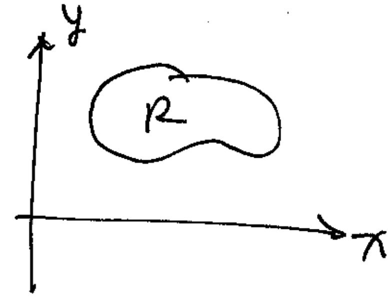
Math 2471 - Calc 3

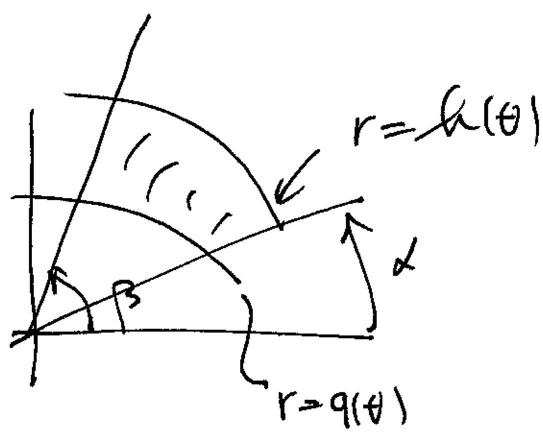
Double Integral - Polar

$$\iint_R f(x,y)\,dA$$



in polar

$$\int_\alpha^\beta \int_{g(\theta)}^{h(\theta)} f(r\cos\theta, r\sin\theta)\, r\,dr\,d\theta$$



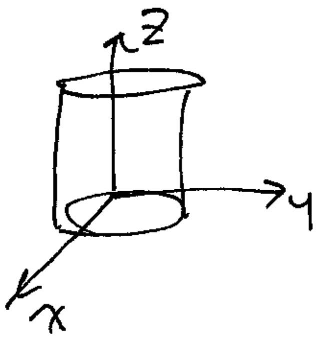
Last 2 class - Triple ∫'s

$$\iiint_V f(x,y,z)\,dV$$

$dV = dx\,dy\,dz$ or some combination of this

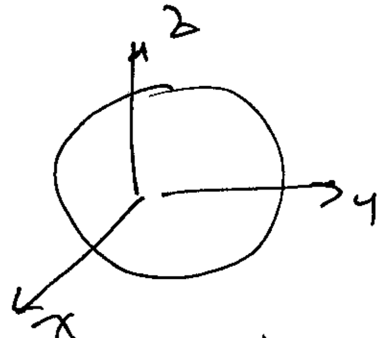
Now often the 3-D volume involve cylinders, cones sphere and a better coordinate works (well better).

Ex



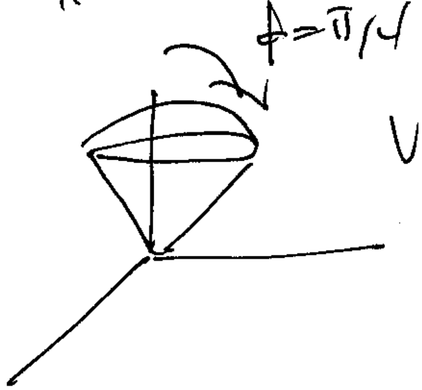
$V: \{(r,\theta,z) \mid 0 \le r \le 1,\ 0 \le z \le 1\}$

Ex



$V: \{(\rho,\theta,\phi) \mid 0 \le \rho \le 1\}$

Ex



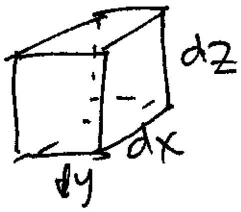
$V = \{(\rho,\theta,\phi) \mid 0 \le \rho \le 1,\ 0 \le \phi \le \pi/4\}$

How do set up triple integrals in

(1) cylindrical polar coords

(2) spherical polar coords

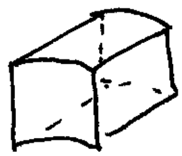
$\underline{\underline{dV}}$



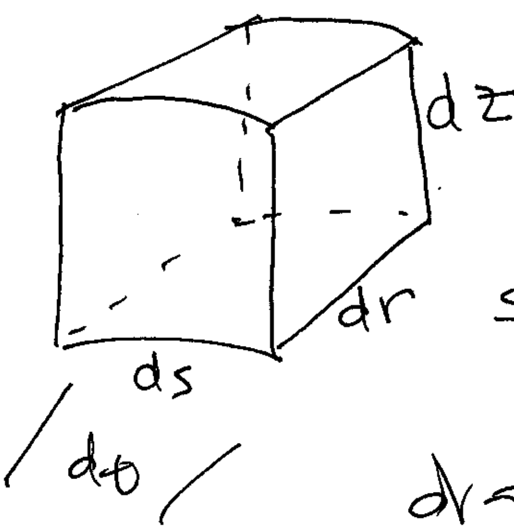
dV - volume of box

$= dx\,dy\,dz$

in cylindrical polars



← dV is volume of this small part of cylindrical shell



$dV = ds\,dr\,dz$

$s = r\theta \quad ds = r\,d\theta$

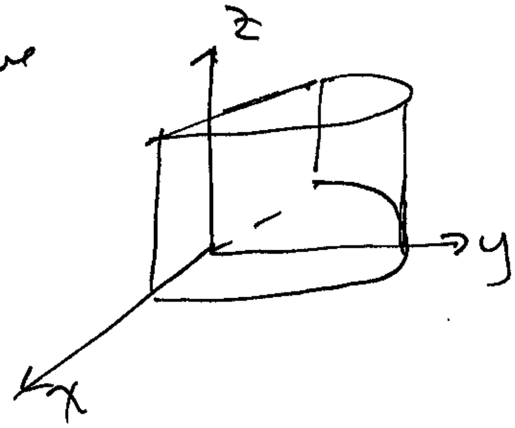
$$dV = \underbrace{r\,dr\,d\theta}_{\text{2D polar}}\,dz$$

