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WINDMILLING CHARACTERISTICS OF PROPELLERS

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FOREWORD

The study reported here was carried out in order to provide necessary information for a proposed experimental water tunnel program and also to permit estimates of drag on full-scale torpedoes when propellers are used as variable drag devices.

The work was performed under the Exploratory and Foundational Research fund title, "Propeller-Stabilized and Controlled Torpedoes," NOTS Local Project 701.

The analytical work presented in the report has not been reviewed outside the Guidance and Control Division. It is hoped that an experimental program in the near future will establish the validity of the analysis.

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ABSTRACT

A simplified method for obtaining the characteristics of windmilling propellers is presented. The equations are equally valid for freewheeling propellers or propellers used as power generators. Once the propeller geometry is known, such characteristics as the drag, torque, slip, horsepower output, and local blade angles of attack can be calculated as outlined.

CONTENTS

INTRODUCTION

Water-tunnel torpedo models are sometimes fitted with windmilling propellers rather than powered propellers for economic reasons. Windmilling propellers may also be used as power generators. In either case, it is necessary to evaluate such characteristics as the drag, torque, slip, horsepower output, and blade angles of attack. This report presents a quick method for estimating these characteristics for engineering purposes.

SYMBOLS AND DEFINITIONS

The following notation is as consistent as possible with both torpedo and aircraft nomenclature.

Æ	Aspect ratio of two opposing blades, d/c
c	Actual chord-length of blade, ft
C_{L_α}	Average lift coefficient derivative over complete wing
$C_{L_{\alpha_X}}$	Lift coefficient derivative at point X
C_d	Drag coefficient of propeller blade
d	Propeller diameter, ft
D	Propeller drag of each propeller, lb
D_s	Drag due to propeller slip, lb
D_d	Drag due to blade-section drag, lb
J	Advance ratio at zero slip, $\frac{V_o}{nd}$
K	Torque (+) clockwise (looking forward), ft-lb

K_s Torque due to propeller slip, ft-lb

K_d Torque due to blade-section drag, ft-lb

K'_{ϕ_o} Torque coefficient of plane wing due to unit rotation

K_b Bearing friction torque (always +), ft-lb

K_o Output torque (+) for positive output power

L_x Lift of a unit blade element at Station X, lb

N Number of blades per propeller

n Rotational speed at zero slip, rps

q_o Free-stream dynamic pressure $(\rho/2V_o^2)$, lb/ft^2

q_x Dynamic pressure of blade at Station X, lb/ft^2

R Propeller radius (d/2), ft

X Ratio of local radius to propeller radius

V_o Free stream velocity, fps

V_x Local blade velocity at Station X, fps

α Angle of attack, rad

α_x Local blade angle of attack, rad

β Local blade-pitch angle at zero lift, rad

β_o Effective average blade-pitch angle, rad

ρ Density of the fluid medium, slugs/ft^3

$\dot{\phi}$ Change in rotational speed from the zero slip condition (referred to as "slip"), rad/sec

ω Rotational speed for the zero slip condition where the propeller blade angle of attack is zero degree, (rad/sec)

$\frac{\dot{\phi}}{\omega}$ Slip ratio, ratio of change in rotation to the rotational speed at the zero slip advance ratio

EFFECT OF SLIP ON PROPELLER CHARACTERISTICS

To obtain the net propeller characteristics, it is convenient to treat the slip effects and the viscous blade-drag effects separately, and then superimpose the results. Consequently, the frictionless case will be considered first, where a slip velocity $\dot{\phi}$ is applied to the propeller. The problem is to obtain the drag, torque, and blade angle of attack as a function of slip.

It will be shown later that the propeller torque characteristics can be related to the wing-damping torque of a straight rectangular wing. Since this problem is basic, the case of a rotating straight wing (Fig. 1) will be considered first.

