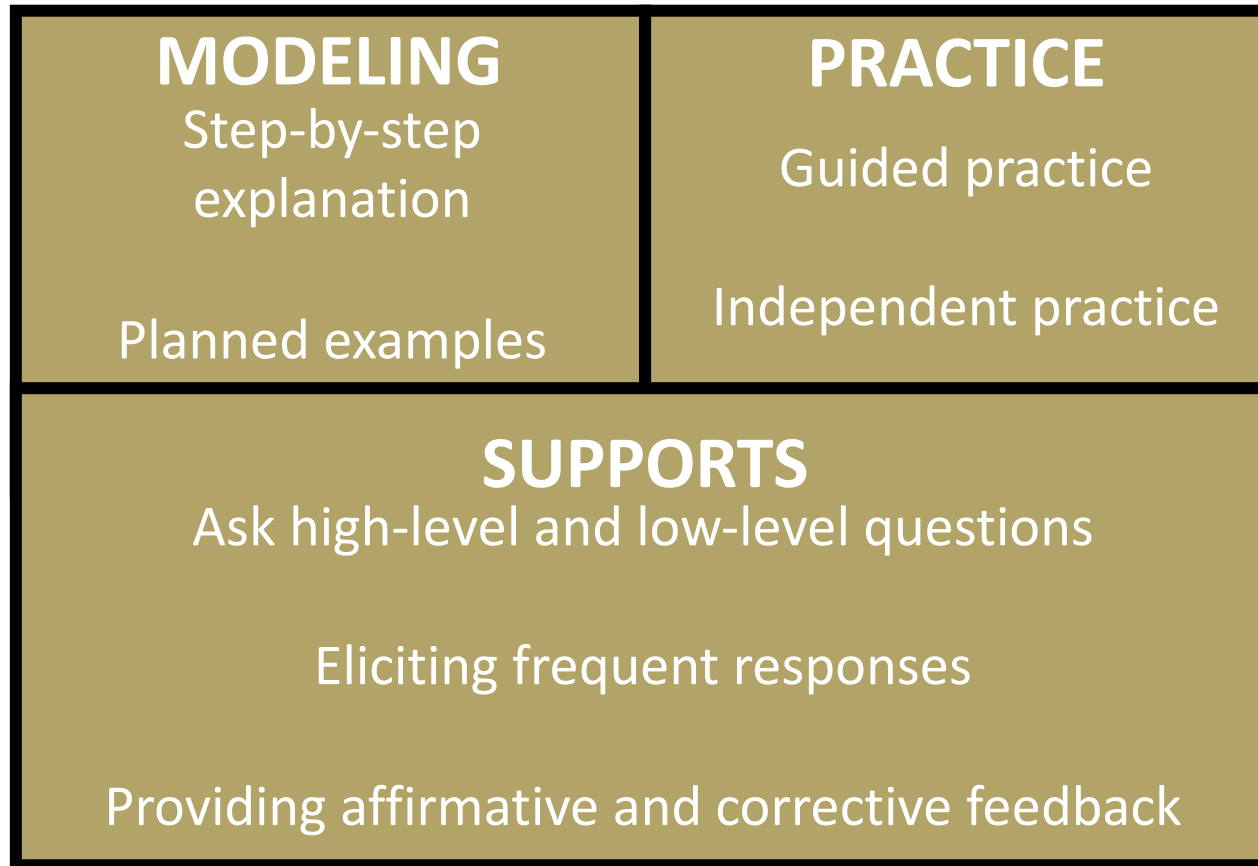
SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

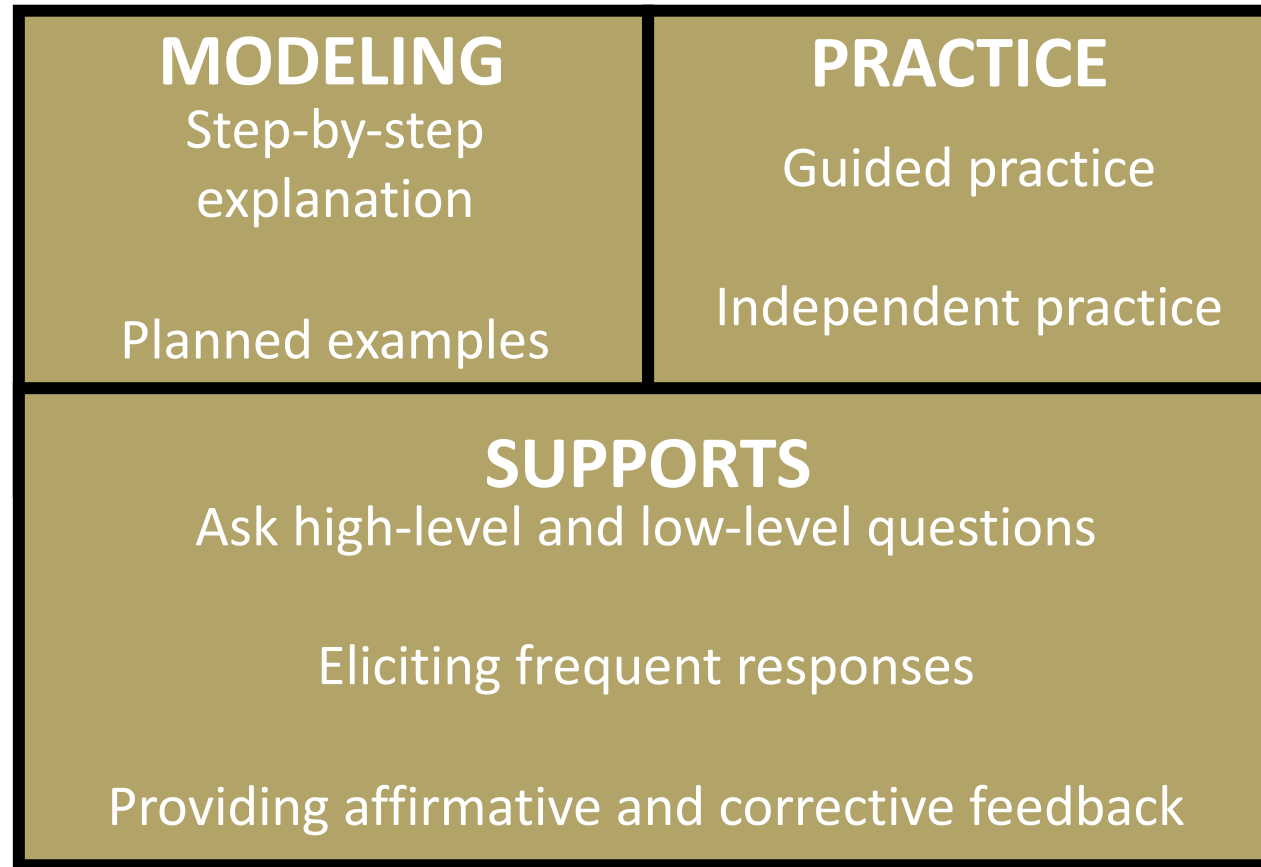
Providing affirmative and corrective feedback

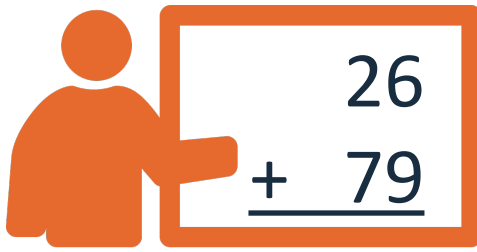
Modeling is a dialogue between the teacher and students.



Modeling includes a step-by-step explanation of how to do a math problem.

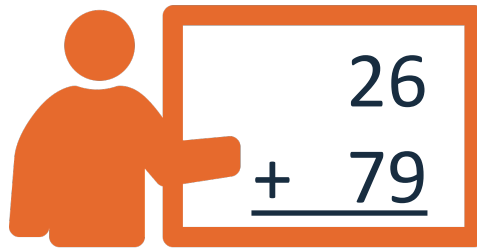
A teacher may do 1 modeled problem or several.





“Today, we are learning about addition. This is important because sometimes you have different amounts – like money – and you want to know how much money you have altogether.”





“Let’s solve this problem. What’s the problem?”

“26 plus 79.”



“To solve 26 plus 79, first decide about the operation. Should we add, subtract, multiply, or divide?”

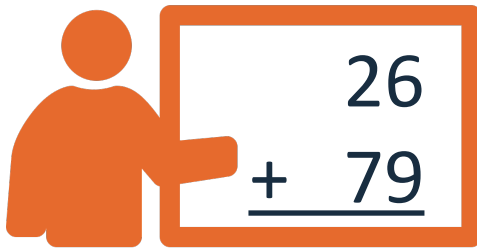
“Add.”



“How did you know we want to add?”

“There’s a plus sign.”





“The plus sign tells us we want to add. To add, let’s use the partial sums strategy. What strategy?”

“Partial sums.”



“What might partial mean?”

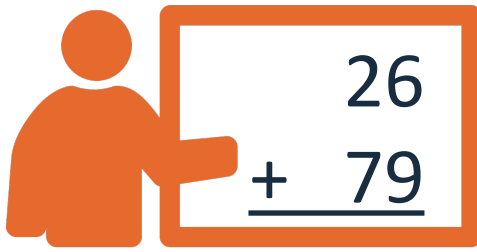
“Part of.”



“We’ll find parts – or partial sums - then add them together. With the partial sums strategy, we start adding in the greatest place value. What’s the greatest place value in this problem?”

“The tens.”





“So, let’s add the tens. What’s 20 plus 70? Use your base-10 blocks or other tools.”

“20 plus 70 equals 90. Let’s write 90 right here below the equal line. What will we write?”

“90 is the partial sum when you add the tens. What does 90 represent?”

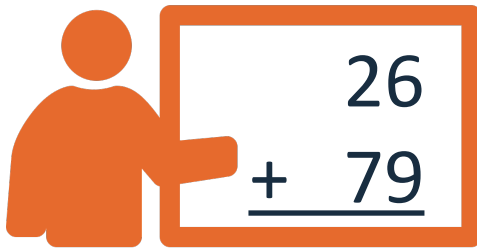
“Now, let’s add the ones. What should we add?”

“90.” 

“90.” 

“It’s the partial sum of adding 20 plus 70.” 

“6 plus 9.” 




“6 plus 9 equals what? Use your base-10 blocks or other tools.”

“How did you get 15?”

“Let’s write 15 below the 90. Where do we write the 15?”

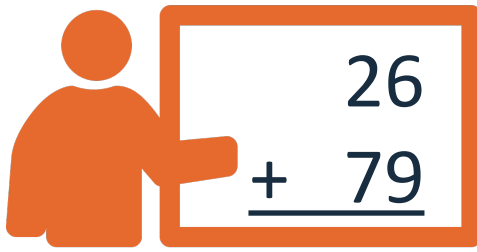
“15 is the partial sum when you add the ones. Now, let’s add the partial sums together. What will we add?”

“15.” 

“We knew we had 9, then we added on 6.” 

“Below the 90.” 

“90 plus 15.” 




“What’s 90 plus 15?
Use your go-to
strategy.”

“105.” 

“How did you add
those addends?”

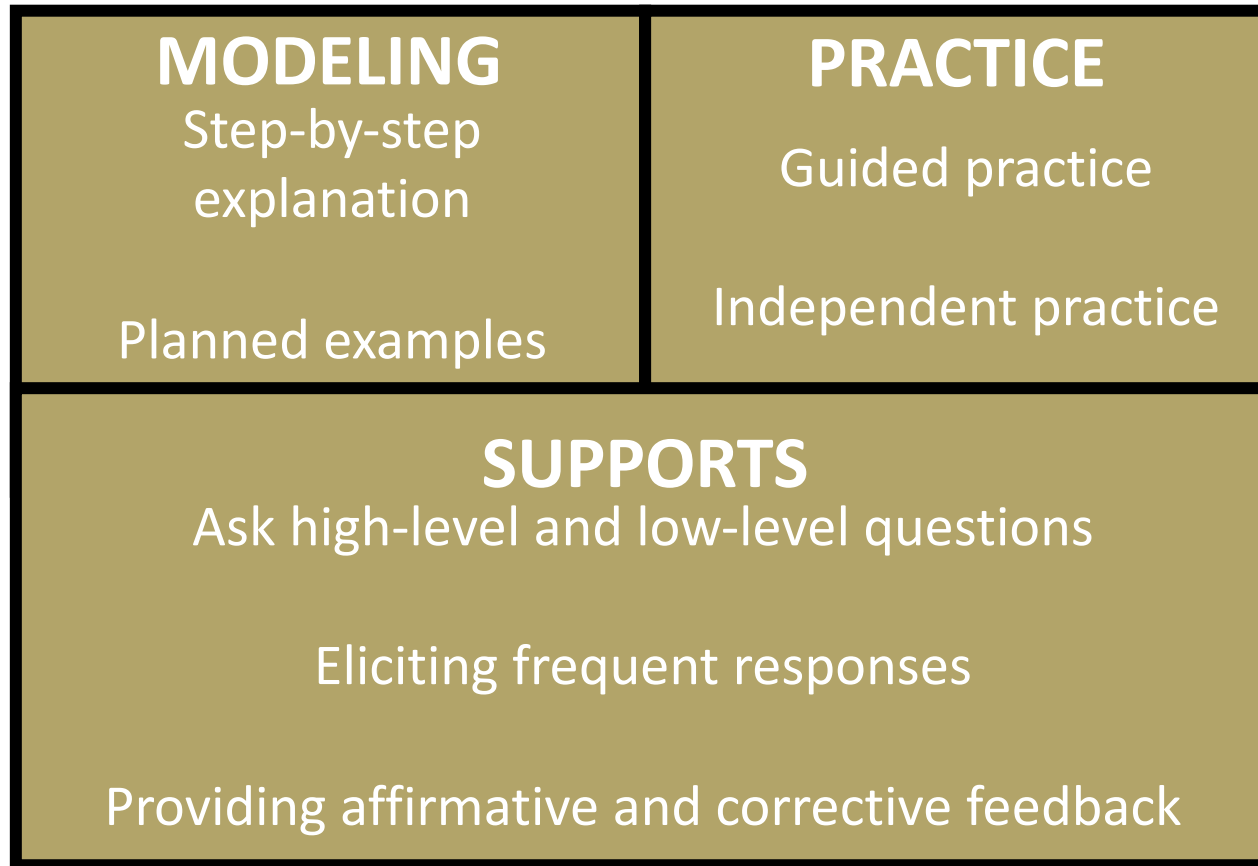
“I added 90 plus
10 then added 5
more.” 

“So, when you add
26 plus 79, the sum
is 105. Who can
share how we
solved this
problem?”

“We used the
partial sums
strategy. We added
the tens then
added the ones.
Then we added the
partial sums.” 

Modeling needs to include planned examples.

These examples should be sequenced so easier skills lead to more difficult skills.



