# Multivariable Directions in the Improvement of Shape Actuator Performance

This article examines a systematic method of determining the combination of settings and adjustments to the mill's mechanical crown, pass schedule and/or shape targets, in the presence of a characterizable incoming strip shape, needed to allow the mill to achieve the pass-to-pass shape target progression, within the constrained shape actuation's capabilities.

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This article is the second of a two-part series by Mark E. Zipf. Part one developed a key method employed in this discussion and was published in the December 2012 issue of *Iron & Steel Technology*.

The classical application of 20-high cluster mills is in a reversing, multi-pass arrangement.[1] The multivariable operating point of each pass is a carefully orchestrated group of settings that achieve the rolling objectives (e.g., reduction, shape, production throughput, etc.) within the capacity of the mill and the shape actuation equipment.

From a shape control perspective, the design of each new pass confronts the issues of scheduling reduction, strip tensions, rolling speed, incoming strip shape, material work hardening, separating force-induced deformation of the roll cluster and the desired shape target progression, all while remaining within the constraints of the shape actuation system.

The pass-by-pass shape targets may follow a progression that accommodates the needs of the material handling/tracking and material protection (e.g., from edge crack-induced strip breaks) and also the final pass delivered shape. Early passes may employ tight edged shape targets (slightly over-rolled center) to promote good strip tracking/payoff loading and a good rewound coil buildup. Depending on the level of edge crack protection and the incoming strip shape requirements of the next downstream process (e.g., side trim and anneal, or ship directly off the mill, etc.), the final pass targeted shape may not be an intuitive flat spatial waveform.

The successful achievement of the shape targets on a pass-by-pass basis is an important aspect of strip quality and mill throughput (i.e., production not slowed or compromised by strip shape problems, edge crack-induced strip breaks or roll changing issues). From a production throughput perspective, it is desirable to achieve the complete rolling of a coil (all passes) with a single roll cluster arrangement (i.e., no roll changes to adjust the cluster's mechanical crown and resulting force-loaded elastic deformation). When this is superseded by strip quality requirements, quick work roll changes may be designed into the process during later/final passes to impart a specified surface finish and final shape criteria (e.g., polished work rolls with possibly less mechanical crown).

**The Design of Pass Schedule/Shape Target Progressions —** When developing pass schedules and shape target progressions, the designer has to confront the multivariable problem of anticipating an incoming strip shape and separating force-induced roll cluster deformation, while considering the shape targeting needs of the process (e.g., edge crack protection) to define a workable series of operating point conditions that can be accommodated by the constrained shape actuators.

The designer's degrees of freedom fall into statically adjustable (off-line) and dynamically adjustable (on-line) parameters.

- **Statically Adjustable (Off-Line) Parameters**
  - Pass Schedule — This involves selecting the reductions, tensions, speeds, etc., which result in a separating force that will induce the desired transverse elastic deformation of the roll cluster. The schedule is usually derived from mathematical models formed along the longitudinal plane of the roll bite. These calculations do not consider the consequences incurred along the transverse plane. It is often necessary to adjust the scheduled deformation to accommodate the shape control needs.
  - Roll Cluster Diameter Profile Setup — This involves the selection of the geometric/mechanical crowns and tapers ground onto the rolls that form the cluster. The transverse diameter profile of each roll within the cluster possesses a different influence factor at the roll bite.[1] The idea of this "black art" is to introduce a specific level of geometric clearances (within the cluster's inter-roll contacts) to create the roll cluster's "effective crown." This crown (the clearances) is specifically absorbed in the cluster's and housing's transverse elastic deformation, creating a desired roll gap profile when loaded at the prescribed/scheduled separating force. The deformation characteristics may be designed to coincide with the pass scheduled progression of the separating force.
  - Shape Target — Although not desirable, adjusting the shape targets (when prudent compromises permit it) can provide an extra degree of freedom in the design of early and intermediate passes. Often, the last pass' shape target cannot be compromised.

- **Dynamically Adjustable (On-Line) Parameters**
  - Top Crown Eccentric Shape Actuators — These actuators provide the ability to adjust the transverse pressure distribution along the roll bite, to dynamically address complex shape distortions in real time. Typically, five to seven (and sometimes more) eccentric actuators are evenly spaced across the width of the mill.[1–4] As shown in Figure 1, the hydraulic actuators apply their adjustments through gear rack interconnections to eccentric rings, turning on needle bearings on the B and C shafts. This arrangement allows these

Figure 1

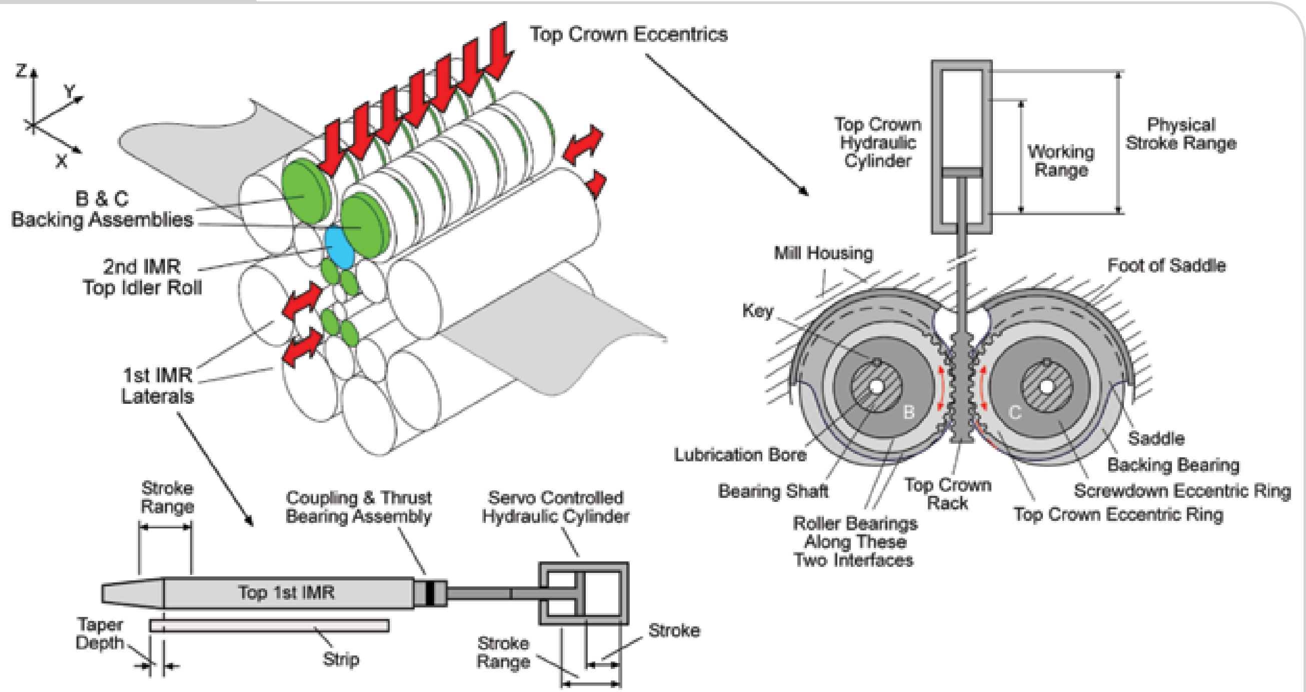



Illustration of the cluster mill shape actuation systems.

eccentric actuators to be adjusted while rolling under separating force loads.[1] Older mills employ hydraulic motors coupled through worm gears to raise/lower the rack extension rods. Modern mills use closed-loop, servo-position-controlled, direct-acting hydraulic cylinders, which provide improved response characteristics and finer control resolution.[2] The absolute and relative positions of these actuators are limited by physical and operational constraints that are imposed to protect the backing assembly shafts and bearings from excess, localized shear stresses and bending moments, which could lead to more rapid bearing wear and/or failure.[3–5] The shape adjustment sensitivities of the actuators are dependent on the flexibility of the B and C shafts, which can range from solid to flexible shaft backing assemblies (FSBA)[1,5,7] and include a segmented second intermediate top idler roll (SIR).[1,5,7]

– <u>Laterally Traversing, First Intermediate Roll (1st IMR) Shape Actuators</u> — These actuators provide independent, controlled pressure relief local to the strip edges. The pressure relief is produced by mechanically ground-in tapers on the operator-side end of the top rolls and the drive side of the bottom rolls. The tapering arrangement can range from single- or double-step conical tapers, to complex curvatures specifically tailored to match the deformation characteristics of the roll cluster for a given strip width and/or situation.[1] As shown in Figure 1, the lateral/axial displacement of these rolls (primarily the location of the tapers with respect to the strip edges) is by closed-loop, servo-position control of direct-acting hydraulic cylinders, which provide improved response characteristics and finer control resolution.[2] The lateral motion is dependent on the inter-roll surface contact frictions (three per roll), which depend on the separating force, rolling speed and roll finishes. At low rolling speeds, these friction loads are too great to allow lateral movement without inducing excessive stresses on the rolls' mechanisms (i.e., couplings, thrust bearings, etc.). It is therefore necessary to impose a rolling speed-based constraint on the actions of the 1st IMRs.[3–5]

The design of the pass schedules and shape target progressions must take into account all of these concerns and available degrees of freedom, and form a sequence of operating points/conditions that simultaneously achieves the reduction and shape control objectives within the range of shape actuation capabilities.

**Black Arts and Realities —** To many outside observers, the pass schedule and shape targeting design process appears as an unfathomable riddle, a black art of secretive selections, tweaking and conjuring. When confronted with curious shape defects or conditions that cause the shape actuators to engage their limits/constraints and not achieve the shape target, questions arise:

- What parameter or group of parameters should be adjusted? Do I adjust the pass schedule or roll cluster setup? Maybe I should adjust the tapers?
- What direction of adjustment to take? Should I increase or decrease the mechanical crown, and which roll(s) need to be considered? Should the tapers be shallower?
- What is the amplitude of adjustment? How much crown should I add?
- Is this shape target practical/reachable? How can I tell, and what might be a more appropriate target waveform?
- And many, many more.…

There are no simple answers to these questions, and a great deal of uncertainty as to how to proceed, may be present. In fact, this design process is really just a complex (but understandable) sequential mechanics problem that can be resolved through mathematical modeling, analytic examination and carefully orchestrated field trials. Analytically resolving this problem involves:

- Forming a mathematical framework[8] from which the individual components can be defined and organized.
- Deriving a sufficiently dexterous mathematical model[8] (within the previously formed framework) of the mill and shape actuation influences.
- Determining the static/setup parameter values, based on required, modeled, empirical and historical evidence. The primary outcomes are the pass-to-pass progression of the separating force and its impact on the nature and extent of the roll cluster deformation influence on the rolled/exit strip shape, and the shape target sequence. In a well-controlled multi-pass scenario, only the first pass entry/incoming strip shape has an element of "mystery." The second and subsequent passes have an incoming strip shape governed by the shape target of the previous pass.

