Math 3331 - ODE Lect 16

ODE $\quad ay'' + by' + cy = 0$

CE. $\quad am^2 + bm + c = 0$

Suppose we have complex roots

$$m = \alpha \pm \beta i$$

so $\quad y = K_1 e^{(\alpha+\beta i)x} + K_2 e^{(\alpha-\beta i)x}$

$$= e^{\alpha x}\left(K_1 e^{\beta i x} + K_2 e^{-\beta i x}\right)$$

Euler's Formula

$$e^{i\theta} = \cos\theta + i\sin\theta$$

so $\quad y = e^{\alpha x}\begin{bmatrix} K_1(\cos\beta x + i\sin\beta x) \\ K_2(\cos\beta x - i\sin\beta x) \end{bmatrix}$

$$y = e^{\alpha x}\{(K_1+K_2)\cos\beta x + i(K_1-K_2)\sin\beta x\}$$

let $c_1 = K_1+K_2, \quad c_2 = i(K_1-K_2)$

G.S.

$$y = c_1 e^{\alpha x}\cos\beta x + c_2 e^{\alpha x}\sin\beta x.$$

Ex Solve

$$y'' + 4y' + 5y = 0$$

$$m^2 + 4m + 5 = 0$$

$$m = \frac{-4 \pm \sqrt{16-20}}{2} = \frac{-4 \pm 2i}{2} = -2 \pm i$$

$$\alpha = -2, \ \beta = 1$$

G.S. $y = c_1 e^{-2x}\cos x + c_2 e^{-2x}\sin x$

Ex 2

$$y'' + 4y = 0$$

$$m^2 + 4 = 0 \qquad m = \pm 3i$$

$$\alpha = 0 \qquad \beta = 3$$

$$y = c_1 \cos 3x + c_2 \sin 3x$$

Ex 3 Soln

$$y'' + 2y' + 17y = 0 \qquad y(0) = 0, \quad y'(0) = -8$$

$$m^2 + 2m + 17 = 0$$

$$m = \frac{2 \pm \sqrt{4 - 68}}{2} = \frac{2 \pm 8i}{2} = 1 \pm 4i$$

$$y = c_1 e^{x} \cos 4x + c_2 e^{x} \sin 4x$$

$$y(0) = c_1 = 0 \qquad \text{so} \qquad y = c_2 e^{x} \sin 4x$$

$$y' = c_2 e^{x} \sin 4x + 4 c_2 e^{x} \cos 4x$$

$$y'(0) = 4c_2 = -8 \Rightarrow \quad c_2 = -2$$

$$y = -2e^{x} \sin 4x.$$