MODELING

Step-by-step
explanation

Planned examples

PRACTICE

Guided practice

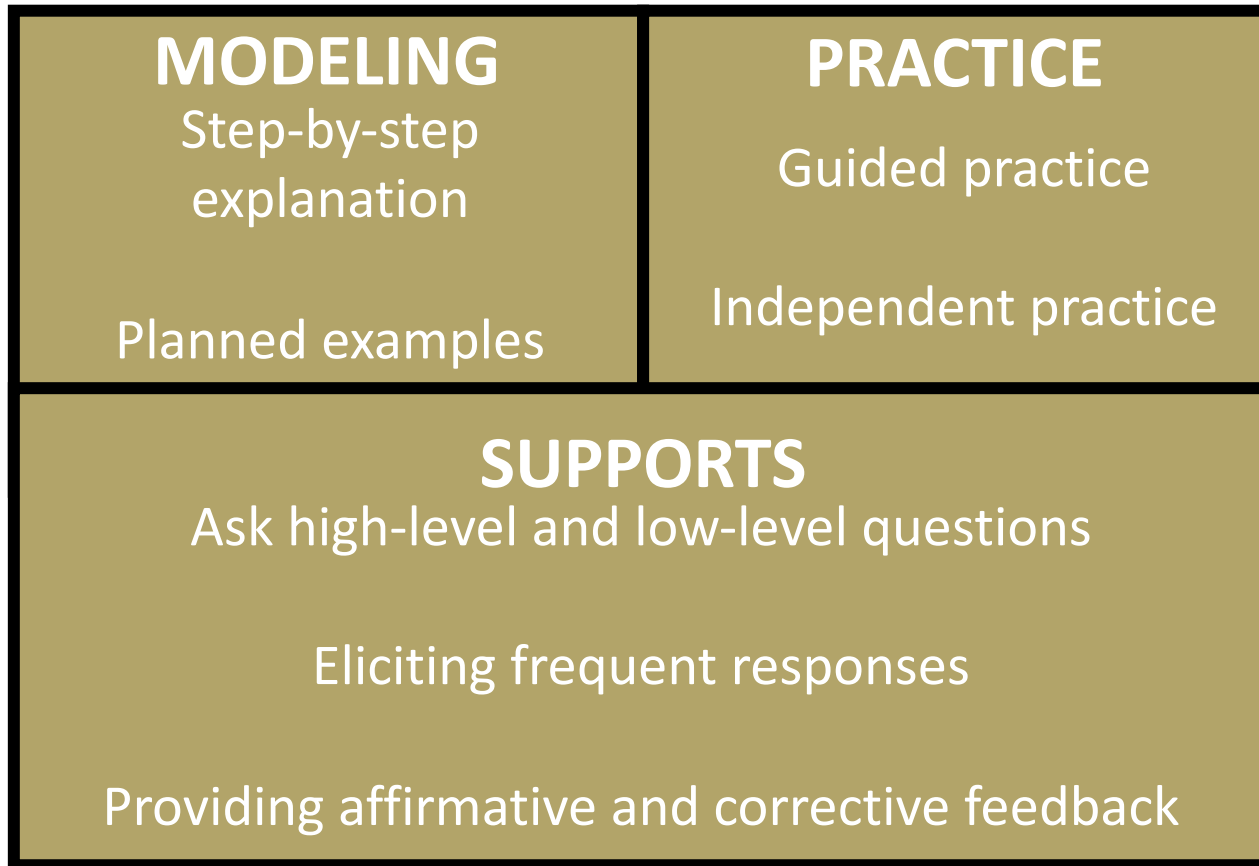
Independent practice

SUPPORTS

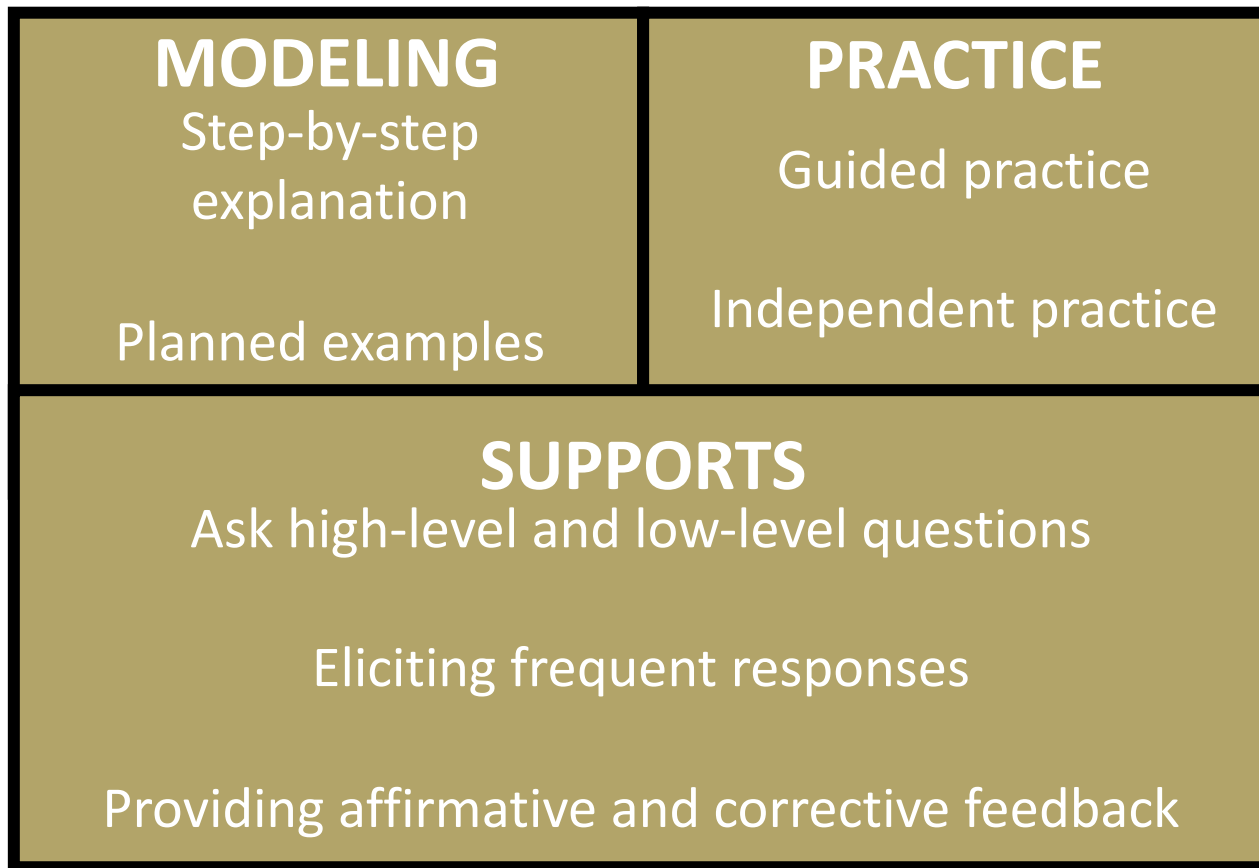
Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback



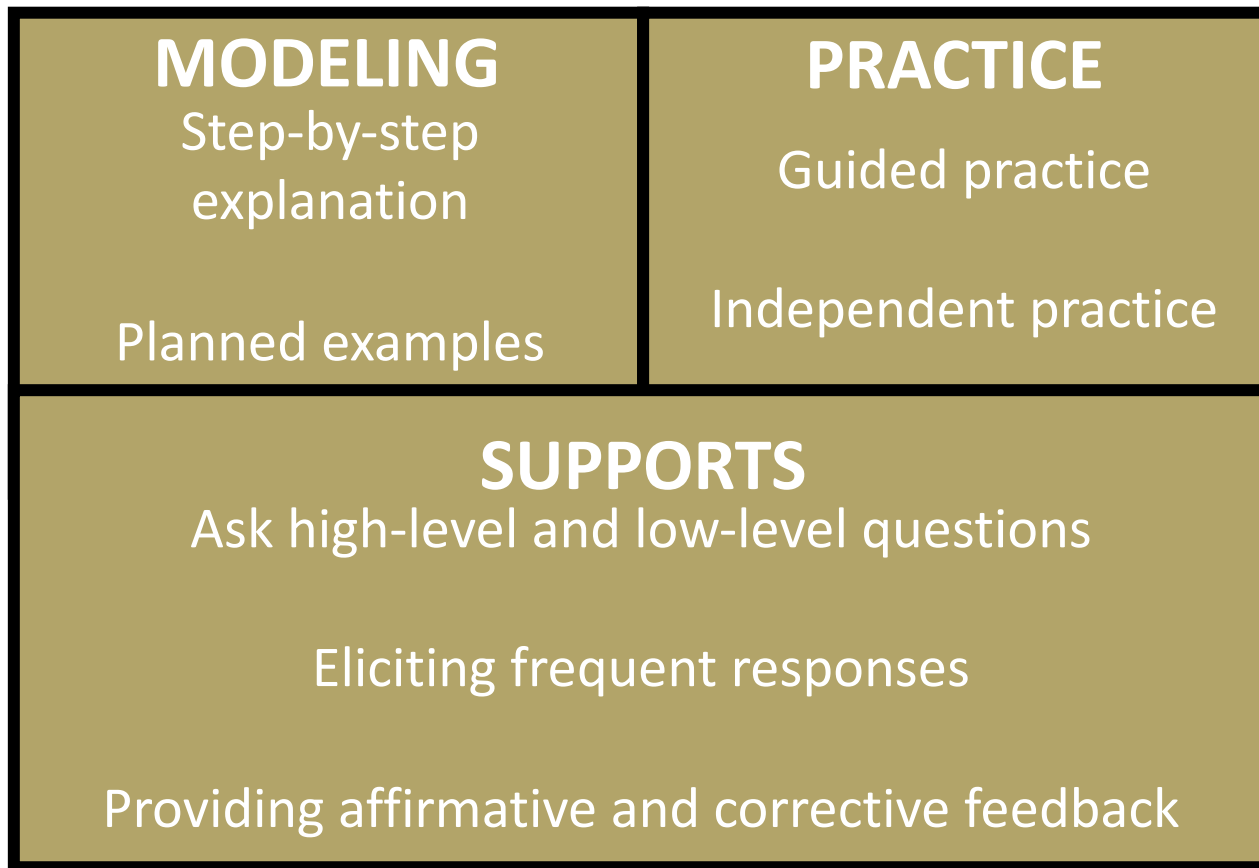
Practice continues as a dialogue between the teacher and students.



Guided practice is practice in which the teacher and students practice problems together.



“Let’s work on a problem together.”



Independent practice is practice in which the students practice independently with teacher support.



“Now, you’ll practice a problem on your own. Use your attack strategy!”

MODELING

Step-by-step
explanation

Planned examples

PRACTICE

Guided practice

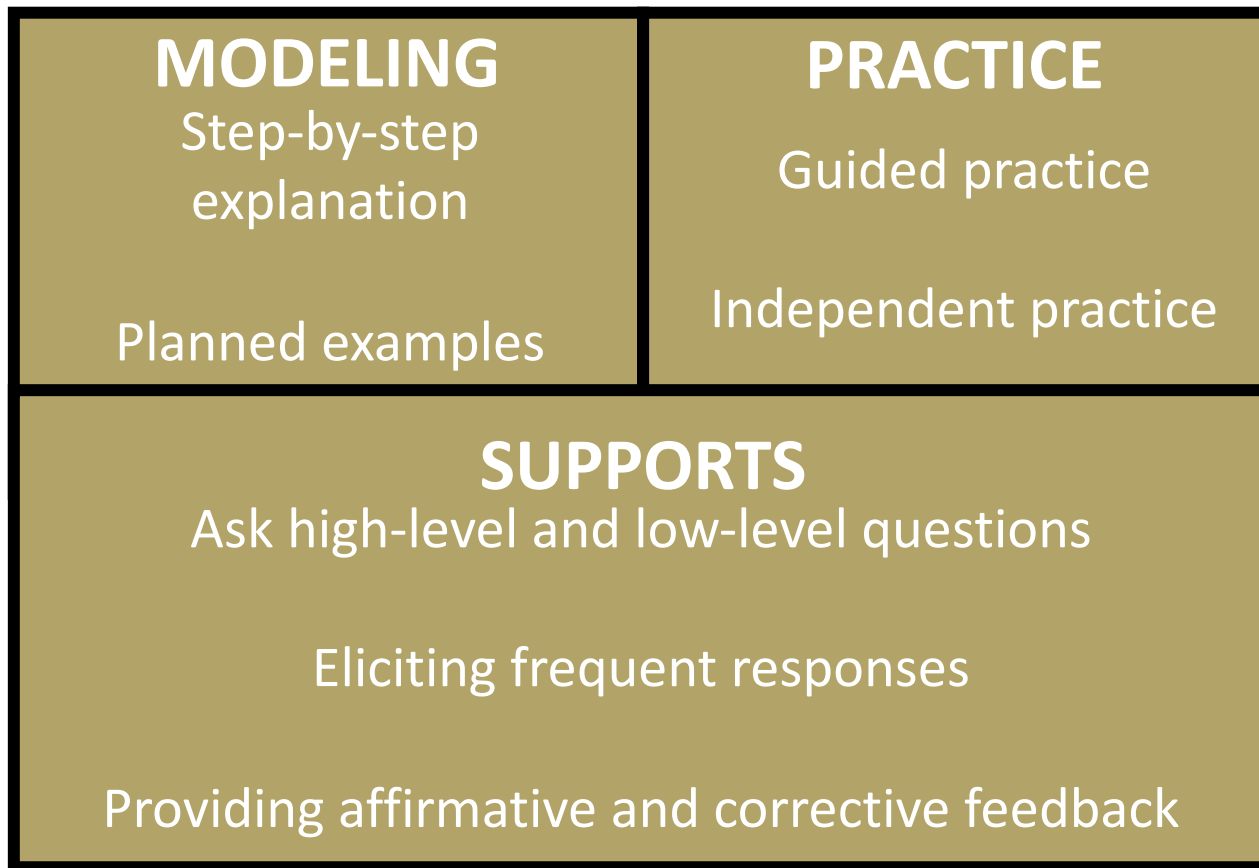
Independent practice

SUPPORTS

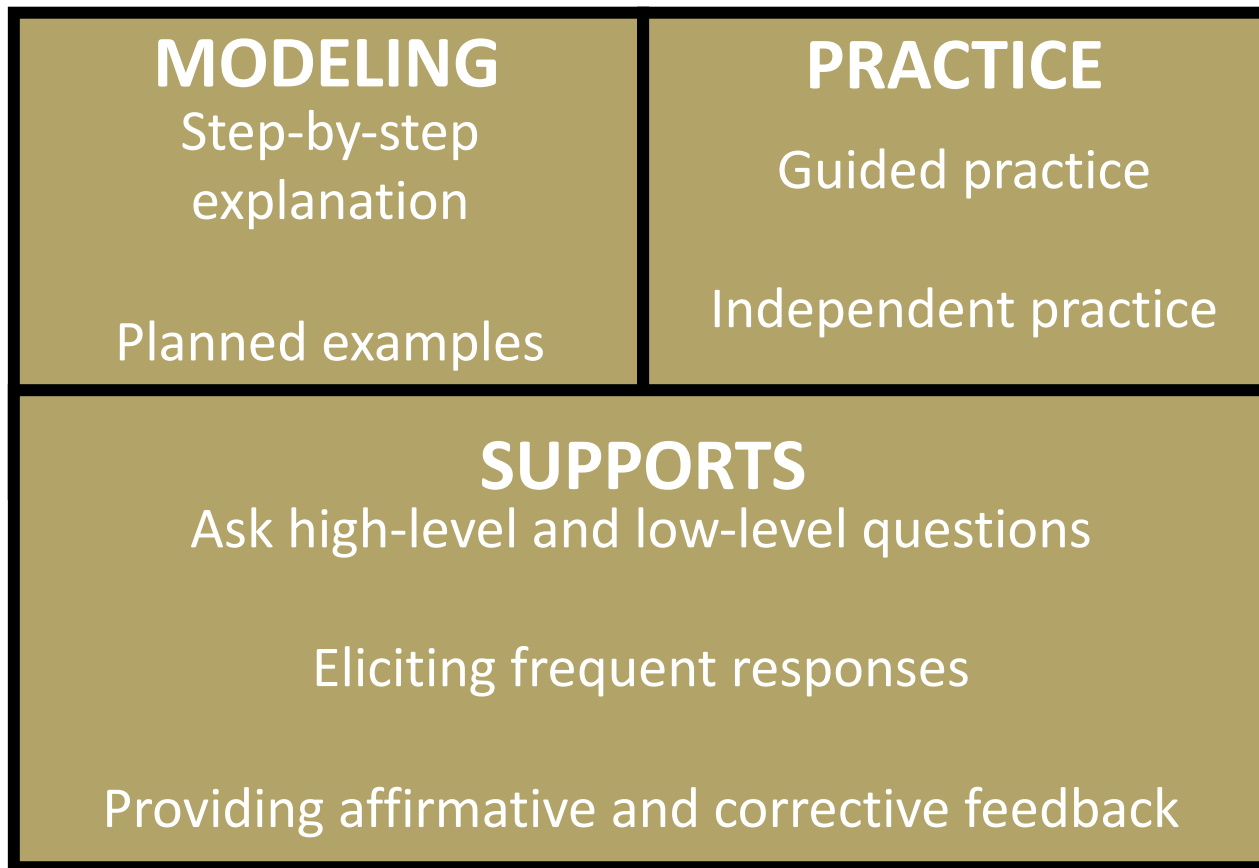
Ask high-level and low-level questions

Eliciting frequent responses

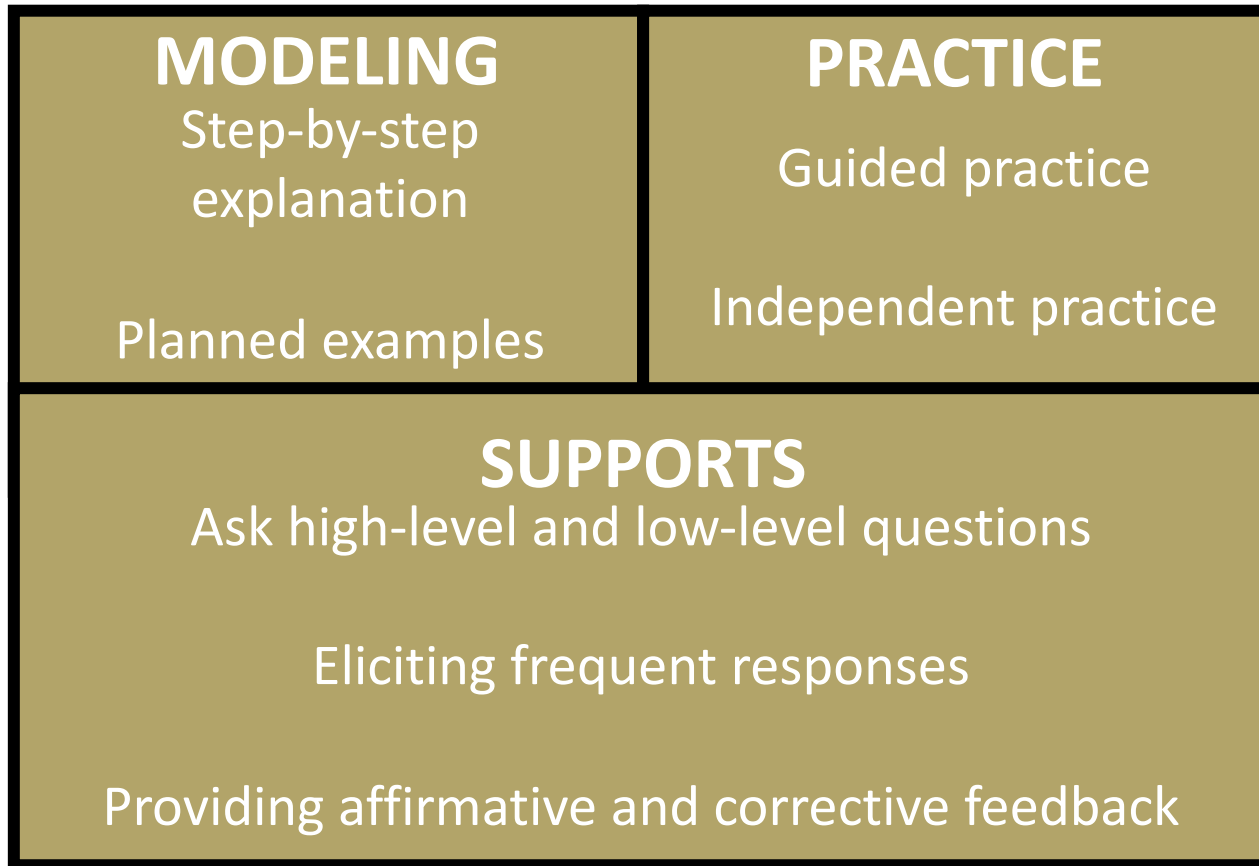
Providing affirmative and corrective feedback



These **Supports** should be used in both **Modeling** and **Practice**.