- Determining the range of the dynamic parameter values (the shape actuation capabilities envelope (SACE))[8–10] for the given conditions and actuator constraints. As shown in Reference 5, the shape and orientation of this envelope will change as the passes progress.
- Using these results, a representation of the expected rolled/exit strip shape is formed. This operating point is over-contained by the extent of the *Offset* SACE (i.e., the envelope is offset from the origin by the vectoral addition of the incoming shape and the roll cluster's elastic deformation contributions).
- The shape target must fall within the *Offset* SACEs on each pass. As the passes evolve, the geometry and orientation of the SACEs change, as do their offsets. The bounding curves defining each *Offset* SACE form a closed, over-containing pass progressing surface within which the set of reachable shape targets forms a volume.
- For a given pass schedule, the sequence of reachable, pass-to-pass shape targets progresses as a mapped trajectory within the reachability volume.
- If the shape target trajectory is not completely contained within the volume of the reachable shape targets, the direction and amplitude of the corrective adjustments to the roll cluster setup, pass schedule and shape target are defined.

Combined, the above components provide an analytic means of resolving the mystery and black art of designing the mill setup, pass scheduling and shape targeting.

**Fundamental Relationships and Concepts —** Before continuing, it is important to review the mathematical constructs that form the framework underlying this methodology. Reference 8 provides a complete derivation.

Spatial Waveform Relationships: As derived from the literature,[3,4,8] the spatial waveform of the rolled/exit strip shape (transverse stress pattern) is a combination of the entry strip shape, roll cluster deformation and the applied shape actuation.

$$\mathbf{S}_T(y_M) \sim \mathbf{S}(y_M) = \mathbf{S}_0(y_M) + \mathbf{S}_R(y_M) + \mathbf{S}_A(y_M)$$

(Eq. 1)

where these components are the discrete, spatial representations of the transverse shape/stress waveform patterns, given by:

$\mathbf{S}(y_M) \triangleq$ Rolled/exist strip shape vector (stress pattern) produced by the mill, given by:

$$\mathbf{S}(y_M) = \left[ S\left(y_M^0\right) \quad S\left(y_M^1\right) \quad \cdots \quad S\left(y_M^{M-1}\right) \right]^T$$

(Eq. 2)

The elements of this vector are the stress amplitudes at the corresponding transverse locations of $y_M$.

$\mathbf{S}_T(y_M) \triangleq$ Shape target vector, indicating the desired shape of the rolled/exit strip.

$\mathbf{S}_0(y_M) \triangleq$ Incoming strip shape vector. This component is static for the evaluated situation.

$\mathbf{S}_R(y_M) \triangleq$ Exit strip shape contribution vector formed by the natural mechanical deformation characteristics of the mill, based on a combination of material geometry and yield stress, applied separating force, roll cluster setup of roll diameter profiles and tapers, roll cluster flexibility,[5,7] etc. This component is static and cannot be modified during on-line/rolling operations.

$\mathbf{S}_A(y_M) \triangleq$ Exit strip shape contribution vector induced by the top crown eccentrics and 1st IMR laterals, as transmitted/distributed to the roll bite through the roll cluster's mechanical characteristics (which function as a form of spatial filter). This component is dynamic and can be modified/adjusted during on-line/rolling operations. The nature of the actuation's spatial influence varies over the operating conditions, requiring a degree of adaptation to describe the full range of rolling conditions.

The discrete spatial variable, $y_M$, is an M-dimensional set of uniformly distributed locations across the strip width, W, which have been mapped to the normalized domain interval [–W/2, +W/2]→[–1, 1] of a Sobolev space.

$$y_M = \left\{ y_M^0, y_M^1, \cdots, y_M^{M-1} \right\} \text{with the requirement of:}$$
$$y_M^0 = -1 \text{ and } y_M^{M-1} = +1$$

(Eq. 3)

In this linear algebraic framework, the spatial waveform characteristics of the mill's transmission of the shape actuator settings is provided through a matrix, $\mathbf{G}_M \in \Re^{M \times N}$, whose columns are evaluations of the individual actuator's spatial influence at the sampling grid associated with $y_M$. This representation provides a specific definition of the actuator's dynamic changes in the shape, $\mathbf{S}_A(y_M)$:

$$\mathbf{S}_A(y_M) = \mathbf{G}_M\mathbf{A} \tag{Eq. 4}$$

where

$\mathbf{A} \in \Re^N$ is the actuation vector, given by:

$$\mathbf{A}=\begin{bmatrix} L_T & C_1 & C_2 & \cdots & C_{N-2} & L_B \end{bmatrix}^T \tag{Eq. 5}$$

where

L elements correspond to the 1st IMRs and
C elements are the top crown eccentrics.

It's important to note that $\mathbf{G}_M$ behaves as a spatial filter, by distributing and attenuating the individual actuator's influence across the transverse width of the strip.

The central relationship of Equation 1 becomes:

$$\mathbf{S}_T(y_M) \sim \mathbf{S}(y_M) = \mathbf{S}_0(y_M) + \mathbf{S}_R(y_M) + \mathbf{G}_M\mathbf{A} \tag{Eq. 6}$$

A key aspect of Equations 1 and 6 is that $\mathbf{S}_R$ is a static adjustment and $\mathbf{G}_M\mathbf{A}$ is a dynamic adjustment. Further, as noted above, $\mathbf{S}_0$ of the second and subsequent passes is the shape target of the previous pass.

$$\mathbf{S}_0(y_M)\big|_k = \mathbf{S}_T(y_M)\big|_{k-1}$$

$$\text{k is the pass index: } k = 1, 2, 3, \ldots \tag{Eq. 7}$$

Parametric Decomposition: As shown in Reference 8, the curvature components (curvature order spectrum/distribution, $\$_S$) of spatial waveforms can be obtained through an algebraic parametric decomposition. This decomposition is based on an orthogonal polynomial basis (Gram polynomials) which employs a linear algebraic/matrix transformation:

$$\mathbf{S} = \tilde{\mathbf{P}}\$_S \iff \$_S = \left(\tilde{\mathbf{P}}^T\tilde{\mathbf{P}}\right)^{-1}\tilde{\mathbf{P}}^T\mathbf{S} = \tilde{\mathbf{P}}^T\mathbf{S} \iff \tilde{\mathbf{P}}^T\tilde{\mathbf{P}} = \mathbf{I}_{Np} \tag{Eq. 8}$$

where the matrix $\tilde{\mathbf{P}}$ is the curvature decomposition transform matrix.[8] Because of the polynomial orthogonality, the inverse transform matrix is its transpose, $\tilde{\mathbf{P}}^T$.

Spatial Curvature Relationships: Using the decomposition method noted in Equation 8, the corresponding spatial curvature relationship of the rolled, exit shape is based on the waveform decomposition of the central relationships of Equations 1, 4 and 6.[3,4,8]

$$\$_T \subseteq \$_S = \$_0 + \$_R + \$_A \tag{Eq. 9a}$$

$$= \$_0 + \$_R + \tilde{\mathbf{P}}^T\mathbf{G}_M\mathbf{A} \tag{Eq. 9b}$$

where

- $\$_S \triangleq$ Rolled/exit strip shape's spatial curvature distribution/spectrum vector.
- $\$_T \triangleq$ Shape target's spatial curvature distribution/spectrum vector, indicating the desired curvatures.
- $\$_0 \triangleq$ Incoming strip shape's spatial curvature distribution/spectrum vector.
- $\$_R \triangleq$ Exit strip shape's spatial curvature distribution/spectrum vector of the contributions formed by the natural mechanical deformation of the mill, based on a combination of material geometry and yield stress, applied separating force, roll cluster setup of roll profiles and tapers, roll cluster flexibility,[5,7] etc. This component is static and cannot be modified during on-line/rolling operations.
- $\$_A \triangleq$ Exit strip shape's spatial curvature distribution/spectrum vector of the contributions induced by the top crown eccentrics and 1st IMR laterals, as transmitted/distributed to the roll bite through the roll cluster's mechanical characteristics (which function as a form of spatial filter). This component is dynamic and can be modified/adjusted during on-line/rolling operations.

Actuator Constraints and the Shape Actuation Capabilities Envelope (SACE): The influences of the shape actuation systems are limited by the imposed operational and physical constraints,[1,5,7] which define the set of constrained actuation vectors, $\bar{\mathbf{A}}$:

$$\bar{\mathbf{A}} = \left\{\mathbf{A}\big|_{\text{Constrained}}\right\} \tag{Eq. 10}$$

This set is a collection of the available actuation inputs that, when applied to the mill, form a limited set of shape adjustment waveforms associated with the spatial influence functions, $\mathbf{G}_M$, and the shape actuation constraints, $\bar{\mathbf{A}}$.

$$\left\{\mathbf{S}_A(y_M)\right\} = \left\{\mathbf{S}_A\right\} = \mathbf{G}_M\bar{\mathbf{A}} \tag{Eq. 11}$$

The corresponding spatial curvature representation is given by:

$$\{\$_A\} = \tilde{\mathbf{P}}^T \mathbf{G}_M \bar{\mathbf{A}}$$

(Eq. 12)

This set of spatial curvatures is contained within the SACE.[8] The shape actuation constraints place limits on the extent of shape targets that can be achieved (reached) by the mill.

$$\{\$_T^{\text{Reachable}}\} = \$_0 + \$_R + \tilde{\mathbf{P}}^T \mathbf{G}_M \bar{\mathbf{A}}$$

(Eq. 13)

This equation is the overall governing relationship. It defines a region, $\tilde{\mathbf{P}}^T \mathbf{G}_M \bar{\mathbf{A}}$ (SACE), that is vector offset by the incoming shape and roll cluster deformation, $\$_0$ and $\$_R$, respectively. This *Offset* SACE is the set of achievable/reachable shape targets.

**Primary Development —** For an arbitrary pass, the shape target spatial curvatures, $\$_T\big|_k$, must reside within the *Offset* SACE of Equation 13.

$$\$_T\big|_k \subseteq \{\$_T^{\text{Reachable}}\}\big|_k = \$_0\big|_k + \$_R\big|_k + \tilde{\mathbf{P}}^T\, \mathbf{G}_M\big|_k\, \bar{\mathbf{A}}\big|_k$$

for k = 1, 2, 3, … (includes payoff/loading pass)

(Eq. 14a)

$$= \$_T\big|_{k\text{-}1} + \$_R\big|_k + \tilde{\mathbf{P}}^T\, \mathbf{G}_M\big|_k\, \bar{\mathbf{A}}\big|_k$$

for k = 2, 3, 4, … (for return/reversing passes)

(Eq. 14b)

As shown in Equations 14a and 14b, it is necessary to consider the pass-to-pass progressions of $\$_0$, $\$_R$, $\mathbf{G}_M$ and $\bar{\mathbf{A}}$ to accommodate Equation 7 and the variations in separating force, which will have an impact on the roll cluster deformation and the actuation constraints.

