Math 2371 Calc III - Sample Test 3 - 2017
Version 2

1. Is the following vector field conservative?

$$\vec{F} =< 2xy + 3z^3, x^2 + 1, 9xz^2 > .$$

If so, determine the potential function f such that $\vec{F} = \vec{\nabla} f$ Use this to evaluate

$$\int_C (2xy + 3z^3)\, dx + (x^2 + 1)\, dy + 9xz^2\, dz$$

where C is any curve joining $(0, 0, 0)$ and $(1, -1, 1)$.

2. Evaluate the following line integral $\int_c x\, y\, ds$ where c is ccw direction around the circle $x^2 + y^2 = 4$ from $(0, 2)$ to $(-2, 0)$.

3. Evaluate the following line integral $\int_c dx + (x + y)\, dy$ where c is the curve $y = x^2$ from $(0, 0)$ to $(1, 1)$.

4. Green's Theorem is

$$\int_C P\, dx + Q\, dy = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA.$$

Verify Green's Theorem where $\vec{F} =< x^2 + 2y, x^2 >$ where R is the region bound by the curves $y = 0$, $x = 1$, and $y = x$ in $Q1$.

5. Evaluate $\iint_S z\, dS$ where S is the surface of the plane $2x + 2y + z = 2$ in Q1.

6. Find the flux $\iint_S \vec{F} \cdot \vec{n} ds$ through the surface of the plane $y + z = 1$, for $0 \leq x \leq 1$ and $0 \leq y \leq 1$ if the vector field is given by $\vec{F} =< y, z, z^2 >$.