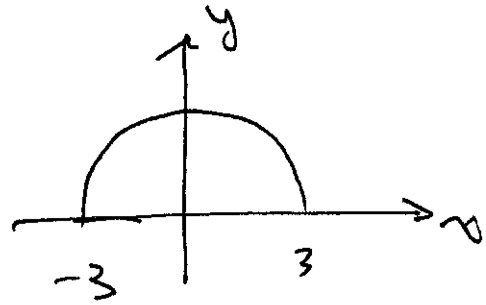
change in z

$$\iiint_V f(x,y,z)\,dV = \iiint_V f(r\cos\theta, r\sin\theta, z)\,r\,dr\,d\theta\,dz$$

Ex 1 #18 $\displaystyle\int_{-3}^{3}\int_{0}^{\sqrt{9-x^2}}\int_{0}^{2}\frac{dz\,dy\,dx}{1+x^2+y^2}$

Volume





so $\displaystyle\int_{\theta=0}^{\pi}\int_{r=0}^{3}\int_{z=0}^{2}\frac{r\,dz\,dr\,d\theta}{1+r^2}$

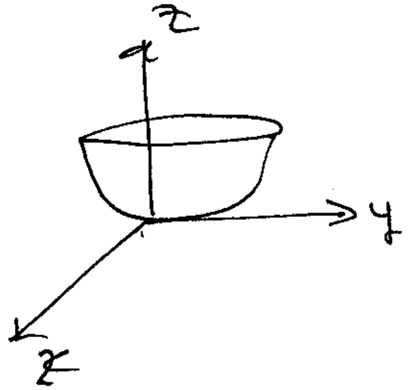
$$\int_{0}^{\pi}\int_{r=0}^{3}\frac{rz}{1+r^2}\Big|_0^2\,dr\,d\theta = \int_0^{\pi}\int_0^3\frac{2r}{1+r^2}\,dr\,d\theta$$

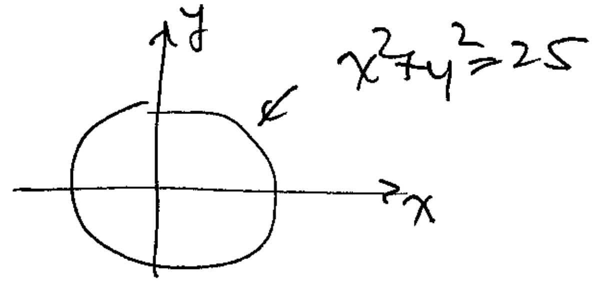
$$= \int_0^{\pi}\ln(1+r^2)\Big|_0^3\,d\theta = \int_0^{\pi}(\ln 10-\ln 1)\,d\theta$$

$$= \ln 10\,\theta\Big|_0^{\pi} = \pi\ln 10$$

#30 Find the volum bound by

$z = 25$ & $z = x^2 + y^2$





$$V = \iiint_V 1\, dV \qquad z = x^2 + y^2 = r^2$$

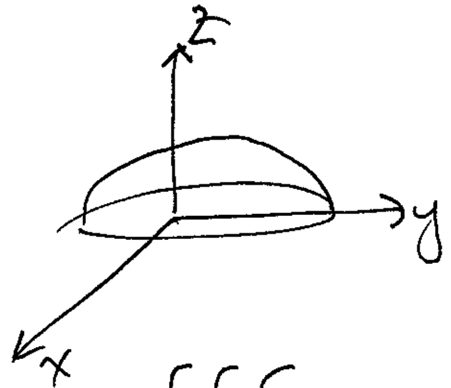
$$= \int_{\theta=0}^{2\pi}\int_{r=0}^{5}\int_{r^2}^{25} r\,dz\,dr\,d\theta = \int_0^{2\pi}\int_0^{5} rz\Big|_{r^2}^{25} dr\,d\theta$$

$$= \int_0^{2\pi}\int_0^{5} 25r - r^3\, dr\,d\theta = \int_0^{2\pi} \frac{25r^2}{2} - \frac{r^4}{4}\Big|_0^5 d\theta$$

$$= \int_0^{2\pi} \frac{625}{2} - \frac{625}{4}\, d\theta = \frac{(625)}{4}\,\theta\Big|_0^{2\pi}$$

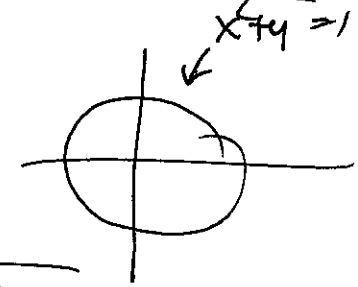
$$= \frac{(625)\cdot 2\pi}{4} = \frac{(625)\pi}{2}$$

Ex 3



$x^2+y^2+z^2=1$

$r^2+z^2=1$



$$\iiint_V 1\,dV = \int_{x=-1}^{1}\int_{y=-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{z=0}^{\sqrt{1-x^2-y^2}} dz\,dy\,dx$$

Hard $\int$

cylindrical polar

$$\int_{\theta=0}^{2\pi}\int_{r=0}^{1}\int_{z=0}^{\sqrt{1-r^2}} r\,dz\,dr\,d\theta$$ ← better

but we can do better

spherical polar coords

HW pg 1019-1020

#17, 29, 31, 43, 45, 47, 49