The Autocrat's Commitment Problem with Endogenous Windows of Opportunity

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Abstract

Why does conflict occur? Why do rulers share power with the opposition? A common explanation is the autocrat's commitment problem; the opposition can extract concessions only during periodic windows of opportunities to revolt. I replace the standard assumption that opportunities arise exogenously. Instead, in each period, the opposition chooses whether to pay a (time-varying) cost to mobilize an anti-regime threat. Modeling endogenous windows of opportunity yields three new findings. First, strategic decisions that reduce the frequency of mobilization do not prompt greater demands by the opposition. Lower average mobilization costs perfectly offset less frequent consumption, which smooths out a key friction in existing models. Second, fully endogenous mobilization eliminates the ruler's commitment problem. Temporary concessions secure peace unless peaceful bargaining is costlier than conflict. Third, costly mobilization can prompt the ruler either to voluntarily share more power than needed to buy off a revolt—or instead refuse to share any power.

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1 INTRODUCTION

Any autocrat faces a commitment problem. Whenever the opposition mobilizes a threat of revolt, the ruler tries to buy them off by offering temporary spoils and policy concessions (e.g., subsidies, higher wages, government jobs). If the opposition posed a constant threat of revolt, there would be no commitment problem—the ruler could respond to the constant threat by making continual concessions. The commitment problem, then, arises specifically because of the standard assumption that windows of opportunity for a revolt cannot last indefinitely. This makes the opposition's position tenuous during its fleeting windows of opportunity. Although they can extract desired concessions today, they may be unable to do so tomorrow. The opposition is subject to the whims of the autocrat, who can temper or reverse temporary policy concessions after the window of opportunity ends. Beyond the autocrat's commitment problem specifically, commitment problems are a pervasive explanation for IR conflict and other outcomes.[1]

The standard logic yields three important implications. First, the lower the frequency of windows of opportunity, **the more the opposition demands**. Less frequent future concessions raise the opposition's opportunity cost of forgoing a revolt.

Second, if windows of opportunity arise very infrequently, then temporary concessions alone cannot buy off the opposition. Confronting a highly unfavorable shadow of the future under the incumbent regime, the opposition prefers a revolt over any amount of temporary concessions. Thus, **conflict** is one possible outcome of infrequent windows of opportunity.

Third, the costliness of conflict creates a preference for the ruler to avoid a revolt, if possible.

[1]Fearon (1995), Powell (2004, 2006), Spaniel (2023), and Little and Paine (2024) provide general theoretical statements about commitment problems and costly conflict. Acemoglu and Robinson (2000, 2001, 2006) develop this mechanism in a window-of-opportunity model with an option of institutional reform to explain the relationship between inequality and democratic transitions. Many scholars have applied variants of these models to distinct outcomes. For democratization, see Ansell and Samuels (2014); Leventoğlu (2014); Castañeda Dower et al. (2018). For authoritarian power sharing and democratic separation of powers, see Helmke (2017); Meng (2019); Paine (2022, 2024*b*); Christensen and Gibilisco (2024); Powell (2024). For civil conflict, see Fearon (2004); Chassang and Padro-i Miquel (2009); Walter (2009); Powell (2012); Gibilisco (2021). For international war, see Powell (1999); Chadefaux (2011); Debs and Monteiro (2014); Krainin (2017); Spaniel (2019). For power consolidation, see Fearon (1996); Powell (2013); Luo and Przeworski (2023); Luo (2023).

An alternative to purely *temporary* policy concessions is to offer *permanent* reforms to political institutions. This would facilitate concessions to the opposition in the future, even during times they cannot mobilize a threat of revolt. Such **power-sharing concessions** include expanding the franchise, conceding positions in the cabinet, holding legislative elections, integrating rebel groups and militias into the state military, and creating autonomous regions. The adverse consequences of loosening political control imply that, absent a credible threat of revolt, the ruler will not make power-sharing concessions. However, the ruler prefers to reform institutions than to incur a revolt.

Periodic windows of opportunity is the key friction that drives each of these three canonical results about commitment problems, conflict, and power sharing. But why, in the first place, does the opposition's anti-regime threat fluctuate over time? Most existing models sidestep this question: windows of opportunities to revolt are assumed to arise exogenously and are uncorrelated with other parameters. That is, in some periods, the opposition costlessly mobilizes a high threat against the government, whereas in other periods, it is infeasible (or restrictively costly) for the opposition to threaten the government. Exogenous, costless threats facilitates a tractable framework that corresponds with the intuitive notion that societal actors face difficulties to perpetually acting collectively (Acemoglu and Robinson 2006, 123–28). Nonetheless, a natural and largely unexplored question is—what if the opposition could strategically decide when to take a costly action to mobilize an anti-regime threat?[2]

In the real world, circumstances for opposition mobilization range between unambiguously "good" (no costs) or "bad" (prohibitive costs). This is a key consideration in the large literature on anti-regime protests. For example, incumbent presidents in electoral authoritarian regimes often claim victory in elections when the best evidence suggests otherwise. Knowledge of a tainted victory helps to lower the costs of collective action for opposition actors, who often engage in mass protests that seek to seat the leading opposition candidate and/or to implement more extensive pro-democratic reforms (Tucker 2007; Beaulieu 2014; Brancati 2016). Nonetheless, although this

[2]Some IR conflict models treat the distribution of power as endogenous, which I discuss below.

setting is favorable for mass anti-regime activity, the costs are not negligible. For example, in Belarus in 2020, the opposition mobilized for months in the capital of Minsk, at their height amassing more than one million participants. However, the incumbent Aleksandr Lukashenko was eventually able to muster sufficient force to suppress the movement (Way 2020).

Other stimulants to mass anti-regime mobilization, in contrast to planned elections, are highly spontaneous. For example, in December 2010, protests broke out across the Middle East in response to a street vendor in Tunisia who lit himself on fire. This action, and the subsequent cascade effect, lowered the costs of mobilizing for subsequent participants. Nonetheless, the costs were not negligible; brutal crackdowns across the region defeated opposition movements in most countries (Brownlee et al. 2015). Finally, in other circumstances, we may not observe any serious mobilization by the opposition, even in cases of known electoral fraud or anti-regime mobilization in neighboring countries. However, this is a strategic choice in response to what are presumably higher (but not infinite) costs of mobilization.

I develop a model that captures these important substantive considerations in a simple and intuitive way. In each period of an infinite-horizon interaction, Nature draws a cost from a continuous distribution. After observing this draw, the opposition chooses whether to mobilize an anti-regime threat in that period.[3] Mobilizing to generate a window of opportunity prompts the ruler to offer a temporary concession, or transfer, to which the opposition can respond by accepting or revolting. This incorporates a standard bargaining interaction following the endogenous mobilization decision.

Replacing costless, exogenous windows of opportunity with costly, endogenous windows reframes the autocrat's commitment problem. The implications are strikingly different for each of the three conventional results described above.

First, strategic decisions that reduce the frequency of windows of opportunity *do not* prompt the

[3] I treat the opposition as a unitary actor throughout. This facilitates a focus on bargaining dynamics amid positive costs to endogenously mobilizing anti-regime threats, and not other aspects of collective action such as coordination problems; and also enables a close connection with existing models of the autocrat's commitment problem. In the conclusion, I discuss connections with formal models of collective action and coordination.

opposition to make greater demands during its fleeting moments in the sun. The frequency of windows of opportunity depends on the endogenous mobilization threshold, the maximum cost the opposition will pay to mobilize. Lowering this threshold implies less frequent windows of opportunity. One effect of lowering the threshold reduces the frequency of periods in which the opposition gains a temporary transfer. This is the standard mechanism that raises the opposition's demand during windows of opportunity. However, because mobilization is costly, lowering the endogenous mobilization threshold also unleashes a second effect: reducing the average cost of mobilizing. These countervailing mechanisms perfectly offset each other. The opposition mobilizes only in periods in which the cost does not exceed the temporary transfer it will receive in return, as the opposition is indifferent at the endogenous mobilization threshold. A marginal reduction in the endogenous mobilization threshold eliminates mobilization in periods in which the opposition's net gain in consumption was zero, anyway. By contrast, with costless, exogenous mobilization, the opposition's net consumption is strictly positive during every window of opportunity.

Second, the ruler *does not face a commitment problem* when mobilization is fully endogenous. Instead, the costs of mobilizing become the only friction in the model that can cause conflict. Mobilization is fully endogenous when the opposition's mobilization choice in every period is non-trivial, which requires that the cost of mobilizing never exceeds total societal output. The opposition can always choose to mobilize, and the ruler can always respond by offering a temporary transfer that yields net positive consumption for the opposition. If mobilization is fully endogenous, then a necessary condition for conflict to occur in equilibrium is costly peace, that is, total surplus under peaceful bargaining is lower than under conflict. This reverses the standard assumption of costly conflict; the reversal is possible in the present model because the opposition pays periodic costs of mobilizing along a peaceful path.

Conversely, under the standard assumption of no-costly peace, conflict cannot happen in equilibrium with fully endogenous mobilization—even if the ruler lacks any institutional ability to commit to deliver concessions in future periods. If no-costly peace holds, then exogenous frictions to mobilization are necessary for conflict. Such frictions reintroduce the ruler's commitment problem present in existing models.

Third, a credible threat of revolt *is neither necessary nor sufficient* to induce power-sharing concessions. Costly mobilization can prompt the ruler either to share more power than needed to buy off a revolt—or instead refuse to share any power, even if this means incurring a revolt. I develop these results in an extension in which the ruler strategically decides how much power to share. Following Meng et al.'s (2023) discussion of the two core elements of authoritarian power-sharing deals, sharing more power (a) facilitates *institutional commitment* by raising the opposition's basement level of per-period spoils, and (b) reallocates power via a *threat-enhancing effect* that increases the opposition's probability of succeeding in a revolt.

Introducing costly, endogenous mobilization qualitatively alters the ruler's power-sharing calculus. On the one hand, a credible threat of revolt is unnecessary to induce power sharing. Costly mobilization creates a novel incentive for the ruler to *voluntarily share power*. Raising the opposition's basement level of spoils reduces the equilibrium frequency of mobilization, and hence the total costs paid to mobilize. Although the ruler does not directly pay these costs, he does so indirectly because he must compensate the opposition to prevent a revolt. Thus, sharing power can make a peaceful path more lucrative for the ruler.

On the other hand, a credible threat of revolt is insufficient to induce power sharing. The threat-enhancing effect bolsters the opposition's bargaining leverage. This can make the ruler unwilling to share power, even if the alternative is to incur a revolt. Costly mobilization makes the *conflictual alternative more tempting* by diminishing the surplus under peace relative to conflict.

In sum, the present model incorporates and builds upon foundational premises about the autocrat's commitment problem. Incorporating an intuitive idea about the costs of strategically mobilizing overturns conventional intuitions about whether periodic windows of opportunity constitute a key friction that triggers commitment problems and conflict. And, paradoxically, the ruler is simultaneously more willing to share high levels of power and less willing to share any power, relative to a baseline with costless mobilization. Collectively, these findings suggest new theoretical and empirical directions for understanding costly conflict and institutional reforms, as the conclusion discusses.

2 Contributions to Related Research

How frequently windows of opportunity arise is a crucial parameter in existing theories connecting commitment problems to outcomes such as conflict, power sharing, and democratization. Some analyses in the most closely related models focus squarely on the frequency of windows of opportunity (e.g., Powell 2004; Castañeda Dower et al. 2018; Paine 2022; Little and Paine 2024). In others, the main comparative statics predictions focus on other parameters, such as the level of societal inequality (Acemoglu and Robinson 2000, 2001, 2006). However, in all these models, restrictions on windows of opportunity are fundamental. If the opposition is never exogenously blocked from posing a threat of revolt, the model lacks a friction that can induce conflict or prompt the ruler to share power.