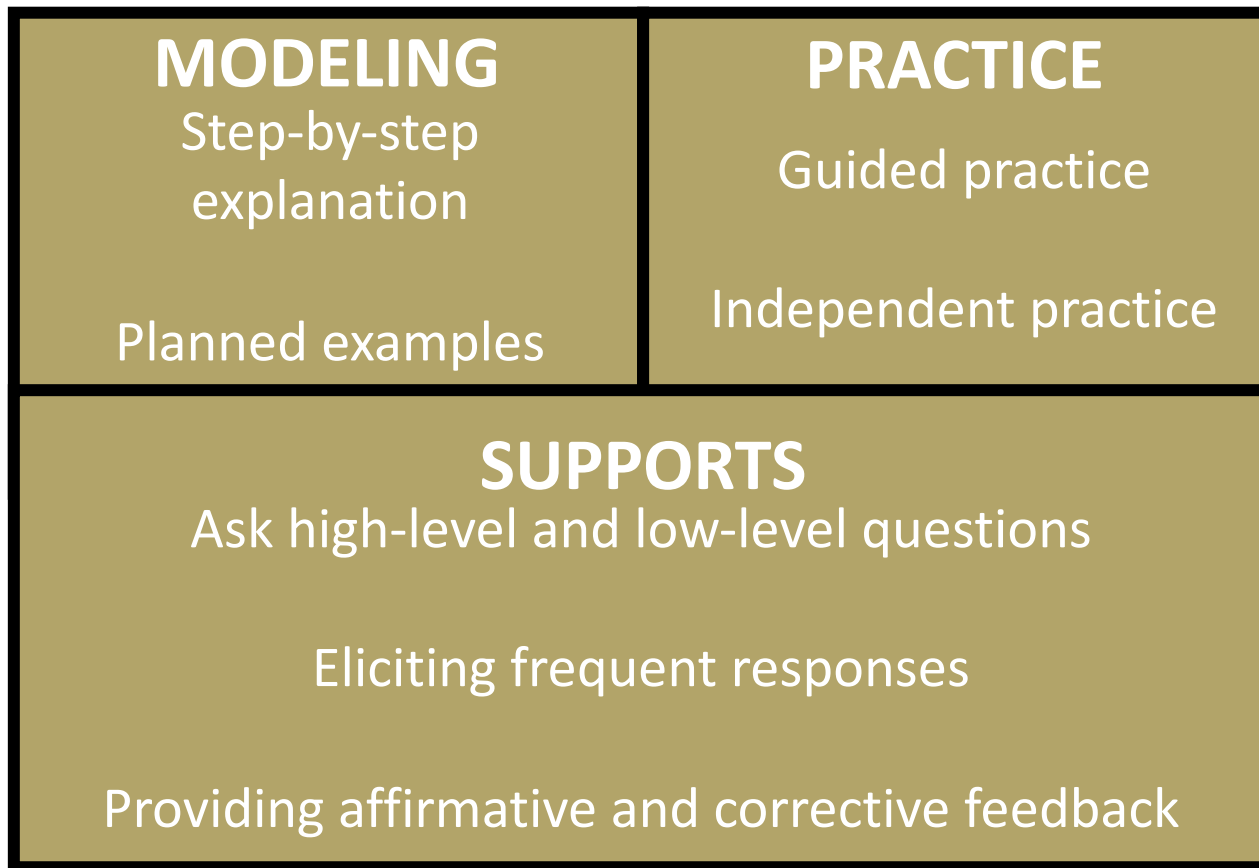


During **Modeling** and **Practice**, it is essential to engage students and check for understanding.



“What is 7 times 9?”

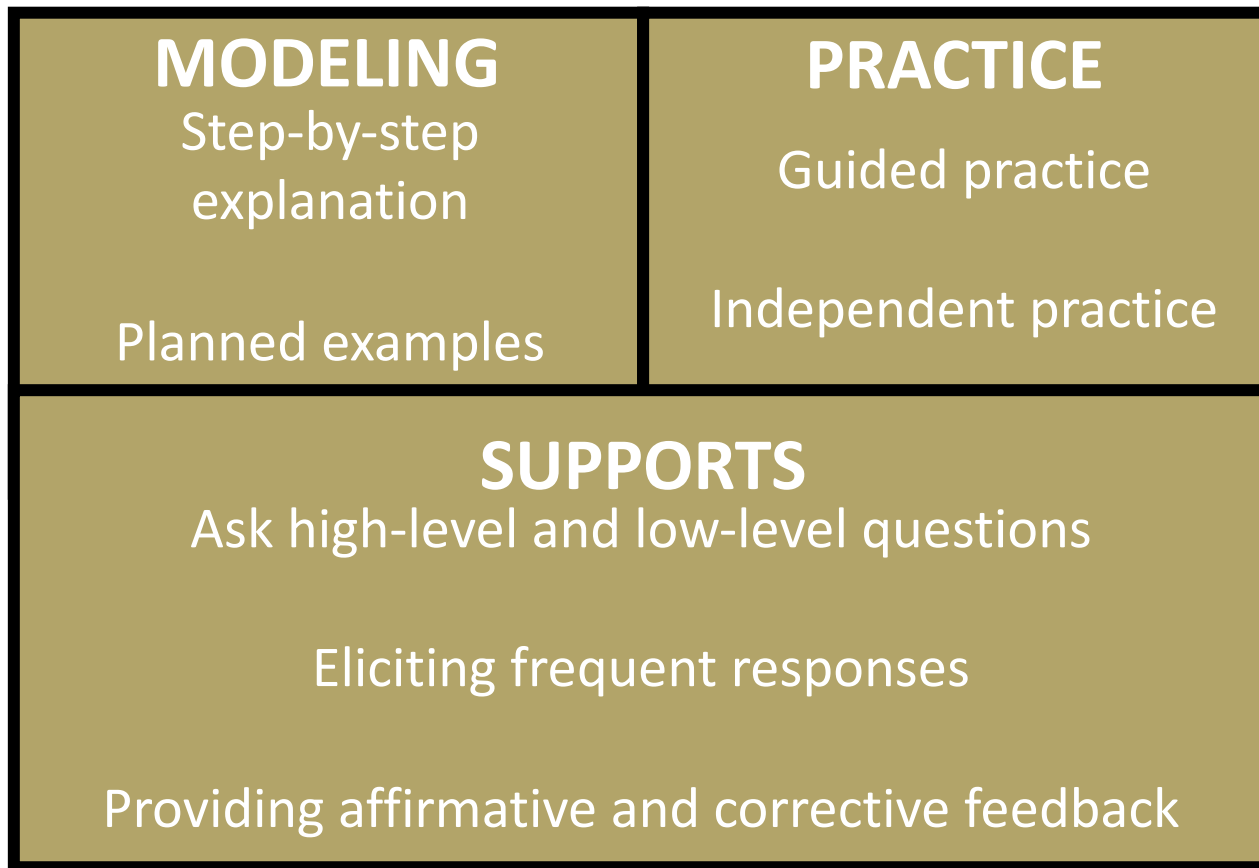
“63.” 



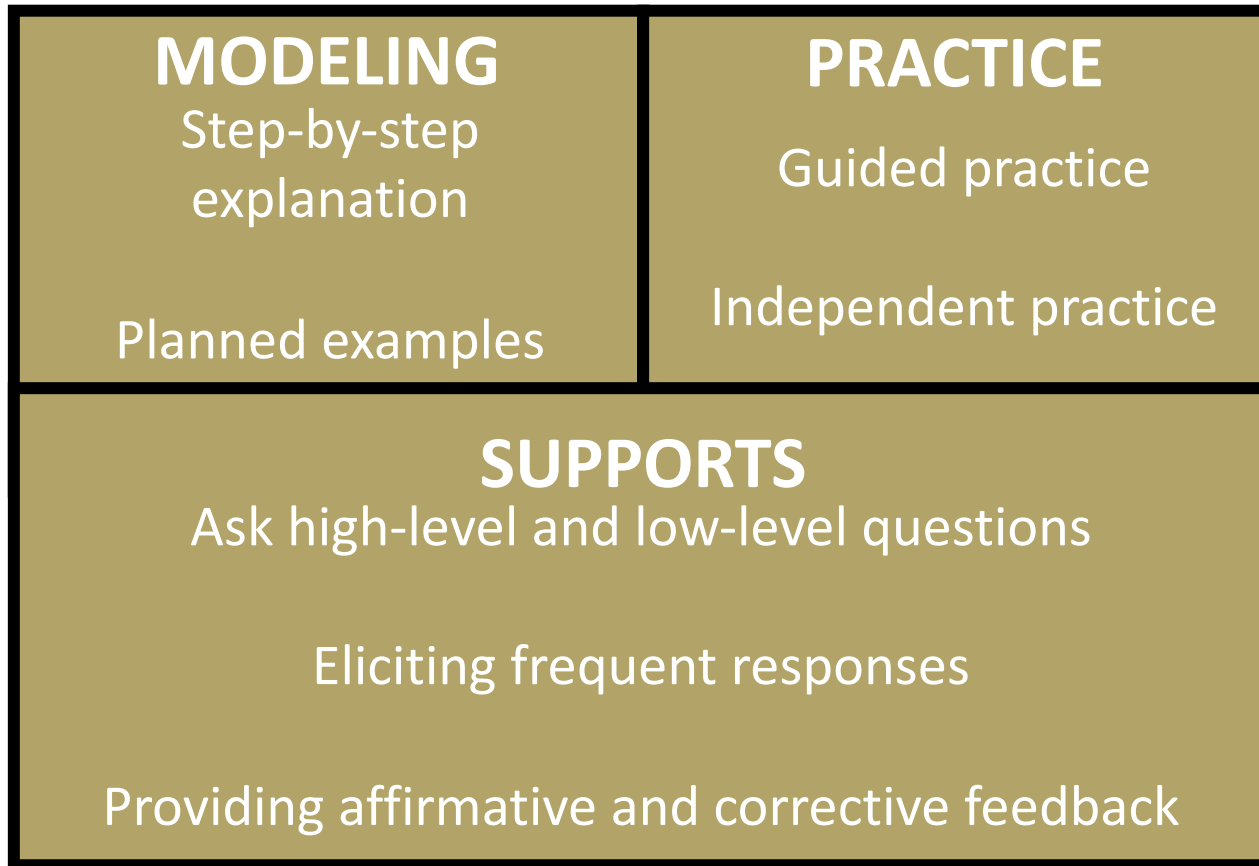
“Why do you use zero pairs?”

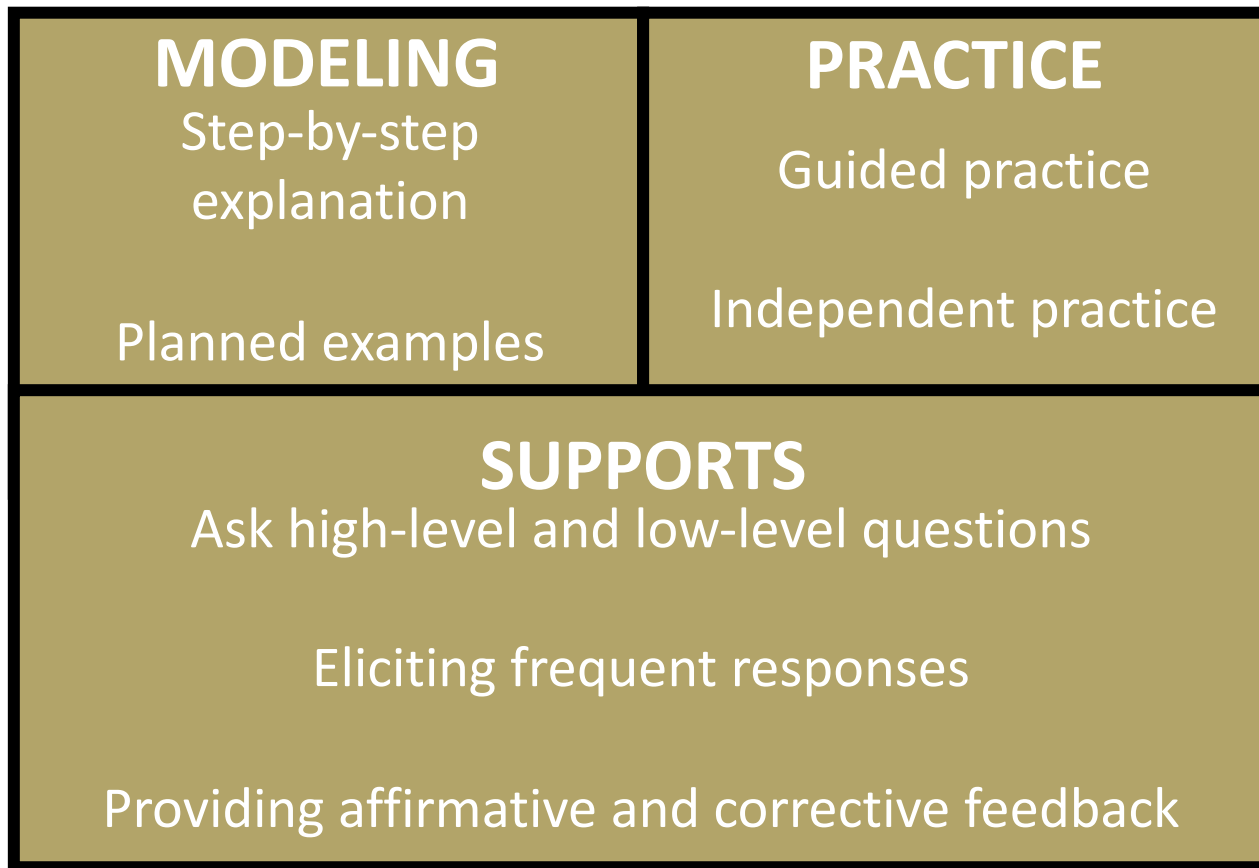
“Because a positive 1 and a negative 1 equal 0. I use the zero pair to help me subtract.”



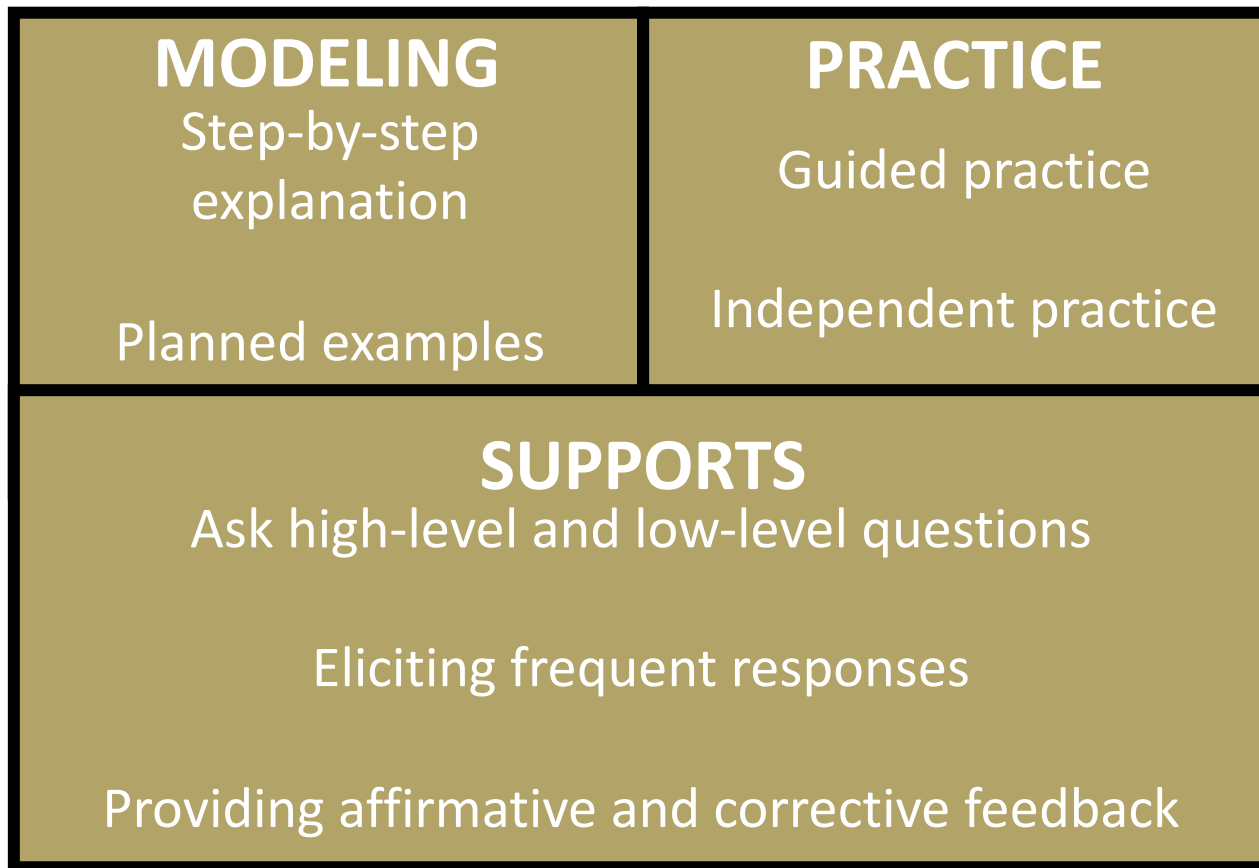


During **Modeling** and **Practice**, students should frequently respond. The frequent responses keeps student attention and keeps student learning active.