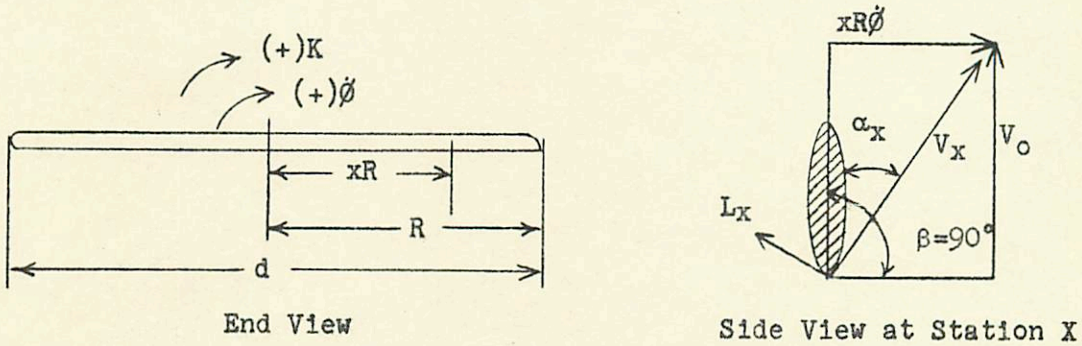

FIG. 1. Velocity Diagram of a Rotating Wing.

The local wing angle of attack in Fig. 1 is

$$\alpha_x \approx \tan \alpha_x = \frac{xR\dot{\phi}}{V_o}$$

since

$$\alpha_X << 1$$

The local dynamic pressure is then

$$q_X = 1/2\rho(V_o^2 + X^2R^2\dot{\phi}^2) = q_o\left[1 + \left(\frac{XR\dot{\phi}}{V_o}\right)^2\right] \approx q_o(1 + \alpha_X^2)$$

where

$$q_o = 1/2\rho V_o^2$$

Therefore, $q_X \approx q_o$ since $\alpha_X << 1$. The local blade lift, L_X, according to thin airfoil theory is

$$L_X = C_{L_{\alpha_X}} \cdot \alpha_X \cdot c \cdot RdX \cdot q_X$$

The total roll torque can be obtained by integration as follows:

$$K_{s_o} = -N\int_o^1 C_{L_{\alpha_X}}\alpha_X cRdXq_X \cdot XR$$

Substituting and simplifying,

$$K_{s_o} = \frac{-NcR^3q_o\dot{\phi}}{V_o}\int_o^1 C_{L_{\alpha_X}}X^2d_X$$

The damping torque of rectangular wings has been shown to be[1]

$$K_{s_o} = -\left(\frac{NcR^3q_o\dot{\phi}}{V_o}\right)\frac{(\pi/8)\,A\!\!R}{\sqrt{A\!\!R^2/16 + 1} + 1}$$

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[1]National Advisory Committee on Aeronautics. A Simple Method of Estimating the Subsonic Lift and Damping Roll of Sweptback Wings, by Edward C. Polhamus. Washington, NACA, April 1949. (NACA Technical Note 1862.) Nomenclature has been adapted to this report.

where

$$Æ = \frac{d}{c} = \frac{2R}{c}$$

Therefore, the value of the integral is

$$\int_0^1 C_{L_{\alpha_X}} X^2 dX = \frac{(\pi/8)\,Æ}{\sqrt{Æ^2/16 + 1} + 1}$$

Substituting $Æ = 2R/c$ and simplifying,

$$\boxed{K_{s_o} = -\frac{\rho \pi N R^4 V_o \dot{\phi}}{8(\sqrt{R^2/4c^2 + 1} + 1)}}$$

The damping torque of a propeller blade can be treated in a similar manner. Figure 2 is the simplified velocity diagram of a rotating blade having a constant pitch where β is the local pitch angle of the zero-lift chord line.

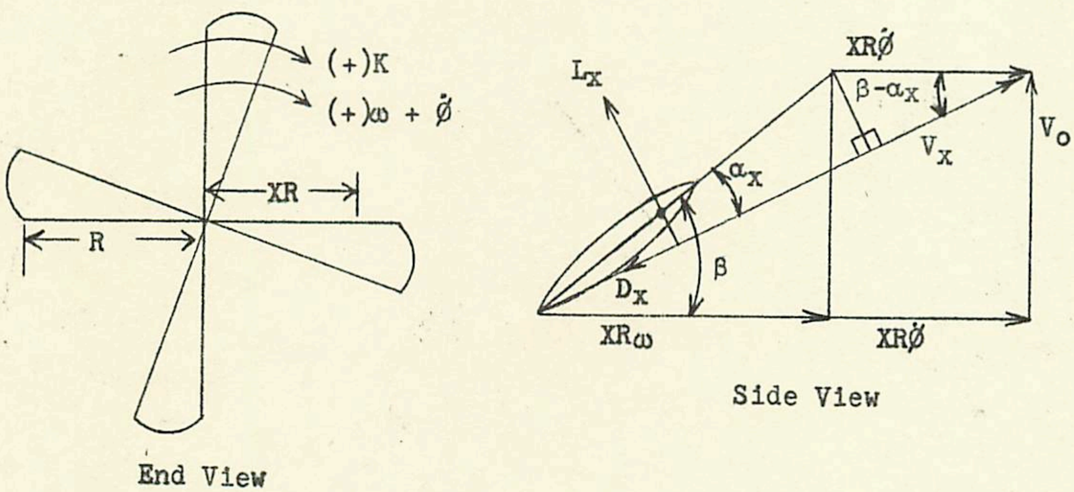

FIG. 2. Rotation of a Set of Propeller Blades.