Throughout, the terminology (e.g., "ruler" and "opposition") refers to domestic actors. This is natural because (a) window-of-opportunity conflict bargaining models and (b) strategic options for institutional reform are more commonly analyzed in the domestic setting. Nonetheless, all the insights about general mechanisms can be applied to the IR setting as well. Moreover, the findings help to connect different strands of the CP and IR literatures.

Endogenous shifts in the distribution of power. Strategic mobilization of anti-regime threats yields endogenous shifts in power, which scholars have studied in other substantive contexts and modeling frameworks. One possible way to avoid conflict, considered in the IR setting, is for a *rising power* to forgo investments that would facilitate a large and rapid rise (Fearon 1996; Chadefaux 2011; Powell 2013; Debs and Monteiro 2014; Spaniel 2019). This could entail agreeing to demilitarized zones or dismantling reactors that could produce plutonium for nuclear weapons.

The rising power instead takes actions that enable it to amass power more incrementally or not at all, which discourages the *declining power* from initiating a game-ending war before the shift in power occurs.[4]

Peaceful bargaining with fully endogenous mobilization in the present model entails a distinct mechanism. Using the IR terminology, in any period the opposition has mobilized, the ruler is the rising power and the opposition is the declining power. The opposition, in expectation, loses strength in the next period because it might not mobilize. However, the standard solution in IR endogenous shifting models is unavailable: the ruler *cannot* strategically choose to limit its rise (e.g., take an action to lower the opposition's cost of mobilizing in the next period). Nonetheless, the opposition's mobilization decisions and the stream of temporary transfers offered by the ruler are strategic responses to each other. Collectively, these smooth out the key friction that yields conflict in standard window-of-opportunity models.

This also implies that the standard solution in existing models of domestic institutional reform—sharing political power—is unnecessary to prevent conflict if mobilization is fully endogenous and peace is not costly. Even if the ruler has limited ability to commit to promises of future concessions, the opposition's ability to endogenously generate windows of opportunity ensures that it can gain at least as much consumption as from revolting.

I am unaware of other window-of-opportunity conflict bargaining models in which the threat of revolt arises endogenously, although some model the frequency of windows of opportunity as correlated with other parameters. In Paine (2022) and Luo (2023), this parameter is positively correlated with the opposition's probability of winning a revolt. Little and Paine (2024) examine a continuous distribution of (exogenously arising) threats, and assume that "strong challengers" have a high maximum *and* average threat.

[4]Nonetheless, conflict can occur in equilibrium because of alternative frictions such as discontinuities in the bargaining space (Powell 2006), contingent spoils from monopolizing power (Powell 2013), or hidden actions (Debs and Monteiro 2014). Gibilisco (2021) models endogenous mobilization differently. Repression by the center causes grievances to accumulate for the periphery. Although higher grievances yield greater prospects for successful mobilization (an endogenous choice), the center will nonetheless pursue a repressive strategy if grievances are already high.

Bottom-up versus top-down incentives for political reform. A standard idea in the most closely related models is that permanent institutional reforms are costly whereas temporary concessions are not (Paine 2024*a*). Consequently, a credible threat of revolt is a *necessary condition* for the ruling elite to share power or allow political transitions. This creates *bottom-up* pressure for reform. In the present model, the threat of revolt creates one, but not the only, source of pressure to reform political institutions. I incorporate the natural idea that non-institutionalized bargaining mechanisms are inefficient, as the opposition must pay costs to mobilize to compel the ruler to make temporary concessions. This creates pressure to share power even absent a credible threat of revolt, and constitutes a *top-down* mechanism of reform.

Other contributions assess alternative ways in which the inherent inefficiency of authoritarian institutions creates top-down pressure for institutional reform. Ansell and Samuels (2014) highlight how insecure property rights discourage producers from making investments that would expand the tax base, which legislative representation (Gailmard 2024) or institutionalized parties (Gehlbach and Keefer 2011) could protect. Similarly, Bates and Donald Lien (1985) and Kenkel and Paine (2023) examine the rise of parliaments in response to credible options for elites with mobile wealth to exit the polity, which creates a source of inefficiency under autocratic rule. Alternatively, a broader franchise could alleviate corruption that distorts the political system (Lizzeri and Persico 2004).

The present model also departs from the standard logic that a credible threat of revolt constitutes a *sufficient condition* to induce power sharing, as in Castañeda Dower et al. (2018, 2020). In Powell (2024), the ruler shares power to stave off a revolt if possible, but institutions may be too weak for such concessions to be credible. The same logic is present in Acemoglu and Robinson (2000, 2001, 2006), albeit with somewhat different comparative statics. The ruling elite prefer expanding the franchise over incurring a revolt. However, for some parameter values, the elites respond to a credible threat of revolt with repression, an asymmetric conflict technology that defeats the masses with probability 1. By contrast, in the present model, the ruler lacks access to an asymmetric conflict technology.

Here, the drawback of permanent power-sharing concessions (from the ruler's perspective) is that sharing power creates a threat-enhancing effect that reallocates power toward the opposition. This effect can make the ruler unwilling to share power, even if the alternative is to incur a revolt (Paine 2024*b*). Costly mobilization exacerbates this effect by reducing the relative benefits of peaceful bargaining. Coupled with the top-down pressures, this result showcases the countervailing effects of costly mobilization on incentives to share power.

Costly peace. The model also contributes to a smaller literature on costly peace motives for war, usually studied in the IR context. Powell (1993) studies the guns-butter tradeoff and explains how the costs of constantly arming against an adversary can prompt a state to initiate war, which would eliminate these costs (see also Coe and Vaynman 2020 and Monteiro and Debs 2020). No existing scholarship, to my knowledge, examines the costly peace mechanism in a domestic context. However, it is inherently inefficient for autocrats to continually compensate the opposition for the costly hurdles faced to gaining concessions. This makes conflict relatively less costly, but also creates a rationale to share power—even if the opposition lacks a credible threat to revolt.

3 Setup of Baseline Model

A ruler and a representative opposition actor bargain over spoils throughout an infinite-horizon interaction. Periods are denoted by $t = 0, 1, 2, \ldots$ and the players share a common discount factor $\delta \in (0, 1)$. Total societal output is 1 in each period, with the opposition controlling a basement level of spoils $\pi \in [0, \overline{\pi}]$, and the ruler controlling the remaining $1 - \pi$. The upper bound $\overline{\pi} < 1$ simplifies the exposition in the text by ruling out corner solutions to the temporary transfer for the opposition, which Appendix A.2 discusses. Later, I extend the model to allow the ruler to endogenously choose π.

Nature moves first in every period, determining a contemporaneous cost c_t that the opposition would pay to mobilize an anti-regime threat—or, equivalently, a window of opportunity. In every period, with probability $r \in (0,1]$, the cost is drawn from an iid distribution $F(c)$ with full support over $[0, c^{\text{max}}]$, for $c^{\text{max}} \in (0,1]$.[5] If the opposition mobilizes in any period such that the cost does not exceed $z \leq c^{\text{max}}$, then the average cost paid in periods with a window of opportunity is

$$c^{\text{avg}}(z) \equiv \frac{\int_0^z c_t dF(c)}{F(z)}.$$

Thus, $c^{\text{avg}}(c^{\text{max}}) = \int_0^{c^{\text{max}}} c_t dF(c)$ corresponds with the average cost if mobilization occurs in every period; with slight abuse of notation, I shorten this to c^{avg}. With complementary probability $1-r$, the cost is degenerate: $c_t = \infty$.

The standard model is, in essence, a special case of this setup in which $r < 1$ and $c^{\text{max}} = 0$. That is, in a fraction r of periods, mobilization is costless; and in the remaining $1-r$ periods, mobilization is not possible. A setup with purely endogenous mobilization is one in which $r = 1$ and $c^{\text{max}} > 0$. That is, the opposition always has agency to mobilize. Because the upper bound of the support for $F(c_t)$ is $c^{\text{max}} \leq 1$, the cost of mobilizing does not exceed total societal output in periods that c_t is drawn from $F(c)$.[6]

In every period, after observing c_t, the opposition decides whether to mobilize.[7] If not, then the ruler and opposition respectively consume $1-\pi$ and π, and then engage in an identical interaction in period $t+1$ with respective continuation values V_R and V_O.

If instead the opposition mobilizes, then it pays the sunk cost c_t and engages in a bargaining interaction amid its window of opportunity. The ruler proposes a one-period transfer $x_t \in [0, 1-\pi]$ to the opposition. The bounds on the temporary transfer capture (a) no transfer of resources from

[5]The corresponding pdf is denoted as f. One result requires assuming that the pdf is non-increasing, $f' \leq 0$; see Proposition 3. The uniform distribution, for example, satisfies this property.

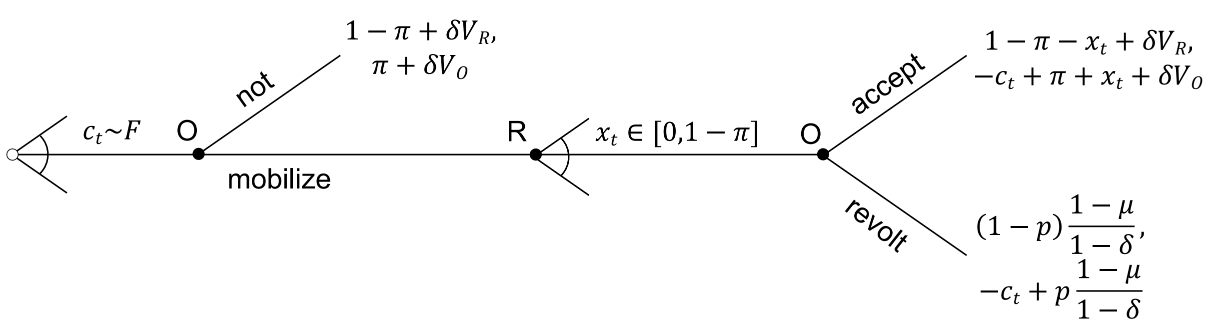
[6]I model the exogenous friction using the parameter r to enable a direct comparison with existing models and to reduce notation. As an alternative way to generate periods in which it is infeasible for the opposition to mobilize, assume the opposition pays a fixed cost of mobilizing $\underline{c} \in [0,1]$ in addition to the variable cost c_t. Then, for any draw such that $c_t > 1 - \underline{c}$, the total cost of mobilizing exceeds societal wealth, and the fraction of such periods equals $1 - F(1-\underline{c})$. This term, however, is clunkier to track than a separate parameter r, on which I take comparative statics (see below).

[7]In Appendix A.5, I extend the model to allow the opposition to choose a continuous level of mobilization.

the opposition to ruler and (b) the offer cannot exceed the total amount controlled by the ruler. If the opposition accepts the transfer, then the ruler and opposition respectively consume $1 - \pi - x_t$ and $-c_t + \pi + x_t$, and they begin an identical interaction in period $t + 1$ with the continuation values stated above.

Alternatively, the opposition can revolt. The winner consumes $1 - \mu$ in the period of the revolt and in perpetuity. Assuming $\mu \in (0, 1)$ implies that conflict permanently reduces total societal output. A revolt succeeds with probability $p \in [0, 1]$, whereas the ruler survives with complementary probability.[8] Figure 1 presents the tree of the stage game, and Appendix A.1 summarizes every variable and threshold presented in the analysis.

Figure 1: Tree of Stage Game



4 Preliminary Analysis with Exogenous Mobilization

Fixing the mobilization threshold enables deriving several preliminary results. Specifically, for an exogenously determined $\hat{c}$, a window of opportunity arises in period t if and only if $c_t \leq \hat{c}$. Consequently, the opposition mobilizes in a fraction $rF(\hat{c})$ of periods and pays an average cost in such periods equaling $c^{\text{avg}}(\hat{c})$.