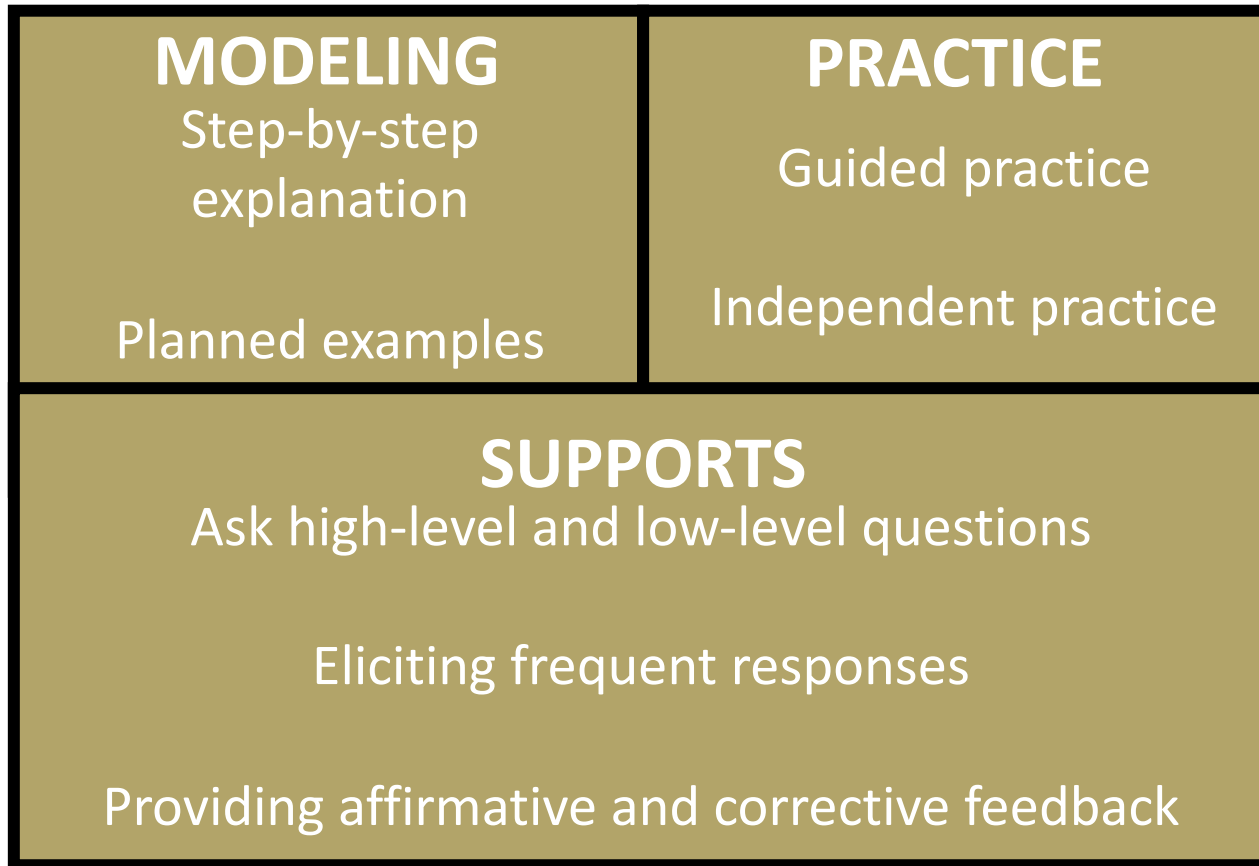




During **Modeling** and **Practice**, students should receive immediate feedback on their responses.



“Nice work using your word
problem attack strategy.”



“Let’s look at that again.
Tell me how you added in
the hundreds column.”

MODELING

Step-by-step
explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

MODEL AND PRACTICE



What are your strengths?



What are your opportunities for growth?



What are your plans for next Monday?
Next month?
Next year?



What are your strengths with modeling and practice?

What are your opportunities for growth?

What are your plans for next Monday?

Next month?

Next year?

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Fluency

FLUENCY

Research and Information

Types of Fluency

Type	Memorization?	
	Yes	No



Instructional Platform

INSTRUCTIONAL DELIVERY

Vocabulary

Representations

Model and Practice

INSTRUCTIONAL STRATEGIES

Fluency



Mathematics fluency, particularly fluency with facts and computation, is related to overall mathematics performance.

(Bailey et al., 2012; Cirino et al., 2019; Koponen et al., 2007, 2017; Vukovic et al., 2014)

Fact fluency practice improves mathematics fact performance.

(Burns et al., 2010; Coddington et al., 2011; McCallum et al., 2004; Nelson et al., 2013; Poncy et al., 2010; Schutte et al., 2015; Stocker & Kubina, 2017)

Fluency is
doing
mathematics
easily and
accurately.

Fluency in
mathematics
makes
mathematics
easier.

Fluency
provides less
stress on
working
memory.

Fluency helps
students build
confidence
with
mathematics.

With fluency, it is important to emphasize both
conceptual learning and procedural learning.

Addition	Subtraction
Multiplication	Division

Counting

Comparison

Fractions

Geometry

Money

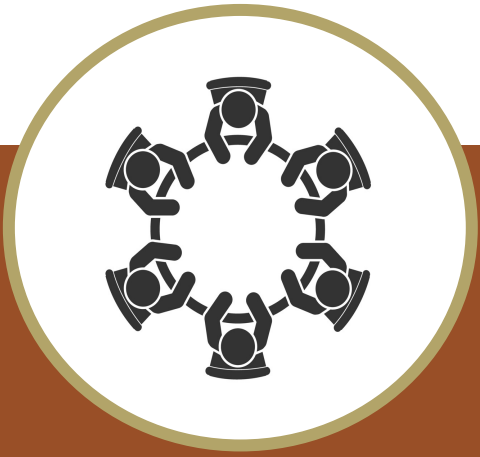
Time

Measurement

Algebra

Ease and
accuracy

Memorization
or automaticity



**List different types of fluency.
Discuss whether they require memorization.**

Addition	Subtraction
Multiplication	Division

Build fluency with math facts.

- Addition: single-digit addends
- Subtraction: single-digit subtrahend
- Multiplication: single-digit factors
- Division: single-digit divisor

$$\begin{array}{r} 5 \\ + 8 \\ \hline \end{array}$$

$$\begin{array}{r} 9 \\ - 4 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \\ \times 7 \\ \hline \end{array}$$

$$\begin{array}{r} 56 \\ \div 8 \\ \hline \end{array}$$

Addition	Subtraction
Multiplication	Division

Build fluency with whole-number computation.

$$\begin{array}{r} 15 \\ + 28 \\ \hline \end{array}$$

$$\begin{array}{r} 23 \\ \times 9 \\ \hline \end{array}$$

$$\begin{array}{r} 1009 \\ - 724 \\ \hline \end{array}$$

$$\begin{array}{r} 7250 \\ \div 15 \\ \hline \end{array}$$

Addition	Subtraction
Multiplication	Division

Build fluency with rational-number computation.

$$\begin{array}{r} 1.4 \\ + 3.9 \\ \hline \end{array}$$

$$\frac{2}{3} \times \frac{3}{4}$$

$$\frac{9}{4} - \frac{3}{8}$$

$$\begin{array}{r} 7.892 \\ \div 0.14 \\ \hline \end{array}$$

Addition	Subtraction
Multiplication	Division

Build fluency with integer computation.

$$\begin{array}{r} 1.4 \\ + -3.9 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \\ \times -12 \\ \hline \end{array}$$

$$-14 - (-7) =$$

$$-135 \div 2 =$$

FLUENCY

Strategies for Building Fluency



What are your strengths?



What are your opportunities for growth?



What are your plans for next Monday?
Next month?
Next year?



DAILY

BRIEF

Work on small sets of facts

Work on unknown facts
(in combination with known facts)

Dice



Roll the Dice



$$\underline{8} + \underline{7} = \underline{15}$$

$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$

$$\underline{\quad} - \underline{\quad} = \underline{\quad}$$

$$\underline{\quad} - \underline{\quad} = \underline{\quad}$$

$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$

$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$

$$\underline{\quad} - \underline{\quad} = \underline{\quad}$$

$$\underline{\quad} - \underline{\quad} = \underline{\quad}$$

Beach Ball

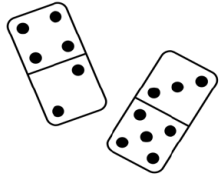


4 plus 6 equals 10.

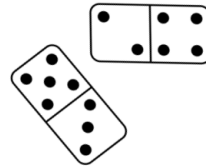
7 plus 6 equals 13.

2 plus 2 equals 4.

Dominoes



Dominoes



$$\underline{4} + \underline{6} = \underline{10}$$

$$\underline{\quad} - \underline{\quad} = \underline{\quad}$$

$$\underline{\quad} \times \underline{\quad} = \underline{\quad}$$

$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$

$$\underline{\quad} - \underline{\quad} = \underline{\quad}$$

$$\underline{\quad} \times \underline{\quad} = \underline{\quad}$$

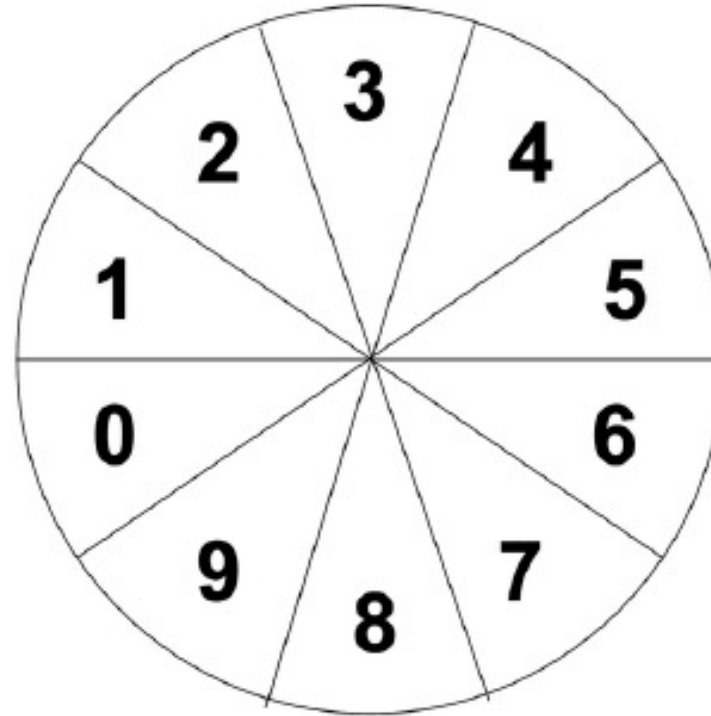


Spinner

2 times 4 equals 8.

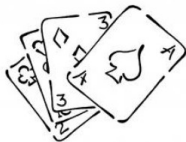
6 times 9 equals 54.

7 times 1 equals 7.



Playing Cards

Cards



$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$

$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$

$$6 - 2 = 4$$

$$\underline{\quad} - \underline{\quad} = \underline{\quad}$$

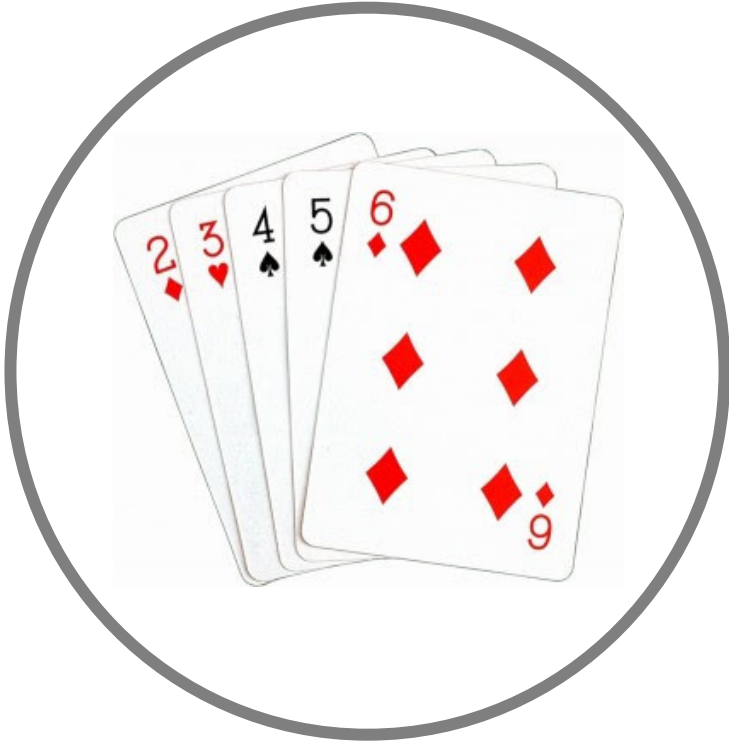
$$\underline{\quad} - \underline{\quad} = \underline{\quad}$$

$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$

$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$

$$\underline{\quad} - \underline{\quad} = \underline{\quad}$$

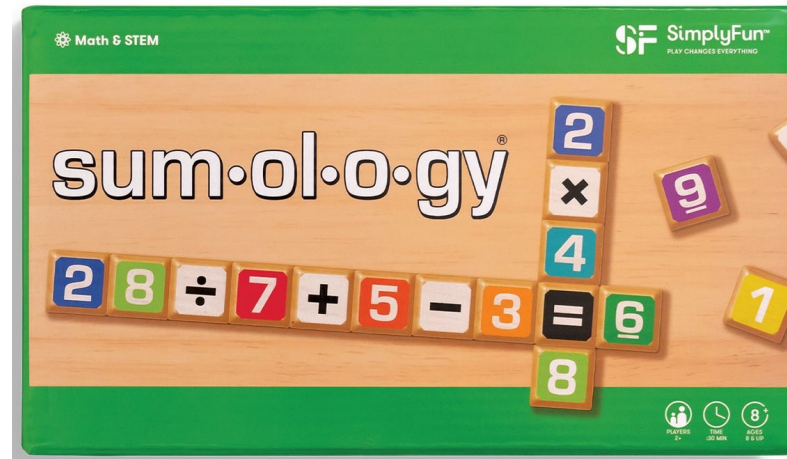
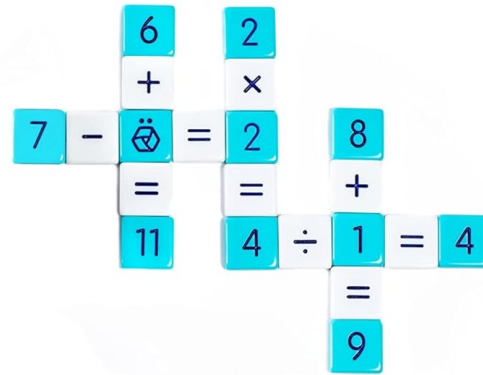
$$\underline{\quad} - \underline{\quad} = \underline{\quad}$$



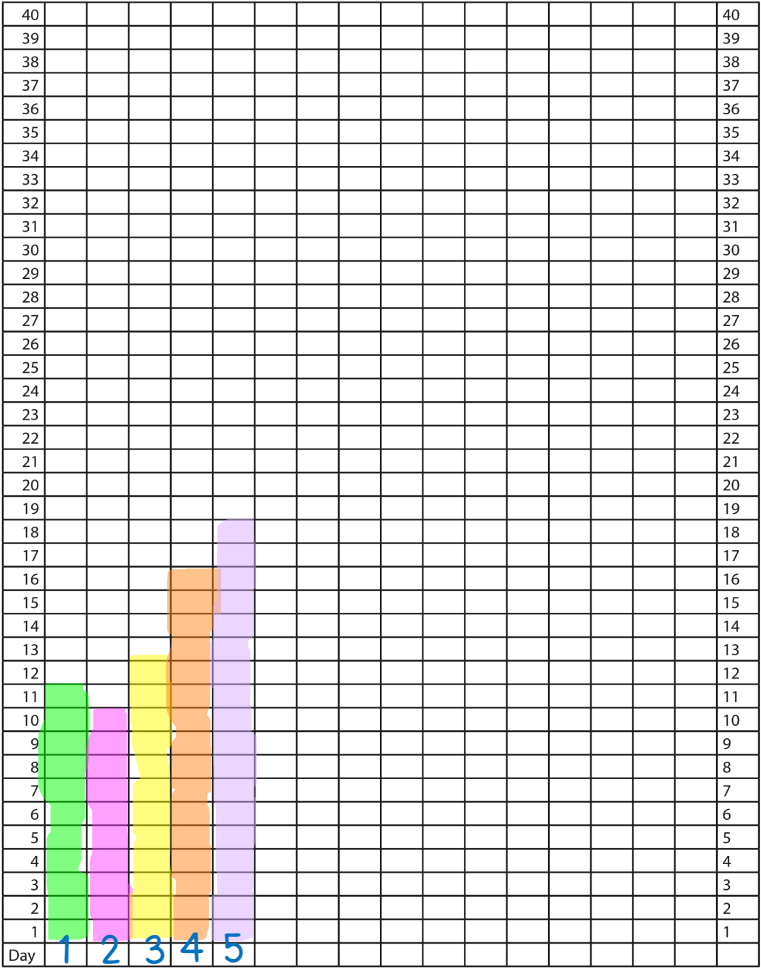
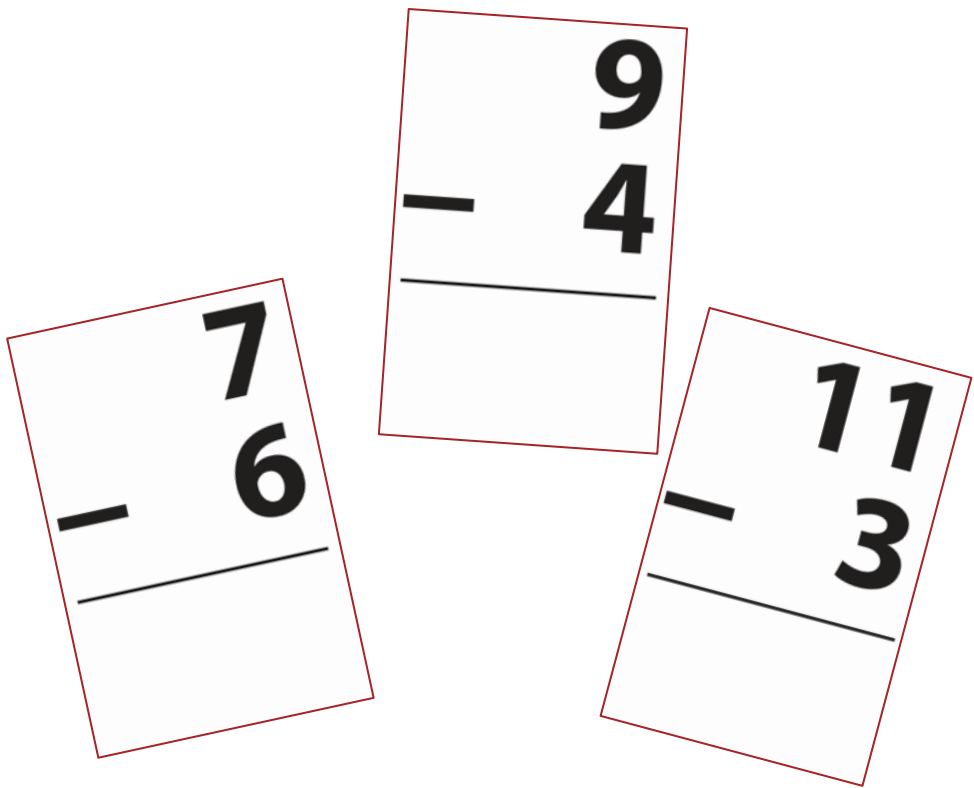
Wrap-Ups



Mobi Math



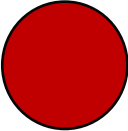
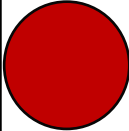
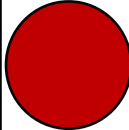
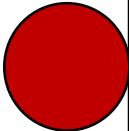
Flash Cards



Bingo

Math Bingo

Your teacher will call out a math problem. Quickly solve the problem. If you have the answer on your bingo card, cover it with a chip. The middle space is free!
The first person to finish the pattern your teacher decides wins!
(5 in a row, 4 corners, etc.)

