

Solutionbank

Edexcel AS and A Level Modular Mathematics

Coordinate geometry in the (x, y) plane

Exercise A, Question 1

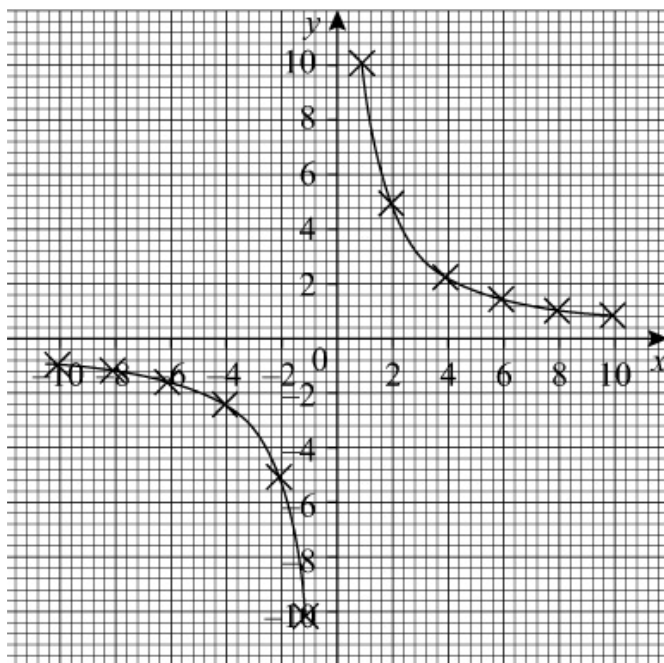
Question:

A curve is given by the parametric equations $x = 2t$, $y = \frac{5}{t}$ where $t \neq 0$. Complete the table and draw a graph of the curve for $-5 \leq t \leq 5$.

t	-5	-4	-3	-2	-1	-0.5	0.5	1	2	3	4	5
$x = 2t$	-10	-8				-1						
$y = \frac{5}{t}$	-1	-1.25					10					

Solution:

t	-5	-4	-3	-2	-1	-0.5	0.5	1	2	3	4	5
$x = 2t$	-10	-8	-6	-4	-2	-1	1	2	4	6	8	10
$y = \frac{5}{t}$	-1	-1.25	-1.67	-2.5	-5	-10	10	5	2.5	1.67	1.25	1



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Coordinate geometry in the (x, y) plane

Exercise A, Question 2

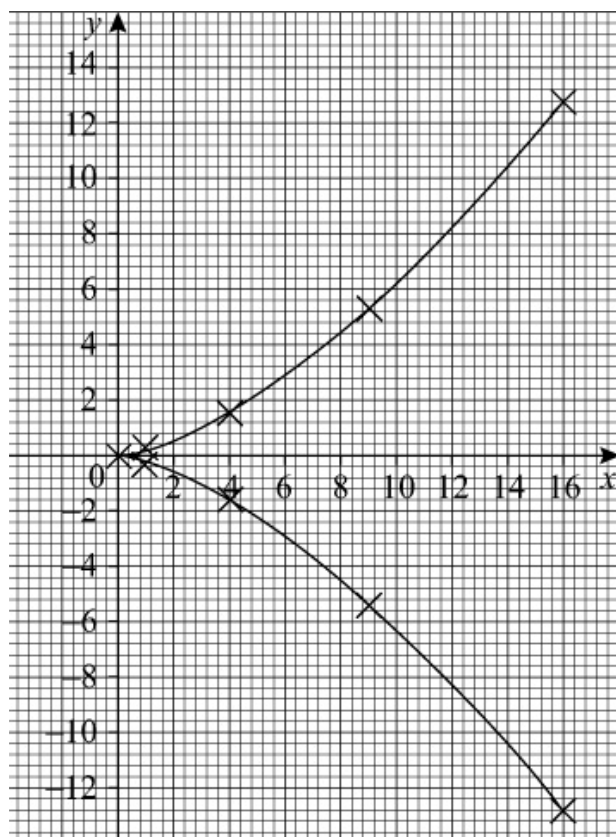
Question:

A curve is given by the parametric equations $x = t^2$, $y = \frac{t^3}{5}$. Complete the table and draw a graph of the curve for $-4 \leq t \leq 4$.

t	-4	-3	-2	-1	0	1	2	3	4
$x = t^2$	16								
$y = \frac{t^3}{5}$	-12.8								

Solution:

t	-4	-3	-2	-1	0	1	2	3	4
$x = t^2$	16	9	4	1	0	1	4	9	16
$y = \frac{t^3}{5}$	-12.8	-5.4	-1.6	-0.2	0	0.2	1.6	5.4	12.8



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Edexcel AS and A Level Modular Mathematics

Coordinate geometry in the (x, y) plane

Exercise A, Question 3

Question:

Sketch the curves given by these parametric equations:

(a) $x = t - 2, y = t^2 + 1$ for $-4 \leq t \leq 4$

(b) $x = t^2 - 2, y = 3 - t$ for $-3 \leq t \leq 3$

(c) $x = t^2, y = t(5 - t)$ for $0 \leq t \leq 5$

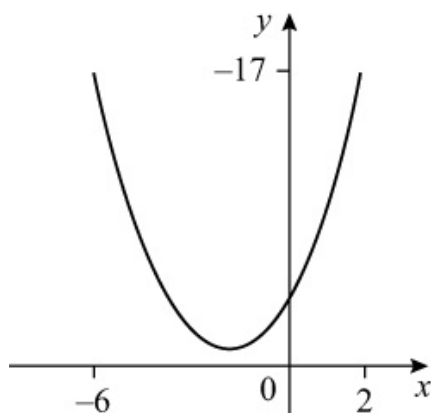
(d) $x = 3\sqrt{t}, y = t^3 - 2t$ for $0 \leq t \leq 2$

(e) $x = t^2, y = (2 - t)(t + 3)$ for $-5 \leq t \leq 5$

Solution:

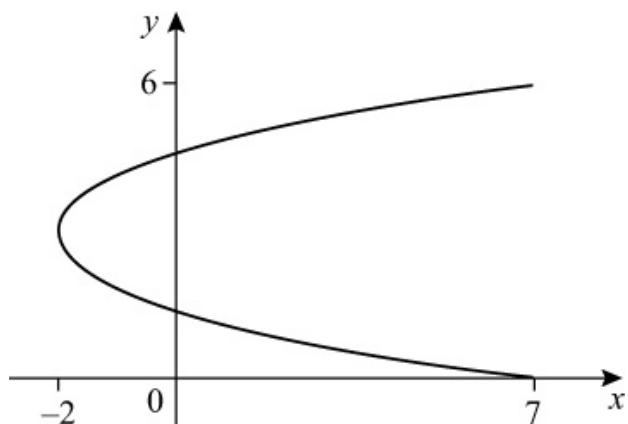
(a)

t	-4	-3	-2	-1	0	1	2	3	4
$x = t - 2$	-6	-5	-4	-3	-2	-1	0	1	2
$y = t^2 + 1$	17	10	5	2	1	2	5	10	17



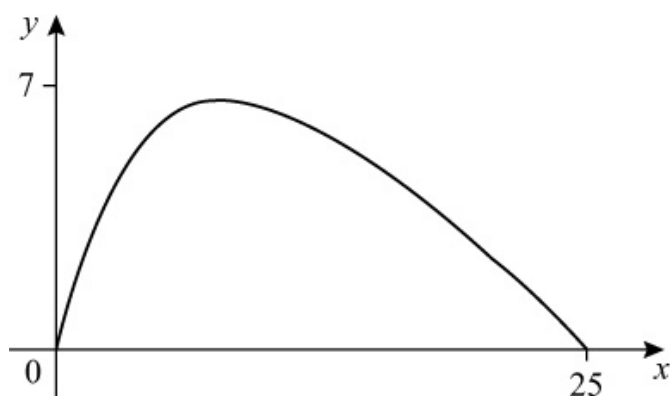
(b)

t	-3	-2	-1	0	1	2	3
$x = t^2 - 2$	7	2	-1	-2	-1	2	7
$y = 3 - t$	6	5	4	3	2	1	0



(c)

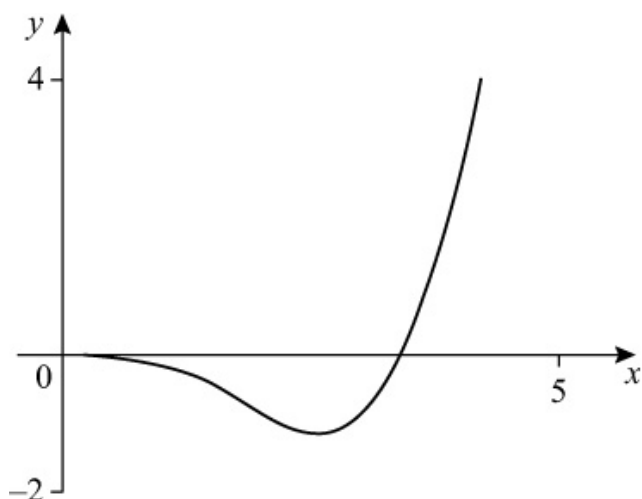
t	0	1	2	3	4	5
$x = t^2$	0	1	4	9	16	25
$y = t(5-t)$	0	4	6	6	4	0



(d)

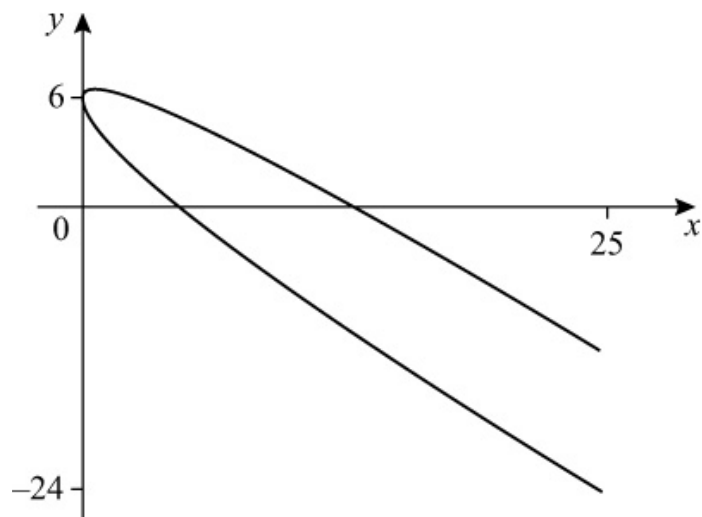
t	0	0.25	0.5	0.75	1	1.25	1.5	1.75	2
$x = 3\sqrt{t}$	0	1.5	2.12	2.60	3	3.35	3.67	3.97	4.24
$y = t^3 - 2t$	0	-0.48	-0.88	-1.08	-1	-0.55	0.38	1.86	4

Answers have been rounded to 2 d.p.



(e)

t	-5	-4	-3	-2	-1	0	1	2	3	4	5
$x = t^2$	25	16	9	4	1	0	1	4	9	16	25
$y = (2-t)(t+3)$	-14	-6	0	4	6	6	4	0	-6	-14	-24



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Edexcel AS and A Level Modular Mathematics

Coordinate geometry in the (x, y) plane
Exercise A, Question 4

Question:

Find the cartesian equation of the curves given by these parametric equations:

(a) $x = t - 2, y = t^2$

(b) $x = 5 - t, y = t^2 - 1$

(c) $x = \frac{1}{t}, y = 3 - t, t \neq 0$

(d) $x = 2t + 1, y = \frac{1}{t}, t \neq 0$

(e) $x = 2t^2 - 3, y = 9 - t^2$

(f) $x = \sqrt{t}, y = t(9 - t)$

(g) $x = 3t - 1, y = (t - 1)(t + 2)$

(h) $x = \frac{1}{t-2}, y = t^2, t \neq 2$

(i) $x = \frac{1}{t+1}, y = \frac{1}{t-2}, t \neq -1, t \neq 2$

(j) $x = \frac{t}{2t-1}, y = \frac{t}{t+1}, t \neq -1, t \neq \frac{1}{2}$

Solution:

(a) $x = t - 2, y = t^2$

$$x = t - 2$$

$$t = x + 2$$

Substitute $t = x + 2$ into $y = t^2$

$$y = (x + 2)^2$$

So the cartesian equation of the curve is $y = (x + 2)^2$.