Throughout, the equilibrium concept is Markov Perfect Equilibrium (MPE). A Markov strategy allows a player to condition its actions only on the current-period state of the world and prior actions in the current period. An MPE is a profile of Markov strategies that is subgame perfect.

[8] In the extension with strategic power sharing, the probability of winning depends on the level of power sharing.

Given the present restrictions on the full game, a Markov strategy requires the ruler to make an offer $x \rightarrow [0, 1-\pi]$ and the opposition to respond with a function $\alpha : [0, 1-\pi] \rightarrow \{0, 1\}$, where $\alpha = 1$ indicates acceptance and $\alpha = 0$ indicates revolt.[9]

4.1 PAYOFFS ALONG A PEACEFUL PATH

Along a peaceful path, the ruler consumes total societal surplus minus the opposition's reservation value to revolting, and the opposition consumes its reservation value.

The opposition's lifetime consumption along a peaceful path, from the perspective of any period in which it has mobilized, is $-c_t + \pi + x + \delta V_O$, for $V_O = \pi + rF(\hat{c})(x - c^{\text{avg}}(\hat{c})) + \delta V_O$.[10] Solving the continuation value and substituting it into the consumption term yields per-period average consumption $\pi + (1-\delta)(x - c_t) + \delta r F(\hat{c})(x - c^{\text{avg}}(\hat{c}))$. The opposition consumes at least π in every period. It also gains an additional transfer x both (a) today, worth $1-\delta$; and (b) in the fraction $rF(\hat{c})$ of future periods in which the opposition will mobilize, worth $\delta r F(\hat{c})$. But the mobilization effort needed to gain these transfers is costly, and entails paying c_t today and an average of $c^{\text{avg}}(\hat{c})$ in future mobilization periods.

The opposition's payoff along a peaceful path is bounded from below by its reservation value to revolting, as it can always choose this outside option amid a window of opportunity. Thus, the consumption stream must satisfy

$$\pi + (1-\delta)(x - c_t) + \delta r F(\hat{c})(x - c^{\text{avg}}(\hat{c})) \geq -(1-\delta)c_t + p(1-\mu). \tag{1}$$

The ruler's lifetime consumption along a peaceful path, from the perspective of any period in which the opposition has mobilized, is $1 - \pi - x + \delta V_R$, for $V_R = 1 - \pi - rF(\hat{c})x + \delta V_R$. Solving the continuation value and substituting it into the consumption term yields per-period average

[9] The continuous distribution of c_t implies that the state space is continuous. However, c_t is payoff irrelevant starting at the ruler's information set, which below I show yields a unique optimal transfer offer. Given the Markov assumption, this must be the transfer in every period.

[10] The continuation value incorporates the Markov assumption by requiring the opposition to receive the same transfer x in every high-threat period.

consumption $1 - \pi - (1 - \delta(1 - rF(\hat{c})))x$. The ruler's consumption strictly decreases in x, but the transfer must satisfy Equation 1 to yield a peaceful path of play. Consequently, the ruler satisfies this constraint with equality to make the opposition indifferent between accepting and revolting.[11] This yields an interior-optimal transfer denoted as x^*.

$$\textbf{Optimal transfer.} \quad \pi + (1 - \delta)x^* + \delta rF(\hat{c})(x^* - c^{\text{avg}}(\hat{c})) - p(1 - \mu) = 0, \tag{2}$$

which can be solved explicitly for

$$x^* = \frac{-\pi + p(1 - \mu) + \delta rF(\hat{c})c^{\text{avg}}(\hat{c})}{1 - \delta(1 - rF(\hat{c}))}. \tag{3}$$

Substituting x^* into the ruler's consumption stream yields

$$\begin{aligned} R(\pi) &= 1 \overbrace{-\pi}^{\text{Direct cost}} - \big(1 - \delta(1 - rF(\hat{c}))\big) \underbrace{\frac{\overbrace{-\pi}^{\text{Indirect benefit}} + p(1 - \mu) + \delta rF(\hat{c})c^{\text{avg}}(\hat{c})}{1 - \delta(1 - rF(\hat{c}))}}_{x^*} \\ &= \underbrace{1 - \delta rF(\hat{c})c^{\text{avg}}(\hat{c})}_{\text{Total surplus}} - \underbrace{p(1 - \mu)}_{\text{Opposition's reservation value}}. \end{aligned} \tag{4}$$

The ruler's lifetime expected average consumption equals total surplus, $1 - \delta rF(\hat{c})c^{\text{avg}}(\hat{c})$,[12] minus the opposition's reservation value to revolting, $p(1 - \mu)$. Conversely, the opposition's lifetime expected average consumption equals its reservation value, $p(1-\mu)$. Notably, the level of basement spoils π does not affect the ruler's consumption along a peaceful path because two countervailing effects cancel out. The ruler loses π in every period, the direct cost of higher basement spoils. However, higher π indirectly benefits the ruler by increasing the opposition's consumption along a peaceful path. By raising the opportunity cost of revolting, the ruler can buy off the opposition

[11] As is standard in these models, any equilibrium strategy profile requires that the opposition accept such an offer with probability 1. Otherwise, the constraint set for the ruler's optimization problem would not be closed.

[12] This term discounts the costs of mobilization by one period because, in the stage game, the opposition sinks the cost of mobilizing prior to the bargaining interaction. Therefore, the ruler does not offer compensation for the present-period cost of mobilizing.

with a lower transfer during windows of opportunity. Thus, the ruler is compensated for higher permanent concessions by giving away fewer temporary transfers. The direct cost and indirect benefit perfectly offset each other because the ruler and opposition weight the stream of transfers identically from the perspective of any mobilization period: a transfer occurs in the current period, worth $1 - \delta$; and a fraction $rF(\hat{c})$ of future periods, worth $\delta r F(\hat{c})$.

Sufficiently high values of π make x^* negative. This yields a unique threshold value such that $x^*(\overline{\pi}) = 0$. This constitutes the upper bound of π assumed in setup, which ensures that x^* has an interior solution. In Appendix A.2, I analyze equilibrium outcomes for all values of π.

4.2 Conditions for Peaceful Bargaining

No-revolt constraint. A necessary condition for peaceful bargaining is that the ruler is able to make a transfer that the opposition will accept rather than revolt. This requires $1 - \pi - x^* \geq 0$. Straightforward algebraic rearrangement of this inequality (after substituting in Equation 3) yields

$$\Theta(\hat{c}, r, \pi) \equiv \underbrace{\pi}_{\text{Basement}} + \underbrace{\big(1 - \delta(1 - rF(\hat{c}))\big)(1 - \pi)}_{\text{Temporary transfers}} - \underbrace{\delta r \int_0^{\hat{c}} c_t dF(c)}_{\text{Future mobilization costs}} - \underbrace{p(1 - \mu)}_{\text{Revolt}} \geq 0. \tag{5}$$

Lemma 1 provides a useful observation when analyzing endogenous mobilization.

Lemma 1 (Mobilization threshold and incentives to revolt)**.**

$$\underset{\hat{c} \in [0,1]}{\arg\max} \; \Theta(\hat{c}, r, \pi) = 1 - \pi.$$

This threshold is natural, as $1 - \pi$ is the highest feasible transfer that the ruler can make amid a window of opportunity. Thus, the opposition's net payoff to peace is highest when it mobilizes in every period for which the cost of mobilization does not exceed the maximum transfer, but never mobilizes when the cost is higher. Formally, the lemma follows from

$$\frac{\partial \Theta(\hat{c}, r, \pi)}{\partial \hat{c}} = \delta r f(\hat{c})(1 - \pi - \hat{c}) = 0.$$

When deriving conditions for peace and conflict with endogenous mobilization, I will refer to the no-revolt threshold with the maximizer $\hat{c} = 1 - \pi$,

$$\Theta^*(r, \pi) \equiv \Theta(1 - \pi, r, \pi). \tag{6}$$

No costly peace. If Equation 5 holds, then a necessary and sufficient condition for an equilibrium with peaceful bargaining is that the ruler prefers to buy off the opposition with the transfer x^* rather than incur a revolt. The ruler's all-else-equal preference for peaceful bargaining over conflict is standard in conflict bargaining models; the cost of conflict induces the ruler to buy off the opposition (Fearon 1995). Here, however, a peaceful path requires the opposition to pay periodic costs of mobilization, which in turn prompts the opposition to demand more from the ruler. If a revolt occurs, the ruler's expected per-period average payoff is $(1 - p)(1 - \mu)$. The ruler prefers peaceful bargaining if and only if total surplus along a peaceful path, $1 - \delta r F(\hat{c}) c^{\text{avg}}(\hat{c})$, exceeds that to a conflictual path,

$$\underbrace{R(\pi)}_{\text{Peace (Eq. 4)}} > \underbrace{(1 - p)(1 - \mu)}_{\text{Conflict}} \implies \frac{1 - \delta r F(\hat{c}) c^{\text{avg}}(\hat{c})}{1 - \mu} > 1. \tag{7}$$

The left-hand side of the latter inequality in Equation 7 reaches its lower bound when setting the total costs of mobilization to their maximum level. This occurs when the opposition mobilizes in every period, which corresponds with $r = 1$ and $\hat{c} = c^{\text{max}}$; which in turn implies $F(\hat{c}) c^{\text{avg}}(\hat{c}) = c^{\text{avg}}$, the average draw from the full distribution $F(c)$. I assume the no-costly peace assumption holds throughout the remainder of the analysis, and Lemma 2 follows directly from this assumption.[13]

[13]The lemma does not hold when allowing values of π high enough not only that the optimal transfer hits a corner solution of 0, but peaceful bargaining entails the ruler consuming less than its reservation value to revolting; see

Assumption 1 (No costly peace)**.**

$$\frac{1-\delta c^{avg}}{1-\mu} > 1.$$

Lemma 2 (Ruler's preference for peaceful bargaining)**.** *The ruler prefers peaceful bargaining over conflict.*

5 Peace and Conflict with Endogenous Mobilization

The full analysis endogenizes the mobilization threshold $\hat{c}$. The addition to the specification of Markov strategies is that the opposition makes a mobilization decision $\beta : [0, c^{\max}] \to \{0,1\}$, where $\beta = 1$ indicates mobilization and $\beta = 0$ indicates not.

5.1 Endogenous Mobilization Threshold

When endogenizing the mobilization threshold $\hat{c}$, the opposition mobilizes only in periods such that the cost does not exceed the transfer it will receive in return. At the threshold, the opposition is indifferent between mobilizing or not. Consequently, lowering $\hat{c}$ and hence the endogenous frequency of mobilization *does not* prompt the opposition to make greater demands, contrary to existing models. Reducing the marginal frequency of windows of opportunity eliminates gaining transfers in periods in which the opposition's net gain in consumption was zero.

Deriving the threshold. The endogenous choice of $\hat{c}$ reflects the following considerations. The value of mobilizing for the opposition is the same regardless of whether the path of play is peaceful or conflictual.[14] If conflictual, the opposition revolts, which yields lifetime expected consumption of $p\frac{1-\mu}{1-\delta}$. If peaceful, the opposition consumes $\pi + x + \delta V_O$. However, as discussed earlier, the ruler holds the opposition to indifference, which yields an identical consumption stream of $p\frac{1-\mu}{1-\delta}$.

Appendix A.2.

[14] If $\pi > \overline{\pi}$, which makes the optimal transfer 0, then the opposition never mobilizes; see Appendix A.2.