12		24	100	
15	0	42	16	20
8	35		6	4
	2	40	27	7
50	10	30	48	14

8 times 10 equals...

3 times 1 equals...

2 plus 3 equals...

Magic Squares

Magic Squares Board

- Place the sum or product in the bottom right corner.
- In the bottom row, create a fact with a sum or product of the bottom right corner.
- In the right column, create a fact with a sum or product of the bottom right corner.
- Create two columns with a sum or product of the bottom number.
- Create two rows with a sum or product of the right column number.
- Write the created facts below.

0	2	2
5	4	9
5	6	11

$$\begin{array}{l} 0 + 2 = 2 \\ 5 + 4 = 9 \end{array}$$

$$\begin{array}{l} 2 - 0 = 2 \\ 9 - 5 = 4 \end{array}$$

4	5	9
2	0	2
6	5	11

7	3	10
1	0	1
8	3	11

6	1	7
3	2	5
9	3	12

4	4	8
2	2	4
6	6	12

5	1	6
4	3	7
9	4	13

5	1	6
3	4	7
8	5	13

6	3	9
2	3	5
8	6	14

1	5	6
6	2	8
7	7	14

6	2	8
3	4	7
9	6	15

Cover, Copy, Compare

Cover, Copy, Compare

	9 <u>x 6</u> 54	8 <u>x 6</u> 48	
7 <u>x 8</u> 56		6 <u>x 5</u> 30	
9 <u>x 9</u> 81		7 <u>x 9</u> 63	
6 <u>x 7</u> 42		8 <u>x 5</u> 40	
8 <u>x 8</u> 64		7 <u>x 7</u> 49	

File Folder

$6 + 3 = 9$

$1 + 7 = 8$

$6 + 4 = 10$

$7 + 3 =$

$2 + 7 =$

$5 + 6 =$

$4 + 7 =$

$7 + 8 =$

$6 + 7 =$

$7 + 9 =$

$7 + 6 =$

$8 + 7 =$

$7 + 0 =$

$9 + 6 =$

$6 + 0 =$

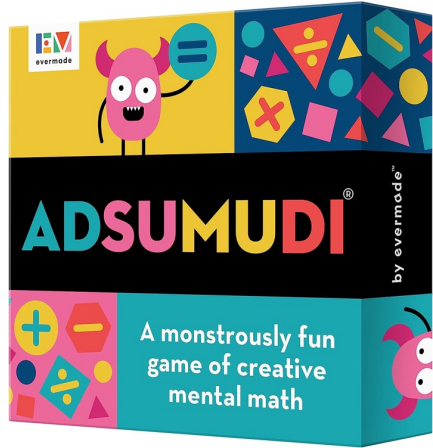
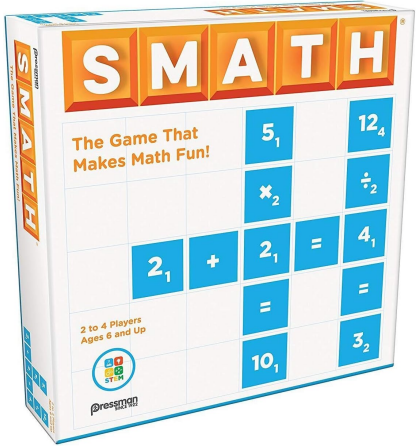
$6 + 8 =$

Taped Problems

Taped Problems

$\begin{array}{r} 8 \\ \times 8 \\ \hline 64 \end{array}$	$\begin{array}{r} 7 \\ \times 7 \\ \hline 49 \end{array}$	$\begin{array}{r} 8 \\ \times 7 \\ \hline \end{array}$
$\begin{array}{r} 6 \\ \times 5 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ \times 7 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ \times 8 \\ \hline \end{array}$
$\begin{array}{r} 5 \\ \times 7 \\ \hline \end{array}$	$\begin{array}{r} 5 \\ \times 5 \\ \hline \end{array}$	$\begin{array}{r} 5 \\ \times 6 \\ \hline \end{array}$
$\begin{array}{r} 6 \\ \times 6 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ \times 6 \\ \hline \end{array}$	$\begin{array}{r} 7 \\ \times 6 \\ \hline \end{array}$
$\begin{array}{r} 7 \\ \times 8 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ \times 5 \\ \hline \end{array}$	$\begin{array}{r} 7 \\ \times 5 \\ \hline \end{array}$

Games



Technology



Third grade > K.7 Division facts up to 10 M8T

Divide:

$$3 \div 3 = \square$$

Submit



Reflex

Get your free 30-day trial

FACT MONSTER

Games / Flashcard

Flashcard

subtraction Level 3 1:51

$$\begin{array}{r} 13 \\ - 6 \\ \hline \end{array}$$

Help your students attain math fact fluency success whether in-person, remote, or through hybrid learning



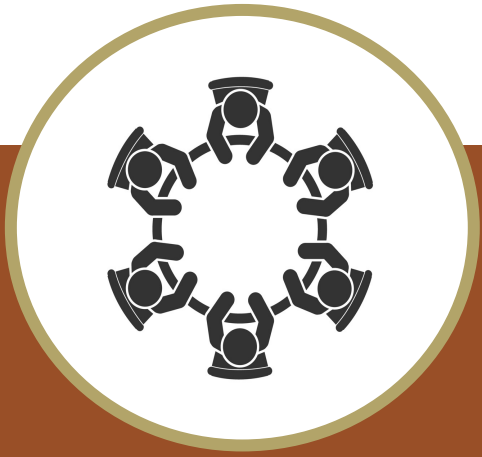
Game-based system to improve math fact fluency for grades 2-6 in less than 30 days!

DAILY

BRIEF

Work on small sets of facts

Work on unknown facts
(in combination with known facts)



What are your strengths with fluency?

What are your opportunities for growth?

What are your plans for next Monday?

Next month?

Next year?

Students require modeling and practice on how to use the language of mathematics.

Students should use hands-on tools, virtual manipulatives, drawings, and other visuals to understand mathematics concepts and procedures.

Teachers should use systematic and explicit instruction to help students develop a strong foundation for specific mathematics skills.

Teachers should use fluency building activities to build counting fluency and fluency with the operations.

Students should learn how to set up and solve word problems by combining an attack strategy with a focus on word-problem schemas.



Word Problems

WORD PROBLEMS

Research and Information

Large empty rectangular box for writing.

Empty rounded rectangular box.

Empty rounded rectangular box.

Empty rounded rectangular box.

Empty rounded rectangular box.

Empty rounded rectangular box.

Empty rounded rectangular box.

Empty rounded rectangular box.



Instructional Platform

INSTRUCTIONAL DELIVERY

Vocabulary

Representations

Model and Practice

INSTRUCTIONAL STRATEGIES

Fluency

Word Problems



Key words tied to operations is an ineffective word-problem strategy.

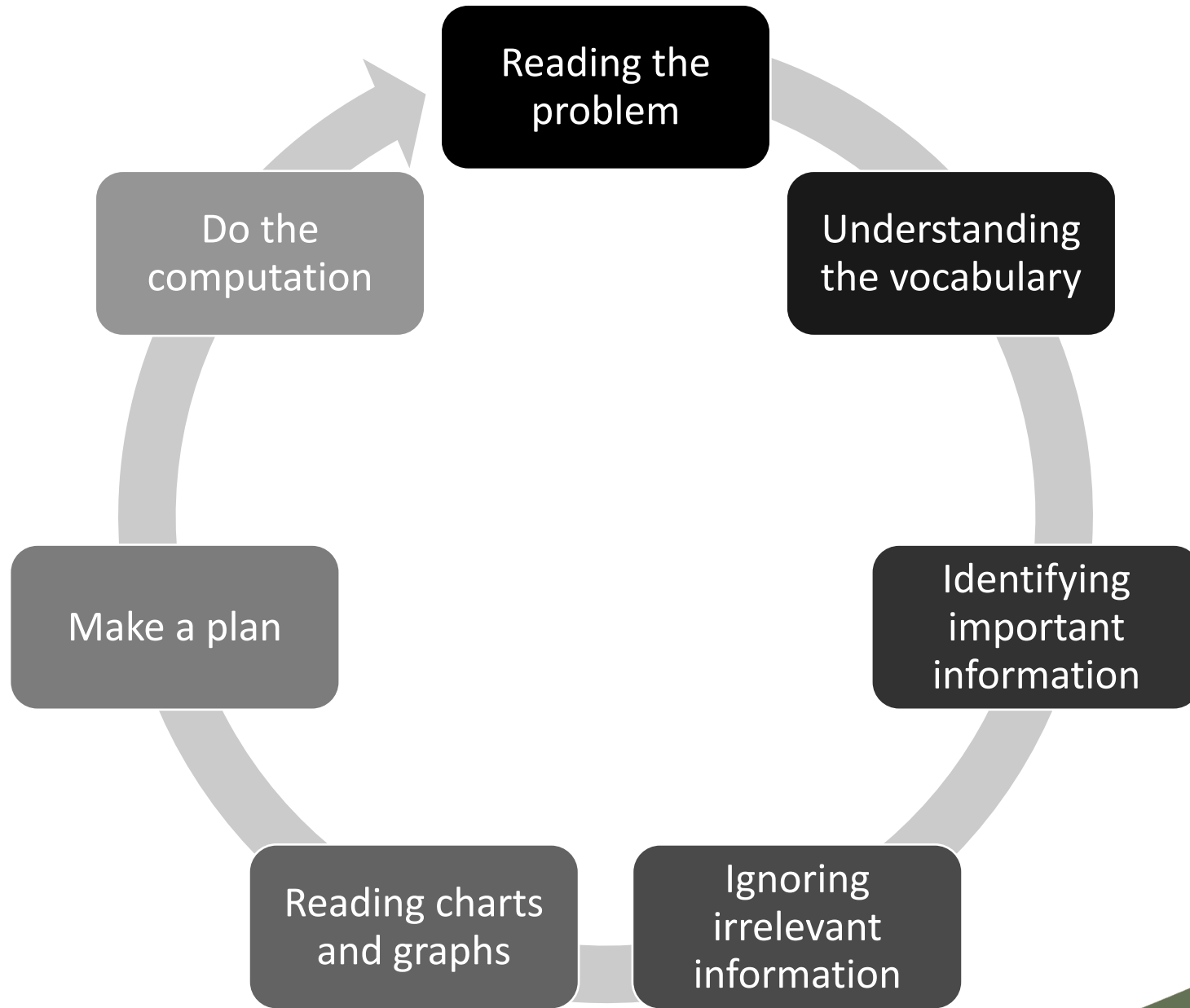
(Karp et al., 2019; Powell et al., 2022)

Using a meta-cognitive strategy improves word-problem performance.

(Freeman-Green et al., 2015; Krawec et al., 2012; Montague et al., 2011; Swanson et al., 2014)

A focus on schemas improves word-problem performance.

(Alghamdi et al., 2020; Cook et al., 2020; Flores et al., 2016; Fuchs et al., 2021; Griffin et al., 2019; Jitendra et al., 2013; Lein et al., 2020; Peltier et al., 2020; Powell et al., 2022; Xin & Xhang, 2009; Zheng et al., 2013)





1. Keywords tied to operations



Carmelita had 8 pencils **fewer** than Jenny. If Jenny had 18 pencils, how many pencils did Carmelita have?

Carmelita had 8 pencils **fewer** than Jenny. If Carmelita had 18 pencils, how many pencils did Jenny have?

Key Words Used In Math Word Problems

<p>Addition Words</p> <ul style="list-style-type: none"> + add + all together or altogether + and + both + combined + how many in all + how much + in all + increased by + plus + sum + together + total <p>Multiplication Words</p> <ul style="list-style-type: none"> x by (dimension) x double x each group x every x factor of x increased by x multiplied by x of x product x times x triple 	<p>Subtraction Words</p> <ul style="list-style-type: none"> - change - decreased by - difference - fewer or fewer than - how many are left (or have left) - how many did not have - how many (or much) more - how much longer (shorter, taller, heavier, etc.) - less or less than - lost - minus - need to - reduce - remain - subtract - take away <p>Division Words</p> <ul style="list-style-type: none"> ÷ as much ÷ cut up ÷ each group has ÷ equal sharing ÷ half (or other fractions) ÷ how many in each ÷ parts ÷ per ÷ percent ÷ quotient of ÷ ratio of ÷ separated ÷ share something equally
---	--

Addition
sum
plus
altogether
perimeter
increased by
together
in all
and

Subtraction
difference
less than
take away
left over
are not
how many more
minus
fewer
exceed
remain

Multiplication
times
in all
product
area
multiplied by
each
twice
factor

Division
half
quotient
equal
group
shared
equally
distributed
cut up

Division
Taking a whole and sharing it into equal parts

Addition
Putting two or more things/amounts together.
Keywords: Total, Altogether, In all, Sum, more than, combined

Addition
add
altogether
both
combined
How many in all?
in all
increase
join
plus
sum
total
together

Subtraction
are not
decrease
difference
fewer, larger, shorter
left
less than
minus
remain
take away

key words

addition: add, together, plus, total, both, perimeter, more than

subtraction: decrease, fewer, remain, take away, less than, how many more...

multiplication: multiply, each, in all, multiple, times, factor, product, double

division: divide, split, equal groups, half, shared equally, each, distribute, average

OPERATION clue words

ADDITION
and, total, join, more than, in all, altogether, sum, increased

SUBTRACTION
less than, decreased, remaining, left, fewer, take away, difference, minus

MULTIPLICATION
product, times, as many as, of, equal groups

DIVISION
quotient, each, broken into, per, cut up, distribute evenly, parts, split

Math Posters!

ADDITION
-sum, -total, -more than, -plus, -both, -combined, -increased by, -perimeter

MULTIPLICATION
-double, -area, -times, -per, -every, -each, -when total is unknown

SUBTRACTION
-difference, -remain, -left, -less than, -minus, -out of, -fewer than, -decrease, -give away, -reduce, -discount, -how many more

DIVISION
-quotient, -into, -minus, -out of, -per, -every, -each, -evenly, -equal groups, -when total is known, -shared

Addition
Total, In all, Sum, more than, combined

Subtraction
Fewer, Less than, Exceed, Remain, Are not, Minus, Difference, How many more, Take away, Left over

Key Words for Addition

added to, plus, join, more than, add, altogether, and, increased by, combined, sum, together, total

Math Key Words

Addition	Subtraction	Multiplication	Division
+	-	x	÷
plus	subtract	times	quotient
sum	minus	product	split
add	difference	factor	share
total	left	double	divide
all together	left over	groups	separate
increase	decrease	each	each
more	take away	area	average
combine	fewer	rows	equal groups



Math Words Poster Set

75%  (4.1)
[7 Reviews](#)
[Write a Review](#)





Description of Single-Step Word Problems (n = 132)

Schema	Occurrence of schema		Any keyword		Schema-specific keywords ^a		Multiple keywords ^a		Keyword(s) led to correct solution ^a	
	n	%	n	%	n	%	n	%	n	%
Total	27	20.5	26	96.3	23	88.5	5	19.2	21	80.8
Difference	17	12.9	17	100.0	14	82.4	2	11.8	12	70.6
Change	11	8.3	7	63.6	5	71.4	5	71.4	2	28.6
Equal groups	29	22.0	26	89.7	22	84.6	18	69.2	8	30.8
Comparison	10	7.6	9	90.0	9	100.0	4	44.4	5	55.6
Ratios or proportions	29	22.0	23	79.3	9	39.1	9	39.1	6	26.1
Product of measures	9	6.8	9	100.0	8	88.9	1	11.1	5	55.6

^aWhen a problem featured a keyword.



Description of Multi-Step Word Problems (n = 84)

Schema	Occurrence of schema ^a		Any keyword		Keyword(s) led to correct solution ^b	
	n	%	n	%	n	%
Total	40	47.6	39	97.5	3	7.7
Difference	11	13.1	11	100.0	1	9.1
Change	21	23.8	19	95.0	1	5.3
Equal groups	49	58.3	48	98.0	1	2.1
Comparison	7	8.3	7	100.0	0	0.0
Ratios or proportions	22	25.0	16	76.2	1	6.3
Product of measures	7	8.3	7	100.0	2	28.6

^aSum across schemas does not equal 100 because each word problem featured more than one schema.