Direction of Shape Control Improvement: If the relationships of Equations 14a and 14b are not achieved, the vector, $\$_P$, defines the direction and amplitude of adjustments that must be made to minimally satisfy Equations 14a and 14b.

$$\$_P = \$_T\big|_k - \{\$_T^{\text{Reachable}}\}\big|_k$$

(Eq. 15)

where $\$_P$ is of minimum magnitude (i.e., $\min\left\|\$_T\big|_k - \{\$_T^{\text{Reachable}}\}\big|_k\right\|_2$) and therefore points along the shortest distance separating the existing shape target from the bounding surface of the *Offset* SACE.

The vector, $\$_P$, provides an explicit indication of the spatial curvature adjustments that resolve a noncompliance with Equations 14a and 14b. The adjustments of $\$_T\big|_k$, $\$_R\big|_k$, $\mathbf{G}_M\big|_k$ and $\bar{\mathbf{A}}\big|_k$ are left to the designer's discretion.

This is the central result. The directions of improvement (indicated by $\$_P$) provide a means of addressing and resolving the uncertainties, and the design problem of the earlier section on Black Arts and Realities.

## Characterizing Pass-to-Pass Progressions

The relationships of Equations 14a and 14b provide the general requirements of the shape target and contributing components. For a multi-pass coil to be successfully rolled, this relationship must be satisfied on a pass-to-pass basis. This involves coordinating the pass-to-pass progressions of the shape target, pass scheduled roll cluster deformation and available range of shape actuation. It is therefore necessary to understand and characterize the pass-to-pass behavior of each of these components.

This section examines the behavior of the components of Equations 14a and 14b as functions of the pass-to-pass progression and properties of the material.

**Shape Target Progression —** A common, intuitive misconception is that the shape target should be perfectly flat, regardless of the pass. In many rolling processes, this approach does not comply with the progressing needs of the process/material. A designed pass-to-pass progression of the shape target must be part of the scheduled operation of the mill.

In many cases, the early passes are rolled with tighter edges (a slightly over-rolled center). The tight edges promote good strip transport tracking (especially on the payoff/loading pass), and a clean buildup of the rewound coil. Intermediate passes progressively move toward a flatter and flatter rolled shape.

The final passes must contend with a high degree of material work hardening, which leads to the development of edge cracks. It is necessary to protect the edge cracks from high tensile stresses, to avoid production diminishing edge-crack-induced strip breaks. Here, the idea is to slightly over-roll the edges (long edges) to reduce the tensile stress they experience. This requires a shape target having looseness local to the edges.

Further, the final pass must provide the prescribed rolled shape associated with the downstream process, which may involve a slightly over-rolled center with tighter edge regions to promote good tracking in process lines (e.g., continuous anneal and pickle, etc.). The nature of the post-side trimming edge tightness must be coordinated with the edge crack

Figure 2

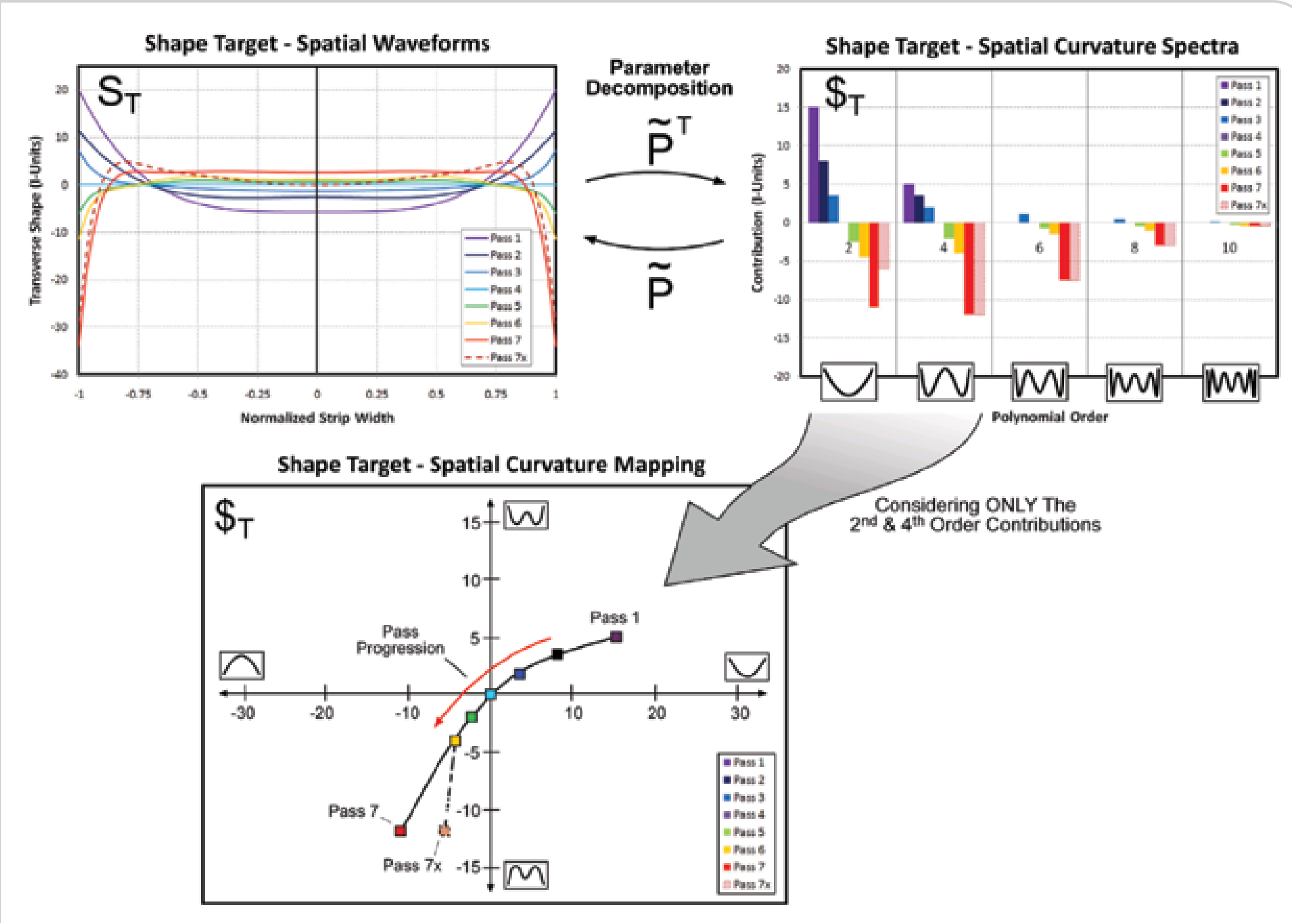



Diagram illustrating the pass-to-pass progression of the shape target in terms of waveform and curvature representations.

compensation. Further, additional center over-rolling may be considered to address shape distortions associated with transverse thermal gradient elongation effects. Lastly, in thin-gauge/foil-class rolling situations, oil film compensation (associated with oil suspension of the strip, which results in larger edge zone measurement uncertainties) may be required, which will induce additional negative edges (looser edges) to the shape target.

Figure 2 provides a typical shape target progression for thin-gauge, high-reduction stainless steels. This diagram shows the pass-to-pass spatial waveforms, their respective spatial curvature spectra and the resulting progression of shape curvatures (in the orthogonal curvature framework developed in Reference 8). It's important to note the high orders of spatial curvature, involving the eighth and tenth orders, are primarily applied to accommodate the strip edge regions.

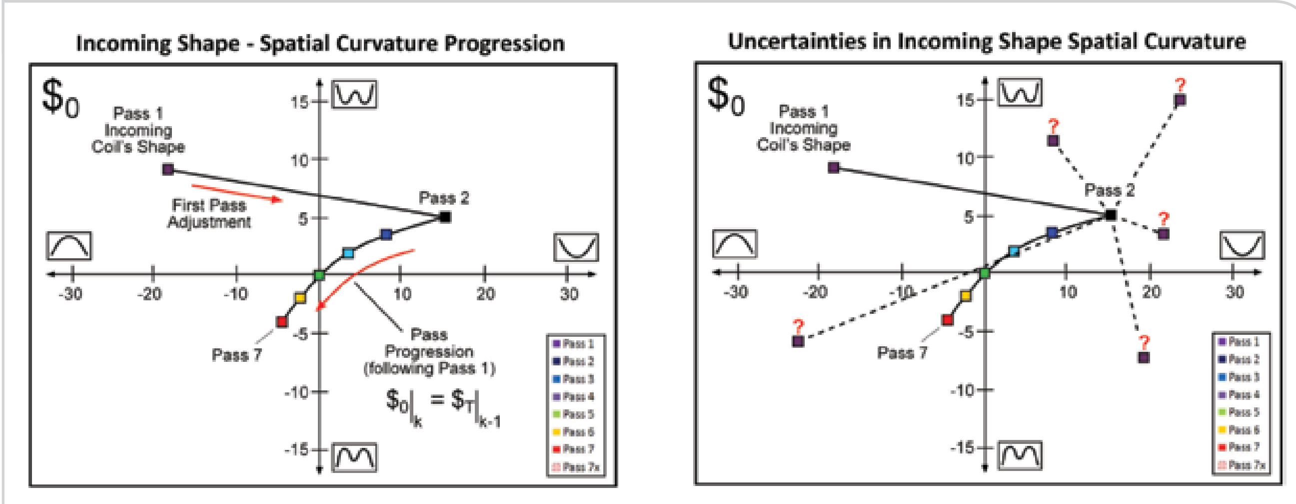
Although possible, it is difficult to depict the higher spatial orders (sixth, eighth, tenth) in a 2-dimensional, graphical manner, via a series of planar slices through the multi-dimensional framework. Therefore, this article will concentrate on only the lowest even-order plane (second, fourth).

**Incoming Shape Progression —** As noted above, in a well-controlled, multi-pass process, only the incoming strip shape of the first/payoff/loading pass has a degree of uncertainty. In general, the incoming shape falls within a certain range of expectations, but variations in the upstream processes must be considered.