The opposition mobilizes only if gaining this consumption stream, minus the cost c_t, exceeds the value of consuming π today and remaining as the opposition tomorrow

$$\underbrace{-c_t + p\frac{1-\mu}{1-\delta}}_{\text{Mobilize}} \geq \underbrace{\pi + \delta V_O}_{\text{Not}}. \tag{8}$$

The cutpoint $\hat{c}$, the *endogenous mobilization threshold*, satisfies this with equality

$$-\hat{c} + p\frac{1-\mu}{1-\delta} = \pi + \delta V_O. \tag{9}$$

Substituting in for V_O (presented earlier) and simplifying yields an intuitive threshold. In any period, the opposition mobilizes if and only if the contemporaneous cost it pays does not exceed the contemporaneous transfer it would gain in return.

> **Lemma 3** (Endogenous mobilization threshold)**.** *In any period, the opposition mobilizes if and only if $c_t \leq \hat{c}$, for $\hat{c} = x^*$.*

Invariance of transfer to endogenous mobilization threshold. This result explains why altering the endogenous mobilization threshold does not prompt the opposition to make greater demands, manifested in a higher equilibrium transfer x^*. Deriving the left-hand side of Equation 2 with respect to $\hat{c}$ yields[15]

$$\frac{\partial}{\partial \hat{c}}\left(\pi + (1-\delta)x^* + \delta r F(\hat{c})\left(x^* - \underbrace{\frac{\int_0^{\hat{c}} c_t dF(c)}{F(\hat{c})}}_{c^{\text{avg}}(\hat{c})}\right) - p(1-\mu)\right)$$

$$= \delta r f(\hat{c})\Big(\underbrace{x^* - c^{\text{avg}}(\hat{c})}_{\text{Consumes less often}} - \underbrace{(\hat{c} - c^{\text{avg}}(\hat{c}))}_{\text{Lower average cost}}\Big) = \delta r f(\hat{c})\underbrace{(x^* - \hat{c})}_{=0} = 0. \tag{10}$$

[15]Denoting the left-hand side of Equation 2 as Ω, by the implicit function theorem,

$$-\frac{dx^*}{d\hat{c}} = \frac{\partial \Omega}{\partial \hat{c}} \Big/ \frac{\partial \Omega}{\partial x^*}, \quad \text{with} \quad \frac{\partial \Omega}{\partial x^*} = 1 - \delta(1 - rF(\hat{c})) > 0.$$

Consequently, the sign of $\frac{\partial \Omega}{\partial \hat{c}}$ determines the sign of the derivative.

Lowering the endogenous mobilization threshold $\hat{c}$ causes the opposition to mobilize less often. This reduces the frequency of consuming transfers. All else equal, this force prompts the opposition to demand more during (rarer) windows of opportunity. However, all else is not equal because of a countervailing effect. Lowering $\hat{c}$ also reduces the average cost $c^{\text{avg}}(\hat{c})$ the opposition pays upon mobilizing. This force raises the opposition's consumption along an equilibrium path, which reduces its transfer demand during windows of opportunity.

The two countervailing forces of lowering $\hat{c}$—less frequent windows of opportunity and lower average costs of mobilizing—perfectly cancel out. A marginal reduction in $\hat{c}$ yields a marginal reduction in consumption of $x^* - \hat{c}$, because the opposition no longer mobilizes when $c_t = \hat{c}$. But, in equilibrium, the opposition mobilizes for any c_t up to the point at which the cost perfectly offsets the gain in consumption from receiving the transfer, yielding $x^* = \hat{c}$ (Equation 9). Consequently, in the marginal period in which the opposition no longer mobilizes, its net consumption was $x^* - \hat{c} = 0$. Lowering the endogenous mobilization threshold therefore does not affect the opposition's demand.

Lemma 4 (Endogenous mobilization threshold and equilibrium transfer)**.**

$$\frac{dx^*}{d\hat{c}} = 0.$$

This logic also yields an envelope theorem-type result: any parameter affects the equilibrium transfer *only* through its direct effects, not indirectly by affecting the endogenous mobilization threshold. This result will help to examine the effects of key parameters on equilibrium prospects for conflict.

Corollary 1 (No indirect effects)**.** *For any parameter z,*

$$\frac{dx^*(z, \hat{c}(z))}{dz} = \frac{\partial x^*}{\partial z} + \underbrace{\frac{\partial x^*}{\partial \hat{c}}}_{=0} \frac{d\hat{c}}{dz} = \frac{\partial x^*}{\partial z}.$$

5.2 EQUILIBRIUM BARGAINING OUTCOMES

The equilibrium path is peaceful whenever $1 - \pi - x^* \geq 0$, which is equivalent to $\Theta^*(r, \pi) \geq 0$ (see Equation 6). If mobilization is fully endogenous, then conflict cannot occur because of the no-costly peace assumption. Exogenous frictions to mobilizing can trigger conflict, but high-enough basement spoils restore peace.

Fully endogenous mobilization prevents conflict. Fully endogenous mobilization eliminates the ruler's commitment problem, even if the ruler cannot guarantee any future consumption, $\pi = 0$. The only friction in the model is the cost of mobilizing. Therefore, assuming no costly peace (Assumption 1) implies conflict *does not occur* in equilibrium.

Fully endogenous mobilization requires that the opposition faces a non-trivial decision regarding whether to mobilize in every period, which means the cost of mobilizing is less than total societal output. Given the assumption $c^{\text{max}} \leq 1$, this is tantamount to setting $r = 1$. The other scope condition for this case, no permanent commitment to concessions, requires $\pi = 0$. The necessary inequality for peaceful bargaining is

$$\Theta^*(0, 1) \geq 0 \implies 1 - \delta c^{\text{avg}} \geq p(1 - \mu).$$

In words, this inequality states that surplus along a peaceful path with windows of opportunity in every period must exceed the opposition's per-period reservation value to revolting. But because peace is assumed to be surplus saving (Assumption 1), this inequality must hold. Thus, conflict does not occur along the equilibrium path.

This result contrasts with existing models with exogenous frictions to mobilizing. Infrequent mobilization triggers conflict because the ruler cannot commit to deliver sufficient concessions along a peaceful path. By contrast, here, in the case of fully endogenous mobilization, the opposition can always choose to mobilize. The ruler does not face a commitment problem, at least as conceptualized in existing models, because he can always offer a transfer at least as large as the opposition's contemporaneous cost of mobilizing. Conversely, any force that causes windows of opportunity to arise less frequently also reduces the average costs of mobilizing by an equivalent amount.

Instead, costly mobilization constitutes the only friction in the present model. If conflict occurs when $r = 1$, it is because Assumption 1 is violated and the costs of perpetually sustaining mobilization along a peaceful path exceed the costs of conflict. But, assuming that conflict is costlier than peace, conflict cannot occur in equilibrium absent an exogenous friction.

Frictions to mobilization trigger conflict. Conflict along the equilibrium path requires exogenous frictions, created by $r < 1$. Whereas strategic decisions that reduce the frequency of mobilization do not affect prospects for conflict (Lemma 4), lowering the exogenous mobilization parameter r does indeed make it harder to buy off the opposition.[16]

Lemma 5 (Exogenous mobilization friction and prospects for conflict)**.**

$$-\frac{d}{dr}\big(1 - \pi - x^*(r, \hat{c}(r))\big) = -\frac{\delta F(\hat{c})}{1 - \delta(1 - rF(\hat{c}))}(x^* - c^{avg}(\hat{c})) < 0.$$

The key difference between marginal changes in $\hat{c}$ and r is that the former alters both (a) the frequency with which the opposition gains an additional consumption amount $x^* - c^{\text{avg}}(\hat{c})$, and (b) how much the opposition consumes in such periods. By contrast, a marginal change in r triggers the first effect, but without changing the consumption amount. Absent an effect to counteract the negative consequences of less frequent transfers, lowering r raises the demand x^*.

As in existing models, low r is a necessary condition for conflict to occur along the equilibrium path, $\Theta(0, r) < 0$. I phrase this in terms of the *opposition's credibility* to revolt amid a window of opportunity. Lacking any basement spoils ($\pi = 0$), the threat of revolt is credible only if r is low enough.

[16] Computing the derivative in Lemma 5 uses Corollary 1.

Lemma 6 (Opposition credibility condition)**.** *A unique threshold $\overline{r} < 1$ exists such that*

$$\Theta^*(0, r) \begin{cases} < 0 & \textit{if } r < \overline{r} \\ = 0 & \textit{if } r = \overline{r} \\ > 0 & \textit{if } r > \overline{r}, \end{cases}$$

for $\overline{r}$ implicitly defined as

$$\Theta^*(0, \overline{r}) = 1 - \delta(1 - \overline{r}(1 - c^{avg})) - p(1 - \mu) = 0.$$

This allows us to characterize equilibrium behavior for any level of r, assuming $\pi = 0$.

Proposition 1 (Equilibrium without basement spoils)**.** *Suppose $\pi = 0$.*

> ***Case 1.** $\boldsymbol{r \geq \overline{r}}$. In every period, the opposition mobilizes if and only if $c_t \leq x^*$. If the opposition mobilizes, the ruler offers $x_t = x^*$ and the opposition accepts if and only if $x_t \geq x^*$. Along the equilibrium path of play, revolts never occur.*
>
> ***Case 2.** $\boldsymbol{r < \overline{r}}$. In every period, the opposition mobilizes if and only if $c_t \leq x^*$. If the opposition mobilizes, the ruler offers any $x_t \in [0, 1]$ and the opposition revolts in response to any offer. Along the equilibrium path of play, a revolt occurs during the first window of opportunity.*

Sharing power yields peace. A higher level of basement spoils for the opposition, alternatively referred to as a higher level of power sharing, improves prospects for peaceful bargaining. Even if the opposition credibility condition (Lemma 6) holds, high-enough π yields peaceful bargaining. The core mechanism reflects existing models: higher basement spoils raise the opportunity cost to revolting, which lowers the transfer needed to secure peace.

However, here, there is a countervailing effect to consider because basement spoils affect the frequency with which the opposition mobilizes. For the same reason that π lowers the optimal transfer, it also reduces $F(\hat{c})$, the endogenous component of windows of opportunity. In existing models, less frequent windows of opportunity create ripe conditions for revolt. However, as seen in Corollary 1, the indirect effect cancels out. Thus, despite affecting the frequency of mobilization, the net effect of greater basement spoils is equivalent to the direct effect.

Lemma 7 (Basement spoils and prospects for conflict)**.**

$$\frac{d}{d\pi}\Big(1-\pi-x^*\big(\pi,\hat{c}(\pi)\big)\Big)=-1-\frac{\partial x^*}{\partial \pi}-\underbrace{\frac{\partial x^*}{\partial \hat{c}}}_{\textit{Indirect}=0}\frac{d\hat{c}}{d\pi}=\frac{\delta(1-rF(\hat{c}))}{1-\delta(1-rF(\hat{c}))}>0.$$

This enables characterizing, for low levels of r in which opposition credibility holds, the effect of raising π.

Proposition 2 (Equilibrium with basement spoils)**.** *Suppose* $r<\overline{r}$*. A unique threshold* $\underline{\pi}\in(0,\overline{\pi})$ *exists,*[17] *implicitly characterized as*

$$\Theta^*(\underline{\pi},r)=\underline{\pi}+\big(1-\delta(1-rF(1-\underline{\pi}))\big)(1-\underline{\pi})-\delta r\int_0^{1-\underline{\pi}}c_t dF(c)-p(1-\mu)=0,$$

with the following properties.

