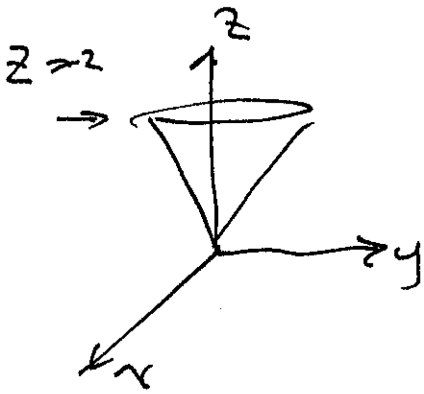
Math 2471 - Calc 3

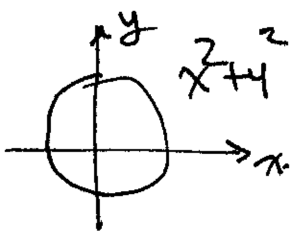
Double Integrals - Polar Coords

Ex 1 Volume of the cone $z=\sqrt{x^2+y^2}$ with top $z=2$



$$\iint_R \left(2-\sqrt{x^2+y^2}\right)dA$$

$$\int_{\theta=0}^{2\pi}\int_{r=0}^{2}(2-r)\,r\,dr\,d\theta$$



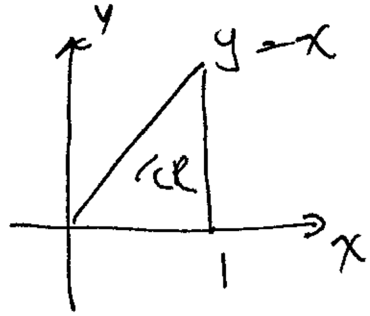
$x^2+y^2=4$

$$\int_0^{2\pi}\left(r^2-\frac{r^3}{3}\Big|_0^2\right)d\theta$$

$$=\int_0^{2\pi}\left(4-\frac{8}{3}\right)d\theta = \frac{4}{3}\int_0^{2\pi}d\theta = \frac{4}{3}\cdot\theta\Big|_0^{2\pi}$$

$$=\frac{8\pi}{3}$$

Ex 2 $\displaystyle\iint_R \frac{dA}{\sqrt{x^2+y^2}}$ $\quad R: \{(x,y) \mid 0 \le y \le x \le 1\}$



so in Cartesian coords

$$\int_0^1 \int_0^x \frac{dy\,dx}{\sqrt{x^2+y^2}}$$ (it's improper but will have a limit)

Switch to polar

Now $y=x$ correspond to $\theta = \pi/4$ so $\theta : 0 \to \pi/4$

Now $r : 0 \to$ line $x=1$ $\quad x = r\cos\theta = 1 \Rightarrow r = \frac{1}{\cos\theta}$

$$\int_0^{\pi/4} \int_0^{\frac{1}{\cos\theta}} \frac{\cancel{r}\,dr\,d\theta}{\cancel{r}} = \int_0^{\pi/4} r\Big|_0^{\sec\theta} d\theta = \int_0^{\pi/4} \sec\theta\,d\theta$$

$$= \ln|\sec\theta + \tan\theta|\Big|_0^{\pi/4}$$

$$= \ln|\sec\pi/4 + 1| - 0$$

$$= \ln(\sqrt{2}+1)$$