Since ω is defined as the rotational velocity at zero slip, the local blade angle of attack, α_X, must then be a function of the slip, $\dot{\phi}$, as shown in Fig. 2.

$$\alpha_X \approx \sin \alpha_X \approx \frac{XR\dot{\phi} \sin (\beta - \alpha_X)}{XR\omega/\cos \beta} \doteq \frac{\dot{\phi}}{\omega} \sin \beta \cos \beta$$

The local dynamic pressure is

$$q_X = \frac{\rho}{2} V_X^2 = \frac{1/2 \rho V_o^2}{\sin^2 (\beta - \alpha_X)} \approx \frac{q_o}{\sin^2 \beta}$$

The local blade lift by thin airfoil theory is

$$L_X = C_{L_{\alpha_X}} \alpha_X \cdot cRdXq_X$$

The torque is therefore

$$K_s = -N \int_0^1 C_{L_{\alpha_X}} \cdot \alpha_X \cdot cRdXq_X RX \sin \beta$$

Substituting in α_X and q_X:

$$K_s = -NcR^2 q_o \dot{\phi} \int_0^1 C_{L_{\alpha_X}} \frac{\cos \beta}{\omega} X \, dX$$

since

$$\tan \beta = \frac{V_o}{XR\omega}$$

$$= \frac{V_o}{XR \tan \beta} = \frac{V_o \cos \beta}{XR \sin \beta}$$

Substituting ω:

$$K_s = -\frac{NcR^3 q_o \dot{\phi}}{V_o} \int_o^1 C_{L_{\alpha_x}} X^2 \sin \beta \, dX$$

With the exception of the sin β term, this expression for K_s is exactly equal to the integral expression for K_{so}. Therefore, K_s can be expressed as follows:

$$\boxed{K_s = K_{so} \sin \beta_o}$$

where β_o is some equivalent average blade-pitch angle, which for engineering purposes is approximated as follows (from Fig. 2):

$$\sin \beta_x = \frac{1}{\sqrt{1 + (x^2 \pi^2)/J^2}}$$

In propeller equations, X = 0.7 is often used as a typical section. Therefore,

$$\sin \beta_o \approx \frac{1}{\sqrt{1 + (\pi^2)/2J^2}}$$

The propeller drag caused by the slip, $\dot{\phi}$, at each section is

$$D_{x_s} = -C_{L_{\alpha_x}} \cdot \alpha_x \cdot cRdXq_x \cos \beta$$

The net drag becomes

$$D_s = -N \int_o^1 C_{L_{\alpha_x}} \alpha_x c R q_x \cos \beta \, dX$$

Substituting α_X and q_X

$$D_s = -NcRq_o\dot{\phi}\int_o^1 C_{L_{\alpha_X}} \frac{\cos^2 \beta \, dX}{\omega \sin \beta}$$

Since from Fig. 2,

$$\frac{\cos \beta}{\sin \beta} = \left(\frac{XR\omega}{V_o}\right)^2$$

so

$$D_s = -\frac{NcR^3 q_o \dot{\phi}\omega}{V_o^2}\int_o^1 C_{L_{\alpha_X}} X^2 \sin \beta \, dX$$

By comparing this integral with that for K_s, it is seen that

$$D_s = \frac{\omega}{V_o} K_s$$

since

$$J = \frac{V_o}{nd} = \frac{2\pi V_o}{\omega 2R} = \frac{\pi V_o}{\omega R}$$

then

$$\frac{\omega}{V_o} = \frac{\pi}{JR}$$

$$\boxed{D_s = \frac{\pi}{JR} K_s = \frac{\pi}{JR} K_{so} \sin \beta_o}$$

BLADE VISCOUS DRAG EFFECTS

It is seen from Fig. 2 that the viscous drag component of the blade forces at each section will contribute to the net propeller torque and drag. Assuming a constant blade-drag coefficient of C_d, the torque due to viscous drag becomes