Case 1. $\boldsymbol{\pi\geq\underline{\pi}}$. *In every period, the opposition mobilizes if and only if* $c_t\leq x^*$*. If the opposition mobilizes, the ruler offers* $x_t=x^*$ *and the opposition accepts if and only if* $x_t\geq x^*$*. Along the equilibrium path of play, revolts never occur.*

Case 2. $\boldsymbol{\pi<\underline{\pi}}$. *In every period, the opposition mobilizes if and only if* $c_t\leq x^*$*. If the opposition mobilizes, the ruler offers any* $x_t\in[0,1]$ *and the opposition revolts in response to any offer. Along the equilibrium path of play, a revolt occurs in the first mobilization period.*

5.3 Frequency of Windows of Opportunity is Epiphenomenal

When mobilization is endogenous, windows of opportunity arise from deeper structural parameters that themselves determine prospects for conflict; the frequency of windows of opportunity is epiphenomenal. To see why, the preceding results highlight manipulations that either (a) do not affect prospects for conflict (lowering the endogenous mobilization threshold $\hat{c}$), (b) raise prospects for conflict (lowering the exogenous friction parameter r), or (c) reduce prospects for

[17] Appendix A.2 characterizes $\overline{\pi}$.

conflict (raising the power-sharing level π). These divergent effects are striking because *all three manipulations* lower the equilibrium frequency of windows of opportunity, $rF(\hat{c})$, as shown in Proposition 3. Thus, of the three manipulations, only that for r recovers the conventional wisdom that the frequency of windows of opportunity and prospects for conflict are inversely related.

Proposition 3 (Effects on frequency of mobilization)**.**

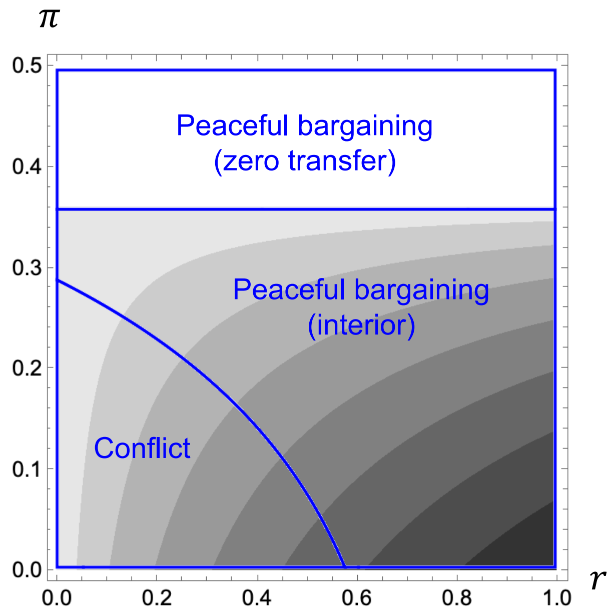
$$\textbf{\textit{Part a.}} \quad -\frac{d}{d\hat{c}}\big(rF(\hat{c})\big) < 0.$$

$$\textbf{\textit{Part b.}} \quad -\frac{d}{dr}\big(rF(\hat{c})\big) < 0.$$

$$\textbf{\textit{Part c.}} \quad \frac{d}{d\pi}\big(rF(\hat{c})\big) < 0.$$

Figure 2 depicts some of these effects. The blue lines separate three distinct regions of bargaining outcomes: peaceful bargaining with an interior-optimal transfer, $x^* \in (0, 1-\pi)$; peaceful bargaining with a zero transfer, $x^* < 0$; and conflict, $x^* > 1-\pi$. The gray bands depict the frequency with which windows opportunity arise; white corresponds with never and darker gray with more frequent. As in existing models, frequent windows of opportunity correspond with peaceful bargaining. This arises in the lower-right corner because r is high (opposition rarely blocked from mobilizing) and π is low (low opportunity cost to mobilizing). Moving either up or to the left of this region lowers the frequency of windows of opportunity, but with divergent consequences for conflict. A horizontal shift that lowers r can cause conflict, similar to the mechanism in existing models. However, a vertical shift that raises π makes it easier to buy off the opposition. When π is high enough, the opposition's basement level of spoils is so high that it never mobilizes—but, for the same reason, the opposition would not revolt given the opportunity to do so.

Figure 2: Frequency of windows of opportunity and conflict



Notes: $\delta = 0.9$, $p = 0.4$, $\mu = 0.1$, $c^{\max} = 1$, $F \sim U[0, c^{\max}]$.

6 Endogenous Power Sharing

This section extends the model to allow the ruler to choose how much power to share. In the most closely related models, a credible threat of revolt by the opposition is necessary for the ruler to choose a positive level of power sharing (Acemoglu and Robinson 2006; Castañeda Dower et al. 2018; Powell 2024; Paine 2024*a*,*b*). Here, however, the costs of mobilizing alter the ruler's calculus. Along a peaceful path, the ruler has incentives to reduce the frequency of mobilization—and, concomitantly, the total costs of mobilizing—by granting basement spoils to the opposition. The inefficiency of perpetual, costly mobilization creates a novel incentive to share power, which can potentially lead to a higher level of power sharing than in a model without mobilization costs. However, another factor is also at play. Mobilization costs lower the ruler's consumption along a peaceful path. Relative to a model without mobilization costs, this force creates a stronger preference for the ruler to refuse to share any power (and instead incur a revolt).

6.1 SETUP

In period 0, the ruler moves first and chooses a power-sharing level $\pi \in [0, 1]$. This choice permanently determines for the opposition (a) its permanent basement level of spoils, π, and (b) its probability of winning a revolt, now denoted as $p(\pi)$. These two effects respectively correspond with what Meng et al. (2023) identify as the two key elements of power-sharing deals: an *institutional commitment* mechanism to deliver spoils; and a coercive mechanism that reallocates power, referred to as the *threat-enhancing effect*. After the initial choice of π at the beginning of period 0, the sequence of play in that and all subsequent periods is the same as in the baseline game.

Now, the opposition's probability of succeeding in a revolt depends on the level of power sharing, $p(\pi) = (1 - \pi)p^{\text{min}} + \pi p^{\text{max}}$.[18] Reflecting the threat-enhancing effect, raising the power-sharing level increases the opposition's probability of winning a revolt, with $p^{\text{min}} \geq 0$ corresponding with the minimum probability at no power sharing and $p^{\text{max}} \in [p^{\text{min}}, 1]$ corresponding with the maximum probability at full power sharing. The threat-enhancing effect is

$$\Delta p(\pi) \equiv p(\pi) - p^{\text{min}} = \pi(p^{\text{max}} - p^{\text{min}}). \tag{11}$$

One way to interpret the one-time power-sharing choice is that critical junctures occur in which a ruler has agency to permanently alter the distribution of spoils and power vis-à-vis the opposition, perhaps because the ruler has recently ascended to power and has open cabinet positions or has newly confiscated land to redistribute. A single such critical juncture occurs in the present model, and I examine the consequences of this choice for the subsequent interaction. Furthermore, I assume $c_0 = 0$ (and this is common knowledge when the ruler sets π) to ensure the ruler chooses the power-sharing level when confronting an imminent threat. Thus, $R(\pi)$ from Equation 4 characterizes the ruler's expected payoff in that period, as $R(\pi)$ is calculated for a period in which the opposition has mobilized. This enables the present model to mimic existing models, in which the ruler only ever chooses to share power during a period with a window of opportunity.

[18] Footnote 22 discusses the technical rationale for assuming the probability-of-winning function is linear.

Most of the preceding equations and formal results are unchanged with this setup. The main difference is that even with an exogenously set π, all the preceding expressions with p must be replaced with $p(\pi)$. The implicit characterization of the minimum power-sharing level that enables peace, originally characterized as $\underline{\pi}$ in Proposition 2 and now referred to as $\underline{\pi}_p$, is

$$\Theta_p^*(\underline{\pi}_p, r) = 0,$$

$$\text{for} \quad \Theta_p^*(\pi, r) = \pi + \big(1 - \delta(1 - rF(1-\pi))\big)(1-\pi) - \delta r \int_0^{1-\pi} c_t dF(c) - \underbrace{p(\pi)}_{\text{Alteration}} (1-\mu), \quad (12)$$

and the subscript p in Θ_p^* indicates that the probability of winning now depends on an endogenous choice.[19]

Finally, to enable a clean characterization of the ruler's endogenous power-sharing choice (Lemma 8), in this section I assume that the costs of mobilizing are uniformly distributed, $F \sim U(0, c^{\text{max}})$.

6.2 Voluntary Power Sharing

Costly mobilization implies that, unlike in a standard setup, sharing more power does not strictly diminish the ruler's consumption along a peaceful path. Deriving the ruler's consumption function with the interior-optimal transfer (Equation 4) yields

$$\frac{dR(\pi)}{d\pi} = \frac{d}{d\pi}\bigg(1 - \delta r F(\hat{c}) c^{\text{avg}}(\hat{c}) - p(\pi)(1-\mu)\bigg) =$$

$$\underbrace{\delta r \hat{c} f(\hat{c}) \Big(- \frac{d\hat{c}}{d\pi}\Big)}_{\text{Lower mobilization costs}} - \underbrace{\overbrace{(p^{\text{max}} - p^{\text{min}})}^{p'(\pi)}(1-\mu)}_{\text{Threat-enhancing effect}}, \quad \text{with} \quad -\frac{d\hat{c}}{d\pi} = -\frac{-1 + (p^{\text{max}} - p^{\text{min}})(1-\mu)}{1 - \delta(1 - rF(\hat{c}))} > 0. \quad (13)$$

This expression reveals two countervailing effects. First, the threat-enhancing effect.[20] Sharing

[19] Appendix Lemma A.2 demonstrates that this alteration does not qualitatively change the existence and uniqueness properties of the $\underline{\pi}$ threshold.

[20] Using Equation 11, $\frac{\Delta p}{\pi} = p'(\pi) = p^{\text{max}} - p^{\text{min}}$.

power raises the opposition's probability of winning, which bolsters its reservation value and thereby detracts from the ruler's consumption. Second, the cost-of-mobilization effect. By creating a basement level of spoils for the opposition, higher π induces the opposition to mobilize less often (Lemma 3). Lowering the total costs of mobilization raises total surplus, which the ruler pockets by virtue of making all the bargaining offers and holding the opposition down to indifference.[21]

When the threat-enhancing effect is sufficiently small in magnitude, the cost-of-mobilization effect implies that the ruler gains greater consumption from setting an interior level $\pi^* > 0$ than $\pi = 0$ when fixing the path of play as peaceful. In this circumstance, contrary to most existing results, the purpose of sharing power is *not* to buy off a revolt, and hence constitutes a *voluntary* element of power sharing.[22]

Lemma 8 (Voluntary power sharing)**.** *Define* $\tilde{\pi} = \underset{\pi \in [0, \overline{\pi}_p]}{\arg\max}\, R(\pi)$*, the level of power sharing that maximizes the ruler's consumption along a peaceful path.*[23] *A unique threshold* $\hat{p}^{max} > p^{min}$ *exists such that*

Case 1. *If* $p^{max} < \hat{p}^{max}$*, then* $\tilde{\pi} = \pi^*$*, for a unique* $\pi^* \in (0, \overline{\pi}_p)$*.*

Case 2. *If* $p^{max} \geq \hat{p}^{max}$*, then* $\tilde{\pi} = 0$*.*

6.3 Costly Mobilization and Ruler Willingness

If the opposition can credibly threaten to revolt if the ruler does not share power (Lemma 6),[24] then sharing enough power is necessary to prevent a revolt. Furthermore, the ruler can always share enough power to prevent a revolt by setting $\pi \geq \underline{\pi}_p$ (see Proposition 2 and Equation 12). However, the ruler is not necessarily willing to do so. He may instead prefer to set $\pi = 0$ and incur a revolt.[25]

[21] The level of basement spoils does not directly affect the ruler's consumption because higher π enables the ruler to lower the transfer by an equivalent amount, as discussed earlier (Equation 4).

[22] Assuming $p(\pi)$ is linear makes the proof of this lemma tractable. A weakly concave function would make the second derivative ambiguous in sign without additional, difficult-to-interpret assumptions. This would disable a clean characterization of the conditions under which the function is negative quadratic over the specified range.

[23] Appendix Equation A.3 defines $\overline{\pi}_p$. The proof demonstrates that choosing $\pi > \overline{\pi}_p$ is never optimal.