(b) $x = 5 - t, y = t^2 - 1$

$$x = 5 - t$$

$$t = 5 - x$$

Substitute $t = 5 - x$ into $y = t^2 - 1$

$$y = (5 - x)^2 - 1$$

$$y = 25 - 10x + x^2 - 1$$

$$y = x^2 - 10x + 24$$

So the cartesian equation of the curve is $y = x^2 - 10x + 24$.

(c) $x = \frac{1}{t}, y = 3 - t$

$$x = \frac{1}{t}$$

$$t = \frac{1}{x}$$

Substitute $t = \frac{1}{x}$ into $y = 3 - t$

$$y = 3 - \frac{1}{x}$$

So the cartesian equation of the curve is $y = 3 - \frac{1}{x}$.

(d) $x = 2t + 1, y = \frac{1}{t}$

$$x = 2t + 1$$

$$2t = x - 1$$

$$t = \frac{x-1}{2}$$

Substitute $t = \frac{x-1}{2}$ into $y = \frac{1}{t}$

$$y = \frac{1}{\left(\frac{x-1}{2}\right)}$$

$$y = \frac{2}{x-1} \quad \left[\text{Note: This uses } \frac{1}{\left(\frac{a}{b}\right)} = \frac{b}{a} \right]$$

So the cartesian equation of the curve is $y = \frac{2}{x-1}$.

$$(e) x = 2t^2 - 3, y = 9 - t^2$$

$$x = 2t^2 - 3$$

$$2t^2 = x + 3$$

$$t^2 = \frac{x+3}{2}$$

Substitute $t^2 = \frac{x+3}{2}$ into $y = 9 - t^2$

$$y = 9 - \frac{x+3}{2}$$

$$y = \frac{18 - (x+3)}{2}$$

$$y = \frac{15-x}{2}$$

So the cartesian equation is $y = \frac{15-x}{2}$.

$$(f) x = \sqrt{t}, y = t(9-t)$$

$$x = \sqrt{t}$$

$$t = x^2$$

Substitute $t = x^2$ into $y = t(9-t)$

$$y = x^2(9-x^2)$$

So the cartesian equation is $y = x^2(9-x^2)$.

$$(g) x = 3t - 1, y = (t-1)(t+2)$$

$$x = 3t - 1$$

$$3t = x + 1$$

$$t = \frac{x+1}{3}$$

Substitute $t = \frac{x+1}{3}$ into $y = (t-1)(t+2)$

$$y = \left(\frac{x+1}{3} - 1 \right) \left(\frac{x+1}{3} + 2 \right)$$

$$y = \left(\frac{x+1}{3} - \frac{3}{3} \right) \left(\frac{x+1}{3} + \frac{6}{3} \right)$$

$$y = \left(\frac{x+1-3}{3} \right) \left(\frac{x+1+6}{3} \right)$$

$$y = \left(\frac{x-2}{3} \right) \left(\frac{x+7}{3} \right)$$

$$y = \frac{1}{9} \begin{pmatrix} x - 2 \\ \end{pmatrix} \begin{pmatrix} x + 7 \\ \end{pmatrix}$$

So the cartesian equation of the curve is $y = \frac{1}{9} \begin{pmatrix} x - 2 \\ \end{pmatrix} \begin{pmatrix} x + 7 \\ \end{pmatrix}$.

(h) $x = \frac{1}{t-2}, y = t^2$

$$x = \frac{1}{t-2}$$

$$x(t-2) = 1$$

$$t-2 = \frac{1}{x}$$

$$t = \frac{1}{x} + 2$$

$$t = \frac{1}{x} + \frac{2x}{x}$$

$$t = \frac{1+2x}{x}$$

Substitute $t = \frac{1+2x}{x}$ into $y = t^2$

$$y = \left(\frac{1+2x}{x} \right)^2$$

So the cartesian equation of the curve is $y = \left(\frac{1+2x}{x} \right)^2$.

(i) $x = \frac{1}{t+1}, y = \frac{1}{t-2}$

$$x = \frac{1}{t+1}$$

$$(t+1)x = 1$$

$$t+1 = \frac{1}{x}$$

$$t = \frac{1}{x} - 1$$

Substitute $t = \frac{1}{x} - 1$ into $y = \frac{1}{t-2}$

$$y = \frac{1}{\left(\frac{1}{x} - 1 \right) - 2}$$

$$y = \frac{1}{\frac{1}{x} - 3}$$

$$y = \frac{1}{\frac{1}{x} - \frac{3x}{x}}$$

$$y = \frac{1}{\left(\frac{1-3x}{x}\right)}$$

$$y = \frac{x}{1-3x} \quad \left[\text{Note: This uses } \frac{1}{\left(\frac{a}{b}\right)} = \frac{b}{a} \right]$$

So the cartesian equation of the curve is $y = \frac{x}{1-3x}$.

$$(j) \quad x = \frac{t}{2t-1}, y = \frac{t}{t+1}$$

$$x = \frac{t}{2t-1}$$

$$x \times (2t-1) = \frac{t}{2t-1} \times (2t-1) \quad \text{Multiply each side by}$$

$$(2t-1)$$

$$x(2t-1) = t \quad \text{Simplify}$$

$$2tx - x = t \quad \text{Expand the brackets}$$

$$2tx = t + x \quad \text{Add } x \text{ to each side}$$

$$2tx - t = x \quad \text{Subtract } 2t \text{ from each side}$$

$$t(2x-1) = x \quad \text{Factorise } t$$

$$\frac{t(2x-1)}{(2x-1)} = \frac{x}{2x-1} \quad \text{Divide each side by } (2x-1)$$

$$t = \frac{x}{2x-1} \quad \text{Simplify}$$

Substitute $t = \frac{x}{2x-1}$ into $y = \frac{t}{t+1}$

$$y = \frac{\left(\frac{x}{2x-1}\right)}{\left(\frac{x}{2x-1} + 1\right)}$$

$$y = \frac{\left(\frac{x}{2x-1}\right)}{\left(\frac{x}{2x-1} + \frac{2x-1}{2x-1}\right)}$$

$$y = \frac{\left(\frac{x}{2x-1}\right)}{\left(\frac{x+2x-1}{2x-1}\right)}$$

$$y = \frac{\left(\frac{x}{2x-1}\right)}{\left(\frac{3x-1}{2x-1}\right)}$$

$$y = \frac{x}{3x-1} \quad \left[\text{Note: This uses } \frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{b}\right)} = \frac{a}{c} \right]$$

So the cartesian equation of the curve is $y = \frac{x}{3x-1}$.

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Edexcel AS and A Level Modular Mathematics

Coordinate geometry in the (x, y) plane
Exercise A, Question 5

Question:

Show that the parametric equations:

(i) $x = 1 + 2t, y = 2 + 3t$

(ii) $x = \frac{1}{2t-3}, y = \frac{t}{2t-3}, t \neq \frac{3}{2}$

represent the same straight line.

Solution:

(i) $x = 1 + 2t, y = 2 + 3t$

$$x = 1 + 2t$$

$$2t = x - 1$$

$$t = \frac{x-1}{2}$$

Substitute $t = \frac{x-1}{2}$ into $y = 2 + 3t$

$$y = 2 + 3 \left(\frac{x-1}{2} \right)$$

$$y = 2 + 3 \left(\frac{x}{2} - \frac{1}{2} \right)$$

$$y = 2 + \frac{3x}{2} - \frac{3}{2}$$

$$y = \frac{3x}{2} + \frac{1}{2}$$

(ii) $x = \frac{1}{2t-3}, y = \frac{t}{2t-3}$

$$\frac{y}{x} = \frac{\left(\frac{t}{2t-3} \right)}{\left(\frac{1}{2t-3} \right)} \quad \text{Note: } \frac{\left(\frac{a}{b} \right)}{\left(\frac{c}{b} \right)} = \frac{a}{c}$$

$$\frac{y}{x} = t$$

Substitute $t = \frac{y}{x}$ into $x = \frac{1}{2t-3}$

$$x = \frac{1}{2\left(\frac{y}{x}\right) - 3}$$

$$x \left[2\left(\frac{y}{x}\right) - 3 \right] = 1$$

$$2y - 3x = 1$$

$$2y = 3x + 1$$

$$y = \frac{3}{2}x + \frac{1}{2}$$

The cartesian equations of (i) and (ii) are the same, so they represent the same straight line.

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Edexcel AS and A Level Modular Mathematics

Coordinate geometry in the (x, y) plane
Exercise B, Question 1

Question:

Find the coordinates of the point(s) where the following curves meet the x -axis:

(a) $x = 5 + t, y = 6 - t$

(b) $x = 2t + 1, y = 2t - 6$

(c) $x = t^2, y = (1 - t)(t + 3)$

(d) $x = \frac{1}{t}, y = \sqrt{(t - 1)(2t - 1)}, t \neq 0$

(e) $x = \frac{2t}{1 + t}, y = t - 9, t \neq -1$

Solution:

(a) $x = 5 + t, y = 6 - t$

When $y = 0$

$$6 - t = 0$$

so $t = 6$

Substitute $t = 6$ into $x = 5 + t$

$$x = 5 + 6$$

$$x = 11$$

So the curve meets the x -axis at (11, 0).

(b) $x = 2t + 1, y = 2t - 6$

When $y = 0$

$$2t - 6 = 0$$

$$2t = 6$$

so $t = 3$

Substitute $t = 3$ into $x = 2t + 1$

$$x = 2(3) + 1$$

$$x = 6 + 1$$

$$x = 7$$

So the curve meets the x -axis at (7, 0).

(c) $x = t^2, y = (1 - t)(t + 3)$

When $y = 0$

$$(1 - t)(t + 3) = 0$$

so $t = 1$ and $t = -3$

(1) Substitute $t = 1$ into $x = t^2$

$$x = 1^2$$

$$x = 1$$

(2) Substitute $t = -3$ into $x = t^2$

$$x = (-3)^2$$

$$x = 9$$

So the curve meets the x -axis at $(1, 0)$ and $(9, 0)$.

$$(d) x = \frac{1}{t}, y = \sqrt{(t-1)(2t-1)}$$

When $y = 0$

$$\sqrt{(t-1)(2t-1)} = 0$$

$$(t-1)(2t-1) = 0$$

so $t = 1$ and $t = \frac{1}{2}$

(1) Substitute $t = 1$ into $x = \frac{1}{t}$

$$x = \frac{1}{(1)}$$

$$x = 1$$

(2) Substitute $t = \frac{1}{2}$ into $x = \frac{1}{t}$

$$x = \frac{1}{\left(\frac{1}{2}\right)}$$

$$x = 2$$

So the curve meets the x -axis at $(1, 0)$ and $(2, 0)$.

$$(e) x = \frac{2t}{1+t}, y = t - 9$$

When $y = 0$

$$t - 9 = 0$$

so $t = 9$

Substitute $t = 9$ into $x = \frac{2t}{1+t}$

$$x = \frac{2(9)}{1+(9)}$$

$$x = \frac{18}{10}$$

$$x = \frac{9}{5}$$

So the curve meets the x -axis at $\left(\frac{9}{5}, 0 \right)$.