The rolled/exit shape of the first pass is controlled to the shape target. Therefore, this shape-targeted result of this first pass becomes the incoming shape of the second pass. As indicated in Equation 7, the incoming shapes of subsequent passes are the shape targets of their preceding passes. Figure 3 provides a pair of spatial curvature plots showing the pass-to-pass progression of the incoming shape, $\$_0$, with an indication of the uncertain/random nature of the incoming shape on the first pass.

**Roll Cluster Deformation Shape Progression —** The applied separating force induces a characterizable roll cluster elastic deformation, $\$_R$. This deformation

Figure 3




Pass-to-pass progression of incoming shape curvatures, $\$_0$.

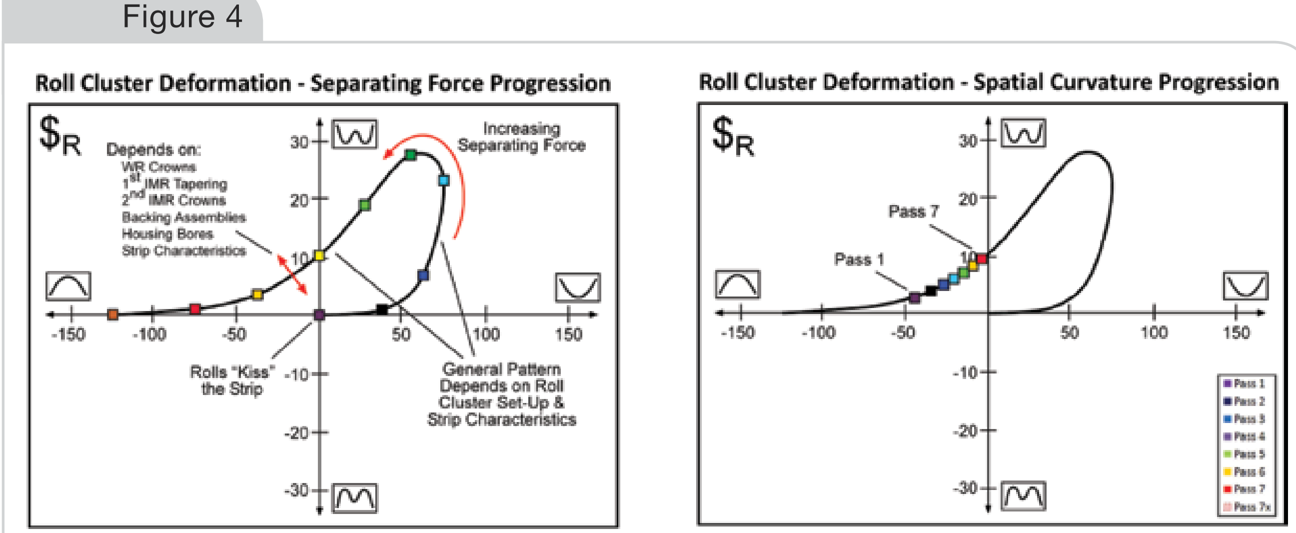
is dominantly a function of the strip width, roll cluster setup (roll diameter profiles) and roll cluster flexibility[5,7] (see the section on Impact of Top Crown Eccentric Shaft-Type and Top Idler Roll). In general, the individual roll diameter profiles (crowns and tapers) are selected to provide an ideal transverse pressure distribution for a given strip width, over the range of separating force conditions expected during the pass scheduled rolling of the coil.

Figure 4 provides a pair of spatial curvature plots showing the behavior of the roll cluster deformation, $\$_R$, over the range of separating force, and also the pass-to-pass progression of the deformation.

Limitations and Variations of Shape Actuation Influence — The spatial waveform adjustments provided by each actuator are defined by their individual spatial influence functions, and therefore, $\mathbf{G}_M$.[8,11–14] These coupled, non-linear transverse waveform patterns are unique to each actuator and vary as functions of the strip geometry, yield stress, roll cluster geometry and setup, separating force, rolling speed, etc.[5,7,8] The shape actuator influence functions (and $\mathbf{G}_M$) are also affected by roll cluster flexibility/shaft type,[5,7] and the applied constraints, $\overline{\mathbf{A}}$.

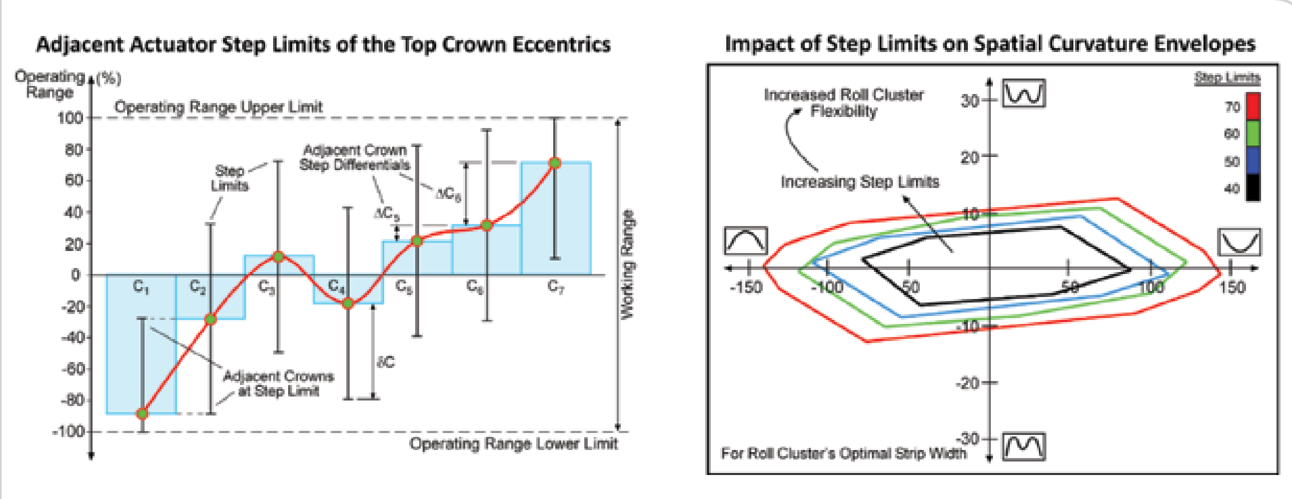
Impact of Shape Actuation Constraints: The shape actuators have specific limitations and operational

Figure 4




Separating force and pass-to-pass progressions of the roll cluster elastic deformation shape curvatures, $\$_R$.

Figure 5




Impact of step limit constraints on the top crown eccentrics' spatial waveforms, $\bar{\mathbf{A}}$, and the geometry/orientation of the SACEs of $\mathbf{\$}_A$, for the roll cluster's optimal width.

constraints. These restrictions limit the extent of the order of spatial curvatures that the shape actuators can impart to the roll cluster (and contribute to achieving the shape target). The top crown eccentric's adjacent crown step limit, the 1st IMR's taper depth limit and the 1st IMR rolling speed restrictions are just a few of the constraints that can be imposed.[1,3–5] Figure 5 illustrates the spatial waveform restrictions of the top crown step limits and the SACEs[8] indicating the extent of spatial curvature (for differing levels of imposed step limit) that can be achieved in the constrained actuation vector, $\bar{\mathbf{A}}$, using a flexible shaft backing assembly (FSBA) and a solid top idler roll (SR), on strip of ideal width for the roll cluster geometry and setup.

As shown in Figure 5, the increase in the step limits causes a discernible expansion of the SACE, meaning that a greater degree of shape adjustment capability is realized with larger step limits. It is interesting to note that this expansion is dominated by the second-order curvatures. This is expected, because the roll cluster's transmission of the actuation patterns (to the roll bite) behaves like a spatial low pass filter,[3,4,8] which suppresses high-order spatial frequencies (i.e., allows the second-order terms to prevail over the fourth-order components).

Impact of Top Crown Eccentric Shaft-Type and Top Idler Roll: The top crown eccentric shape actuators operate on the B and C shafts.[1,2,5] These shafts can have two forms:

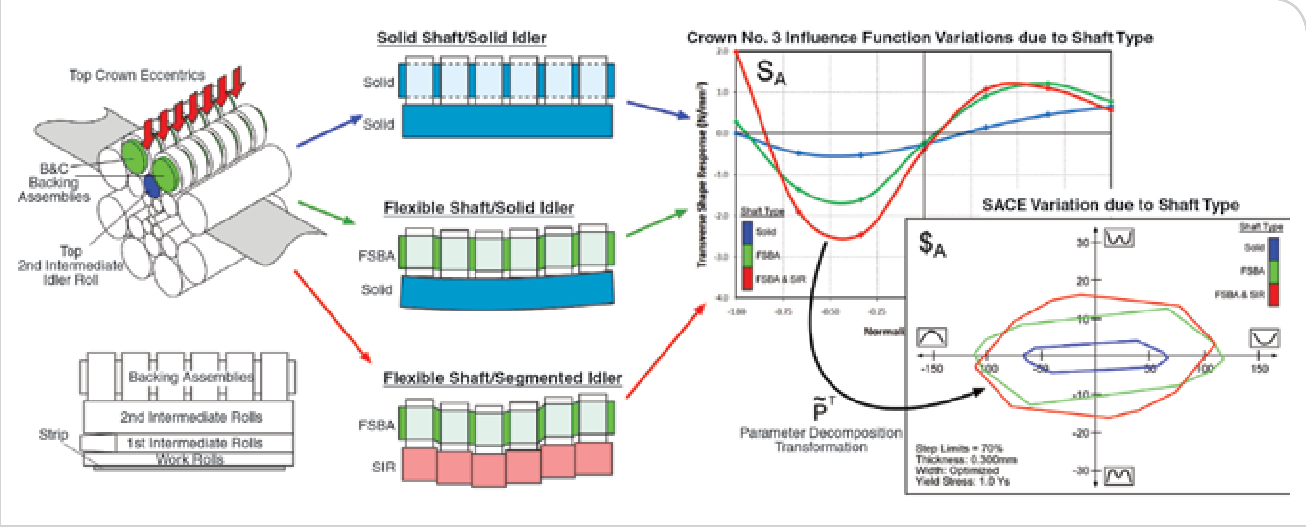
- **Solid Shaft (SS) —** As the name implies, the backing assembly shafts are solid. This arrangement offers the least flexibility and can implement only low-order spatial curvatures.
- **Flexible Shaft (FSBA) —** Strategically located and precision-machined relief slots are cut into the shaft to reduce the localized bending resistance (in the vicinity of the saddles/eccentrics). This radically increases the flexibility of the shaft and allows higher orders of spatial curvature to be applied by the top crown eccentric shape actuators.