[24] With $p(\pi)$, the inequality for opposition credibility must hold at $p(\pi) = p^{\text{min}}$.

[25] Recall that $c_0 = 0$, and thus the opposition surely mobilizes in period 0 when the ruler sets the power-sharing level.

The incentive-compatibility constraint for the ruler to share a positive level of power is

$$\underbrace{1-\delta r F(\hat{c}(\tilde{\pi}))c^{\text{avg}}(\hat{c}(\tilde{\pi}))-p(\tilde{\pi})(1-\mu)}_{\text{Share power and consume } R(\pi) \text{ (Eq. 4)}} \geq \underbrace{(1-p^{\text{min}})(1-\mu)}_{\text{Incur revolt}}, \tag{14}$$

for $\tilde{\pi}$ defined in Lemma 8. This inequality simplifies to

$$\textbf{Ruler willingness.} \quad \underbrace{\tilde{\pi}(p^{\text{max}}-p^{\text{min}})}_{\text{Threat-enhancing effect}}(1-\mu) \leq \underbrace{\mu-\delta r F(\hat{c}(\tilde{\pi}))c^{\text{avg}}(\hat{c}(\tilde{\pi}))}_{\text{Net surplus destroyed by revolt}}. \tag{15}$$

The main force that pushes toward ruler willingness holding is the (net) cost of conflict. As suggested by canonical results on conflict bargaining, more destructive conflict harms the ruler. By virtue of making all the bargaining offers and holding the opposition down to indifference, the ruler consumes the entire surplus saved by preventing a revolt. The costliness of peace induced by costly mobilization tempers this benefit, but it is nonetheless net positive because of Assumption 1.

However, despite this benefit of sharing power, the threat-enhancing effect can cause the ruler willingness condition to fail.[26] Upon sharing power, the ruler holds the opposition down to indifference only *after power has shifted in the opposition's favor*. Consequently, the ruler might prefer costly conflict over buying off a stronger opposition. Similar to a first-strike advantage, the ruler moves first and can induce a revolt (by setting $\pi = 0$) that the opposition wins with probability p^{min}, as opposed to sharing $\pi = \tilde{\pi}$ and buying off an opposition who wins with probability $p(\tilde{\pi})$.[27] Finally, as before, the level of basement spoils cancels out in equilibrium (Equation 4), and therefore does not affect ruler willingness.

The shift in the distribution of power implies that ruler willingness can fail even with no costs to mobilizing. Nonetheless, costly mobilization diminishes the relative benefits of a peaceful path, which undermines incentives for ruler willingness.

[26] The threat-enhancing term is multiplied by post-conflict surplus because this amount affects both players' reservation values to fighting.

[27] Powell (2006) conceptualizes first-strike advantages as a subset of conflicts triggered by commitment problems.

Lemma 9 (Costly mobilization and ruler willingness)**.** *Suppose $r < \overline{r}$ (see Lemma 6). The set of parameter values in which ruler willingness holds is smaller if mobilization is costly than if not.*

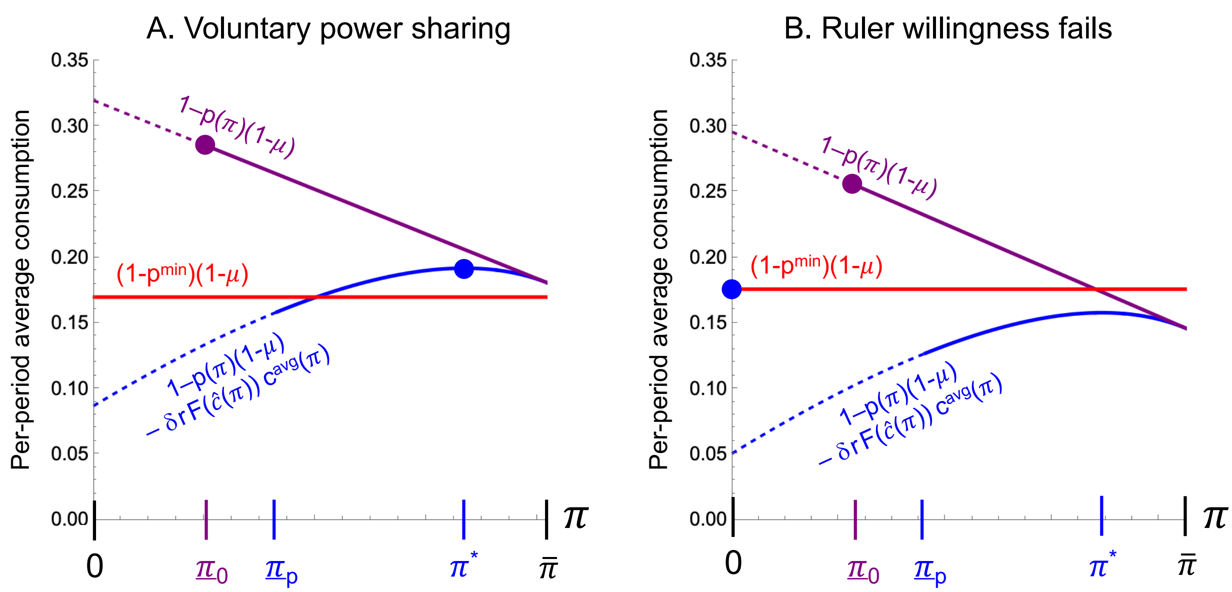
6.4 Equilibrium Power Sharing

Figure 3 illustrates how costly mobilization affects equilibrium power sharing. The blue curve represents the ruler's per-period average consumption along a peaceful path. The solid portion indicates parameter values in which $\pi > \underline{\pi}_p$, and thus the equilibrium path of play is peaceful for those values of π. By contrast, the dashed portions represent values of π in which conflict occurs. The purple curve conveys the same information for the ruler's consumption while setting $c^{\max} = 0$, and hence the costs of mobilization are 0. This is the conventional model in which the opposition costlessly mobilizes in every period that Nature permits. The red line is the ruler's consumption if $\pi = 0$ and a revolt occurs. The large dots indicate the ruler's equilibrium choice of power sharing $\tilde{\pi}$, depending on whether mobilization is costly (blue) or not (purple).

In both panels, the slopes of the blue and purple curves diverge. The purple curves slope downward because of the threat-enhancing effect. This force is also present in the blue curves, but lowering the costs of mobilization (see Equation 13) dominates this effect for most of the parameter values depicted. This creates the long segments with positive slopes.

In both panels, ruler willingness holds if mobilization is costless. Because the ruler's sole motive for sharing power is to prevent a revolt, he shares the minimum amount of power needed to achieve this objective. This is why the purple dots are located where the dashed curves turn to solid. In Panel A, ruler willingness also holds if mobilization is costly. And because the ruler faces incentives to lower the costs of mobilizing, rather than solely to prevent a revolt, the ruler sets a higher-than-needed level of power sharing located at the apex of the blue curve. By contrast, in Panel B, costly mobilization undermines ruler willingness. Despite the upward slope of the blue curve, the low absolute level of consumption along a peaceful path implies the ruler prefers to incur a revolt. Thus, the ruler shares no power—but he would have were mobilization not costly. Proposition 4 summarizes the equilibrium strategy profile and outcomes.

Figure 3: Costly mobilization and endogenous power sharing



Notes: $\delta = 0.7$, $p^{\min} = 0.8$, $p^{\max} = 1$, $r = 0.3$, $c^{\max} = 1$, $F \sim U[0, c^{\max}]$. In Panel A, $\mu = 0.12$. In Panel B, $\mu = 0.15$. Assumption 1 is met for all parameter values at which the blue curves coincide with a peaceful path of play. Proposition 2 and Equation 12 characterize $\underline{\pi}_p$, Lemma 8 characterizes π^*, the proof for Lemma 9 characterizes $\underline{\pi}_0$, and Appendix A.2 characterizes $\overline{\pi}$.

Proposition 4 (Equilibrium with endogenous power sharing)**.** *Suppose $r < \overline{r}$.*

- *If ruler willingness holds (Equation 15), then the ruler sets $\pi = \tilde{\pi}$.*[28] *In every period, the opposition mobilizes if and only if $c_t \leq x^*(\tilde{\pi})$. If the opposition mobilizes, the ruler offers $x_t = x^*(\tilde{\pi})$ and the opposition accepts if and only if $x_t \geq x^*(\tilde{\pi})$. Along the equilibrium path of play, revolts never occur.*
- *If ruler willingness fails, then the ruler sets $\pi = 0$. In every period, the opposition mobilizes if and only if $c_t \leq x^*(0)$. If the opposition mobilizes, the ruler offers any $x_t \in [0, 1 - \pi]$ and the opposition revolts in response to any offer. Along the equilibrium path of play, a revolt occurs during the first window of opportunity.*

[28] This power-sharing level maximizes the ruler's consumption along a peaceful path among all $\pi \geq \underline{\pi}_p$. The proof of Lemma 9 provides a formal characterization; see also Equation 14.

7 Conclusion

This paper embeds endogenous, costly mobilization into a model that otherwise resembles the canonical framework for analyzing commitment problems, conflict, and power sharing. I conclude by offering interpretations of each of the three main insights and highlight relevant considerations for future theoretical and empirical research.

First, the frequency with which windows of opportunity arise tells us little about prospects for conflict without knowing their underlying cause.[29] More stringent exogenous blocks against mobilization and higher levels of basement spoils each reduce the equilibrium frequency of windows of opportunity. However, the former stimulus raises prospects for conflict whereas the latter lowers them (see Proposition 3 and Figure 2). Moreover, lowering the endogenous mobilization threshold does not affect prospects for conflict (Lemma 4). This observation simplifies tasks in empirical research. Parameters such as the frequency of mobilization are notoriously difficult to measure.[30] However, endogenous drivers of windows opportunity should affect outcomes only through their direct effects, and thus the researcher needs only to measure more tangible indicators such as the amount and importance of cabinet positions that are distributed (e.g., Arriola 2009; Francois et al. 2015), not the opposition's ability to leverage such positions to create windows of opportunity.

Second, fully endogenous mobilization eliminates the ruler's commitment problem, even if the opposition lacks a basement level of spoils. The opposition can always choose to mobilize frequently enough to compel a sufficient amount of concessions. Consequently, the ruler's inability to *commit* to future transfers (independent of the opposition's revolt threat) is not *problematic* because the ruler can, nonetheless, peacefully buy off the opposition. Thus, the standard mechanism connecting commitment problems to conflict *requires* exogenous frictions that prevent the opposition from mobilizing in some periods (Proposition 1). Absent exogenous frictions, if conflict occurs in

[29] Specifically, "prospects for conflict" refers either to a revolt occurring along the equilibrium path or to the ruler needing to share power to prevent this outcome.

[30] For an exception that takes measurement of this parameter seriously, see Castañeda Dower et al.'s (2018) analysis Imperial Russia's Great Reforms in the 1860s.

equilibrium, it is because peace is costlier than conflict—*not* a commitment problem. This forces us to rethink the logic of commitment problems while also highlighting a previously unrecognized source of overlap with costly peace mechanisms for conflict.

Showcasing the centrality of exogenous frictions for the canonical commitment problem story prompts important questions about the empirical sources of exogenous frictions. Although the present model opens up one important black box, it maintains the standard assumption in this literature that the opposition is a unitary actor (or, equivalently, all opposition actors have identical preferences and are represented by a single agent). A rich literature examines problems of coordination and free riding that emerge when agents have divergent preferences and sources of information, and externalities are present.[31] Moreover, free-riding incentives and the difficulty of identifying focal points for coordination create a natural friction against anti-government mobilization. Authoritarian governments exacerbate this natural tension by engaging in varied forms of preventive repression to make it restrictively difficult for the opposition to organize (Ritter 2014; Dragu and Przeworski 2019). What may seem like a contented population might occasionally rise in unexpected bursts of revolutionary dissent (Kuran 1991). Such microfoundations are crucial because an exogenous block against the opposition mobilizing is a *necessary friction* for limited commitment ability to trigger conflict or compel institutional reform. Thus, although the collective action literature has stood largely separate from bargaining models of conflict,[32] the present model offers one step in the direction of combining these literatures. Enriching the model with coordination problems for the opposition could represent an important avenue for future theoretical advances.