^bWhen a problem featured a keyword.

Keywords are important to identify and understand

Keywords are the mathematical vocabulary that help an students understand what the story is about and what they need to do

Talk about keywords ("What does *more than* tell you about?")



But, *do not* tie a keyword to a specific operation!



2. Presenting Problems by Operation

Name: _____

Date: _____

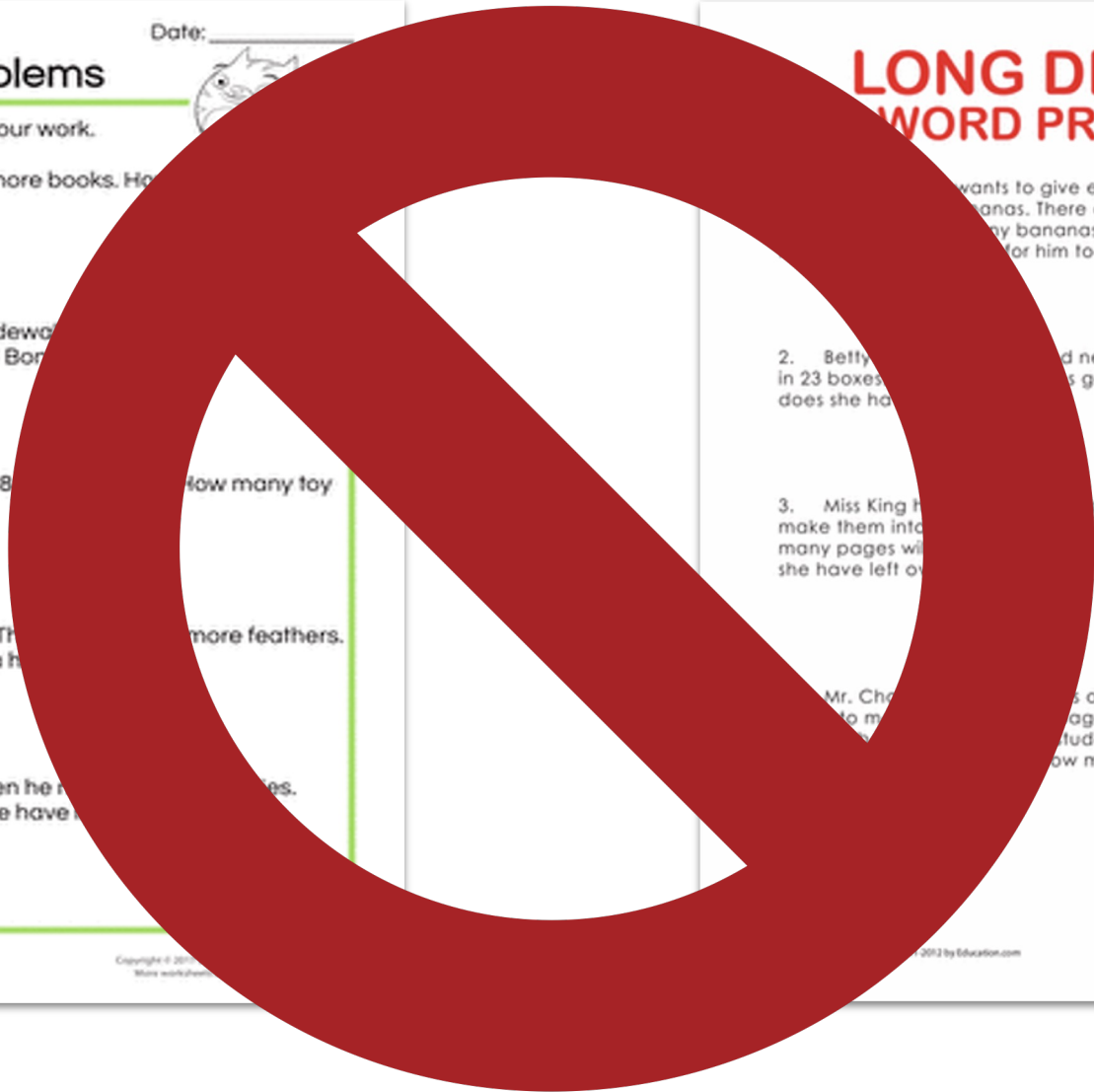
Addition Word Problems

Solve the word problems. Show your work.

1. Noah had 12 books. He got 5 more books. How many books did Noah have in all?
2. Bonnie found 8 rocks on her sidewalk and 15 rocks in her backyard. How many rocks did Bonnie find in all?
3. Edward had 5 toy cars. He got 8 more toy cars. How many toy cars did Edward have in all?
4. Mariela collected 11 feathers. Then she collected 15 more feathers. How many feathers did Mariela have in all?
5. LaMonte made 14 cookies. Then he made 18 more cookies. How many cookies did LaMonte have in all?

LONG DIVISION WORD PROBLEMS

- Mr. King wants to give each monkey at the zoo an equal number of bananas. There are 37 monkeys in the zoo and he has 148 bananas. How many bananas does each monkey get? And how many bananas are left over for him to eat himself?
2. Betty has 148 stickers and needs to pack them up equally in 23 boxes. How many stickers go in each box and how much does she have left over?
 3. Miss King has 148 sheets of scrap paper. She wants to make them into booklets for her 32 students. How many pages will she have left over? How many extra pages will she have left over?
 4. Mr. Cho has 148 sheets of scrap paper. He instead makes booklets for 32 students. Will there be enough pages for each student? How much more scrap paper does he need?



WORD PROBLEMS

Attack Strategies

SOLVE

- Study the problem
- Organize the information
- Line up a plan
- Verify the plan
- Examine the answer

R-CUBES

- Read the problem
- Circle key numbers
- Underline the question
- Box action words
- Evaluate steps
- Solve and check

UPS Check

- Understand
- Plan
- Solve
- Check



Teach an attack strategy

Teach about schemas

RIDE

Read the problem.

Identify the relevant information.

Determine the operation and unit for the answer.

Enter the correct numbers and calculate, then check the answer.

RICE

Read and record the problem.

Illustrate your thinking.

Compute.

Explain your thinking.

RIDGES

Read the problem.

I know statement.

Draw a picture.

Goal statement.

Equation development.

Solve the equation.

SUPER

Slowly read the story problem twice.

Underline the question and circle the numbers you need.

Picture it. Draw the scenario to show what is happening.

Explain the problem with a number sentence.

Rewrite the answer in a sentence.

SHINES

Slowly and carefully read the problem.

Highlight or underline key information.

Identify the question by drawing a circle around it.

Now solve the problem. Show your work.

Examine your work for precision, accuracy, and clarity.

Share your answer by writing a sentence.

STAR

Stop and read the problem carefully.

Think about your plan and the strategy you will use.

Act. Follow your plan and solve the problem.

Review your answer.

SOLVE

Study the problem.

Organize the information.

Line up the plan.

Verify the plan with computation.

Examine the answer.

R-CUBES

Read the problem.

Circle key numbers.

Underline the question.

Box action words.

Evaluate steps.

Solve and check.

UPS✓

UNDERSTAND

Read and explain.

PLAN

How will you solve the problem?

SOLVE

Set up and do the math!

✓CHECK

Does your answer make sense?

Created by: Sarah Powell (srpowell@austin.utexas.edu)



Share your favorite attack strategy.

Teach an attack strategy

Teach about schemas

Total

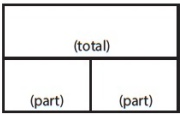
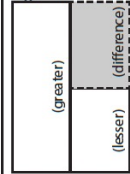
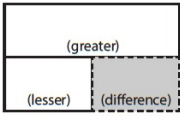
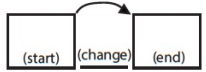
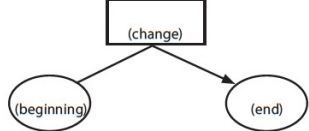
Difference

Change

Equal Groups

Comparison

Ratios/Proportions

Schema and Definition	Equations and Graphic Organizers	Examples	Variations
<p>Total (Combine; Part-part- whole) Parts combined for a sum</p>	$P1 + P2 = T$ (part + part = total) 	<p>Sum unknown: Lyle has 11 red apples and 18 green apples. How many apples does Lyle have altogether?</p> <p>Part unknown: Lyle has 29 red and green apples. If 11 of the apples are red, how many green apples does Lyle have?</p>	<p>More than two parts: Lyle has 34 apples. Of the apples, 11 are red, 18 are green, and the rest are yellow. How many yellow apples does Lyle have?</p>
<p>Difference (Compare) Sets compared for a difference</p>	$B - s = D$ (bigger - smaller = difference)  $G - L = D$ (greater - less = difference) 	<p>Difference unknown: Sasha wrote 85 words in her essay, and Tabitha wrote 110 words. How many fewer words did Sasha write than Tabitha?</p> <p>Bigger/greater unknown: Tabitha wrote 25 more words than Sasha. If Sasha wrote 85 words, how many words did Tabitha write?</p> <p>Smaller/lesser unknown: Tabitha wrote 110 words in her essay. Sasha wrote 25 words fewer than Tabitha. How many words did Sasha write?</p>	<p>(None)</p>
<p>Change (Join; Separate) An amount that increases or decreases</p>	$ST +/- C = E$ (start +/- change = end)  	<p>End (increase) unknown: Jorge had \$52. Then, he earned \$16 babysitting. How much money does Jorge have now?</p> <p>Change (increase) unknown: Jorge had \$52. Then, he earned some money babysitting. Now, Jorge has \$68. How much did Jorge earn babysitting?</p> <p>Start (increase) unknown: Jorge has some money, and then he earned \$16 for babysitting. Now, Jorge has \$68. How much money did he have to start with?</p> <p>End (decrease) unknown: Jorge had \$52. Then, he spent \$29 at the ballpark. How much money does Jorge have now?</p> <p>Change (decrease) unknown: Jorge had \$52 but spent some money when he went to the ballpark. Now, Jorge has \$23. How much did Jorge spend at the ballpark?</p> <p>Start (decrease) unknown: Jorge had some money. Then, he spent \$29 at the ballpark and has \$23 left. How much money did Jorge have before going to the ballpark?</p>	<p>Multiple changes: Jorge had \$78. He stopped and bought a pair of shoes for \$42 and then he spent \$12 at the grocery. How much money does Jorge have now?</p>



Powell & Fuchs (2018).
Material collected from: Griffin & Jitendra, 2009; Fuchs et al., 2014; Fuchs, Seethaler, et al., 2008; Fuchs et al., 2010; Jitendra, 2002; Kintsch & Greeno, 1985; Van de Walle, Karp, & Bay-Williams, 2013.



Total

Part-part-whole
Combine

Parts put together into a **total**

Mandy saw **3** canoes and **8** kayaks. How many boats did Mandy see?

Total

Mandy saw **11** boats. If **3** of the boats were canoes, how many were kayaks?

Part

Mandy saw **11** boats. **8** of the boats were kayaks, how many were canoes?

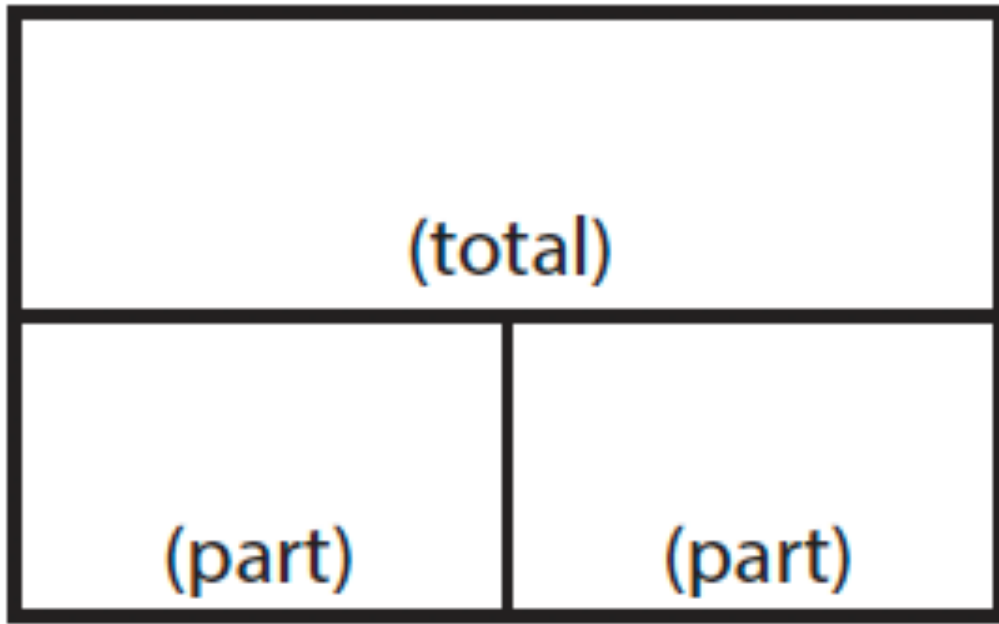
Part

Total

“Are parts put together for a total?”

Total

$$P1 + P2 = T$$



Total

WORD PROBLEMS: TOTAL

A.
An artist poured 6.09 kilograms of orange sand and 14.26 kilograms of blue sand into a mixing container for a project. What was the total amount of sand the artist poured into the container in kilograms?

B.
A movie theater has 710 seats.
• 158 seats are red.
• 247 seats are black.
• 119 seats are yellow.
• The rest of the seats are green.
How many seats are green?

C.
The frequency table shows the number of visitors a park had on three different days.

Park Visitors

Day	Number of Visitors
Friday	
Saturday	
Sunday	

What was the total number of visitors the park had over those three days?

Your TOTAL problem:



Difference

Compare

Greater and **lesser** amounts compared for a **difference**

L'Tanya has **10** pencils. Vickie has **4** pencils.
How many more pencils does L'Tanya have?

Difference

L'Tanya has **6** more pencils than Vickie. If
Vickie has **4** pencils, how many does L'Tanya
have?

Greater
amount

Vickie has **6** fewer pencils than L'Tanya.
L'Tanya has **10** pencils. How many pencils
does Vickie have?

Lesser
amount

Total

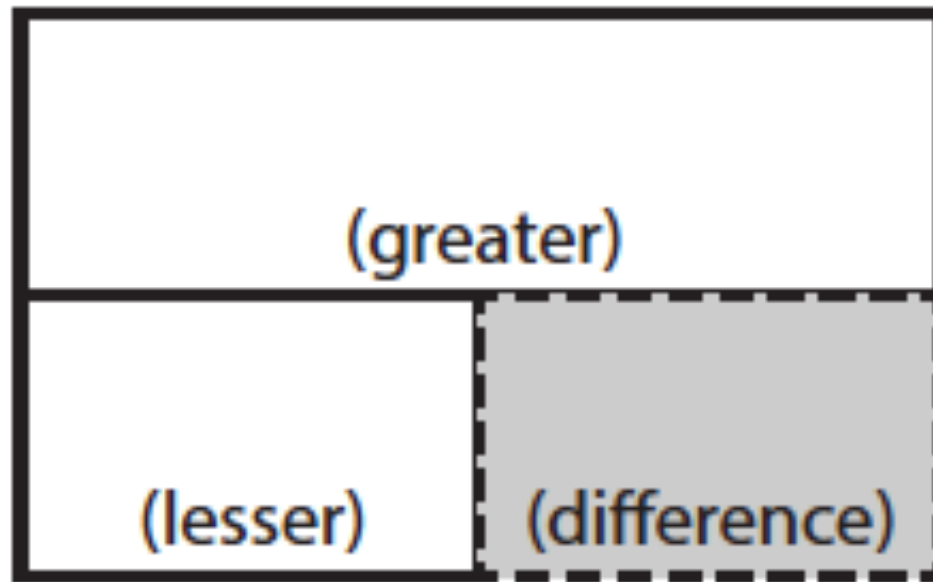
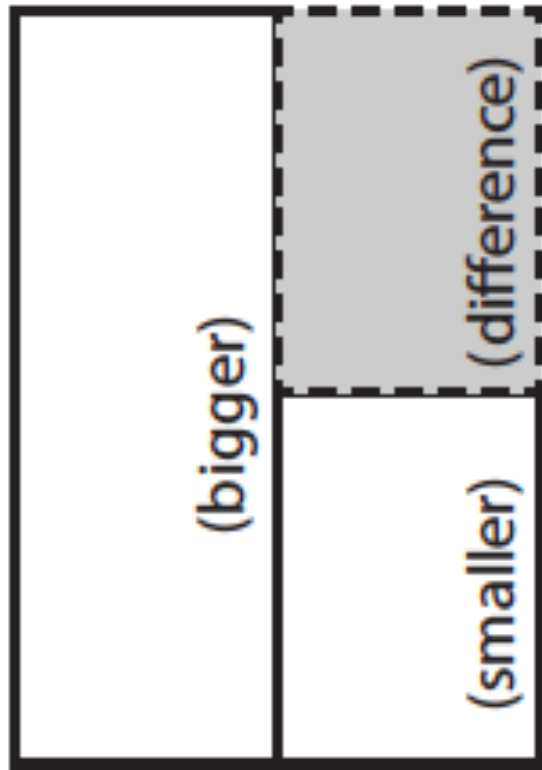
“Are parts put together for a total?”

Difference

“Are amounts compared for a difference?”

Difference

$$G - L = D$$



Difference

WORD PROBLEMS: DIFFERENCE

D.
There are two lions at a zoo. The weight of the younger lion is 379 pounds. The weight of the older lion is 514 pounds. What is the difference in pounds between these two weights?

E.
The frequency table shows the number of movies watched last month by each student in Mr. Westley's class.

Movies Watched Last Month

Number of Movies	Number of Students
1	III II
2	III III II
3	III
4	II

What is the difference between the number of students who watched 2 movies last month and the number of students who watched one movie last month?