From the perspective of the top crown eccentric actuators, the top idler roll is a crucial transmission medium. The top idler roll can have two forms:

- **Solid Roll (SR) —** This is a standard, solid idler roll. This roll arrangement provides the lowest transmission efficiency and is a major contributor to the roll cluster's spatial filtering effects.
- **Segmented Roll (SIR) —** This roll is composed of a series of free-floating segments assembled on an arbor (whose cross-section is an annulus and whose width is matched to the backing assembly bearing faces). Broad reliefs are cut into the outer extents of the segments' interiors, to allow clearances for the individual segments' angular orientation (to match the bearing face orientation). This roll arrangement provides a highly efficient transmission of the top crown eccentric actuator influences, thereby reducing the roll cluster's spatial filtering effects.

Common backing assembly shaft/idler roll combinations (listed in increasing orders of flexibility) include:

## Figure 6



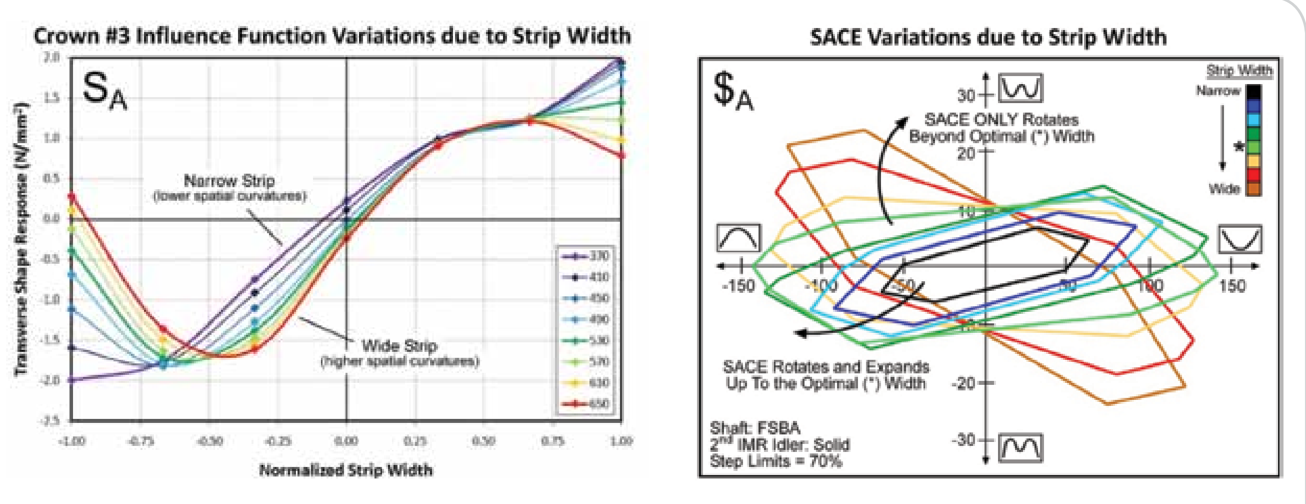

Comparison of the B and C shaft-type influence on the spatial waveform induced by crown No. 3 (changes in $\mathbf{G}_M$) and the resulting overall shape curvature envelopes {$\mathbf{\$}_A$}.

- **Solid Shaft/Solid Roll (SS/SR)** — least flexible — highest attenuation and spatial filtering (poorest flexibility).
- **Flexible Shaft/Solid Roll (FSBA/SR)** — radically improved flexibility — good actuation transmission with some spatial filtering (this is the most common arrangement).
- **Flexible Shaft/Segmented Idler Roll (FSBA/SIR)** — maximized roll cluster flexibility — best actuation transmission with minimal spatial filtering.

Figure 6 provides an illustration and plots showing the SACEs (based on the method described in Reference 8), that describe the shape actuation contributions of crown No. 3 to the rolled strip stress pattern, $\mathbf{S}_A$, for different types of backing assembly shafts (SS, FSBA) and second intermediate top idler rolls (SR, SIR).

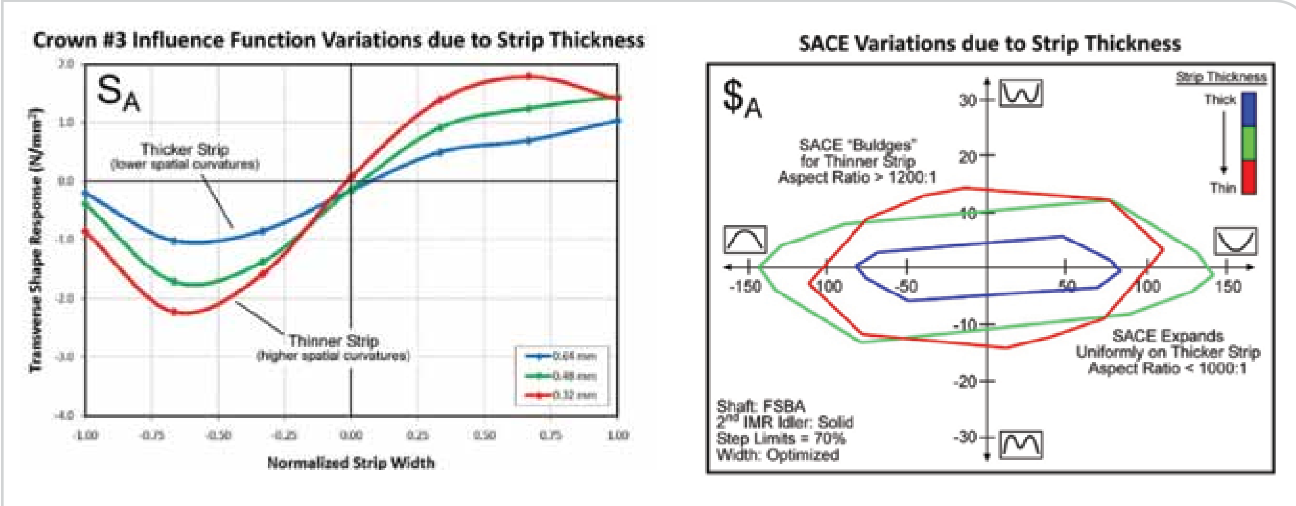
The results of Figure 6 show the changes in the mill's actuator influence transmission matrix, $\Delta\mathbf{G}_M$, associated with different shaft types, and also shows how the SACE expands/bulges (in the higher orders of curvature) as the roll cluster becomes more flexible. The solid shaft arrangement has a very rigid, non-responsive behavior, which forms a very restricted/condensed SACE. The incorporation of FSBAs radically changes the spatial dynamics and expands

## Figure 7




Impact of strip width on the spatial influence waveforms of crown No. 3 and the resulting shape curvature envelopes {$\mathbf{\$}_A$}.

Figure 8



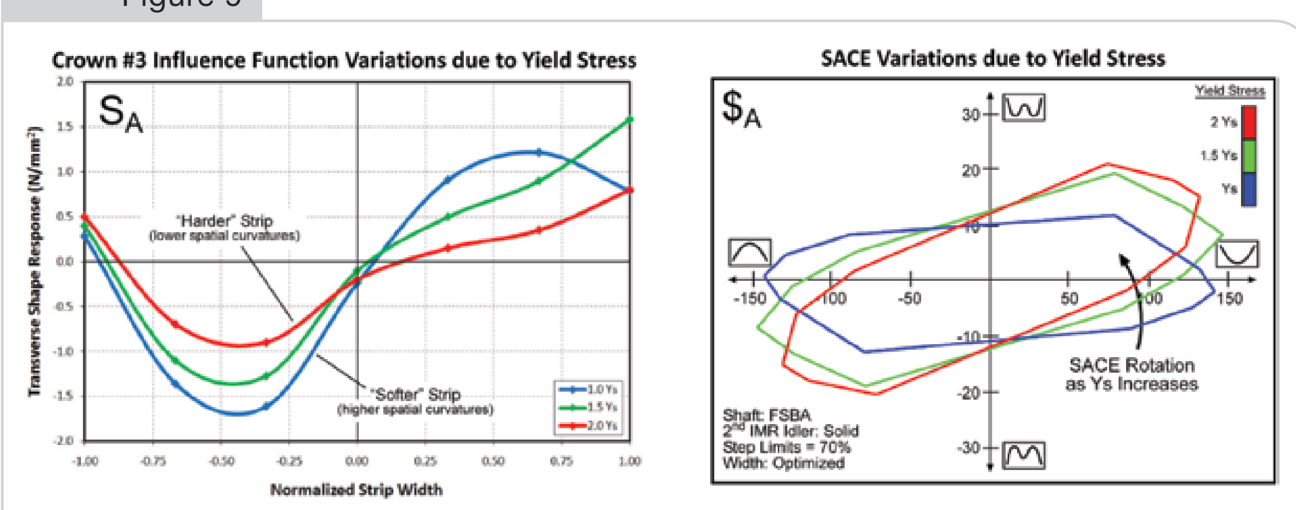

Impact of strip thickness on the top crown's spatial waveform and shape curvatures.

the SACE, which provides for a vast improvement in the ability to apply complex shape adjustment patterns. The incorporation of an SIR further localizes the influence function waveform and expands the SACE with the increase in fourth (and higher)-order curvatures (increased flexibility).

Impact of Strip Width: The spatial influence functions of the top crown eccentric shape actuators are sensitive to the strip width. This is primarily a consequence of the roll cluster's geometric relationship to the strip and the location/distribution of the actuator array over the strip. Figure 7 shows the variations of crown No. 3's spatial influence functions, for changes in the strip width. It also shows the resulting SACEs for varying strip widths, with the step limits set at 70%.

As the strip width increases, the SACEs expand and rotate clockwise. The expansion is associated with the increasing actuator resolution, as the strip widens to the optimal width of the roll cluster (the nearly horizontal orientation of the SACE's dominant axis). Beyond the optimal width, no expansion is noted, but the rotation accelerates. This is due to the higher orders of induced curvature associated with the individual actuator's influence becoming more localized as the strip widens (approaches an ideal Hertzian reaction). The clockwise rotation results in the SACE's dominant axis forming across quadrants II

Figure 9




Impact of strip yield stress on the top crown's spatial waveform and shape curvatures.

and IV. This indicates that an increase in strip width will tend to invert the general shape content, because of the opposing nature of quadrants II and IV (i.e., the second- and fourth-order curvatures in these quadrants are of opposing signs, and therefore narrow strip edge waves will become wider strip quarter buckles, which is supported by the spatial waveforms of Figure 7 and depicted in Figure 13 of Reference 8).