Third, costly mobilization alters the ruler's power-sharing calculus. Existing explanations for institutional concessions typically divide into either bottom-up or top-down approaches. The present model highlights an important way in which these two intersect. The ruler faces bottom-up pressure from the opposition, who can mobilize threats of revolt. However, if mobilization is fully

[31] For a recent review of formal models in this vast literature, see Paine and Tyson (2024).

[32] For an exception, see Chassang and Padro-i Miquel (2009).

endogenous and peace is not (net) costly, anti-regime threats will never manifest into actual revolts. Nonetheless, the ruler faces a previously unrecognized incentive. Sharing power reduces the frequency of windows of opportunity, which raises total societal surplus—consumed by the ruler. This mechanism motivates the ruler to share power to prevent costly windows of opportunity from arising, even if the consequent threats would not topple the regime. Thus, what may appear to be purely voluntarily (top-down) transitions may in fact have a latent bottom-up motive that manifests only off the equilibrium path. These types of insights should help to motivate further theoretical development of the interaction between bottom-up and top-down motives, and provide novel interpretations of empirical cases.

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Online Appendix for *The Autocrat's Commitment Problem with Endogenous Windows of Opportunity*

A Online Appendix

A.1 Summary of Parameters and Thresholds

Primitives

- R: Ruler
- O: Opposition
- V_R: Ruler's continuation value
- V_O: Opposition's continuation value

Parameters and choice variables

- t: Time indicator
- δ: Discount factor
- c_t: Cost of mobilizing
- $c^{\max}$: Maximum cost of mobilizing
- F: Distribution function for cost of mobilizing
- $c^{\text{avg}}(z)$: Average cost paid in a period with a window of opportunity if the opposition mobilizes whenever $c_t \leq z$
- $1 - r$: Frequency with which opposition is exogenously blocked from mobilizing
- π: Basement spoils for opposition
- x_t: Temporary transfer
- μ: Destructiveness of a revolt
- p: Opposition's probability of winning a revolt; exogenous parameter in baseline model and equals $p(\pi) = (1 - \pi)p^{\min} + \pi p^{\max}$ in power-sharing extension
- $p^{\max}$: Opposition's maximum probability of winning a revolt (power-sharing extension)
- $p^{\min}$: Opposition's minimum probability of winning a revolt (power-sharing extension)
- $\Delta p(\pi) = \pi(p^{\max} - p^{\min})$: Magnitude of the threat-enhancing effect (power-sharing extension)

Threshold values

- $\hat{c}$: Endogenous mobilization threshold such that opposition mobilizes if and only if $c_t \leq \hat{c}$; see Equation 9
- $rF(\hat{c})$: Equilibrium frequency of windows of opportunity
- x^*: Interior optimal transfer; see Equations 2 and 3
- $\overline{r}$: Minimum value of r at which peaceful bargaining is possible (if $\pi = 0$); see Proposition 1
- $\underline{\pi}$: Minimum value of π at which peaceful bargaining is possible (if $r < \overline{r}$); see Proposition 2
- $\overline{\pi}$: Minimum value of π at which interior-optimal transfer is non-positive; see Appendix A.2
- $\underline{\pi}_p$: Analog of $\underline{\pi}$ for the power-sharing extension; see Equation 12

A.2 Corner Solution for Equilibrium Transfer

In the baseline model with exogenous π, the analysis in the paper assumes a low-enough value of π that the interior-optimal transfer is positive, $x^* > 0$ (see Equation 3). However, large-enough π yields $x^* < 0$. In the latter circumstance, the non-negativity constraint implies the equilibrium transfer is 0. The threshold is $\overline{\pi} = p(1-\mu)$, characterized below in Lemma A.1. This threshold is intuitive: if the opposition's consumption in every period is at least as great as its per-period average expected utility to revolting, then the ruler does not need to offer an additional transfer to secure peaceful bargaining. This, in turn, ensures that the opposition never mobilizes.

The bargaining interaction can differ when allowing all values $\pi \in [0,1]$. If π is high enough that $1-\pi < (1-p)(1-\mu) \implies \pi > p(1-\mu)+\mu$, then the ruler prefers to incur a revolt rather than peacefully consume $1-\pi$. Thus, similar to how the opposition's reservation value to revolting forms a lower bound for its payoffs (Equation 1), the ruler's reservation value to revolting does the same, at least after modifying the baseline model as follows. Suppose the ruler has an additional strategic option in the stage game to provoke a revolt (e.g., commit an atrocity or attempt to directly govern the opposition's territory in a manner that would necessarily provoke armed resistance). He would exercise this option if $\pi > p(1-\mu)+\mu$. Furthermore, the additional strategic option is necessary to trigger a revolt because, for any $\pi > p(1-\mu)$, the opposition accepts any temporary transfer.

Lemma A.1 (Corner solution for optimal transfer)**.** *A unique threshold $\overline{\pi} \in (0,1)$ exists such that*

$$x^* \begin{cases} > 0 & \text{if } \pi < \overline{\pi} \\ = 0 & \text{if } \pi = \overline{\pi} \\ < 0 & \text{if } \pi > \overline{\pi}, \end{cases}$$

for x^* defined in Equation 3. The explicit characterization is $\overline{\pi} = p(1-\mu)$.

Proof. Rearranging Equation 3 yields an implicit characterization of $\overline{\pi}$

$$\overline{\pi} - p(1-\mu) - \delta r \int_0^{\hat{c}(\overline{\pi})} c_t dF(c) = 0.$$

Given Lemma 3, $x^*(\overline{\pi}) = 0$ implies $\hat{c}(\overline{\pi}) = 0$, and therefore the integral term in the preceding expression equals 0. This leaves $\overline{\pi} = p(1-\mu)$, which is unique and satisfies the bounds $\overline{\pi} \in (0,1)$. ■

Corollary A.1 (Corner solution for endogenous mobilization threshold)**.**

If $\pi \geq p(1-\mu)$, then $\hat{c} = 0$.

A.3 Proofs for Baseline Model

The following presents proofs for all formal statements that do not follow immediately from the surrounding text in the paper.

Proof of Lemma 3. Starting with Equation 9, substituting in V_O, and multiplying through by $1-\delta$ yields

$$-(1-\delta)\hat{c}+p(1-\mu)=\pi+\delta rF(\hat{c})(x^*-c^{\text{avg}}(\hat{c})).$$

Slightly rearranging and dividing both sides by $1-\delta(1-rF(\hat{c}))$ yields

$$\underbrace{\frac{-\pi+p(1-\mu)+\delta rF(\hat{c})c^{\text{avg}}(\hat{c})}{1-\delta(1-rF(\hat{c}))}}_{x^*}=\frac{(1-\delta)\hat{c}+\delta rF(\hat{c})x^*}{1-\delta(1-rF(\hat{c}))}.$$

Further reducing this expression leads easily to $\hat{c}=x^*$. ■

Proof of Proposition 1. At the upper bound, $\Theta^*(0,1)=1-\delta c^{\text{avg}}-p(1-\mu)>0$ follows from Assumption 1. The unique threshold follows from strict monotonicity: $\frac{d\Theta^*(0,\overline{r})}{dr}=\delta(1-c^{\text{avg}})>0$. This can also be written as

$$\overline{r}=\frac{p(1-\mu)-(1-\delta)}{\delta(1-c^{\text{avg}}(\hat{c}(\overline{r})))}. \tag{A.1}$$

■

Proof of Proposition 2. Applying the intermediate value theorem establishes existence

- Lower bound: $\Theta^*(0,r)<0$. Follows from the present assumption $r<\overline{r}$.
- Upper bound

  $$\Theta^*(\overline{\pi},r)=(1-\delta(1-rF(1-\overline{\pi})))(1-\overline{\pi})+\underbrace{\overline{\pi}-p(1-\mu)-\delta r\int_0^{\hat{c}(\overline{\pi})}c_t dF(c)}_{=0\ (\text{Lemma A.1})}>0.$$

- Each constituent term in $\Theta^*(\pi,r)$ containing π is continuous, and therefore the overall term is continuous.

Uniqueness follows from strict monotonicity

$$\frac{d\Theta^*(\pi,r)}{d\pi}=\delta(1-rF(1-\pi))\underbrace{\overbrace{-\delta rf(1-\pi)(1-\pi)}^{\text{Less frequent mobilization}}+\overbrace{\delta rf(1-\pi)(1-\pi)}^{\text{Lower avg. cost of mobilizing}}}_{=0}>0. \tag{A.2}$$

■

Proof of Proposition 3, part a.

$$-\frac{d}{d\hat{c}}\big(rF(\hat{c})\big) = -rf(\hat{c}) < 0.$$

Proof of part b.

$$-\frac{d}{dr}\big(rF(\hat{c})\big) = -F(\hat{c}) - rf(\hat{c})\frac{d\hat{c}}{dr},$$

$$\text{for} \quad \frac{d\hat{c}}{dr} = -\frac{\delta F(\hat{c})}{1-\delta(1-rF(\hat{c}))}(x^* - c^{\text{avg}}(\hat{c})) < 0$$

$$\Longrightarrow \; -\frac{F(\hat{c})}{1-\delta(1-rF(\hat{c}))}\left(1-\delta+\delta rf(\hat{c})c^{\text{avg}} + \delta r\big(F(\hat{c}) - f(\hat{c})\underbrace{x^*}_{\hat{c}}\,)\right).$$

To prove the claim, it suffices to demonstrate $F(z) - f(z)z \geq 0$ for any z within the support. This left-hand side reaches its lower bound at $z = 0$ because $\frac{d}{dz}\big(F(z) - f(z)z\big) = -zf'(z) \geq 0$, which incorporates the assumption $f'(z) \leq 0$. But $F(0) - f(0) \cdot 0 = 0$, which satisfies the inequality.

Proof of part c.

$$\frac{d}{d\pi}\big(rF(\hat{c})\big) = rf(\hat{c})\underbrace{\left(\frac{-1}{1-\delta(1-rF(\hat{c}))}\right)}_{\frac{d\hat{c}}{d\pi}} < 0.$$

■

A.4 Proofs for Strategic Power Sharing

In the extension with strategic power sharing, the constant p is replaced with a function $p(\pi) = (1-\pi)p^{\text{min}} + \pi p^{\text{max}}$. Consequently, the thresholds $\underline{\pi}$ (minimum power-sharing level that enables peace) and $\overline{\pi}$ (minimum power-sharing level at which the interior-optimal transfer equals 0) require modification. For the latter, replace the expression from Lemma A.1 with

$$\overline{\pi}_p = ((1-\overline{\pi}_p)p^{\text{min}} + \overline{\pi}_p p^{\text{max}})(1-\mu), \tag{A.3}$$

which solves explicitly to $\overline{\pi}_p = \frac{p^{\text{min}}(1-\mu)}{1-(p^{\text{max}}-p^{\text{min}})(1-\mu)}$. It is straightforward to verify that this term is unique and satisfies the bounds $\overline{\pi}_p \in (0,1)$.

Equation 12 presents a new version of $\underline{\pi}$, now denoted as $\underline{\pi}_p$. The proof of uniqueness for $\underline{\pi}$ in Proposition 2 is no longer valid because Equation A.2 does not account for the effect of π on $p(\pi)$. The following lemma establishes the same strictly monotonic relationship even when accounting for the additional effect of π on $p(\pi)$, which enables applying the proof for Proposition 2 to establish the uniqueness of $\underline{\pi}_p$.

