The mimetic multiscale method for electromagnetics at a borehole with casing

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Summary

A cased borehole setting is a good example of a geophysical problem that involves different length scales: casing that is a few millimeters thick but is highly conductive as well as length scales of interest in meters and kilometers. This leads to a large model where including fine scale information to an adequate level of detail comes with the risk of computationally too expensive or even intractable simulations.

The mimetic multiscale method is developed to overcome the impracticably large linear systems that result from large models and their very detailed meshes. Multiscale methods provide a framework to forward model features on finer scales than the simulation mesh. Mimetic methods mimic the behavior of the continuous operators in the discrete setting, for instance magnetic fields on the coarse mesh are discretely divergence-free. Thus, spurious modes are prevented and the solutions are physical. Maxwell's equations are used to interpolate between the fine mesh, where the electrical conductivity is discretized on, and a nested coarse mesh on which the fields are simulated. This yields a natural homogenization that accounts for fine scale conductivity changes. It reduces the number of degrees of freedom enormously.

We demonstrate the effectiveness of the mimetic multiscale method at a vertical borehole with casing by simulating electric fields on the coarse mesh and comparing the results with the reference solution on the fine mesh.

Keywords: mimetic multiscale method, forward modeling, finite elements, borehole, casing, electromagnetics

Introduction

The motivation for the development of a mimetic multiscale method in geophysics is that many surveys nowadays require large fine-scaled meshes. This can be due to large transmitter and receiver setups that are spread over vast areas. Another example is a borehole with metal casing. There, fine-scale information of the very thin and highly conductive casing needs to be incorporated but at the same time the mesh needs to be kept at a reasonable size. To model this with the necessary amount of detail, fine meshes are required that involve large computational costs. Applying multiscale methods reduces the size of the mesh significantly and so impracticably large models become manageable.

While mimetic multiscale methods were considered for solving problems that evolve from flow in porous media (Efendiev & Hou, 2009), there is little work done directly for Maxwell's equations (Haber & Ruthotto, 2014) and the developed method was not mimetic (for mimetic methods see Haber (2014)). We advanced the multiscale method for Maxwell's equations to become mimetic. That means it keeps the properties of the solution on coarse meshes. Our mimetic methods extend the ideas of multilevel multiscale mimetic (m^3) (Lipnikov, Moulton, & Svyatskiy, 2008) to 3D and to Maxwell's equations when the electrical conductivity σ is discontinuous.

Mimetic multiscale method

Multiscale methods aim for accurate simulation results by deriving a new coarse-scale discrete problem reduced in size, but still accounting for the underlying fine-scale features. Typically, two nested meshes are involved. The first mesh is the fine one on which the electrical conductivity resides and the second mesh is the coarse mesh on which the electric and magnetic fields reside. This construction is depicted in Figure 1.

The intention is to use a discretization of Maxwell's equations on the coarse mesh that generates a similar result to the one obtained by solving the fine-mesh problem. The multiscaling method requires three major steps. First, the fine mesh needs to be partitioned into coarse-mesh cells. Second, the main step is the special interpolation between the fine and the coarse mesh. Therefore, local versions of Maxwell's equations are solved in order to construct σ based basis functions for each coarse-mesh cell. That is why multiscaling is referred to as an operator-induced method. The basis functions transfer fine-mesh information to the coarse-mesh system in order to approximate the fields. As the third step, the coarse-mesh system is solved and the basis functions are used to reach sub-mesh resolution.

This construction does not mandate the discretization of the system on a very fine mesh and allows for coarse-mesh discretization directly. So, the linear system is set up only for the coarse mesh and therefore we overcome the computational challenges of too large meshes. To improve the interpolation, not only fine-mesh conductivity cells inside each coarse cell are used for constructing the basis functions but also fine cells in the direct neighborhood. This is referred as oversampling (Caudillo-Mata, Haber, & Schwarzbach, 2016). As a result, we accurately approximate the fine-mesh solution.

Using a mimetic discretization provides superior characteristics. It means the behavior of the continuous operators is kept in the discrete setting. This implies that, for instance, the discrete divergence of the magnetic field is zero everywhere. Therefore, mimetic methods provide physical solutions. For time stepping problems, the mimetic discretization enables to compute consistent initial conditions.

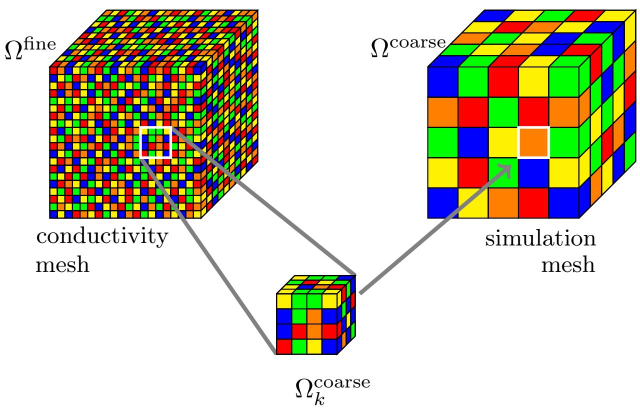

Figure 1: A fine conductivity mesh (left), a nested coarse simulation mesh (right) and one coarse cell with nonconstant σ (middle).

Vertical borehole example

We demonstrate the performance of the mimetic multiscale method using a synthetic 3D example of a vertical borehole in a homogeneous halfspace. The model is discretized on an OcTree mesh where the vertical borehole is situated in the center. We compute a reference solution on the fine conductivity mesh to compare different coarse mesh solutions with.