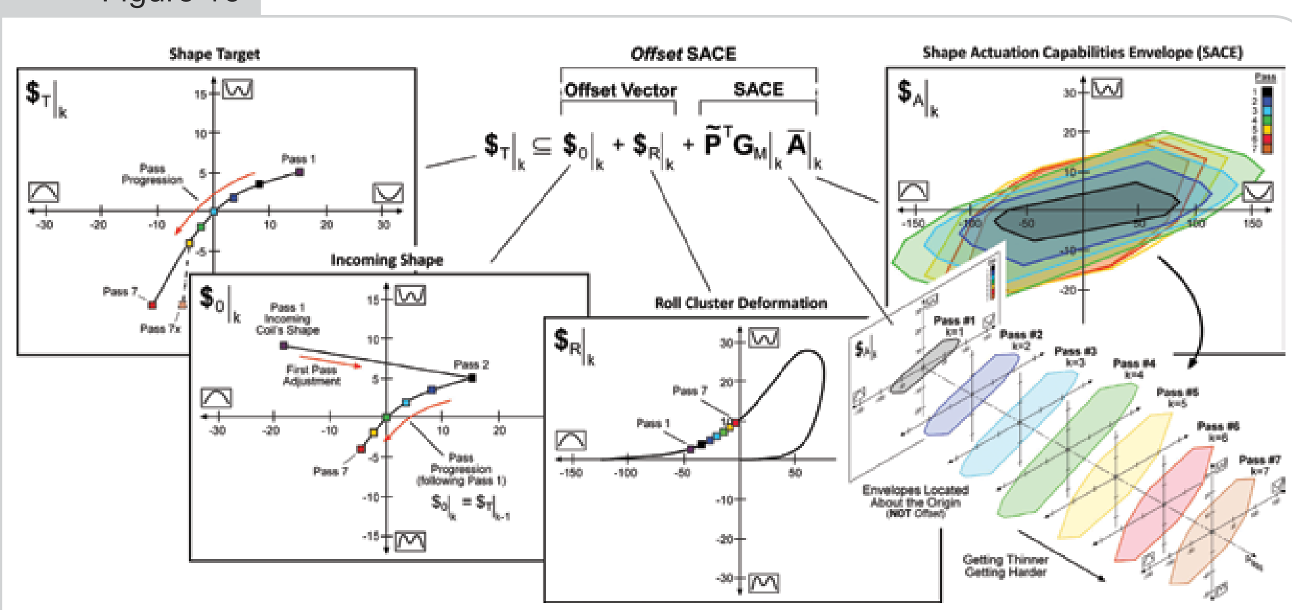
Impact of Strip Thickness: The spatial influence functions of the top crown eccentric shape actuators are sensitive to the strip thickness; however, this is more of an issue with the ability of the strip to store and react to internal stress patterns. The strip's stress storage/reaction behavior is related to approximately the square of the aspect ratio.[1,10] As the thickness (and aspect ratio) is reduced, the strip's sensitivity to variations in the transverse pressure distribution increases, which can be observed as an increase in the shape actuator's spatial influence function sensitivity. Figure 8 shows the variations of crown No. 3's spatial influence functions, for changes in the strip thickness. It also shows the resulting SACEs for varying strip thicknesses, with step limits set at 70%.

Essentially, thicker strip is more resistant to shape actuation corrections. As the strip thickness is reduced, the SACE expands in a uniform manner. During thin strip rolling, the increased sensitivity of the reduced aspect ratio causes second-order terms to be suppressed, while the higher-order terms are expanded (as noted by the bulbous form associated with the increased fourth-order contributions). Similar to the Hertzian contact reactions of the wider strip (Figure 7), the thinner material exposes the higher-order deformation characteristics of the roll cluster.

Impact of Strip Yield Stress: The spatial influence functions of the top crown eccentric shape actuators are sensitive to the material's yield stress and the pass-to-pass work hardening. It's clear that the material's resistance to deformation increases with each additional reduction; however, the issue here is primarily one of changes in the roll cluster's deformation. Figure 9 shows the variations of crown No. 3's spatial influence functions, for changes in the material yield stress. It also shows the resulting SACEs for varying yield stress, with step limits set at 70%.

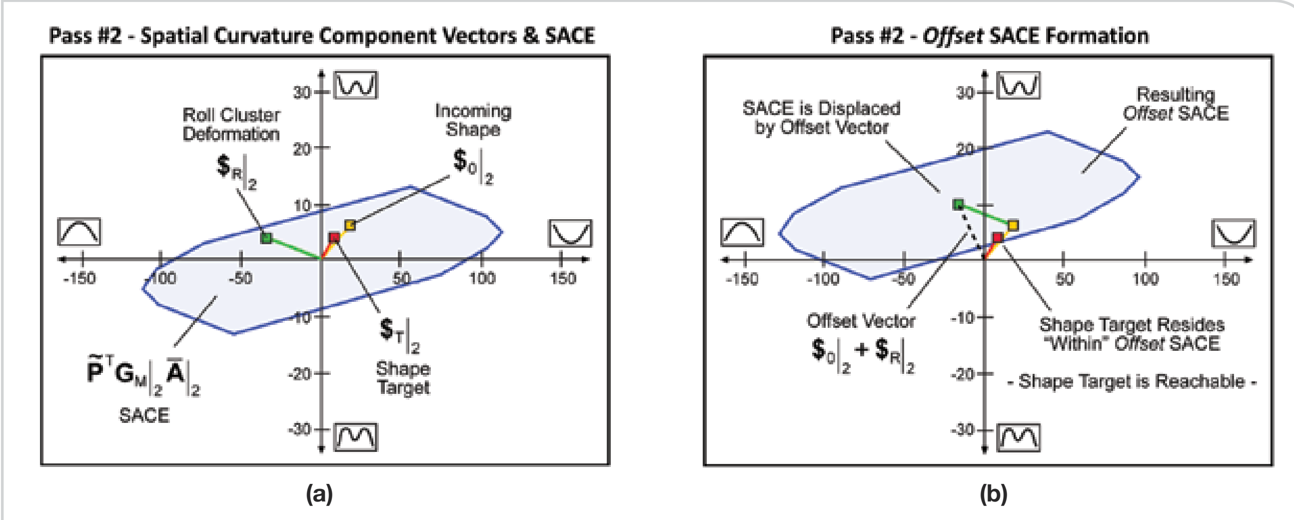
As the yield stress increases, the SACEs show no form of expansion, and rotate only counter-clockwise (with the dominant axis across quadrants I and III). This indicates that an increase in yield stress will tend to preserve the general shape content, because of the complementary nature of quadrants I and III (i.e., the second- and fourth-order curvatures in these quadrants are of the same sign).

Figure 10




Collection of the pass-by-pass shape curvatures, including an illustration of the evolution of the SACE.

Figure 11




Development of the *Offset* SACE: (a) spatial curvature component vectors and SACE, and (b) SACE displaced by the offset vector to form the *Offset* SACE.

## Shape Target Reachability

The relationship of Equations 14a and 14b provides the general requirements of the shape target and contributing components. For a multi-pass coil to be successfully rolled, these relationships must be satisfied on a pass-to-pass basis. This involves coordinating the pass-to-pass progressions of the shape target, pass scheduled roll cluster deformation and available range of shape action.

**Components and Pass-to-Pass Progression of the *Offset* SACE —** Consider the combined behavior of the mill, roll cluster, strip and pass schedule on a pass-by-pass basis. Returning to the primary relationship of Equations 14a and 14b, Figure 10 provides an overview of the component contributions for a typical seven-pass reduction plan for thin-gauge stainless steel using a FSBA and solid second IMR top idler roll (SR).

For a given pass, the *Offset* SACE is determined by the vectoral summation of the incoming strip shape and roll cluster deformation spatial curvatures, $\$_0|_k$, and $\$_R|_k$, respectively, to form an offset vector. The origin of the SACE ($\tilde{\mathbf{P}}^T \mathbf{G}_M|_k \bar{\mathbf{A}}|_k$) is then displaced by the offset vector. Figure 11a shows the individual vector components for an example pass No. 2. Figure 11b shows the formation of the *Offset* SACE through the vector summation and displacement of the SACE.

**Evaluation of Shape Target Reachability —** As indicated in Figure 11, assessing whether a given pass' shape target is reachable reduces to determining if the shape target resides with the bounds of the *Offset* SACE. This is the fundamental essence of satisfying the conditions of Equations 14a and 14b. Figure 12 provides an illustration of the entire pass-to-pass progression of the *Offset* SACE and the sequence of desired shape targets.

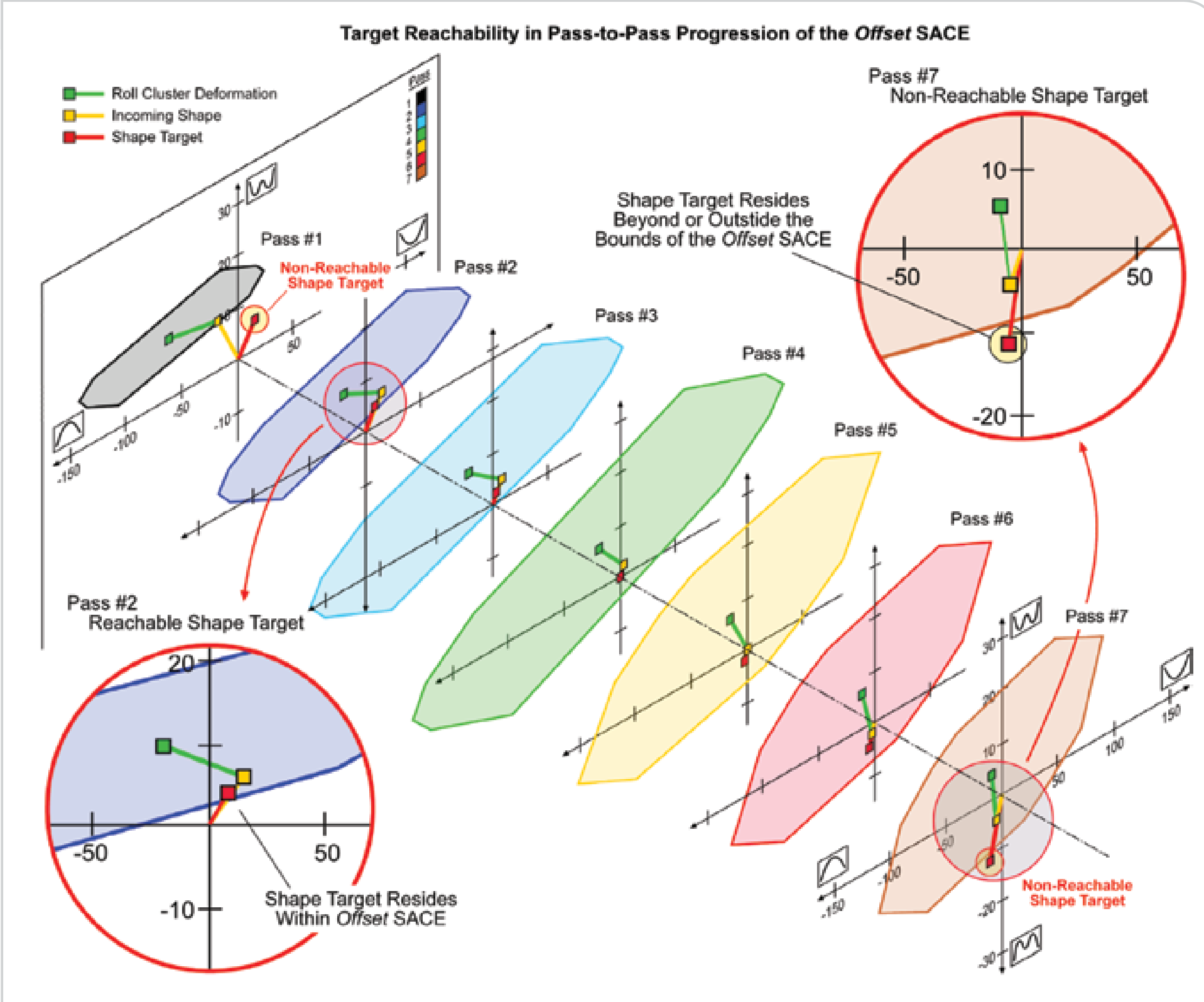
As noted in Figure 12, the shape targets of passes 1 and 7 are not over-contained by the *Offset* SACE, and therefore are not reachable by the constrained actuation (i.e., they don't satisfy Equations 14a and 14b). While rolling, the top crown eccentrics will engage the working range and step limits, the 1st IMR will engage the edge and penetration limits, and the desired shape targets will not be achieved.