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Edexcel AS and A Level Modular Mathematics

Coordinate geometry in the (x, y) plane
Exercise B, Question 2

Question:

Find the coordinates of the point(s) where the following curves meet the y-axis:

(a) $x = 2t, y = t^2 - 5$

(b) $x = \sqrt{(3t - 4)}, y = \frac{1}{t^2}, t \neq 0$

(c) $x = t^2 + 2t - 3, y = t(t - 1)$

(d) $x = 27 - t^3, y = \frac{1}{t-1}, t \neq 1$

(e) $x = \frac{t-1}{t+1}, y = \frac{2t}{t^2+1}, t \neq -1$

Solution:

(a) When $x = 0$

$$2t = 0$$

$$\text{so } t = 0$$

Substitute $t = 0$ into $y = t^2 - 5$

$$y = (0)^2 - 5$$

$$y = -5$$

So the curve meets the y-axis at $(0, -5)$.

(b) When $x = 0$

$$\sqrt{3t - 4} = 0$$

$$3t - 4 = 0$$

$$3t = 4$$

$$\text{so } t = \frac{4}{3}$$

Substitute $t = \frac{4}{3}$ into $y = \frac{1}{t^2}$

$$y = \frac{1}{\left(\frac{4}{3}\right)^2}$$

$$y = \frac{1}{\left(\frac{16}{9}\right)}$$

$$y = \frac{9}{16} \quad \left[\text{Note: This uses } \frac{1}{\left(\frac{a}{b}\right)} = \frac{b}{a} \right]$$

So the curve meets the y-axis at $\left(0, \frac{9}{16}\right)$.

(c) When $x = 0$

$$t^2 + 2t - 3 = 0$$

$$(t + 3)(t - 1) = 0$$

so $t = -3$ and $t = 1$

(1) Substitute $t = -3$ into $y = t(t - 1)$

$$y = (-3)[(-3) - 1]$$

$$y = (-3) \times (-4)$$

$$y = 12$$

(2) Substitute $t = 1$ into $y = t(t - 1)$

$$y = 1(1 - 1)$$

$$y = 1 \times 0$$

$$y = 0$$

So the curve meets the y-axis at $(0, 0)$ and $(0, 12)$.

(d) When $x = 0$

$$27 - t^3 = 0$$

$$t^3 = 27$$

$$t = \sqrt[3]{27}$$

so $t = 3$

Substitute $t = 3$ into $y = \frac{1}{t-1}$

$$y = \frac{1}{(3) - 1}$$

$$y = \frac{1}{2}$$

So the curve meets the y-axis at $\left(0, \frac{1}{2} \right)$.

(e) When $x = 0$

$$\frac{t-1}{t+1} = 0$$

$$t - 1 = 0 \quad \left[\text{Note: } \frac{a}{b} = 0 \Rightarrow a = 0 \right]$$

So $t = 1$

Substitute $t = 1$ into $y = \frac{2t}{t^2 + 1}$

$$y = \frac{2(1)}{(1)^2 + 1}$$

$$y = \frac{2}{2}$$

$$y = 1$$

So the curve meets the y-axis at $(0, 1)$.

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Edexcel AS and A Level Modular Mathematics

Coordinate geometry in the (x, y) plane
Exercise B, Question 3

Question:

A curve has parametric equations $x = 4at^2$, $y = a(2t - 1)$, where a is a constant. The curve passes through the point (4, 0). Find the value of a .

Solution:

When $y = 0$

$$a(2t - 1) = 0$$

$$2t - 1 = 0$$

$$2t = 1$$

$$t = \frac{1}{2}$$

When $t = \frac{1}{2}$, $x = 4$

So substitute $t = \frac{1}{2}$ and $x = 4$ into $x = 4at^2$

$$4a \left(\frac{1}{2} \right)^2 = 4$$

$$4a \times \frac{1}{4} = 4$$

$$a = 4$$

So the value of a is 4.

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Edexcel AS and A Level Modular Mathematics

Coordinate geometry in the (x, y) plane
Exercise B, Question 4

Question:

A curve has parametric equations $x = b(2t - 3)$, $y = b(1 - t^2)$, where b is a constant. The curve passes through the point $(0, -5)$. Find the value of b .

Solution:

When $x = 0$

$$b(2t - 3) = 0$$

$$2t - 3 = 0$$

$$2t = 3$$

$$t = \frac{3}{2}$$

When $t = \frac{3}{2}$, $y = -5$

So substitute $t = \frac{3}{2}$ and $y = -5$ into $y = b(1 - t^2)$

$$b \left[1 - \left(\frac{3}{2} \right)^2 \right] = -5$$

$$b \left(1 - \frac{9}{4} \right) = -5$$

$$b \left(\frac{-5}{4} \right) = -5$$

$$b = \frac{-5}{\left(\frac{-5}{4} \right)}$$

$$b = 4$$

So the value of b is 4.

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Edexcel AS and A Level Modular Mathematics

Coordinate geometry in the (x, y) plane

Exercise B, Question 5

Question:

A curve has parametric equations $x = p(2t - 1)$, $y = p(t^3 + 8)$, where p is a constant. The curve meets the x -axis at $(2, 0)$ and the y -axis at A .

(a) Find the value of p .

(b) Find the coordinates of A .

Solution:

(a) When $y = 0$

$$p(t^3 + 8) = 0$$

$$t^3 + 8 = 0$$

$$t^3 = -8$$

$$t = \sqrt[3]{-8}$$

$$t = -2$$

When $t = -2$, $x = 2$

So substitute $t = -2$ and $x = 2$ into $x = p(2t - 1)$

$$p[2(-2) - 1] = 2$$

$$p(-4 - 1) = 2$$

$$p(-5) = 2$$

$$p = -\frac{2}{5}$$

(b) When $x = 0$

$$p(2t - 1) = 0$$

$$2t - 1 = 0$$

$$2t = 1$$

$$t = \frac{1}{2}$$

When the curve meets the y -axis $t = \frac{1}{2}$

So substitute $t = \frac{1}{2}$ into $y = p(t^3 + 8)$

$$y = p \left[\left(\frac{1}{2} \right)^3 + 8 \right]$$

$$\text{but } p = -\frac{2}{5}$$

$$\text{So } y = -\frac{2}{5} \left[\left(\frac{1}{2} \right)^3 + 8 \right] = -\frac{2}{5} \left(\frac{1}{8} + 8 \right) = -\frac{2}{5} \times \frac{65}{8} = -\frac{13}{4}$$

$$\text{So the coordinates of } A \text{ are } \left(0, -\frac{13}{4} \right).$$

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Edexcel AS and A Level Modular Mathematics

Coordinate geometry in the (x, y) plane
Exercise B, Question 6

Question:

A curve is given parametrically by the equations $x = 3qt^2$, $y = 4(t^3 + 1)$, where q is a constant. The curve meets the x -axis at X and the y -axis at Y . Given that $OX = 2OY$, where O is the origin, find the value of q .

Solution:

(1) When $y = 0$

$$4(t^3 + 1) = 0$$

$$t^3 + 1 = 0$$

$$t^3 = -1$$

$$t = \sqrt[3]{-1}$$

$$t = -1$$

Substitute $t = -1$ into $x = 3qt^2$

$$x = 3q(-1)^2$$

$$x = 3q$$

So the coordinates of X are $(3q, 0)$.

(2) When $x = 0$

$$3qt^2 = 0$$

$$t^2 = 0$$

$$t = 0$$

Substitute $t = 0$ into $y = 4(t^3 + 1)$

$$y = 4[(0)^3 + 1]$$

$$y = 4$$

So the coordinates of Y are $(0, 4)$.

(3) Now $OX = 3q$ and $OY = 4$

As $OX = 2OY$

$$(3q) = 2(4)$$

$$3q = 8$$

$$q = \frac{8}{3}$$

So the value of q is $\frac{8}{3}$.

Solutionbank

Edexcel AS and A Level Modular Mathematics

Coordinate geometry in the (x, y) plane
Exercise B, Question 7

Question:

Find the coordinates of the point of intersection of the line with parametric equations $x = 3t + 2$, $y = 1 - t$ and the line $y + x = 2$.

Solution:

(1) Substitute $x = 3t + 2$ and $y = 1 - t$ into $y + x = 2$

$$(1 - t) + (3t + 2) = 2$$

$$1 - t + 3t + 2 = 2$$

$$2t + 3 = 2$$

$$2t = -1$$

$$t = -\frac{1}{2}$$

(2) Substitute $t = -\frac{1}{2}$ into $x = 3t + 2$

$$x = 3 \left(-\frac{1}{2} \right) + 2 = -\frac{3}{2} + 2 = \frac{1}{2}$$

(3) Substitute $t = -\frac{1}{2}$ into $y = 1 - t$

$$y = 1 - \left(-\frac{1}{2} \right) = 1 + \frac{1}{2} = \frac{3}{2}$$

So the coordinates of the point of intersection are $\left(\frac{1}{2}, \frac{3}{2} \right)$.

Solutionbank

Edexcel AS and A Level Modular Mathematics

Coordinate geometry in the (x, y) plane
Exercise B, Question 8

Question:

Find the coordinates of the points of intersection of the curve with parametric equations $x = 2t^2 - 1$, $y = 3(t + 1)$ and the line $3x - 4y = 3$.

Solution:

(1) Substitute $x = 2t^2 - 1$ and $y = 3(t + 1)$ into $3x - 4y = 3$

$$3(2t^2 - 1) - 4[3(t + 1)] = 3$$

$$3(2t^2 - 1) - 12(t + 1) = 3$$

$$6t^2 - 3 - 12t - 12 = 3$$

$$6t^2 - 12t - 15 = 3$$

$$6t^2 - 12t - 18 = 0 \quad (\div 6)$$

$$t^2 - 2t - 3 = 0$$

$$(t - 3)(t + 1) = 0$$

so $t = 3$ and $t = -1$

(2) Substitute $t = 3$ into $x = 2t^2 - 1$ and $y = 3(t + 1)$

$$x = 2(3)^2 - 1 = 17$$

$$y = 3(3 + 1) = 12$$

(3) Substitute $t = -1$ into $x = 2t^2 - 1$ and $y = 3(t + 1)$

$$x = 2(-1)^2 - 1 = 1$$

$$y = 3(-1 + 1) = 0$$

So the coordinates of the points of intersection are (17, 12) and (1, 0).

Solutionbank

Edexcel AS and A Level Modular Mathematics

Coordinate geometry in the (x, y) plane
Exercise B, Question 9

Question:

Find the values of t at the points of intersection of the line $4x - 2y - 15 = 0$ with the parabola $x = t^2$, $y = 2t$ and give the coordinates of these points.

Solution:

(1) Substitute $x = t^2$ and $y = 2t$ into $4x - 2y - 15 = 0$

$$4(t^2) - 2(2t) - 15 = 0$$

$$4t^2 - 4t - 15 = 0$$

$$(2t + 3)(2t - 5) = 0$$

$$\text{So } 2t + 3 = 0 \Rightarrow 2t = -3 \Rightarrow t = \frac{-3}{2} \text{ and}$$

$$2t - 5 = 0 \Rightarrow 2t = 5 \Rightarrow t = \frac{5}{2}$$

(2) Substitute $t = -\frac{3}{2}$ into $x = t^2$ and $y = 2t$

$$x = \left(-\frac{3}{2}\right)^2 = \frac{9}{4}$$

$$y = 2\left(-\frac{3}{2}\right) = -3$$

(3) Substitute $t = \frac{5}{2}$ into $x = t^2$ and $y = 2t$

$$x = \left(\frac{5}{2}\right)^2 = \frac{25}{4}$$

$$y = 2\left(\frac{5}{2}\right) = 5$$

So the coordinates of the points of intersection are $\left(\frac{9}{4}, -3\right)$ and $\left(\frac{25}{4}, 5\right)$.