Lemma A.2 (Higher π relaxes the no-revolt constraint)**.** *If $r < \overline{r}$, then $\frac{d\Theta^*(\pi,r)}{d\pi} > 0$, for $\Theta^*(\pi,r)$ introduced in Equation 12.*

Proof of Lemma A.2.

$$\frac{d\Theta^*(\pi,r)}{d\pi} = \delta(1 - rF(1-\pi)) - \underbrace{(p^{\text{max}} - p^{\text{min}})}_{p'(\pi)}(1-\mu)$$

This term reaches a lower bound at $r = \overline{r}$ and $\pi = 0$. Thus, substituting in $\overline{r}$ from Equation A.1, it suffices to establish

$$\delta\Bigg(1 - \underbrace{\frac{p^{\text{min}}(1-\mu) - (1-\delta)}{\delta(1 - c^{\text{avg}}(\hat{c}(\overline{r})))}}_{\overline{r}}\Bigg) > (p^{\text{max}} - p^{\text{min}})(1-\mu).$$

Algebraic rearranging yields

$$\frac{1-\delta c^{\text{avg}}(\hat{c}(\overline{r}))}{1-\mu} > c^{\text{avg}}(\hat{c}(\overline{r}))p^{\text{min}} + (1 - c^{\text{avg}}(\hat{c}(\overline{r})))p^{\text{max}}. \tag{A.4}$$

Because $c^{\text{avg}}(\hat{c}(\overline{r})) < c^{\text{avg}}$, incorporating Assumption 1 yields $\frac{1-\delta c^{\text{avg}}(\hat{c}(\overline{r}))}{1-\mu} > \frac{1-\delta c^{\text{avg}}}{1-\mu} > 1$. Thus, it suffices to prove $c^{\text{avg}}(\hat{c}(\overline{r}))p^{\text{min}} + (1 - c^{\text{avg}}(\hat{c}(\overline{r})))p^{\text{max}} \leq 1$. Because $p^{\text{max}} > p^{\text{min}}$ and $c^{\text{avg}}(\hat{c}(\overline{r})) \leq 1$, the maximum value of the expression is $p^{\text{max}} \leq 1$, and thus Equation A.4 is true. ∎

Proof of Lemma 8, Step 1. For all $\pi \in [0, \overline{\pi}_p]$, $R(\pi)$ from Equation 4 characterizes the ruler's payoff. Demonstrating $\frac{d^2R(\pi)}{d\pi^2} < 0$ proves (generically) that the maximizer is unique and limits the set of possible maximizers over the domain $[0, \overline{\pi}_p]$ to the set $\{0, \pi^*, \overline{\pi}_p\}$, where π^* is an interior maximizer characterized below in Equation A.5.

$$\frac{d^2R(\pi)}{d\pi^2} = -\delta r f(\hat{c})\Big(\frac{d\hat{c}}{d\pi}\Big)^2 - \delta r\hat{c}\underbrace{f'(\hat{c})}_{=0}\Big(\frac{d\hat{c}}{d\pi}\Big)^2 - \delta r\hat{c}f(\hat{c})\frac{d^2\hat{c}}{d\pi^2} - \underbrace{p''(\pi)}_{=0}(1-\mu),$$

$$\text{with} \quad \frac{d^2\hat{c}}{d\pi^2} = \frac{1}{1-\delta(1-rF(\hat{c}))}\Bigg(\underbrace{p''(\pi)}_{=0}(1-\mu) - \delta r f(\hat{c})\Big(\frac{d\hat{c}}{d\pi}\Big)^2\Bigg),$$

where $f'(\hat{c}) = 0$ follows from assuming F is uniform and $p''(\pi) = 0$ follows from the linear functional form. *NB: This is the only part of any proof in which the uniform assumption, and hence $f'(z) = 0$, is used; all others require only $f'(z) \leq 0$ for all z within the support.*

The entire expression simplifies to

$$-\delta r f(\hat{c})\Big(\frac{d\hat{c}}{d\pi}\Big)^2\Bigg(1 - \frac{\delta r\hat{c}f(\hat{c})}{1-\delta(1-rF(\hat{c}))}\Bigg).$$

It suffices to demonstrate

$$1 > \frac{\delta r\hat{c}f(\hat{c})}{1-\delta(1-rF(\hat{c}))} \implies 1 - \delta + \delta r\big(F(\hat{c}) - \hat{c}f(\hat{c})\big) > 0.$$

The proof of part b of Proposition 3 demonstrated that $f'(z) \leq 0$ implies $F(z) - f(z)z \geq 0$ for any z within the support, which proves the claim.

Step 2. At the upper bound $\pi = \overline{\pi}_p$, $\hat{c}(\overline{\pi}_p) = 0$. This is not a maximizer because

$$\frac{dR(\pi)}{d\pi}\bigg|_{\pi=\overline{\pi}_p} = -(p^{\text{max}} - p^{\text{min}})(1-\mu) < 0.$$

Step 3. The first of the following two expressions establishes the lower bound for $\hat{p}^{\text{max}}$ at p^{min}, and the second establishes uniqueness.

$$\frac{dR(\pi)}{d\pi}\bigg|_{\pi=0, p^{\text{max}}=p^{\text{min}}} = \frac{\delta r\hat{c}f(\hat{c})}{1-\delta(1-rF(\hat{c}))} > 0$$

$$\frac{d^2R(\pi)}{d\pi dp^{\text{max}}}\bigg|_{\pi=0} = -(1-\mu)\Bigg(1 + \frac{\delta r\hat{c}f(\hat{c})}{1-\delta(1-rF(\hat{c}))}\Bigg) < 0.$$

Step 4. The unique maximizer is the interior value π^* that satisfies

$$\frac{Z(\pi^*)}{1+Z(\pi^*)} = (p^{\text{max}} - p^{\text{min}})(1-\mu), \quad \text{for} \quad Z(\pi^*) \equiv \frac{\delta r\int_0^{\hat{c}(\pi^*)} c_t dF(c)}{1-\delta(1-rF(\hat{c}(\pi^*)))}. \tag{A.5}$$

Step 5. No choice $\pi > \overline{\pi}_p$ maximizes the ruler's payoff. Over this domain, his per-period consumption equals $1-\pi$, which strictly decreases in π. Finally, to rule out jumps at $\overline{\pi}_p$, the ruler's utility is continuous at $\pi = \overline{\pi}_p$ because

$$\lim_{\pi \to \overline{\pi}_p^+} 1-\pi = 1-\overline{\pi}_p = 1 - \underbrace{\lim_{\pi \to \overline{\pi}_p^-} p(\pi)(1-\pi)}_{=\overline{\pi}_p} - \underbrace{\lim_{\pi \to \overline{\pi}_p^-} \delta r \int_0^{\hat{c}(\pi)} c_t dF(c)}_{=0} = \lim_{\pi \to \overline{\pi}_p^-} R(\pi).$$

■

Proof of Lemma 9, Step 1. Optimal power sharing with costless mobilization. If mobilization is costless, then the ruler's consumption strictly decreases in π. Thus, the maximizer for $R(\pi)$ is the level that makes the opposition indifferent between accepting and revolting,

$$\underline{\pi}_0 + \big(1-\delta(1-rF(1-\underline{\pi}_0))\big)(1-\underline{\pi}_0) - p(\underline{\pi}_0)(1-\mu) = 0. \tag{A.6}$$

The same proof as for Proposition 2 establishes existence/uniqueness for $\underline{\pi}_0$.

Step 2. Optimal power sharing with costly mobilization. Lemma 8 established $\tilde{\pi} \in \{0, \pi^*\}$, and a peaceful path requires $\pi \geq \underline{\pi}_p$. Thus, the set of possible optimal choices of π along a peaceful path is $\{\underline{\pi}_p, \pi^*\}$. If $\pi^* < \underline{\pi}_p$, then setting $\pi = \pi^*$ does not yield a peaceful path, which makes this value irrelevant for assessing ruler willingness. Thus, without loss of generality, in the following, assume $\pi^* \geq \underline{\pi}_p$.

Step 3. Prove the lemma. The needed inequality is

$$\underbrace{\mu - \underline{\pi}_0(p^{\max} - p^{\min})(1-\mu)}_{\text{Costless mobilization}} > \underbrace{\mu - \delta r \int_0^{\hat{c}(\tilde{\pi})} c_t dF(c) - \tilde{\pi}(p^{\max} - p^{\min})(1-\mu)}_{\text{Costly mobilization}}.$$

This easily rearranges to

$$(\tilde{\pi} - \underline{\pi}_0)(p^{\max} - p^{\min})(1-\mu) + \delta r \int_0^{\hat{c}(\tilde{\pi})} c_t dF(c) > 0.$$

Thus, it suffices to demonstrate $\tilde{\pi} > \underline{\pi}_0$; and, because of Step 2, $\underline{\pi}_p > \underline{\pi}_0$ suffices to establish that inequality. The implicit definitions of each are presented in Equations 12 and A.6. Setting the left-hand side of each of those equations equal to each other and rearranging yields

$$\underline{\pi}_p + \big(1-\delta(1-rF(1-\underline{\pi}_p))\big)(1-\underline{\pi}_p) - p(\underline{\pi}_p)(1-\mu)$$
$$-\Big(\underline{\pi}_0 + \big(1-\delta(1-rF(1-\underline{\pi}_0))\big)(1-\underline{\pi}_0) - p(\underline{\pi}_0)(1-\mu)\Big) = \delta r \int_0^{1-\underline{\pi}_p} c_t dF(c).$$

The inequality $\underline{\pi}_p > \underline{\pi}_0$ follows from $\frac{d}{d\pi}\Big(\pi + \big(1-\delta(1-rF(1-\pi))\big)(1-\pi) - p(\pi)(1-\mu)\Big) > 0$ and $\delta r \int_0^{1-\underline{\pi}_p} c_t dF(c) > 0$. ■

A.5 Extension: Continuous Distribution of Threats

Most models assume a binary distribution of threats (see Little and Paine 2024 for an exception). I adopt this convention in the baseline model by assuming the opposition either mobilizes and wins a revolt with probability p, or refrains from mobilizing and wins (implicitly) with probability 0. This setup is less restrictive than it may seem. The following shows how the binary structure can be recovered endogenously when allowing the opposition to choose from a continuum of mobilization levels.

Assume that, in any period, the opposition's mobilization decision entails choosing a probability of winning $p_t \in [0, \overline{p}]$, for an upper bound $\overline{p} \leq 1$. Achieving a higher probability of winning requires the opposition to pay a higher cost, but the distribution of costs varies across periods depending on a Nature draw. Formally, the cost is $\kappa(p_t, c_t) = c_t \frac{p_t}{\overline{p}}$, with c_t drawn from the same distribution F as in the baseline model. By construction, $\kappa(0, c_t) = 0$ and $\kappa(\overline{p}, c_t) = c_t$.

In equilibrium, in each period, the opposition chooses either $p_t = 0$ or $p_t = \overline{p}$, as in the baseline model. The characterization of the optimal transfer (Equation 3) is unchanged, except replacing p with p_t and explicitly writing the transfer as a function of the contemporaneous probability of winning, $x^*(p_t)$. Given this, the opposition's optimal mobilization choice solves

$$\max_{p_t \in [0, \overline{p}]} \quad - c_t \frac{p_t}{\overline{p}} + \underbrace{p_t \frac{1-\mu}{1-\delta}}_{\pi + x^*(p_t) + \delta V_O} .$$

This objective function is linear, and therefore is maximized either at the lower bound (no mobilization) or upper bound (full mobilization). Moreover, the threshold at which the opposition chooses full mobilization is identical to Equation 9 in the baseline game. Thus, allowing for endogenous mobilization choices is the key alteration relative to existing models, whereas a continuous distribution of threats does not qualitatively change the insights.