## Directions of Shape Actuation Improvement

In the example of Figure 12, the shape targets of passes 1 and 7 are not reachable. The pass 1 incoming shape conditions (moderate quarter buckle) exceed the mill's ability to induce the extent of the desired tight edges. There is not enough mechanical crown in the roll cluster, and the constrained top crown eccentric and 1st IMR shape actuators do not have enough dynamic range to resolve the situation. From a practical point of view, and considering the random nature of the incoming shape, this limitation is tolerable if a sufficiently tight edge can be rolled to promote a reasonably good rewound coil buildup (which can be provided by overriding the 1st IMR taper knee penetration limits).

Pass 7 is a problem. This is a light reduction, final/finishing pass that has very specific shape target requirements. The thin strip has experienced a total reduction of more than 75%, it is very work hardened

Figure 12




Pass-to-pass progression showing the relationships of the *Offset* SACE and the shape target.

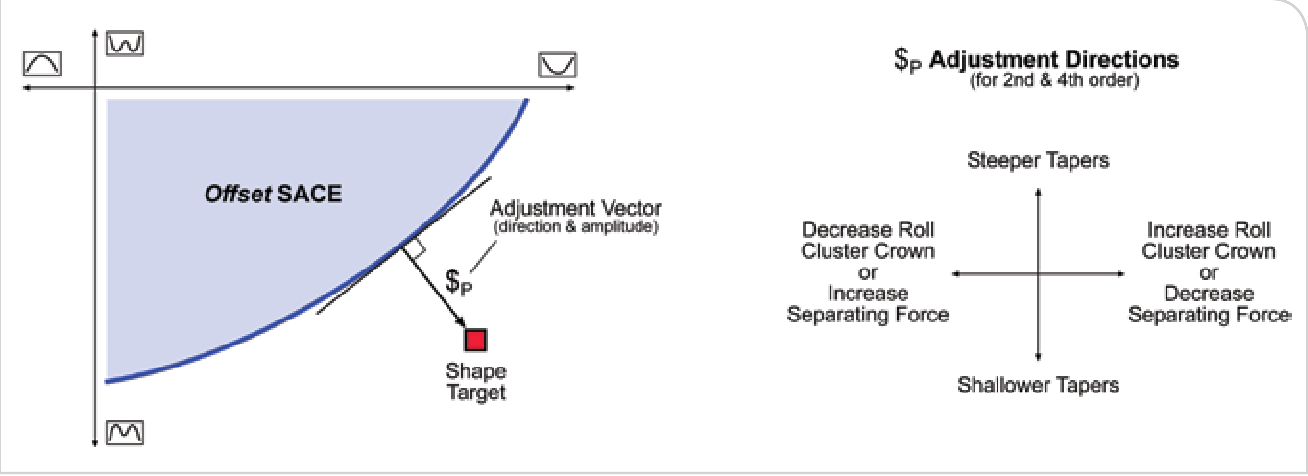
and potentially destructive edge cracks are present. It is necessary to protect the strip edges by reducing the tension stresses along the edges (by over-rolling and lengthening those regions). The Pass 7 schedule prescribed separating force is light and coupled with the roll cluster setup (mechanical crown), induces a relatively strong tight-edged quarter-buckle effect on the thin, work-hardened material (as indicated by the location of the offset vector, $\$_0 + \$_R$). The Pass 7 SACE is bulged and counter-clockwise rotated by the strip aspect ratio and higher yield stress.

To approach the shape target, the top crown eccentrics operate in a maximum anti-quarter-buckle pattern, causing both their step and working range limits to be engaged. The 1st IMR tapers are steeper than the separating force/deformation and top crown eccentric induced roll gap geometry, which requires their taper knees to operate very close to the strip edges (possibly at their strip edge limits). These taper knee locations induce a very large “effective flat”.[1]

This large “effective flat,” coupled with the force and top crown actuator deformed roll gap, causes the strip edge over-rolling to occur deeper into the strip’s interior than desired (which will produce an undesirable wavy edge following downstream side trimming). Essentially, the strip edges cannot be over-rolled with the necessary sharpness. The tapers are operating so close to the edge that they provide no real compensation for the natural profile of the roll gap. The 1st IMR shape control activity may oscillate due to the high sensitivity of the steep tapers.

In some regards, the nature of the steep tapers’ behavior is counterintuitive. The very large “effective flat” causes a deceiving impression that the tapers

## Figure 13




Diagrams showing the relationship of the $\$_P$ vector and the shape target, and the indicated changes to the roll cluster setup (the deformation curvatures of $\$_R$) associated with the directions of the $\$_P$ vector.

are not steep enough because of the deep edge overrolling. When in actuality, it's the steep tapers' knees being too close to the strip edges.

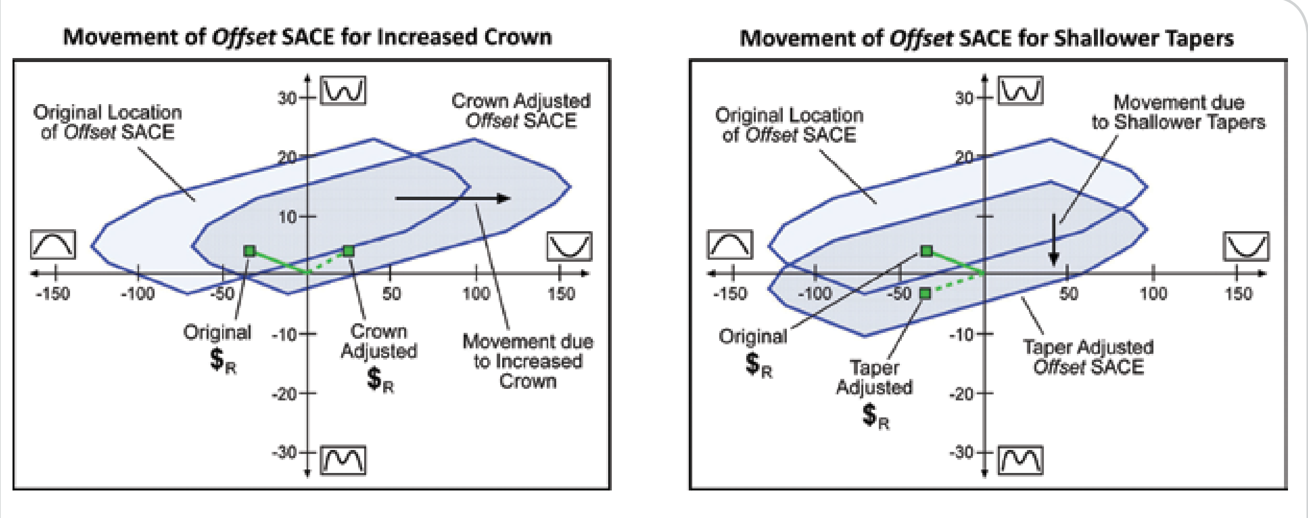
This is a difficult situation in which to operate, because there is no dynamic margin for the actuators to maneuver within, and a limited/stable condition (in a Lyapunov sense) will persist.

It is therefore necessary to look at alternatives in the selections of the roll cluster setup, the pass schedule and the shape targets.

**Determining the Direction of Shape Control Improvement —** As shown in the earlier section on Direction of Shape Control Improvement, the vector, $\$_P$, defines the direction and amplitude of the adjustments that must be made to satisfy Equations 14a and 14b. Figure 13 provides an illustration of the $\$_P$ vector with respect to an *Offset* SACE and shape target, and the changes to the roll cluster setup or pass schedule (that define the deformation curvatures of $\$_R$) associated with the direction of $\$_P$.