Solutionbank

Edexcel AS and A Level Modular Mathematics

Coordinate geometry in the (x, y) plane
Exercise B, Question 10

Question:

Find the points of intersection of the parabola $x = t^2$, $y = 2t$ with the circle $x^2 + y^2 - 9x + 4 = 0$.

Solution:

(1) Substitute $x = t^2$ and $y = 2t$ into $x^2 + y^2 - 9x + 4 = 0$

$$(t^2)^2 + (2t)^2 - 9(t^2) + 4 = 0$$

$$t^4 + 4t^2 - 9t^2 + 4 = 0$$

$$t^4 - 5t^2 + 4 = 0$$

$$(t^2 - 4)(t^2 - 1) = 0$$

$$\text{So } t^2 - 4 = 0 \Rightarrow t^2 = 4 \Rightarrow t = \sqrt{4} \Rightarrow t = \pm 2 \text{ and}$$

$$t^2 - 1 = 0 \Rightarrow t^2 = 1 \Rightarrow t = \sqrt{1} \Rightarrow t = \pm 1$$

(2) Substitute $t = 2$ into $x = t^2$ and $y = 2t$

$$x = (2)^2 = 4$$

$$y = 2(2) = 4$$

(3) Substitute $t = -2$ into $x = t^2$ and $y = 2t$

$$x = (-2)^2 = 4$$

$$y = 2(-2) = -4$$

(4) Substitute $t = 1$ into $x = t^2$ and $y = 2t$

$$x = (1)^2 = 1$$

$$y = 2(1) = 2$$

(5) Substitute $t = -1$ into $x = t^2$ and $y = 2t$

$$x = (-1)^2 = 1$$

$$y = 2(-1) = -2$$

So the coordinates of the points of intersection are (4, 4), (4, -4), (1, 2) and (1, -2).

Solutionbank

Edexcel AS and A Level Modular Mathematics

Coordinate geometry in the (x, y) plane

Exercise C, Question 1

Question:

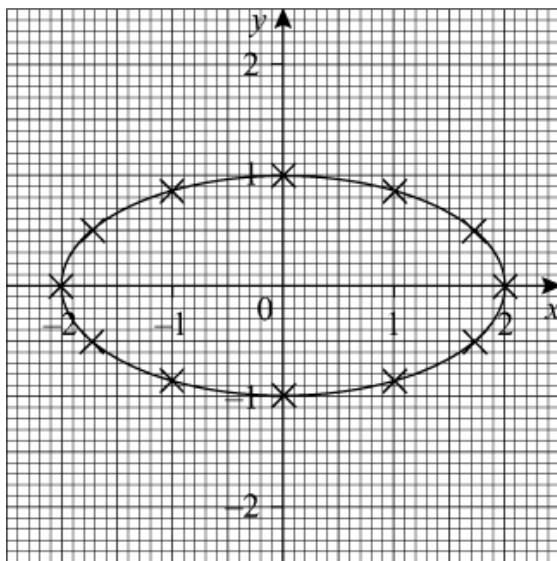
A curve is given by the parametric equations $x = 2 \sin t$, $y = \cos t$.

Complete the table and draw a graph of the curve for $0 \leq t \leq 2\pi$.

t	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{11\pi}{6}$	2π
$x = 2\sin t$			1.73		1.73			-1		-2			0
$y = \cos t$		0.87					-1		-0.5		0.5		

Solution:

t	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{11\pi}{6}$	2π
$x = 2\sin t$	0	1	1.73	2	1.73	1	0	-1	-1.73	-2	-1.73	-1	0
$y = \cos t$	1	0.87	0.5	0	-0.5	-0.87	-1	-0.87	-0.5	0	0.5	0.87	1



Solutionbank

Edexcel AS and A Level Modular Mathematics

Coordinate geometry in the (x, y) plane
Exercise C, Question 2

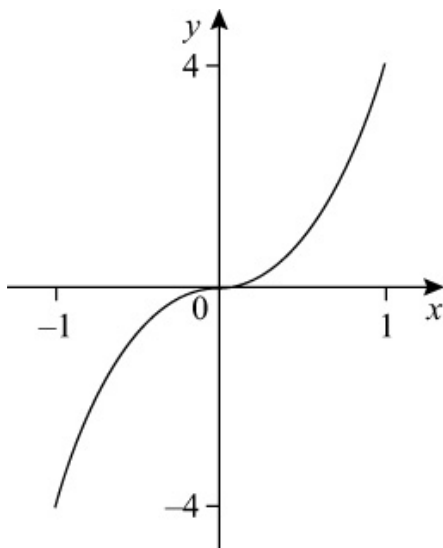
Question:

A curve is given by the parametric equations $x = \sin t$, $y = \tan t$, $-\frac{\pi}{2} < t < \frac{\pi}{2}$. Draw a graph of the curve.

Solution:

t	$\frac{-4\pi}{10}$	$\frac{-3\pi}{10}$	$\frac{-2\pi}{10}$	$\frac{-\pi}{10}$	0	$\frac{\pi}{10}$	$\frac{2\pi}{10}$	$\frac{3\pi}{10}$	$\frac{4\pi}{10}$
$x = \sin t$	-0.95	-0.81	-0.59	-0.31	0	0.31	0.59	0.81	0.95
$y = \tan t$	-3.08	-1.38	-0.73	-0.32	0	0.32	0.73	1.38	3.08

Answers are given to 2 d.p.



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Edexcel AS and A Level Modular Mathematics

Coordinate geometry in the (x, y) plane
Exercise C, Question 3

Question:

Find the cartesian equation of the curves given by the following parametric equations:

(a) $x = \sin t, y = \cos t$

(b) $x = \sin t - 3, y = \cos t$

(c) $x = \cos t - 2, y = \sin t + 3$

(d) $x = 2 \cos t, y = 3 \sin t$

(e) $x = 2 \sin t - 1, y = 5 \cos t + 4$

(f) $x = \cos t, y = \sin 2t$

(g) $x = \cos t, y = 2 \cos 2t$

(h) $x = \sin t, y = \tan t$

(i) $x = \cos t + 2, y = 4 \sec t$

(j) $x = 3 \cot t, y = \operatorname{cosec} t$

Solution:

(a) $x = \sin t, y = \cos t$

$$x^2 = \sin^2 t, y^2 = \cos^2 t$$

$$\text{As } \sin^2 t + \cos^2 t = 1$$

$$x^2 + y^2 = 1$$

(b) $x = \sin t - 3, y = \cos t$

$$\sin t = x + 3$$

$$\sin^2 t = (x + 3)^2$$

$$\cos t = y$$

$$\cos^2 t = y^2$$

$$\text{As } \sin^2 t + \cos^2 t = 1$$

$$(x + 3)^2 + y^2 = 1$$

(c) $x = \cos t - 2, y = \sin t + 3$

$$\cos t = x + 2$$

$$\sin t = y - 3$$

As $\sin^2 t + \cos^2 t = 1$

$$(y - 3)^2 + (x + 2)^2 = 1 \quad \text{or} \quad (x + 2)^2 + (y - 3)^2 = 1$$

(d) $x = 2 \cos t, y = 3 \sin t$

$$\sin t = \frac{y}{3}$$

$$\cos t = \frac{x}{2}$$

As $\sin^2 t + \cos^2 t = 1$

$$\left(\frac{y}{3}\right)^2 + \left(\frac{x}{2}\right)^2 = 1 \quad \text{or} \quad \left(\frac{x}{2}\right)^2 + \left(\frac{y}{3}\right)^2 = 1$$

(e) $x = 2 \sin t - 1, y = 5 \cos t + 4$

$$2 \sin t - 1 = x$$

$$2 \sin t = x + 1$$

$$\sin t = \frac{x + 1}{2}$$

and

$$5 \cos t + 4 = y$$

$$5 \cos t = y - 4$$

$$\cos t = \frac{y - 4}{5}$$

As $\sin^2 t + \cos^2 t = 1$

$$\left(\frac{x + 1}{2}\right)^2 + \left(\frac{y - 4}{5}\right)^2 = 1$$

(f) $x = \cos t, y = \sin 2t$

As $\sin 2t = 2 \sin t \cos t$

$$y = 2 \sin t \cos t = (2 \sin t) x$$

Now $\sin^2 t + \cos^2 t = 1$

So $\sin^2 t + x^2 = 1$

$$\Rightarrow \sin^2 t = 1 - x^2$$

$$\Rightarrow \sin t = \sqrt{1 - x^2}$$

$$\text{So } y = (2\sqrt{1 - x^2}) x \quad \text{or} \quad y = 2x\sqrt{1 - x^2}$$

(g) $x = \cos t, y = 2 \cos 2t$

As $\cos 2t = 2 \cos^2 t - 1$

$$y = 2 (2 \cos^2 t - 1)$$

But $x = \cos t$

So $y = 2 (2x^2 - 1)$

$$y = 4x^2 - 2$$

(h) $x = \sin t, y = \tan t$

As $\tan t = \frac{\sin t}{\cos t}$

$$y = \frac{\sin t}{\cos t}$$

But $x = \sin t$

So $y = \frac{x}{\cos t}$

Now $\cos t = \sqrt{1 - \sin^2 t} = \sqrt{1 - x^2}$ (from $\sin^2 t + \cos^2 t = 1$)

So $y = \frac{x}{\sqrt{1 - x^2}}$

(i) $x = \cos t + 2, y = 4 \sec t$

As $\sec t = \frac{1}{\cos t}$

$$y = 4 \times \frac{1}{\cos t} = \frac{4}{\cos t}$$

Now $x = \cos t + 2 \Rightarrow \cos t = x - 2$

So $y = \frac{4}{x - 2}$

(j) $x = 3 \cot t, y = \operatorname{cosec} t$

As $\sin^2 t + \cos^2 t = 1$

$$\frac{\sin^2 t}{\sin^2 t} + \frac{\cos^2 t}{\sin^2 t} = \frac{1}{\sin^2 t} \quad \left(\div \sin^2 t \right)$$

$$1 + \left(\frac{\cos t}{\sin t} \right)^2 = \left(\frac{1}{\sin t} \right)^2$$

$$1 + \cot^2 t = \operatorname{cosec}^2 t$$

Now $x = 3 \cot t \Rightarrow \cot t = \frac{x}{3}$

and $y = \operatorname{cosec} t$

So $1 + \left(\frac{x}{3} \right)^2 = (y)^2$ (using $1 + \cot^2 t = \operatorname{cosec}^2 t$)

$$\text{or } y^2 = 1 + \left(\frac{x}{3} \right)^2$$

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Solutionbank

Edexcel AS and A Level Modular Mathematics

Coordinate geometry in the (x, y) plane
Exercise C, Question 4

Question:

A circle has parametric equations $x = \sin t - 5$, $y = \cos t + 2$.

- (a) Find the cartesian equation of the circle.
- (b) Write down the radius and the coordinates of the centre of the circle.

Solution:

(a) $x = \sin t - 5$, $y = \cos t + 2$
 $\sin t = x + 5$ and $\cos t = y - 2$
As $\sin^2 t + \cos^2 t = 1$
 $(x + 5)^2 + (y - 2)^2 = 1$

- (b) This is a circle with centre $(-5, 2)$ and radius 1.