F.
Samantha, Gordon, and Diego each brought an ice chest to a picnic.

- The weight of Samantha's ice chest was 83 pounds.
- The weight of Gordon's ice chest was 28 pounds.
- The weight of Diego's ice chest was 37 pounds.

What was the difference in pounds between the weight of Samantha's ice chest and the combined weight of Gordon's and Diego's ice chests?

Your DIFFERENCE problem:



Change

Join

An amount that **increases** or decreases

Annette had 6 notebooks. Then, she bought 3 notebooks. How many notebooks does Annette have now?

Annette had 6 notebooks. Then, she bought a few more notebooks. Now, Annette has 9 notebooks. How many notebooks did she buy?

Annette had some notebooks. Then, she bought 3 notebooks. Now, Annette has 9 notebooks. How many notebooks did she have to start with?

End
amount

Change
amount

Start
amount

Change

Separate

An amount that increases or **decreases**

Jenny baked **9** cookies. Then, she ate **3** of the cookies. How many cookies does Jenny have now?

End
amount

Jenny baked **9** cookies. Then, she ate some of the cookies. Now, she has **6** cookies. How many cookies did Jenny eat?

Change
amount

Jenny baked some cookies. She ate **3** of the cookies and has **6** cookies left. How many cookies did Jenny bake?

Start
amount

Total

“Are parts put together for a total?”

Difference

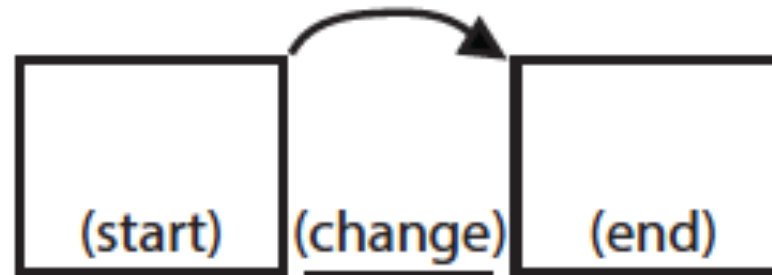
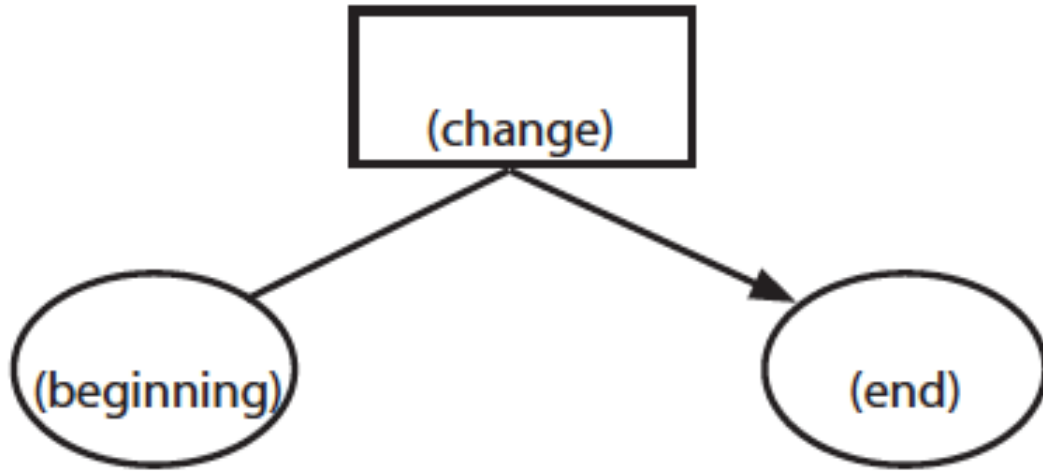
“Are amounts compared for a difference?”

Change

“Does an amount increase or decrease?”

Change

$$ST + / - C = E$$



Change

WORD PROBLEMS: CHANGE

G.
Landon had one string that was 10 meters long. He used 6.275 meters of this string for a project. What was the length of string in meters that Landon had left?

H.
A parade began at 11:30 a.m. and ended at 2:18 p.m. How long did the parade last?

I.
Ms. Fitzgerald had gallons of fruit punch. She served gallon of the fruit punch to her family at lunch. How many gallons of fruit punch did Ms. Fitzgerald have left after lunch?

Your CHANGE problem:



Total

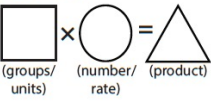
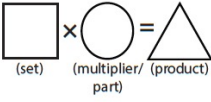
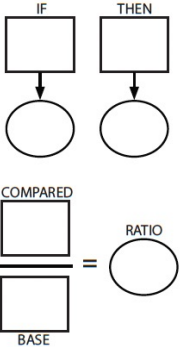
Difference

Change

Equal Groups

Comparison

Ratios/Proportions

Schema and Definition	Graphic Organizers	Examples	Variations
<p>Equal Groups (Vary)</p> <p>A number of equal sets or units</p>	$GR \times N = P$ 	<p>Product unknown:</p> <p>Maria bought 5 cartons of eggs with 12 eggs in each carton. How many eggs did Maria buy?</p> <p>Groups unknown:</p> <p>Maria bought 60 eggs. The eggs were sold in cartons with 12 eggs each. How many cartons of eggs did Maria buy?</p> <p>Number unknown:</p> <p>Maria bought 5 cartons of eggs for a total of 60 eggs. How many eggs were in each carton?</p>	<p>With rate:</p> <p>Maria bought 5 cartons of eggs. Each carton cost \$2.95. How much did Maria spend on eggs?</p>
<p>Comparison</p> <p>One set as a multiple or part of another set</p>	$S \times T = P$ 	<p>Product unknown:</p> <p>Malik picked 7 flowers. Danica picked 3 times as many flowers. How many flowers did Danica pick?</p> <p>Set unknown:</p> <p>Danica picked 3 times as many flowers as Malik. If Danica picked 21 flowers, how many flowers did Malik pick?</p> <p>Times unknown:</p> <p>Malik picked 7 flowers. Danica picked 21 flowers. How many times more flowers did Danica pick?</p>	<p>With fraction:</p> <p>Malik picked 25 red and yellow flowers. If 1/5 of the flowers were yellow, how many were red?</p>
<p>Ratios/Proportions (Percentages; Unit Rate)</p> <p>Relationships among quantities</p> <p>Ratio</p>	<p>IF THEN</p> 	<p>Subject unknown:</p> <p>Sally typed 56 words in 2 minutes. How many words could Sally type in 7 minutes?</p> <p>Object unknown:</p> <p>Sally typed 56 words in 2 minutes. How many minutes would it take Sally to type 192 words?</p> <p>Base unknown:</p> <p>Justin baked cookies and brownies. The ratio of cookies to brownies was 3:5. If he baked 15 cookies, how many brownies did he bake?</p> <p>Compared unknown:</p> <p>Justin baked cookies and brownies. The ratio of cookies to brownies was 3:5. If he baked 25 brownies, how many cookies did he bake?</p> <p>Ratio unknown:</p> <p>Justin baked 15 cookies and 25 brownies. What's the ratio of cookies to brownies?</p>	<p>With percentage:</p> <p>Watson received an 80% on his science quiz. If the test had 40 questions, how many questions did Watson answer correctly?</p> <p>With unit rate:</p> <p>Paula bought 5 boxes of markers. She spent \$9.75. What is the price of one box of markers?</p>

Material collected from: Jitendra, DiPipi, & Perron-Jones, 2002; Jitendra & Star, 2011; Jitendra et al., 2009; Van de Walle et al., 2013; Xin, Jitendra, & Deatline-Buchman, 2005; Xin & Zhang, 2009.



Equal Groups

Array
Vary

Groups multiplied by **number in each group** for a **product**

Evan has **2** boxes. There are **6** muffins in each box.
How many muffins does Evan have?

Evan has **12** muffins. They want to place them equally
into **2** boxes. How many muffins will Evan place in
each box?

Evan has **12** muffins. They put them into boxes with **6**
muffins each. How many boxes did Evan use?

Product

Number in
each group

Groups

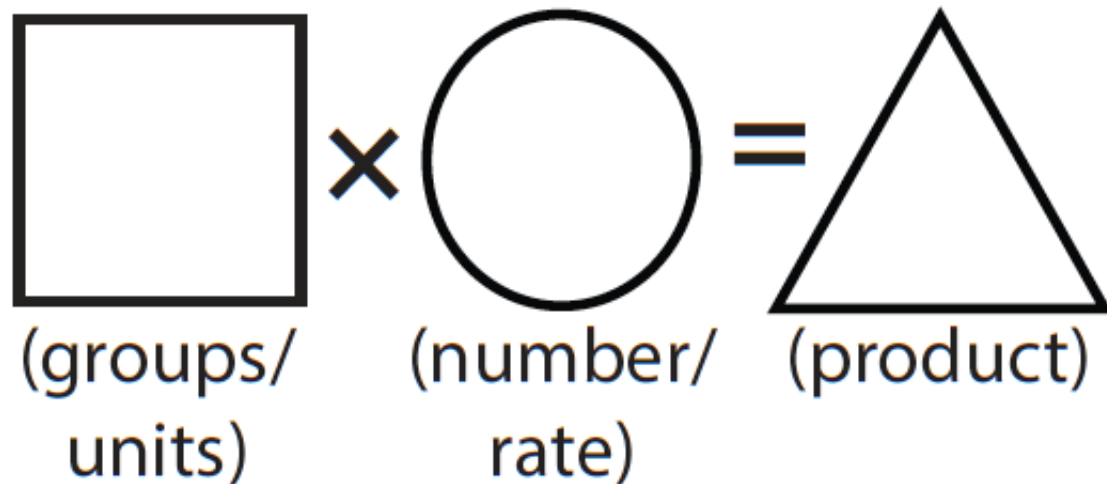
Equal Groups

“Are there groups with an equal number in each group?”

Equal Groups

$$\mathbf{GR} \times \mathbf{N} = \mathbf{P}$$

$$\mathbf{GR} \times \mathbf{E} = \mathbf{P}$$



Equal Groups

WORD PROBLEMS: EQUAL GROUPS

J.
There are 4 erasers on each table in a classroom.
There are 5 tables in the classroom. What is the total number of erasers on all of the tables in this classroom?

K.
Carmine has 291 balloons. She put the same number of balloons into 3 groups. What is the best estimate of the number of balloons in each group?
A. 90
B. 100
C. 75
D. 85

L.
Kelsi spends \$6.75 every Saturday for breakfast.
What is the total amount of money Kelsi spends on breakfast for 14 Saturdays?

Your EQUAL GROUPS problem:

Comparison

Set multiplied by a number of **times** for a **product**

Joan ran **6** minutes. L'Tanya ran **4** times longer than Joan. How many minutes did L'Tanya run?

Set

Number of
times

Product

Equal Groups

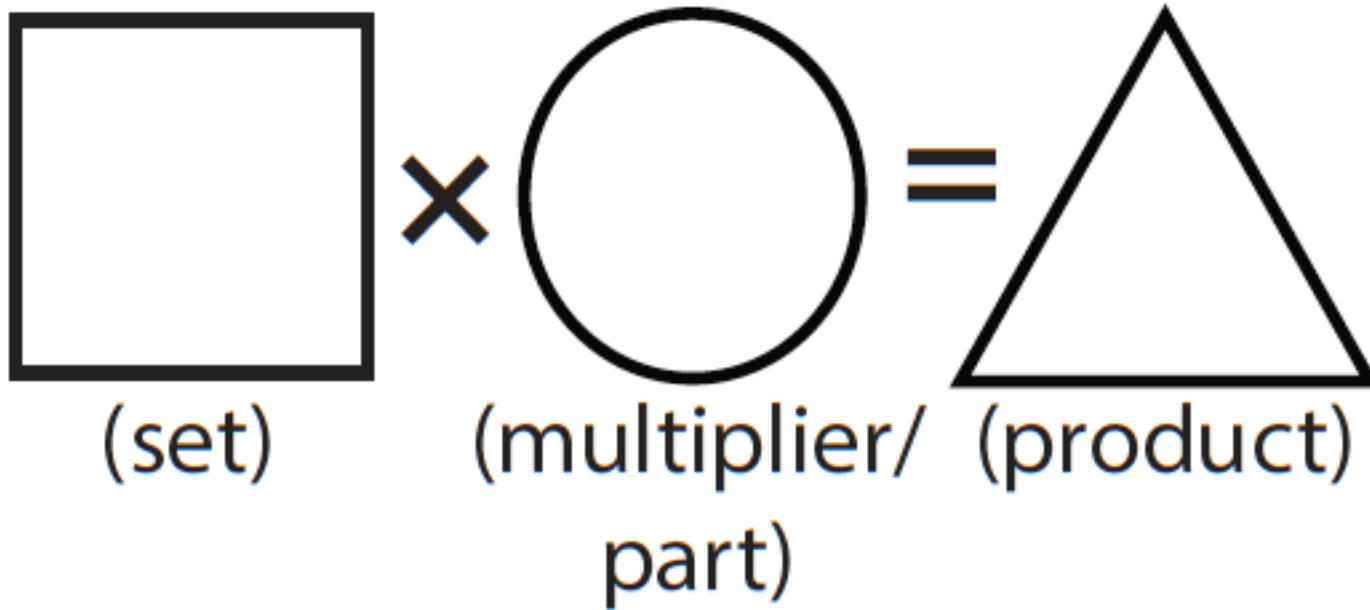
“Are there groups with an equal number in each group?”

Comparison

“Is a set compared a number of times?”

Comparison

$$S \times T = P$$



Comparison

WORD PROBLEMS: COMPARISON

M.
Jonathan and Elizabeth are comparing the masses of their rocks.
•Jonathan's rock has a mass of 0.2 kilogram.
•Elizabeth's rock has a mass 8 times the mass of Jonathan's rock.

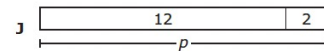
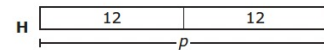
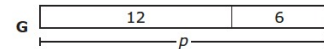
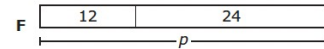
What is the mass of Elizabeth's rock in kilograms??

N.
Haruko did 9 sit-ups in P.E. class. The number of sit-ups Tom did can be represented by this expression. 2×9

Which statement is true?

- F. Tom did 2 times as many sit-ups as Haruko.
- G. Haruko did 2 times as many sit-ups as Tom.
- H. Tom did 2 more sit-ups than Haruko.
- J. Haruko did 2 more sit-ups than Tom.

O.
Erin has 12 pictures from a field trip and some pictures from a vacation. She has twice as many pictures from the vacation as from the field trip. Which strip diagram represents p , the total number of pictures Erin has?



Your COMPARISON problem:

Ratio/Proportion

Description of **relationships** among quantities

Melissa baked cookies and brownies. The ratio of cookies to brownies was 3:5. If she baked 25 brownies, how many cookies did she bake?

Emma typed 56 words in 2 minutes. At this rate, how many words could Emma type in 7 minutes?

Equal Groups

“Are there groups with an equal number in each group?”

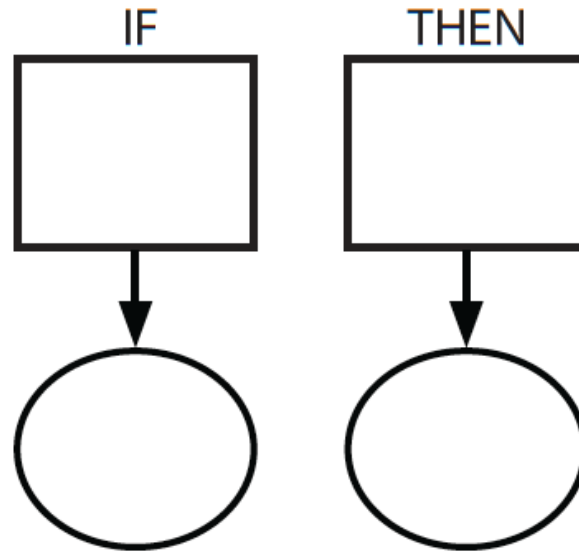
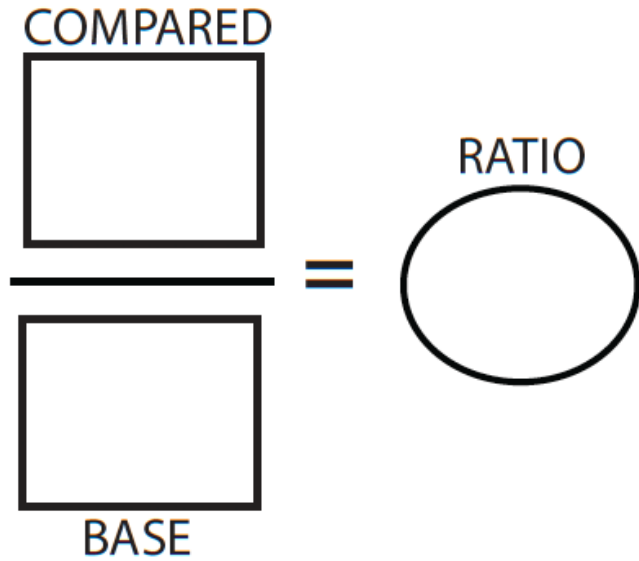
Comparison

“Is a set compared a number of times?”

Ratio/Proportion

“Are there relationships among quantities - if this, then this?”

Ratio/Proportion



Ratio/Proportion

WORD PROBLEMS: RATIO or PROPORTION

P.
Dennis made an extra \$245.00 for selling furniture. The extra \$245.00 was 7% of the total value of the furniture he sold. What was the total value of the furniture Dennis sold?

Q.
A baseball traveled 330 feet in 5 seconds. Which rate is equivalent to the rate at which the baseball traveled?
A. 55 feet per second
B. 66 feet per second
C. 55 seconds per foot
D. 66 seconds per foot

R.
During a 90-minute school play, the main character was on stage 80% of the time. What amount of time in minutes was the main character on stage?