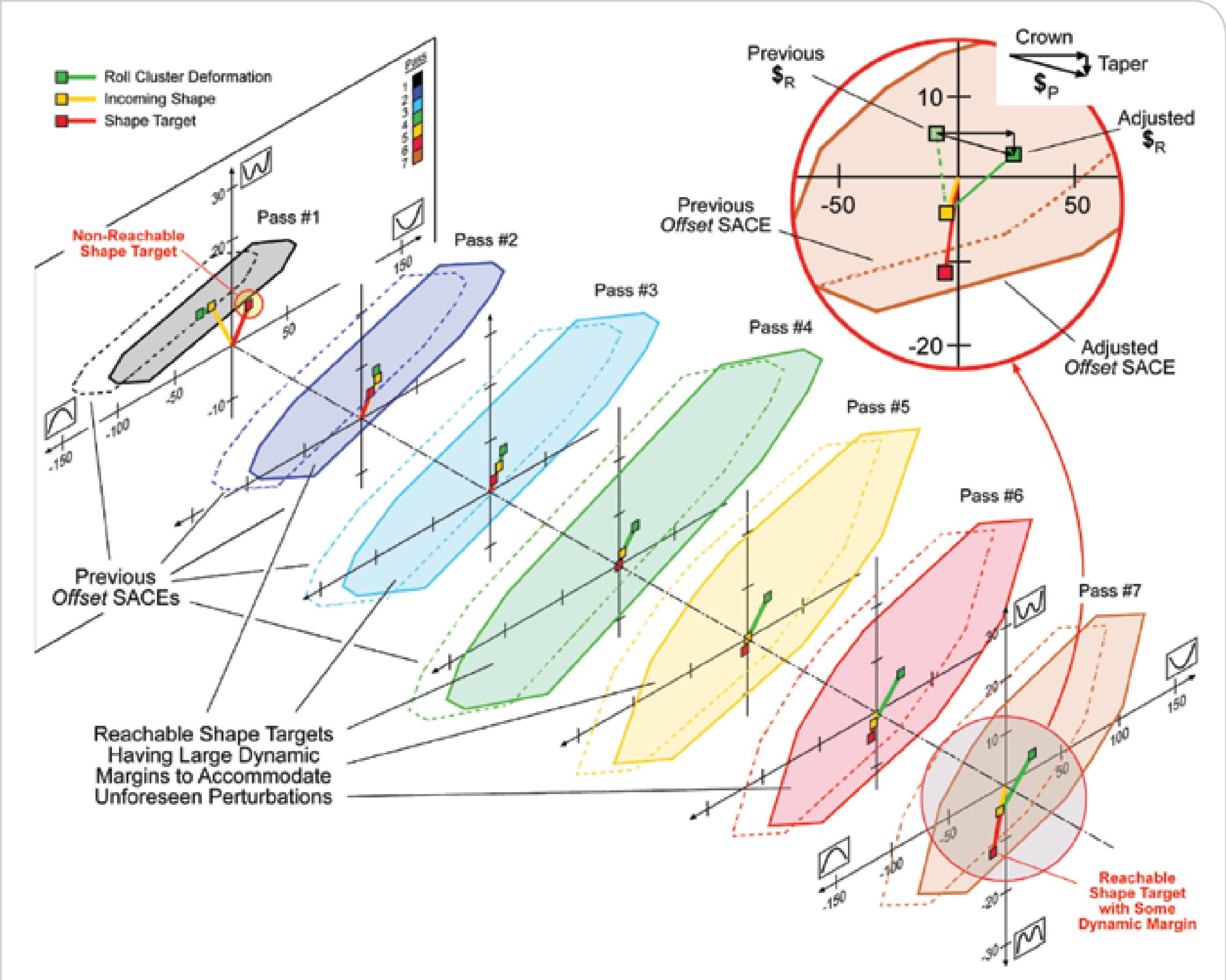
The key aspect of the $\$_P$ vector is that it indicates how to move the *Offset* SACE by changing the roll cluster roll grinds or the pass schedule. It also suggests how the shape target could be modified to reside with the *Offset* SACE. The question of what to do is really a decision to be made by the operations designer.

## Figure 14




Diagrams showing how changes to the roll cluster setup (diameter profile crowns and 1st IMR tapers) move the *Offset* SACE.

## Figure 15




Pass-to-pass progression showing the relocation of the *Offset* SACE associated with changes to the roll cluster.

Using these directional conventions, it can be seen how modifications to the roll cluster setup (crowns and/or 1st IMR tapers) impact the location of the *Offset* SACE. Figure 14 provides a pair of examples of the *Offset* SACE movement.

**Correcting the Location of the *Offset* SACE to Achieve Shape Target Reachability —** Returning to the example of Figure 12, and considering how the directional indications of the section on Determining the Direction of Shape Control Improvement provide guidance in moving the *Offset* SACE to improve the prospects of shape target reachability, the pass 7 problem can now be resolved. The pass 7 $\$_P$ vector (defined in Equation 15 and described in Figure 13) indicates that an increase in the overall roll cluster crown and/or a decrease in the slope of the 1st IMR tapers will relocate the *Offset* SACE toward a condition of over-containing the shape target. Figure 15 provides an illustration of the entire pass-to-pass progression of the *Offset* SACE employing this combined roll cluster modification.

The increase in the roll cluster's mechanical crown suppresses the quarter-buckle effect and creates more of a broad, flat-based center-buckle (as indicated by the location of the adjusted offset vector, $\$_0 + \$_R$). The shallower tapers more closely match the separating force/deformation induced roll gap geometry, which provides improved strip edge control and allows the shape target's edge conditions to be achieved. Combined, these changes will sufficiently shift the *Offset* SACE to over-contain the pass 7 shape target.

In this new arrangement, to approach the shape target, the top crown eccentrics operate in a smooth, flat-topped crown-out arrangement that is within the step and working range limits. The shallower 1st IMR tapers operate at more desirable depths of penetration from the strip edge and can control a narrow edge over-rolling.

The results of Figure 15 show an improvement in the pass 1 conditions, but more importantly, it shows that all subsequent passes meet the shape target reachability criteria. Further, those passes also have an enhanced level of dynamic margin, allowing the shape actuation systems to better accommodate potential, unforeseen perturbations (i.e., by following the guidance of the modified $\$_P$ vector, these changes to the roll cluster crown and 1st IMR tapers that make the mill's shape control more robust).

**Discussion** — In some respects, these dual adjustments seem oddly counterintuitive. In one respect, the roll cluster crown is increased (which inherently increases the inter-roll clearances in the edge regions of the strip). Simultaneously, the $\$_P$ vector is calling for a reduction in the taper slope (which reduces certain inter-roll clearances at the strip edges). What's going on here?

The key is a coordinated balancing of the seemingly opposing adjustment actions. Increasing the mill's mechanical crown reduces the deformation along the strip edges. Making the tapers shallower more closely matches this new deformation profile, which allows the taper knees to retract into the strip interior, reducing their control sensitivity. The top crown eccentric actuators operate away from their limits/constraints, allowing the outboard crowns to impart edge adjustments that complement the 1st IMR tapers in achieving the shape targeted edge over-rolling. The reduced 1st IMR control sensitivity creates a more stable/robust condition.

Making drastic roll cluster changes in one dimension (only crown or only tapers) may resolve the shape target reachability concerns, but they may also induce unsatisfactory shape control behavior. If the 1st IMR taper slopes are made radically steeper, the heightened control sensitivity of these actuators may cause overshoot or oscillatory behavior in the closed-loop dynamic response. Alternatively, making the taper slopes radically shallower will cause a large reduction in the control sensitivity, causing a sluggish response (which can be detrimental in the highly dynamic conditions experienced during accelerations/decelerations). So, a combined approach to changing the roll cluster often is advisable.

When considering the increase or decrease of the roll cluster crown, there are several ways to proceed. The most direct method is to apply the crown adjustment to the work rolls. The relatively small, chockless work rolls of the 20-high cluster mill, are easily/quickly changed and therefore, the needs of the pass 7 adjustments (as indicated in Figure 12) can also be addressed with a work roll change during the inter-pass reversal (prior to initiating pass 7). In many cases (e.g., rolling BA class stainless steels, etc.), highly polished work rolls are applied to the last pass (and sometimes the last several passes) to impart a specific strip surface quality. This rolling practice is well suited for accommodating crowning conventions on a pass-by-pass basis.

In some rolling practices, there is a focus on maintaining flat (non-crowned) work rolls. Here, the roll cluster crown is generated by specifically crowning other rolls within the cluster. Typically, attention is paid to the 2nd IMR idler rolls; however, the 2nd IMR drive rolls can also be considered (but this is not too practical). When viewed from the roll bite, mechanical crowns applied to the idler rolls are attenuated by the contact geometry of the roll cluster and the elastic characteristics of the adjacent rolls (e.g., high-speed steel or carbide work rolls, etc.). As noted by Duprez and Turley,[1] an equal crowning of the idler rolls is transmitted to the roll bite with an attenuation factor of 0.9 (and consequently, crowning a single idler has an attenuation factor of 0.45). These attenuation factors are general rules of thumb, as they are more precisely functions of the roll cluster's inter-roll force angle transmission geometries and the resulting backing assembly saddle loads (which defined the mode/geometry of the housing deflections). These characteristics are governed by the settings of the side eccentrics (A, D, E, H), along with the diameters of the other rolls within the cluster. From a practical point of view, crowning the bottom idler roll is often an action of choice. The roll is retained by gravity and not suspended (like the top idler), making the changing of this roll somewhat easier. However, the crowning of these rolls is usually reserved for campaign rolling of specific strip widths.

It is important to remember that the diagrams of Figures 13 and 14 (and others shown herein) are simplistic depictions of the true situation. The higher-order curvatures (sixth, eighth, etc.) are not shown or considered; therefore, the true complexities in the adjustment directions and changes to the roll cluster crowns, 1st IMR tapers and pass schedule are not fully described by these diagrams. However, these complexities are fully accounted for by the concepts presented here and the framework developed in Reference 8.

As a final note, the section on Correcting the Location of the *Offset* SACE to Achieve Shape Target Reachability shows that this mathematical approach provides some much-needed guidance into what roll cluster and/or pass schedule adjustments need to be considered. However, it does not fully embrace all of the encompassing issues that confront the operations designer (e.g., closed-shape control sensitivities, process-related requirements, etc.). Yes, the recommended directions of roll cluster adjustments will achieve the shape target reachability, but this is really the first step in developing a comprehensive means of guiding the operations/process design, and recommending solutions when confronted with complex shape distortions.

The current, ongoing research involves incorporating higher degrees of embedded knowledge/expertise[15] and process-directed optimization to more accurately align this method to the practical needs of the mill, process and operations design. The ultimate objective is a comprehensive commissioning and operations design assistance analytic tool kit that will offer guidance and suggestions. However, in the end, this is not a total solution, since it's really the decisions and actions of the operations designer that achieve the solution.

## Conclusion

This paper has examined a method of assisting 20-high cluster mill operations designers in contending with situations where a pass-to-pass sequence of shape targets cannot be achieved within the extents of the constrained shape actuation (i.e., cases where certain shape targets are not reachable). The approach first develops the sequence of shape actuation capabilities envelopes (SACEs) that describe the range of shape adjustments that can be provided by the constrained shape actuation systems. These SACEs are functions of the B and C backing assembly shafts, strip's width, thickness and yield stress. The SACEs are displaced by the pass-to-pass incoming shape and the pass scheduled roll cluster elastic deformation (based on the roll cluster's roll diameter profiles and elastic deflection characteristics) to form the *Offset* SACEs. The scheduled progression of shape targets are then overlaid to determine whether they can be achieved by the mill and prevailing conditions. In cases where the shape targets are not reachable (i.e., they reside outside the bounds of the *Offset* SACEs), an adjustment vector is determined. This adjustment vector provides insight/guidance into how the pass schedule, shape targets and roll cluster setup (roll cluster diameter profiles, taper selections, etc.) can be modified to achieve the reachability criteria. A final discussion is provided to consider the methods of implementing these modifications and the general methods associated with choosing the best course of action, with respect to the operational practices of the mill and rolling process.

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