Solutionbank

Edexcel AS and A Level Modular Mathematics

Coordinate geometry in the (x, y) plane
Exercise C, Question 5

Question:

A circle has parametric equations $x = 4 \sin t + 3$, $y = 4 \cos t - 1$. Find the radius and the coordinates of the centre of the circle.

Solution:

$$x = 4 \sin t + 3$$

$$4 \sin t = x - 3$$

$$\sin t = \frac{x-3}{4}$$

and

$$y = 4 \cos t - 1$$

$$4 \cos t = y + 1$$

$$\cos t = \frac{y+1}{4}$$

As $\sin^2 t + \cos^2 t = 1$

$$\left(\frac{x-3}{4} \right)^2 + \left(\frac{y+1}{4} \right)^2 = 1$$

$$\frac{(x-3)^2}{4^2} + \frac{(y+1)^2}{4^2} = 1$$

$$\frac{(x-3)^2}{16} + \frac{(y+1)^2}{16} = 1$$

$(x-3)^2 + (y+1)^2 = 16$ Multiply throughout by 16
So the centre of the circle is $(3, -1)$ and the radius is 4.

Solutionbank

Edexcel AS and A Level Modular Mathematics

Coordinate geometry in the (x, y) plane
Exercise D, Question 1

Question:

The following curves are given parametrically. In each case, find an expression for $y \frac{dx}{dt}$ in terms of t .

(a) $x = t + 3, y = 4t - 3$

(b) $x = t^3 + 3t, y = t^2$

(c) $x = (2t - 3)^2, y = 1 - t^2$

(d) $x = 6 - \frac{1}{t}, y = 4t^3, t > 0$

(e) $x = \sqrt{t}, y = 6t^3, t \geq 0$

(f) $x = \frac{4}{t^2}, y = 5t^2, t < 0$

(g) $x = 5t^{\frac{1}{2}}, y = 4t^{-\frac{3}{2}}, t > 0$

(h) $x = t^{\frac{1}{3}} - 1, y = \sqrt{t}, t \geq 0$

(i) $x = 16 - t^4, y = 3 - \frac{2}{t}, t < 0$

(j) $x = 6t^{\frac{2}{3}}, y = t^2$

Solution:

(a) $x = t + 3, y = 4t - 3$

$$\frac{dx}{dt} = 1$$

$$\text{So } y \frac{dx}{dt} = \left(4t - 3 \right) \times 1 = 4t - 3$$

$$(b) x = t^3 + 3t, y = t^2$$

$$\frac{dx}{dt} = 3t^2 + 3$$

$$\text{So } y \frac{dx}{dt} = t^2 \left(3t^2 + 3 \right) = 3t^2 \left(t^2 + 1 \right) \quad \text{Factorise 3}$$

$$(c) x = (2t - 3)^2, y = 1 - t^2$$

$$x = 4t^2 - 12t + 9$$

$$\frac{dx}{dt} = 8t - 12$$

$$\text{So } y \frac{dx}{dt} = \left(1 - t^2 \right) \left(8t - 12 \right) = 4 \left(1 - t^2 \right) \left(2t - 3 \right)$$

Factorise 4

$$(d) x = 6 - \frac{1}{t}, y = 4t^3$$

$$x = 6 - t^{-1}$$

$$\frac{dx}{dt} = t^{-2}$$

$$\text{So } y \frac{dx}{dt} = 4t^3 \times t^{-2} = 4t$$

$$(e) x = \sqrt{t}, y = 6t^3$$

$$x = t^{\frac{1}{2}}$$

$$\frac{dx}{dt} = \frac{1}{2}t^{-\frac{1}{2}}$$

$$\text{So } y \frac{dx}{dt} = 6t^3 \times \frac{1}{2}t^{-\frac{1}{2}} = 3t^{3-\frac{1}{2}} = 3t^{\frac{5}{2}}$$

$$(f) x = \frac{4}{t^2}, y = 5t^2$$

$$x = 4t^{-2}$$

$$\frac{dx}{dt} = -8t^{-3}$$

$$\text{So } y \frac{dx}{dt} = 5t^2 \times -8t^{-3} = -40t^{2-3} = -40t^{-1} = -\frac{40}{t}$$

$$(g) x = 5t^{\frac{1}{2}}, y = 4t^{-\frac{3}{2}}$$

$$\frac{dx}{dt} = 5 \times \frac{1}{2}t^{-\frac{1}{2}} = \frac{5}{2}t^{-\frac{1}{2}}$$

$$\text{So } y \frac{dx}{dt} = 4t^{-\frac{3}{2}} \times \frac{5}{2}t^{-\frac{1}{2}} = 10t^{-\frac{3}{2}-\frac{1}{2}} = 10t^{-2}$$

$$\text{(h) } x = t^{\frac{1}{3}} - 1, y = \sqrt{t}$$

$$\frac{dx}{dt} = \frac{1}{3}t^{\frac{1}{3}-1} = \frac{1}{3}t^{-\frac{2}{3}}$$

$$\text{So } y \frac{dx}{dt} = \sqrt{t} \times \frac{1}{3}t^{-\frac{2}{3}} = t^{\frac{1}{2}} \times \frac{1}{3}t^{-\frac{2}{3}} = \frac{1}{3}t^{\frac{1}{2}-\frac{2}{3}} = \frac{1}{3}t^{-\frac{1}{6}}$$

$$\text{(i) } x = 16 - t^4, y = 3 - \frac{2}{t}$$

$$\frac{dx}{dt} = -4t^3$$

$$\begin{aligned} \text{So } y \frac{dx}{dt} &= \left(3 - \frac{2}{t}\right) \left(-4t^3\right) \\ &= 3 \times \left(-4t^3\right) + \frac{2}{t} \times 4t^3 \\ &= -12t^3 + 8t^2 \quad [\text{or } 8t^2 - 12t^3 \text{ or } 4t^2(2 - 3t)] \end{aligned}$$

$$\text{(j) } x = 6t^{\frac{2}{3}}, y = t^2$$

$$\frac{dx}{dt} = 6 \times \frac{2}{3}t^{\frac{2}{3}-1} = 4t^{-\frac{1}{3}}$$

$$\text{So } y \frac{dx}{dt} = t^2 \times 4t^{-\frac{1}{3}} = 4t^{2-\frac{1}{3}} = 4t^{\frac{5}{3}}$$

Solutionbank

Edexcel AS and A Level Modular Mathematics

Coordinate geometry in the (x, y) plane
Exercise D, Question 2

Question:

A curve has parametric equations $x = 2t - 5$, $y = 3t + 8$. Work out $\int_0^4 y \frac{dx}{dt} dt$.

Solution:

$$x = 2t - 5, y = 3t + 8$$

$$\frac{dx}{dt} = 2$$

$$\text{So } y \frac{dx}{dt} = (3t + 8) \times 2 = 6t + 16$$

$$\begin{aligned} \int_0^4 y \frac{dx}{dt} dt &= \int_0^4 6t + 16 \, dt \\ &= [3t^2 + 16t]_0^4 \\ &= [3(4)^2 + 16(4)] - [3(0)^2 + 16(0)] \\ &= (3 \times 16 + 16 \times 4) - 0 \\ &= 48 + 64 \\ &= 112 \end{aligned}$$

Solutionbank

Edexcel AS and A Level Modular Mathematics

Coordinate geometry in the (x, y) plane
Exercise D, Question 3

Question:

A curve has parametric equations $x = t^2 - 3t + 1$, $y = 4t^2$. Work out $\int_{-1}^5 y \frac{dx}{dt} dt$.

Solution:

$$x = t^2 - 3t + 1, y = 4t^2$$

$$\frac{dx}{dt} = 2t - 3$$

$$\text{So } y \frac{dx}{dt} = 4t^2 (2t - 3) = 8t^3 - 12t^2$$

$$\begin{aligned} \int_{-1}^5 y \frac{dx}{dt} dt &= \int_{-1}^5 (8t^3 - 12t^2) dt \\ &= [2t^4 - 4t^3]_{-1}^5 \\ &= [2(5)^4 - 4(5)^3] - [2(-1)^4 - 4(-1)^3] \\ &= 750 - 6 \\ &= 744 \end{aligned}$$

Solutionbank

Edexcel AS and A Level Modular Mathematics

Coordinate geometry in the (x, y) plane
Exercise D, Question 4

Question:

A curve has parametric equations $x = 3t^2$, $y = \frac{1}{t} + t^3$, $t > 0$. Work out $\int_{0.5}^3 y \frac{dx}{dt} dt$.

Solution:

$$x = 3t^2, y = \frac{1}{t} + t^3$$

$$\frac{dx}{dt} = 6t$$

$$\text{So } y \frac{dx}{dt} = \left(\frac{1}{t} + t^3 \right) \times 6t = \frac{1}{t} \times 6t + t^3 \times 6t = 6 + 6t^4$$

$$\int_{0.5}^3 y \frac{dx}{dt} dt = \int_{0.5}^3 6 + 6t^4 dt$$

$$= \left[6t + \frac{6}{5}t^5 \right]_{0.5}^3$$

$$= \left[6 \left(3 \right) + \frac{6}{5} \left(3 \right)^5 \right] - \left[6 \left(0.5 \right) + \frac{6}{5} \left(0.5 \right)^5 \right]$$

5]

$$= 309.6 - 3.0375$$

$$= 306.5625 \quad \left(\text{or } 306 \frac{9}{16} \right)$$

Solutionbank

Edexcel AS and A Level Modular Mathematics

Coordinate geometry in the (x, y) plane
Exercise D, Question 5

Question:

A curve has parametric equations $x = t^3 - 4t$, $y = t^2 - 1$. Work out $\int_{-2}^2 y \frac{dx}{dt} dt$.

Solution:

$$x = t^3 - 4t, y = t^2 - 1$$

$$\frac{dx}{dt} = 3t^2 - 4$$

$$\text{So } y \frac{dx}{dt} = (t^2 - 1) \times (3t^2 - 4) = 3t^4 - 4t^2 - 3t^2 + 4 = 3t^4 - 7t^2 + 4$$

$$\begin{aligned} \int_{-2}^2 3t^4 - 7t^2 + 4 \, dt &= \left[\frac{3}{5}t^5 - \frac{7}{3}t^3 + 4t \right]_{-2}^2 \\ &= \left[\frac{3}{5}(2)^5 - \frac{7}{3}(2)^3 + 4(2) \right] - \left[\frac{3}{5}(-2)^5 - \frac{7}{3}(-2)^3 + \right. \\ &\quad \left. (-2) \right] \\ &= 8\frac{8}{15} - \left(-8\frac{8}{15} \right) \\ &= 17\frac{1}{15} \end{aligned}$$

Solutionbank

Edexcel AS and A Level Modular Mathematics

Coordinate geometry in the (x, y) plane
Exercise D, Question 6

Question:

A curve has parametric equations $x = 9t^{\frac{4}{3}}$, $y = t^{-\frac{1}{3}}$, $t > 0$.

(a) Show that $y \frac{dx}{dt} = a$, where a is a constant to be found.

(b) Work out $\int_3^5 y \frac{dx}{dt} dt$.

Solution:

(a) $x = 9t^{\frac{4}{3}}$, $y = t^{-\frac{1}{3}}$

$$\frac{dx}{dt} = 9 \times \frac{4}{3} t^{\frac{4}{3} - 1} = 9 \times \frac{4}{3} t^{\frac{1}{3}} = 12t^{\frac{1}{3}}$$

$$\text{So } y \frac{dx}{dt} = t^{-\frac{1}{3}} \times 12t^{\frac{1}{3}} = 12t^{-\frac{1}{3} + \frac{1}{3}} = 12t^0 = 12$$

$$\text{So } a = 12$$

$$(b) \int_3^5 y \frac{dx}{dt} dt = \int_3^5 12 dt = \left[12t \right]_3^5 = 12 \left(5 \right) - 12 \left(3 \right) = 24$$

Solutionbank

Edexcel AS and A Level Modular Mathematics

Coordinate geometry in the (x, y) plane
Exercise D, Question 7

Question:

A curve has parametric equations $x = \sqrt{t}$, $y = 4\sqrt{t^3}$, $t > 0$.

