Math 1497—Calc 2

Test #4 Direct Comparison Test (DCT)

if $0 < a_n \leq b_n$

if $\sum a_n$ diverges $\sum b_n$ diverges

if $\sum b_n$ converges $\sum a_n$ converges

Ex! $\sum_{n=1}^{\infty} \frac{1}{1+n^2}$ (again)

which converges (by $\int$ test & LCT w/ $\sum \frac{1}{n^2}$)

Now if we thought it would converge

$0 < \frac{1}{1+n^2} < ?$ need to come up with this

If we thought it would diverge

$0 < ? < \frac{1}{1+n^2}$

come up with this

go with the first

$$\frac{1}{1+n^2} \overset{?}{<} \frac{1}{n^2}$$

so $n^2 < n^2+1$ a $0<1$ obvious!

So $\sum_{n=1}^{\infty} \frac{1}{1+n^2} < \sum_{n=1}^{\infty} \frac{1}{n^2}$ "it's easy"

$\therefore \sum \frac{1}{n^2}$ conv $(p=2)$ by DCT our series conv.

Ex 2 $\sum_{n=1}^{\infty} \frac{1}{2^n+n}$ Note: 2^n gets bigger faster than n

think conv.

$$\frac{1}{2^n+n} \overset{?}{<} \frac{1}{2^n}$$

$2^n < 2^n+n$ a $0<n$ ✓

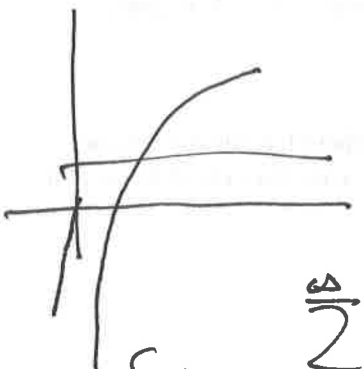
so $\frac{1}{2^n+n} < \frac{1}{2^n}$

Since $\sum \frac{1}{2^n}$ converges geometric w/ $r = 1/2$

then by DCT our series converges

Ex 3 $\displaystyle\sum_{n=3}^{\infty} \frac{\ln n}{n}$ Can use $\int$ test

$1 < \ln n$ for $n > e = 2.7$



so $\frac{1}{n} < \frac{\ln n}{n}$

So $\displaystyle\sum_{n=3}^{\infty} \frac{1}{n} < \sum_{n=3}^{\infty} \frac{\ln n}{n}$

this diverges so by DCT diverges

Ex 4 $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n^2 - \sin^2 n}$

when n gets large $\frac{n}{n^2 - \sin^2 n} \approx \frac{n}{n^2} = \frac{1}{n}$

for DCT

$$\frac{1}{n} \overset{?}{<} \frac{n}{n^2 - \sin^2 n}$$

$$n^2 - \sin^2 n \cdot n \overset{?}{<} n^2$$

$$-n\sin^2 n < 0 \text{ yes}$$

so $\sum_{n=1}^{\infty} \frac{1}{n} < \sum_{n=1}^{\infty} \frac{n}{n^2 - \sin^2 n}$

since this diverges by DCT our series diverges

Ex5 $\sum_{n=1}^{\infty} \frac{1}{n!}$ we will introduce another ~~test~~ for this next class

1st - do we think it will converge

$$0 < \frac{1}{n!} < ?$$

or diverges

$$0 < ? < \frac{1}{n!}$$

the terms are getting small - fast!

15
5

So I think converges. So come up with b_n

when

$$0 < \frac{1}{n!} < b_n \quad \text{where } \sum b_n \text{ conv}$$

Here we think of our special series

(1) geometric

(2) p series

$$\frac{1}{n!} = \frac{1}{1\cdot 2\cdot 3\cdot 4\cdots} < \frac{1}{1\cdot 2\cdot 2\cdot 2} = \frac{1}{2^3}$$

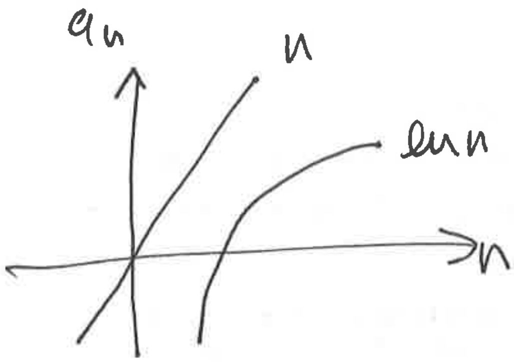
$$\frac{1}{1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6} < \frac{1}{1\cdot 2\cdot 2\cdot 2\cdot 2\cdot 2} = \frac{1}{2^5}$$

So $\frac{1}{n!} < \frac{1}{2^{n-1}}$

So $\sum_{n=1}^{\infty} \frac{1}{n!} < \sum_{n=1}^{\infty} \frac{1}{2^{n-1}} \leftarrow$ this conv $r = 1/2$

So by DCT our series converges.

Ex $\sum_{n=2}^{\infty} \frac{1}{\ln n}$ terms are going to zero but not as slow as $\frac{1}{n}$



$$\frac{1}{n} \overset{?}{<} \frac{1}{\ln n}$$

$\ln n \overset{?}{<} n$ ✓ (see picture)

$\therefore \sum_{n=2}^{\infty} \frac{1}{n} < \sum_{n=2}^{\infty} \frac{1}{\ln n}$ series diverges by DCT

Ex $\sum_{n=2}^{\infty} \frac{1}{n+1}$ LCT with $\sum \frac{1}{n}$ shows div

Also Harmonic series with $1, \frac{1}{2}$ missing — still diverge

then

$$\frac{1}{n} \overset{?}{<} \frac{1}{n+1}$$

but $n+1 \overset{?}{<} n$

$1 \overset{?}{<} 0$ which is not true

so here a DCT with $\sum \frac{1}{n}$ doesn't work!