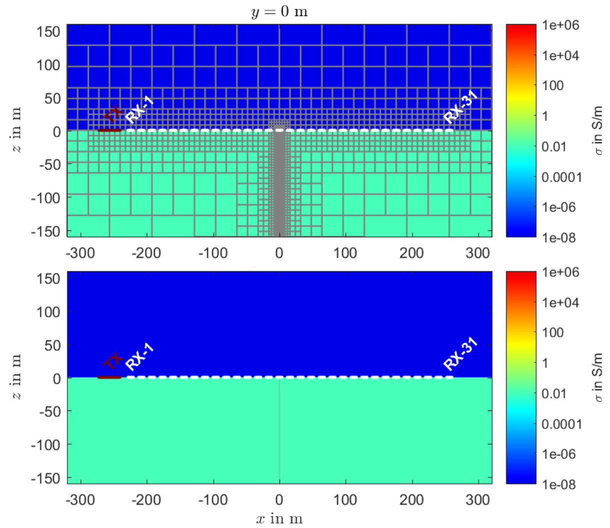

Figure 2: Vertical borehole in a homogeneous halfspace: with mesh (top) and without (bottom).

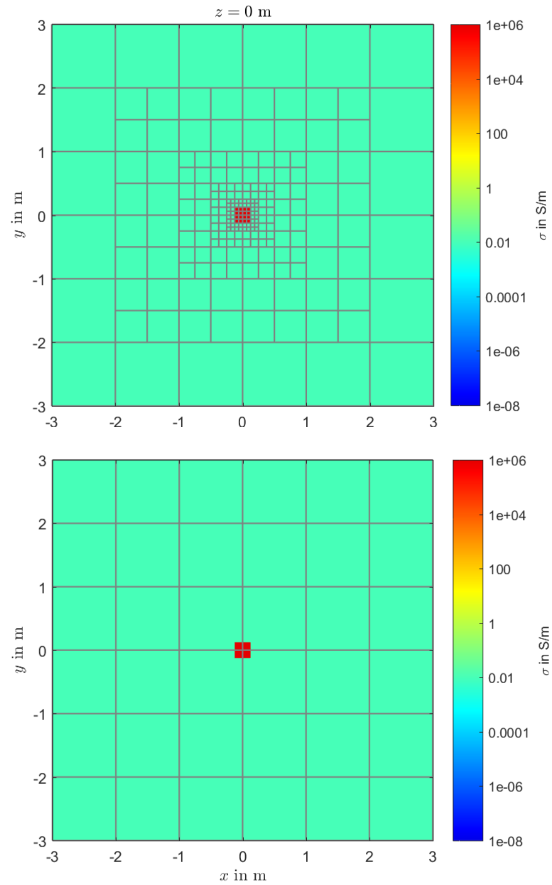

Figure 3: Borehole in plan view: fine conductivity mesh (top), coarse simulation mesh (bottom).

The fine mesh consists of very fine cells along the borehole and fine cells along the profile where the transmitter and the receivers are located on the Earth's surface. The mesh coarsens further away from the survey area and the borehole. Figure 2 shows the conductivity model in the survey area with and without the mesh.

The experimental setup consists of a 32 m line source pointing in x-direction. It is placed on the Earth's surface 256 m away from the borehole and it is transmitting at 21 frequencies logarithmically spaced between 100 and $10,000$ Hz. Electric field components E_x are measured at 31 receivers of 16 m length each. They are located along a profile in x-direction as indicated by white bars in Figure 2.

Figure 3 shows a plan view of the borehole plotted with the fine conductivity mesh (top) and the coarse simulation mesh (bottom). The coarse mesh contains $129,312$ cells and its number of degrees of freedom is $333,150$ (compared to 4.7 million for the fine mesh).

We compare forward modeling results (electric fields E_x at receivers) of three different methods: homogenization (volume weighted arithmetic average of conductivity), mimetic multiscaling and mimetic multiscaling with oversampling. In Figure 4 the maximum relative error with respect to the reference solution is plotted over frequency for all three methods. The mimetic multiscaling is more precise than homogenization which is expressed by a 10 % smaller maximum relative error. Mimetic multiscaling with oversampling is the most accurate method of all three. It is differing the least from the reference solution for all frequencies with a maximum relative error of less than 10 %.

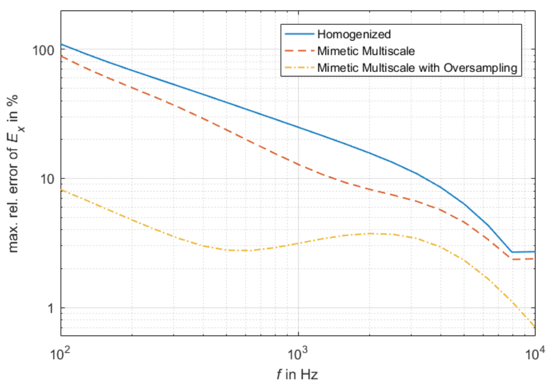

Figure 4: Maximum relative data error with respect to the reference solution on the fine grid for electric fields E_x recorded at 31 receiver locations.

Conclusions

The numerical example of a vertical borehole exhibits the potential of the mimetic multiscale method. By applying mimetic multiscaling with oversampling we achieve coarse mesh accuracy that is similar to the fine mesh.

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