(a) Show that $y \frac{dx}{dt} = pt$, where p is a constant to be found.

(b) Work out $\int_1^6 y \frac{dx}{dt} dt$.

Solution:

(a) $x = \sqrt{t}$, $y = 4\sqrt{t^3}$

$$x = t^{\frac{1}{2}}$$

$$\frac{dx}{dt} = \frac{1}{2}t^{\frac{1}{2}-1} = \frac{1}{2}t^{-\frac{1}{2}}$$

$$y \frac{dx}{dt} = 4\sqrt{t^3} \times \frac{1}{2}t^{-\frac{1}{2}}$$

$$= 4t^{\frac{3}{2}} \times \frac{1}{2}t^{-\frac{1}{2}}$$

$$= 2t^{\frac{3}{2}-\frac{1}{2}}$$

$$= 2t^1$$

$$= 2t$$

$$\text{So } p = 2$$

(b) $\int_1^6 y \frac{dx}{dt} dt = \int_1^6 2t dt = [t^2]_1^6 = (6)^2 - (1)^2 = 35$

Solutionbank

Edexcel AS and A Level Modular Mathematics

Coordinate geometry in the (x, y) plane
Exercise D, Question 8

Question:

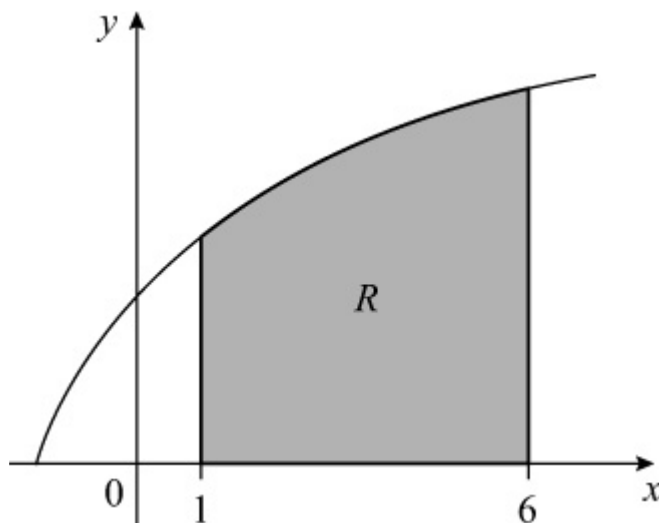
The diagram shows a sketch of the curve with parametric equations $x = t^2 - 3$, $y = 3t$, $t > 0$. The shaded region R is bounded by the curve, the x -axis and the lines $x = 1$ and $x = 6$.

(a) Find the value of t when

(i) $x = 1$

(ii) $x = 6$

(b) Find the area of R .



Solution:

(a) Substitute $x = 1$ into $x = t^2 - 3$

$$t^2 - 3 = 1$$

$$t^2 = 4$$

$$t = 2 \quad (\text{as } t > 0)$$

Substitute $x = 6$ into $x = t^2 - 3$

$$t^2 - 3 = 6$$

$$t^2 = 9$$

$$t = 3 \quad (\text{as } t > 0)$$

(b) $\int_1^6 y dx = \int_2^3 y \frac{dx}{dt} dt$

$$\frac{dx}{dt} = 2t$$

$$\text{So } y \frac{dx}{dt} = 3t \times 2t = 6t^2$$

$$\begin{aligned}\int_2^3 y \frac{dx}{dt} dt &= \int_2^3 6t^2 dt \\ &= [2t^3]_2^3 \\ &= 2(3)^3 - 2(2)^3 \\ &= 54 - 16 \\ &= 38\end{aligned}$$

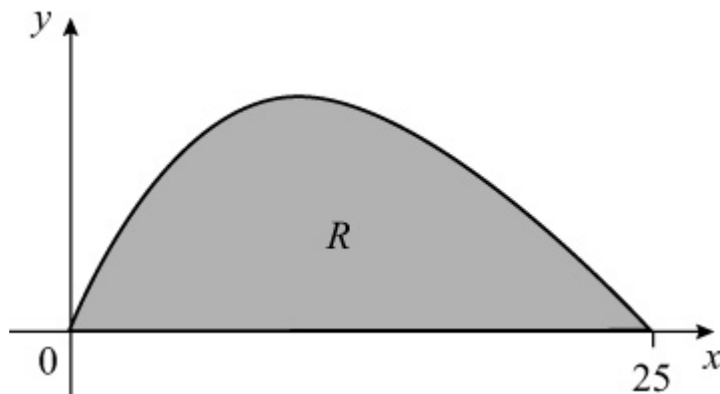
Solutionbank

Edexcel AS and A Level Modular Mathematics

Coordinate geometry in the (x, y) plane
Exercise D, Question 9

Question:

The diagram shows a sketch of the curve with parametric equations $x = 4t^2$, $y = t(5 - 2t)$, $t \geq 0$. The shaded region R is bounded by the curve and the x -axis. Find the area of R .



Solution:

When $x = 0$

$$4t^2 = 0$$

$$t^2 = 0$$

$$t = 0$$

When $x = 25$

$$4t^2 = 25$$

$$t^2 = \frac{25}{4}$$

$$t = \sqrt{\frac{25}{4}}$$

$$t = \frac{5}{2} \quad (\text{as } t \geq 0)$$

$$\text{So } \int_0^{25} y dx = \int_0^{\frac{5}{2}} \frac{5}{2} y \frac{dx}{dt} dt$$

$$\frac{dx}{dt} = 8t$$

$$\text{So } y \frac{dx}{dt} = t \left(5 - 2t \right) \times 8t = 8t^2 \left(5 - 2t \right) = 40t^2 - 16t^3$$

$$\begin{aligned}\int_0^{\frac{5}{2}} y \frac{dx}{dt} dt &= \int_0^{\frac{5}{2}} 40t^2 - 16t^3 dt \\ &= \left[\frac{40}{3} t^3 - 4t^4 \right]_0^{\frac{5}{2}} \\ &= \left[\frac{40}{3} \left(\frac{5}{2} \right)^3 - 4 \left(\frac{5}{2} \right)^4 \right] - \left[\frac{40}{3} (0)^3 - 4(0)^4 \right] \\ &= 52 \frac{1}{12} - 0 \\ &= 52 \frac{1}{12}\end{aligned}$$

Solutionbank

Edexcel AS and A Level Modular Mathematics

Coordinate geometry in the (x, y) plane
Exercise D, Question 10

Question:

The region R is bounded by the curve with parametric equations $x = t^3$, $y = \frac{1}{3t^2}$, the x -axis and the lines $x = -1$ and $x = -8$.

(a) Find the value of t when

- (i) $x = -1$
- (ii) $x = -8$

(b) Find the area of R .

Solution:

(a) (i) Substitute $x = -1$ into $x = t^3$

$$\begin{aligned} t^3 &= -1 \\ t &= \sqrt[3]{-1} \\ t &= -1 \end{aligned}$$

(ii) Substitute $x = -8$ into $x = t^3$

$$\begin{aligned} t^3 &= -8 \\ t &= \sqrt[3]{-8} \\ t &= -2 \end{aligned}$$

(b) $R = \int_{-8}^{-1} y dx = \int_{-2}^{-1} y \frac{dx}{dt} dt$

$$\frac{dx}{dt} = 3t^2$$

$$\text{So } y \frac{dx}{dt} = \frac{1}{3t^2} \times 3t^2 = 1$$

$$\int_{-2}^{-1} y \frac{dx}{dt} dt = \int_{-2}^{-1} 1 dt = \left[t \right]_{-2}^{-1} = \begin{pmatrix} -1 \\ \end{pmatrix} - \begin{pmatrix} -2 \\ \end{pmatrix}$$

$$= -1 + 2 = 1$$

Solutionbank

Edexcel AS and A Level Modular Mathematics

Coordinate geometry in the (x, y) plane

Exercise E, Question 1

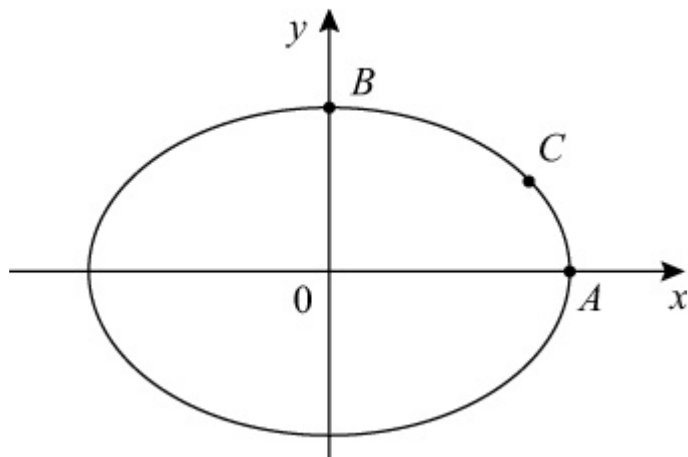
Question:

The diagram shows a sketch of the curve with parametric equations $x = 4 \cos t$, $y = 3 \sin t$, $0 \leq t < 2\pi$.

(a) Find the coordinates of the points A and B.

(b) The point C has parameter $t = \frac{\pi}{6}$. Find the exact coordinates of C.

(c) Find the cartesian equation of the curve.



Solution:

(a) (1) At A, $y = 0 \Rightarrow 3 \sin t = 0 \Rightarrow \sin t = 0$

So $t = 0$ and $t = \pi$

Substitute $t = 0$ and $t = \pi$ into $x = 4 \cos t$

$$t = 0 \Rightarrow x = 4 \cos(0) = 4 \times 1 = 4$$

$$t = \pi \Rightarrow x = 4 \cos \pi = 4 \times (-1) = -4$$

So the coordinates of A are (4, 0).

(2) At B, $x = 0 \Rightarrow 4 \cos t = 0 \Rightarrow \cos t = 0$

$$\text{So } t = \frac{\pi}{2} \text{ and } t = \frac{3\pi}{2}$$

Substitute $t = \frac{\pi}{2}$ and $t = \frac{3\pi}{2}$ into $y = 3 \sin t$

$$t = \frac{\pi}{2} \Rightarrow y = 3 \sin \left(\frac{\pi}{2} \right) = 3 \times 1 = 3$$

$$t = \frac{3\pi}{2} \Rightarrow y = 3 \sin \left(\frac{3\pi}{2} \right) = 3 \times -1 = -3$$

So the coordinates of B are $(0, 3)$

(b) Substitute $t = \frac{\pi}{6}$ into $x = 4 \cos t$ and $y = 3 \sin t$

$$x = 4 \cos \left(\frac{\pi}{6} \right) = 4 \times \frac{\sqrt{3}}{2} = 2\sqrt{3}$$

$$y = 3 \sin \left(\frac{\pi}{6} \right) = 3 \times \frac{1}{2} = \frac{3}{2}$$

So the coordinates of C are $\left(2\sqrt{3}, \frac{3}{2} \right)$

(c) $x = 4 \cos t$, $y = 3 \sin t$

$$\cos t = \frac{x}{4} \text{ and } \sin t = \frac{y}{3}$$

As $\sin^2 t + \cos^2 t = 1$

$$\left(\frac{y}{3} \right)^2 + \left(\frac{x}{4} \right)^2 = 1 \quad \text{or} \quad \left(\frac{x}{4} \right)^2 + \left(\frac{y}{3} \right)^2 = 1$$

Solutionbank

Edexcel AS and A Level Modular Mathematics

Coordinate geometry in the (x, y) plane
Exercise E, Question 2

Question:

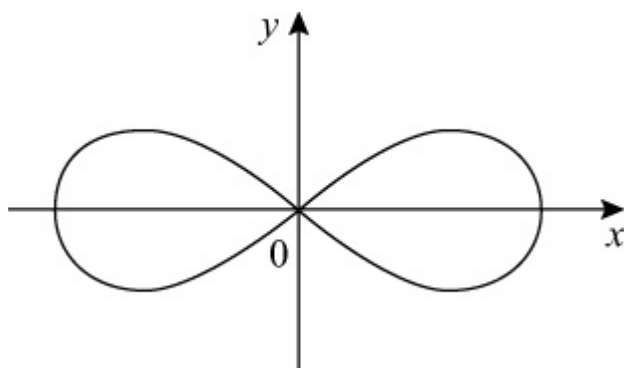
The diagram shows a sketch of the curve with parametric equations $x = \cos t$,

$$y = \frac{1}{2} \sin 2t.$$

$0 \leq t < 2\pi$. The curve is symmetrical about both axes.