Your RATIO or PROPORTION problem



Total

Difference

Change

Equal Groups

Comparison

Ratios/Proportions

WORD PROBLEMS



What are your strengths?



What are your opportunities for growth?



What are your plans for next Monday?
Next month?
Next year?



What are your strengths with word-problem solving?

What are your opportunities for growth?

What are your plans for next Monday?

Next month?

Next year?

Students require modeling and practice on how to use the language of mathematics.

Students should use hands-on tools, virtual manipulatives, drawings, and other visuals to understand mathematics concepts and procedures.

Teachers should use systematic and explicit instruction to help students develop a strong foundation for specific mathematics skills.

Teachers should use fluency building activities to build counting fluency and fluency with the operations.

Students should learn how to set up and solve word problems by combining an attack strategy with a focus on word-problem schemas.

Instructional Platform

INSTRUCTIONAL DELIVERY

Vocabulary

Representations

Model and Practice

INSTRUCTIONAL STRATEGIES

Fluency

Word Problems

Resources



Pirate Math Equation Quest

About

Research

Individual

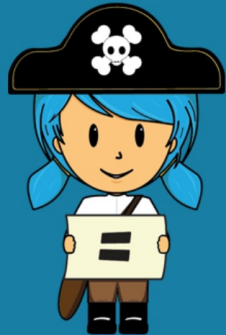
Small Group

STAAR

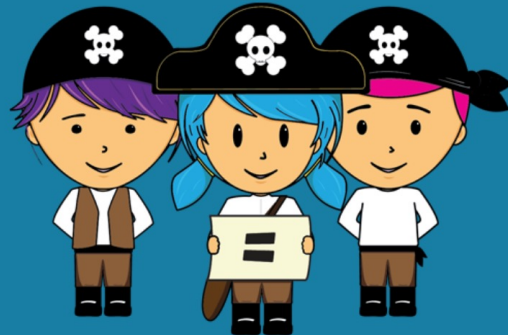
Videos

Welcome to Pirate Math Equation Quest!

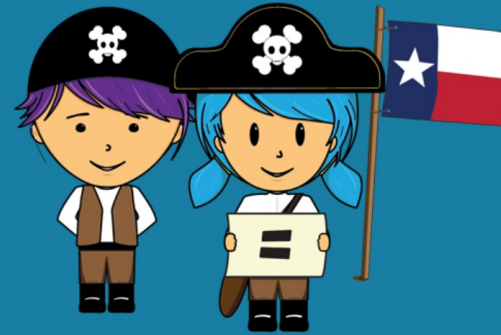
Individual Word-Problem Intervention



Small-Group Word-Problem Intervention



Small-Group Word-Problem Intervention for STAAR



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<https://intensiveintervention.org>

National Center on
INTENSIVE INTERVENTION
at American Institutes for Research

Search


Intensive Intervention ▾ Tools Charts ▾ Implementation Support ▾ Intervention Materials ▾ Information For... ▾

Intensive Intervention in Mathematics Course Content

NCII, through a collaboration with the University of Connecticut, developed a set of course content focused on developing educators' skills in designing and delivering intensive mathematics instruction. This content is designed to support faculty and professional development providers with instructing pre-service and in-service educators who are developing and/or refining their implementation of intensive mathematics intervention.


Intensive instruction was recently identified as a [high-leverage practice in special education](#), and DBI is a research based approach to delivering intensive instruction across content areas (NCII, 2013). This course provides learners with an opportunity to extend their understanding of intensive instruction through in-depth exposure to DBI in mathematics, complete with exemplars from actual classroom teachers.

NCII, through a collaboration with the University of Connecticut and the [National Center on Leadership in Intensive Intervention](#) and with support from the [CEEDAR Center](#), developed course content focused on enhancing educators' skills in intensive mathematics intervention. The course includes eight modules that can support faculty and professional development providers with instructing pre-service and in-service educators who are learning to implement intensive mathematics intervention through data-based individualization (DBI). The content in this course complements concepts covered in the [Features of Explicit Instruction Course](#) and so we suggest that users complete both courses.



MODULE 5: INTENSIVE MATHEMATICS INTERVENTION: INSTRUCTIONAL STRATEGIES





Instructional Routines for Mathematics Intervention

The purpose of these mathematics instructional routines is to provide educators with materials to use when providing intervention to students who experience difficulty with mathematics. The routines address content included in the grades 2-8 Texas Essential Knowledge and Skills (TEKS). There are 23 modules that include routines and examples – each focused on different mathematical content. Each of the 23 modules include vocabulary cards and problem sets to use during instruction. These materials are intended to be implemented explicitly with the aim of improving mathematics outcomes for students.

TEA
Texas Education Agency

Module 9: Subtraction of Rational Numbers

Problem Sets

- A. [Proper fractions with like denominators and sums <1 \(20\)](#)
- B. [Improper fractions with like denominators and sums >1 \(10\)](#)
- C. [Mixed numbers with like denominators and sums >1 \(10\)](#)
- D. [Proper fractions with unlike denominator and sums <1 \(20\)](#)
- E. [Improper fractions with unlike denominator and sums >1 \(10\)](#)
- F. [Mixed numbers with unlike denominator and sums >1 \(10\)](#)
- G. [Decimals with tenths; no regrouping \(20\)](#)
- H. [Decimals with tenths; regrouping \(20\)](#)
- I. [Decimals with hundredths; no regrouping \(20\)](#)
- J. [Decimals with hundredths; regrouping \(20\)](#)
- K. [Decimals with tenths and hundredths; mix of regrouping \(20\)](#)



numerator

The term in a fraction that tells how many parts of a fraction.

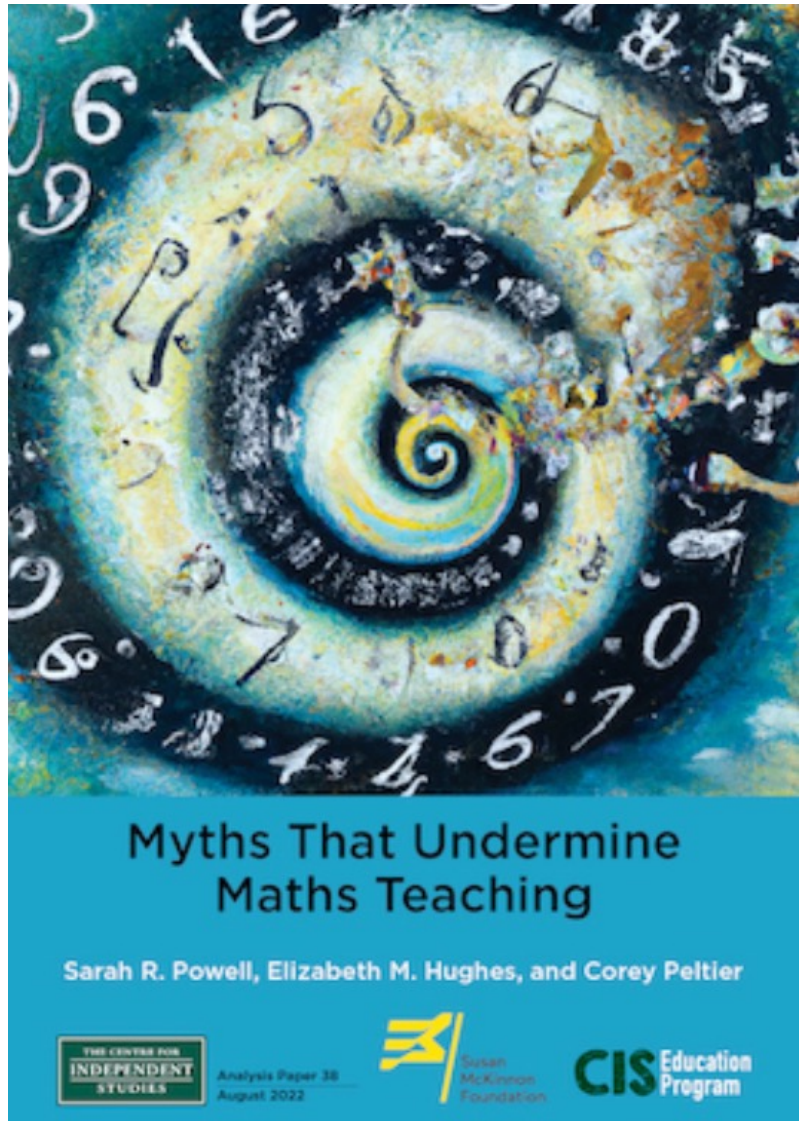
$\frac{2}{3}$ $\frac{2}{3}$ In these fractions, 2 is the numerator.

ones

The digit representing 1.

In the number 4.23, 4 is in the ones place.

<https://spedsupportstage.tea.texas.gov/resource-library/instructional-routines-mathematics-intervention>



Centre for Independent Studies



MATHS PRACTICES YOU CAN COUNT ON

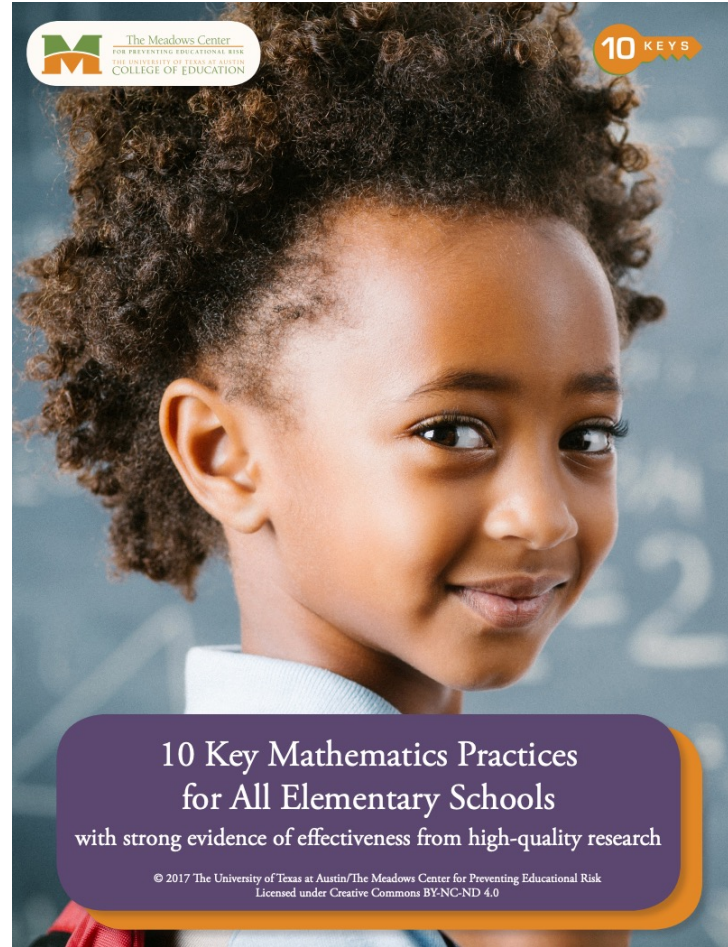
A Guide to Five Research-Validated
Practices in Mathematics

Sarah R. Powell, Sarah G. King, and Sarah A. Benz



Centre for Independent Studies





<https://meadowscenter.org/resource/10-key-mathematics-practices-for-all-elementary-schools-with-strong-evidence-of-effectiveness-from-high-quality-research/>

#CEC2024





10 KEY MATH PRACTICES for All Middle and High Schools

with strong evidence of effectiveness from high-quality research

All middle and high school students can become proficient in mathematics if:

1. Teachers help students to solve mathematics problems by using manipulatives and tools to bridge concrete to abstract and symbolic understandings of mathematics.
2. Students are asked to make their mathematics thinking transparent by talking about their solution process, drawing a picture, or making a graph and using mathematically correct language (for example, using the terms "numerator" and "denominator" rather than "top number" and "bottom number" for fractions, telling how many groups of a divisor are in the dividend for division rather than saying 5 "goes into" 20 four times, or using the term "zero pairs" rather than "canceling out").
3. Students are asked to read and critique one another's written responses to problems.
4. Teachers present "real-life" word problems for students to solve daily.
5. Students are expected to solve multiplication and division facts regularly as a basis for working on rational numbers and algebraic problems.
6. Students are expected to master the properties of operations (order of operations; commutative, associative, and distributive properties; multiplicative identity property; multiplicative inverse property).
7. Students are given solved problems (correctly solved and incorrectly solved using common misconceptions) to analyze and discuss how the problems were solved and where the solution strategy broke down for incorrectly solved problems.
8. Teachers differentiate mathematics instruction for diverse learners (for example, struggling learners, English language learners, gifted students, and average achievers).
9. Teachers verbalize (think aloud, describe steps for a strategy) explanations of concepts and steps for solving problems.
10. Teachers collect data regularly to determine whether their students are benefiting from instruction and use the data to make informed instructional decisions for subsequent lessons.

10 KEYS

The Meadows Center
FOR PREVENTING EDUCATIONAL RISK
THE UNIVERSITY OF TEXAS AT AUSTIN
COLLEGE OF EDUCATION

—www.meadowscenter.org—

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<https://meadowscenter.org/resource/10-key-math-practices-for-all-middle-and-high-schools-with-strong-evidence-of-effectiveness-from-high-quality-research/>

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<https://ies.ed.gov/ncee/wwc/PracticeGuide/26>



What Works
Clearinghouse™

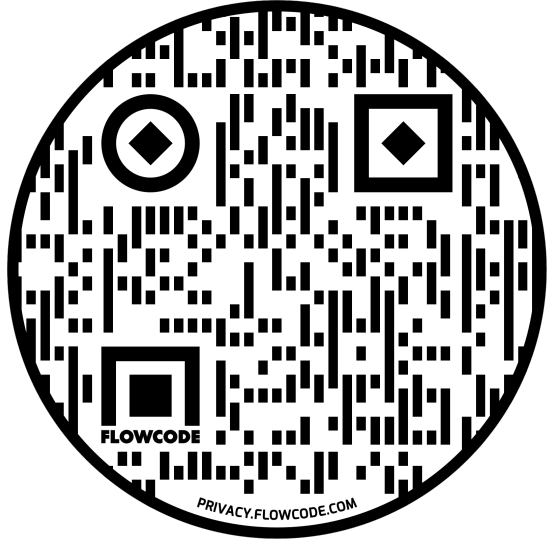
Assisting Students Struggling with Mathematics: Intervention in the Elementary Grades

Educator's Practice Guide

WWC 2021006
U.S. DEPARTMENT OF EDUCATION

A publication of the National Center for Education Evaluation and Regional Assistance (NCEE) at IES





bit.ly/srpowell

Virtual Manipulatives

Help students see and learn math using different tools!

Number & Operations

Fractions & Decimals

Geometry

Data & Probability

Place Value

Integers & Algebra

Time & Money

Extras

Sarah R. Powell, Ph.D.
srpowell@utexas.edu
www.sarahpowellphd.com
@sarahpowellphd

Fractions & Decimals	<p>fraction strips</p>	<p>fraction strips</p>	<p>fraction strips</p>	<p>Cuisenaire rods</p>
<p>fraction circles</p>	<p>geoboard</p>	<p>geoboard</p>	<p>geoboard</p>	<p>pattern blocks</p>
<p>two-color counters</p>	<p>decimal strips</p>	<p>place value disks</p>	<p>percentage strips</p>	

Resources



Instructional Platform



Explicit Instruction



Language



Multiple Representations



Fluency



Problem Solving



Additional Resources



Coaching



<https://www.mathspiral.com>



<p>STAIR Tailored: Culturally Responsive Teaching Part 1: What is it?</p> <p>6 videos</p>	<p>STAIR Tailored: Multiplying Linear Expressions – Part 1: Using Algebra Blocks</p> <p>9 videos</p>	<p>STAIR Tailored: Introduction to the Coordinate Plane Using CPA and Measurement</p> <p>5 videos</p>	<p>STAIR Tailored Practice to Research... and Back to Practice: Concrete Learning</p> <p>3 videos</p>
<p>Culturally Responsive Teaching</p> <p>View full playlist</p>	<p>Teaching Quadratic Expressions</p> <p>View full playlist</p>	<p>Coordinate Plane/Grids</p> <p>View full playlist</p>	<p>Practice to Research & Back to Practice</p> <p>View full playlist</p>
<p>STAIR Tailored: Exploring Properties of Quadrilaterals Using the CPA Framework and AngLegs</p> <p>5 videos</p>	<p>STAIR Tailored: Defining Data-Based Individualization (DBI)</p> <p>17 videos</p>	<p>Project STAIR: One-Step Equations with an Addition Operator using Cups and Counters</p> <p>16 videos</p>	<p>Project STAIR: Adding Integers Using a Positive and Negative Mat</p> <p>13 videos</p>
<p>Introduction to Geometry</p> <p>View full playlist</p>	<p>Data-Based Individualization</p> <p>View full playlist</p>	<p>Introduction to Equations</p> <p>View full playlist</p>	<p>Integers</p> <p>View full playlist</p>
<p>Project STAIR: Representing Fractions with the Area Model</p> <p>42 videos</p>	<p>STAIR Tailored: Do Not Use Key Words</p> <p>10 videos</p>	<p>Project STAIR: Whole-Number Computation: Addition with Partial Sums</p> <p>9 videos</p>	<p>Project STAIR: Explicit Instruction</p> <p>8 videos</p>
<p>Fraction Fundamentals</p> <p>View full playlist</p>	<p>Word Problem Instruction</p> <p>View full playlist</p>	<p>Whole Number Computation</p> <p>View full playlist</p>	<p>Best Practices for Math Teachers</p> <p>View full playlist</p>

<https://www.youtube.com/channel/UCE2puwDtUSNXFONIOhmYmvA>

Sarah R. Powell, Ph.D.

Professor
College of Education
The University of Texas at Austin



srpowell@utexas.edu



[@sarahpowellphd](https://www.instagram.com/sarahpowellphd)