(a) Copy the diagram and label the points having parameters $t = 0$, $t = \frac{\pi}{2}$, $t = \pi$ and $t = \frac{3\pi}{2}$.

(b) Show that the cartesian equation of the curve is $y^2 = x^2(1 - x^2)$.



Solution:

(a) (1) Substitute $t = 0$ into $x = \cos t$ and $y = \frac{1}{2} \sin 2t$

$$x = \cos 0 = 1$$

$$y = \frac{1}{2} \sin \left(2 \times 0 \right) = \frac{1}{2} \sin 0 = \frac{1}{2} \times 0 = 0$$

So when $t = 0$, $(x, y) = (1, 0)$

(2) Substitute $t = \frac{\pi}{2}$ into $x = \cos t$ and $y = \frac{1}{2} \sin 2t$

$$x = \cos \frac{\pi}{2} = 0$$

$$y = \frac{1}{2} \sin \left(2 \times \frac{\pi}{2} \right) = \frac{1}{2} \sin \pi = \frac{1}{2} \times 0 = 0$$

So when $t = \frac{\pi}{2}$, $(x, y) = (0, 0)$

(3) Substitute $t = \pi$ into $x = \cos t$ and $y = \frac{1}{2} \sin 2t$

$$x = \cos \pi = -1$$

$$y = \frac{1}{2} \sin \left(2\pi \right) = \frac{1}{2} \times 0 = 0$$

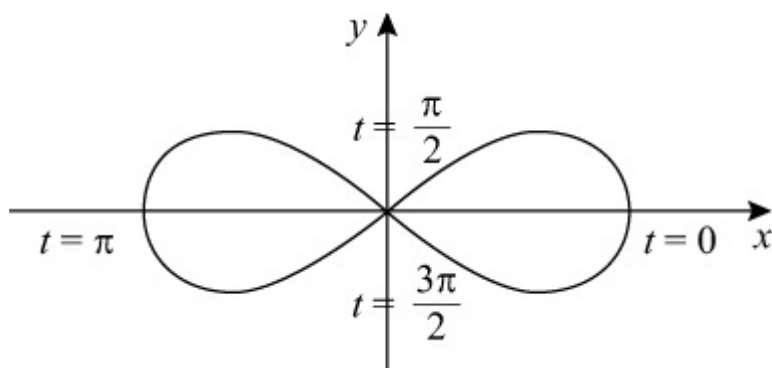
So when $t = \pi$, $(x, y) = (-1, 0)$

(4) Substitute $t = \frac{3\pi}{2}$ into $x = \cos t$ and $y = \frac{1}{2} \sin 2t$

$$x = \cos \frac{3\pi}{2} = 0$$

$$y = \frac{1}{2} \sin \left(2 \times \frac{3\pi}{2} \right) = \frac{1}{2} \sin \left(3\pi \right) = \frac{1}{2} \times 0 = 0$$

So when $t = \frac{3\pi}{2}$, $(x, y) = (0, 0)$



(b) $y = \frac{1}{2} \sin 2t = \frac{1}{2} \times 2 \sin t \cos t = \sin t \cos t$

As $x = \cos t$

$$y = \sin t \times x$$

$$y = x \sin t$$

Now $\sin^2 t + \cos^2 t = 1$

So $\sin^2 t + x^2 = 1$

$$\Rightarrow \sin^2 t = 1 - x^2$$

$$\Rightarrow \sin t = \sqrt{1 - x^2}$$

So $y = x\sqrt{1 - x^2}$ or $y^2 = x^2(1 - x^2)$

Solutionbank

Edexcel AS and A Level Modular Mathematics

Coordinate geometry in the (x, y) plane
Exercise E, Question 3

Question:

A curve has parametric equations $x = \sin t$, $y = \cos 2t$, $0 \leq t < 2\pi$.

(a) Find the cartesian equation of the curve.
The curve cuts the x -axis at $(a, 0)$ and $(b, 0)$.

(b) Find the value of a and b .

Solution:

(a) $x = \sin t$, $y = \cos 2t$

$$\text{As } \cos 2t = 1 - 2 \sin^2 t$$

$$y = 1 - 2x^2$$

(b) Substitute $y = 0$ into $y = 1 - 2x^2$

$$0 = 1 - 2x^2$$

$$2x^2 = 1$$

$$x^2 = \frac{1}{2}$$

$$x = \pm \sqrt{\frac{1}{2}} = \pm \frac{\sqrt{1}}{\sqrt{2}} = \pm \frac{1}{\sqrt{2}}$$

$$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

So the curve meets the x -axis at $\left(\frac{\sqrt{2}}{2}, 0 \right)$ and $\left(-\frac{\sqrt{2}}{2}, 0 \right)$

Solutionbank

Edexcel AS and A Level Modular Mathematics

Coordinate geometry in the (x, y) plane
Exercise E, Question 4

Question:

A curve has parametric equations $x = \frac{1}{1+t}$, $y = \frac{1}{(1+t)(1-t)}$, $t \neq \pm 1$.

Express t in terms of x . Hence show that the cartesian equation of the curve is

$$y = \frac{x^2}{2x-1}.$$

Solution:

$$(1) x = \frac{1}{1+t}$$

$$x \times (1+t) = \frac{1}{(1+t)} \times (1+t) \quad \text{Multiply each side by } (1+t)$$

$$x(1+t) = 1 \quad \text{Simplify}$$

$$\frac{x(1+t)}{x} = \frac{1}{x} \quad \text{Divide each side by } x$$

$$1+t = \frac{1}{x} \quad \text{Simplify}$$

$$\text{So } t = \frac{1}{x} - 1$$

$$\text{Substitute } t = \frac{1}{x} - 1 \text{ into } y = \frac{1}{(1+t)(1-t)}$$

$$y = \frac{1}{\left(1 + \frac{1}{x} - 1\right) \left[1 - \left(\frac{1}{x} - 1\right)\right]}$$

$$= \frac{1}{\frac{1}{x} \left(1 - \frac{1}{x} + 1\right)}$$

$$= \frac{1}{\frac{1}{x} \left(2 - \frac{1}{x}\right)}$$

$$= \frac{1}{\frac{1}{x} \left(\frac{2x}{x} - \frac{1}{x} \right)}$$

$$= \frac{1}{\frac{1}{x} \left(\frac{2x-1}{x} \right)}$$

$$= \frac{1}{\left(\frac{2x-1}{x^2} \right)}$$

$$= \frac{x^2}{2x-1} \quad \left(\text{Remember } \frac{1}{\left(\frac{a}{b} \right)} = \frac{b}{a} \right)$$

So the cartesian equation of the curve is $y = \frac{x^2}{2x-1}$.

Solutionbank

Edexcel AS and A Level Modular Mathematics

Coordinate geometry in the (x, y) plane
Exercise E, Question 5

Question:

A circle has parametric equations $x = 4 \sin t - 3$, $y = 4 \cos t + 5$.

- (a) Find the cartesian equation of the circle.
- (b) Draw a sketch of the circle.
- (c) Find the exact coordinates of the points of intersection of the circle with the y-axis.

Solution:

(a) $x = 4 \sin t - 3$, $y = 4 \cos t + 5$

$$4 \sin t = x + 3$$

$$\sin t = \frac{x+3}{4}$$

and

$$4 \cos t = y - 5$$

$$\cos t = \frac{y-5}{4}$$

As $\sin^2 t + \cos^2 t = 1$

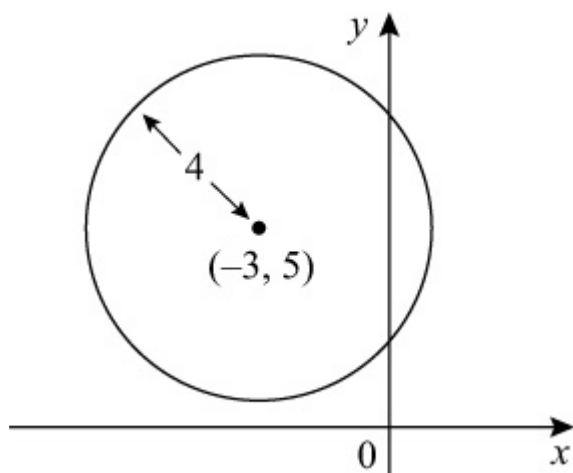
$$\left(\frac{x+3}{4} \right)^2 + \left(\frac{y-5}{4} \right)^2 = 1$$

$$\frac{(x+3)^2}{4^2} + \frac{(y-5)^2}{4^2} = 1$$

$$\frac{(x+3)^2}{4^2} \times 4^2 + \frac{(y-5)^2}{4^2} \times 4^2 = 1 \times 4^2$$

$$(x+3)^2 + (y-5)^2 = 4^2 \quad \text{or} \quad (x+3)^2 + (y-5)^2 = 16$$

- (b) The circle $(x+3)^2 + (y-5)^2 = 4^2$ has centre $(-3, 5)$ and radius 4.



(c) Substitute $x = 0$ into $(x + 3)^2 + (y - 5)^2 = 4^2$

$$(0 + 3)^2 + (y - 5)^2 = 4^2$$

$$3^2 + (y - 5)^2 = 4^2$$

$$9 + (y - 5)^2 = 16$$

$$(y - 5)^2 = 7$$

$$y - 5 = \pm\sqrt{7}$$

$$y = 5 \pm \sqrt{7}$$

So the circle meets the y-axis at $(0, 5 + \sqrt{7})$ and $(0, 5 - \sqrt{7})$.

Solutionbank

Edexcel AS and A Level Modular Mathematics

Coordinate geometry in the (x, y) plane

Exercise E, Question 6

Question:

Find the cartesian equation of the line with parametric equations $x = \frac{2-3t}{1+t}$, $y = \frac{3+2t}{1+t}$, $t \neq -1$.

Solution:

$$x = \frac{2-3t}{1+t}$$

$$x(1+t) = \frac{2-3t}{(1+t)} \times (1+t)$$

$$x(1+t) = 2-3t$$

$$x + xt = 2 - 3t$$

$$x + xt + 3t = 2$$

$$xt + 3t = 2 - x$$

$$t(x+3) = 2-x$$

$$t \frac{(x+3)}{(x+3)} = \frac{2-x}{x+3}$$

$$t = \frac{2-x}{x+3}$$

Substitute $t = \frac{2-x}{x+3}$ into $y = \frac{3+2t}{1+t}$

$$y = \frac{3+2\left(\frac{2-x}{x+3}\right)}{1+\left(\frac{2-x}{x+3}\right)}$$

$$= \frac{3+2\left(\frac{2-x}{x+3}\right)}{1+\left(\frac{2-x}{x+3}\right)} \times \frac{(x+3)}{(x+3)}$$

$$\begin{aligned} & 3 \times (x+3) + 2 \left(\frac{2-x}{x+3} \right) \times (x+3) \\ = & \frac{3 \times (x+3) + 2 \left(\frac{2-x}{x+3} \right) \times (x+3)}{1 \times (x+3) + \left(\frac{2-x}{x+3} \right) \times (x+3)} \\ = & \frac{3(x+3) + 2(2-x)}{(x+3) + (2-x)} \\ = & \frac{3x+9+4-2x}{x+3+2-x} \\ = & \frac{x+13}{5} \end{aligned}$$

$$\text{So } y = \frac{x}{5} + \frac{13}{5}$$

Solutionbank

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Coordinate geometry in the (x, y) plane
Exercise E, Question 7

Question:

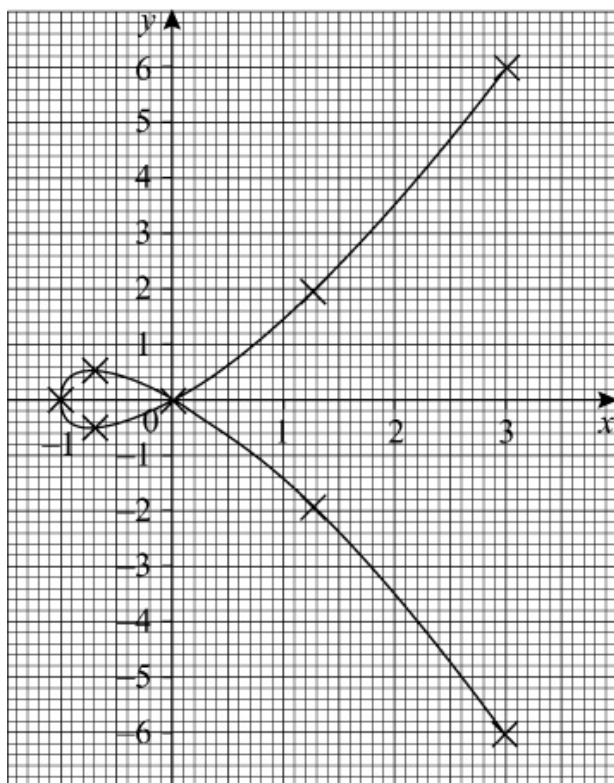
A curve has parametric equations $x = t^2 - 1$, $y = t - t^3$, where t is a parameter.

(a) Draw a graph of the curve for $-2 \leq t \leq 2$.

(b) Find the area of the finite region enclosed by the loop of the curve.

Solution:

t	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2
$x = t^2 - 1$	3	1.25	0	-0.75	-1	-0.75	0	1.25	3
$y = t - t^3$	6	1.875	0	-0.375	0	0.375	0	-1.875	-6



$$(b) A = 2 \int_{-1}^0 y dx = 2 \int_0^1 y \frac{dx}{dt} dt, \text{ When } x = -1, t^2 - 1 = -1, \text{ So } t = 0$$

$$\text{When } x = 0, t^2 - 1 = 0, \text{ So } t = 1$$

$$\frac{dx}{dt} = 2t$$

$$\text{So } y \frac{dx}{dt} = (t - t^3) \times 2t = 2t^2 - 2t^4$$

$$\text{Therefore } A = 2 \int_0^1 2t^2 - 2t^4 dt$$

$$\begin{aligned} &= 2 \left[\frac{2}{3}t^3 - \frac{2}{5}t^5 \right]_0^1 \\ &= 2 \left(\left[\frac{2}{3}(1)^3 - \frac{2}{5}(1)^5 \right] - \left[\frac{2}{3}(0)^3 - \frac{2}{5}(0)^5 \right] \right) \\ &= 2 \left[\left(\frac{2}{3} - \frac{2}{5} \right) - 0 \right] \\ &= 2 \times \frac{4}{15} \\ &= \frac{8}{15} \end{aligned}$$

So the area of the loop is $\frac{8}{15}$.

Solutionbank

Edexcel AS and A Level Modular Mathematics

Coordinate geometry in the (x, y) plane
Exercise E, Question 8

Question:

A curve has parametric equations $x = t^2 - 2$, $y = 2t$, where $-2 \leq t \leq 2$.

(a) Draw a graph of the curve.

(b) Indicate on your graph where

(i) $t = 0$

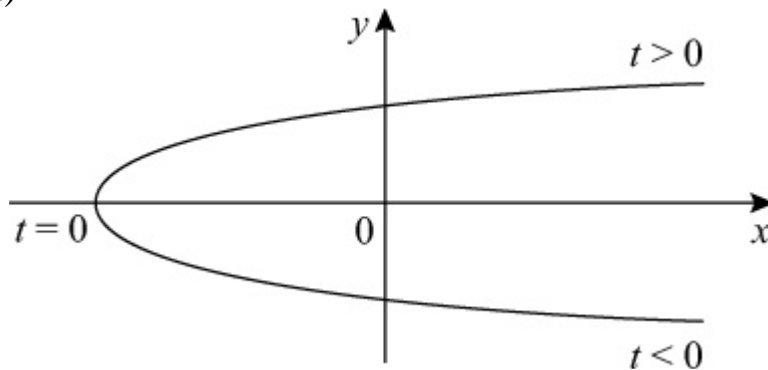
(ii) $t > 0$

(iii) $t < 0$

(c) Calculate the area of the finite region enclosed by the curve and the y-axis.

Solution:

(a)



(b) (i) When $t = 0$, $y = 2(0) = 0$.

This is where the curve meets the x -axis.

(ii) When $t > 0$, $y > 0$.

This is where the curve is above the x -axis.

(iii) When $t < 0$, $y < 0$.

This is where the curve is below the x -axis.

(c) $A = 2 \int_{-2}^0 y dx = 2 \int_0^{\sqrt{2}} y \frac{dx}{dt} dt$, When $x = -2$, $t^2 - 2 = -2$, so $t = 0$

When $x = 0$, $t^2 - 2 = 0$, so $t = \sqrt{2}$

$$\frac{dx}{dt} = 2t$$

$$\text{So } y \frac{dx}{dt} = 2t \times 2t = 4t^2$$

$$\begin{aligned} \text{Therefore } A &= 2 \int_0^{\sqrt{2}} 4t^2 dt \\ &= 2 \left[\frac{4}{3} t^3 \right]_0^{\sqrt{2}} \\ &= 2 \left[\frac{4}{3} (\sqrt{2})^3 - \frac{4}{3} (0)^3 \right] \\ &= 2 \times \frac{4}{3} (\sqrt{2})^3 \\ &= \frac{8}{3} (\sqrt{2})^3 \\ &= \frac{16}{3} \sqrt{2}, \quad \text{As } (\sqrt{2})^3 = (\sqrt{2} \times \sqrt{2}) \times \sqrt{2} = 2\sqrt{2} \end{aligned}$$

Solutionbank

Edexcel AS and A Level Modular Mathematics

Coordinate geometry in the (x, y) plane
Exercise E, Question 9

Question:

Find the area of the finite region bounded by the curve with parametric equations $x = t^3$, $y = \frac{4}{t}$, $t \neq 0$, the x -axis and the lines $x = 1$ and $x = 8$.

Solution:

(1) When $x = 1$, $t^3 = 1$, so $t = \sqrt[3]{1} = 1$

When $x = 8$, $t^3 = 8$, so $t = \sqrt[3]{8} = 2$

$$(2) A = \int_1^8 y dx = \int_1^2 y \frac{dx}{dt} dt$$

$$(3) \frac{dx}{dt} = 3t^2$$

$$\text{So } y \frac{dx}{dt} = \frac{4}{t} \times 3t^2 = 12t$$

$$\begin{aligned} \text{Therefore } A &= \int_1^2 12t dt \\ &= [6t^2]_1^2 \\ &= 6(2)^2 - 6(1)^2 \\ &= 24 - 6 \\ &= 18 \end{aligned}$$

Solutionbank

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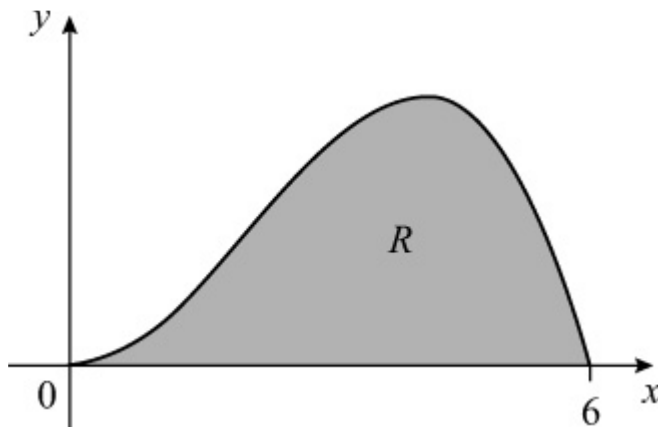
Coordinate geometry in the (x, y) plane
Exercise E, Question 10

Question:

The diagram shows a sketch of the curve with parametric equations $x = 3\sqrt[3]{t}$, $y = t(4 - t)$, where $0 \leq t \leq 4$. The region R is bounded by the curve and the x -axis.

(a) Show that $y \frac{dx}{dt} = 6t^{\frac{1}{2}} - \frac{3}{2}t^{\frac{3}{2}}$.

(b) Find the area of R .



Solution:

(a) $x = 3\sqrt[3]{t} = 3t^{\frac{1}{3}}$

$$\frac{dx}{dt} = \frac{1}{3} \times 3t^{\frac{1}{3}-1} = \frac{1}{3}t^{-\frac{2}{3}}$$

$$y \frac{dx}{dt} = t(4 - t) \times \frac{1}{3}t^{-\frac{2}{3}}$$

$$= (4t - t^2) \times \frac{1}{3}t^{-\frac{2}{3}}$$

$$= 4t \times \frac{1}{3}t^{-\frac{2}{3}} - t^2 \times \frac{1}{3}t^{-\frac{2}{3}}$$

$$= 4t^{1-\frac{2}{3}} - \frac{1}{3}t^{2-\frac{2}{3}}$$

$$= 4t^{\frac{1}{3}} - \frac{1}{3}t^{\frac{4}{3}}$$

$$\begin{aligned}
 \text{(b) } A &= \int_0^4 y \frac{dx}{dt} dt \\
 &= \int_0^4 6t^{\frac{1}{2}} - \frac{3}{2}t^{\frac{3}{2}} dt \\
 &= \left[\frac{6t^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} - \frac{\frac{3}{2}t^{\frac{5}{2}}}{\left(\frac{5}{2}\right)} \right]_0^4 \\
 &= \left[4t^{\frac{3}{2}} - \frac{3}{5}t^{\frac{5}{2}} \right]_0^4 \\
 &= \left[4(4)^{\frac{3}{2}} - \frac{3}{5}(4)^{\frac{5}{2}} \right] - \left[4(0)^{\frac{3}{2}} - \frac{3}{5}(0)^{\frac{5}{2}} \right] \\
 &= \left(4 \times 8 - \frac{3}{5} \times 32 \right) - 0 \\
 &= 32 - 19\frac{1}{5} \\
 &= 12\frac{4}{5}
 \end{aligned}$$