$$K_d = -N\int_0^1 C_d cRdXq_x XR \cos\beta$$

or

$$K_d = -NC_d cR^2 q_o \int_0^1 \frac{X\cos\beta}{\sin^2\beta}\, dX$$

Since, from Fig. 2,

$$\cos\beta = \frac{RX\omega}{V_o}\sin\beta$$

and

$$\sin\beta = \frac{V_o}{\sqrt{V_o^2 + X^2R^2\omega^2}}$$

Substituting,

$$K_d = -NC_d cR^2 q_o \int_0^1 \frac{RX^2\omega}{V_o^2}\sqrt{V_o^2 + X^2R^2\omega^2}\, dX$$

or

$$K_d = -\frac{NC_d cR^4\omega^2 q_o}{V_o^2}\int_0^1 X^2\sqrt{\frac{V_o^2}{R^2\omega^2} + X^2}\, dX$$

Upon integration,

$$K_d = -\frac{NC_d cR^4\omega^2 q_o}{V_o^2}\left[1/4\sqrt{\left(1+\frac{V_o^2}{R^2\omega^2}\right)^3} - \frac{V_o^2}{8R^2\omega^2}\sqrt{1+\frac{V_o^2}{R^2\omega^2}}\right.$$

$$\left. - \frac{V_o^4}{8R^4\omega^4}\log\left(1+\sqrt{1+\frac{V_o^2}{R^2\omega^2}}\right) + \frac{V_o^4}{8R^4\omega^4}\log\left(\frac{V_o}{R\omega}\right)\right]$$

Simplifying,

$$K_d = -\frac{NC_d cR^4\omega^2 q_o}{8V_o^2}\left[2\left(1+\frac{V_o^2}{R^2\omega^2}\right)^{3/2} - \frac{V_o^2\left(1+\frac{V_o^2}{R^2\omega^2}\right)^{1/2}}{R^2\omega^2}\right.$$

$$\left. - \frac{V_o^4}{R^4\omega^4}\log\left(\frac{R\omega}{V_o}+\sqrt{\frac{R^2\omega^2}{V_o^2}+1}\right)\right]$$

The propeller drag caused by the blade viscous drag is

$$D_d = N\int_o^1 C_d cRdXq_x \sin\beta$$

or

$$D_d = NC_d cRq_o\int_o^1 \frac{R\omega}{V_o}\sqrt{\frac{V_o^2}{R^2\omega^2}+X^2}\,dX$$

Integrating,

$$D_d = \frac{NC_d cR^2\omega q_o}{2V_o}\left[\sqrt{\frac{V_o^2}{R^2\omega^2}+1} + \frac{V_o^2}{R^2\omega^2}\log\left(\frac{R\omega}{V_o}+\sqrt{\frac{R^2\omega^2}{V_o^2}+1}\right)\right]$$

Setting $V/\omega R = J/\pi$, the final expressions for the torque and drag become

$$K_d = -\frac{NC_d cR^2 q_o \pi^2}{8J^2}\left[2\left(1 + \frac{J^2}{\pi^2}\right)^{3/2} - \frac{J^2}{8\pi^2}\sqrt{1 + \frac{J^2}{\pi^2}} - \frac{J^4}{8\pi^4}\log\left(\frac{\pi}{J} + \sqrt{\frac{\pi^2}{J^2} + 1}\right)\right]$$

$$D_d = \frac{NC_d cRq_o \pi}{2J}\left[\sqrt{1 + \frac{J^2}{\pi^2}} + \frac{J^2}{\pi^2}\log\left(\frac{\pi}{J} + \sqrt{\frac{\pi^2}{J^2} + 1}\right)\right]$$

The drag coefficient, C_d, can generally be assumed equal to 0.015, unless the local blade angles of attack are greater than 5 degrees. The net torque and drag is

$$\boxed{K = K_s + K_d}$$

and

$$\boxed{D = D_s + D_d}$$

The local blade angles of attack are

$$\alpha_x = \frac{\dot{\phi}}{\omega}\sin\beta\cos\beta$$

substituting

$$\sin\beta = \frac{V_o}{\sqrt{X^2R^2\omega^2 + V_o^2}} \text{ and } \cos\beta = \frac{XR\omega}{\sqrt{X^2R^2\omega^2 + V_o^2}}$$

and

$$J = \frac{\pi V_o}{\omega R}$$

α_x becomes

$$\boxed{\alpha_x = \frac{\dot{\phi}}{\omega}\left(\frac{X\pi/J}{1 + (X^2\pi^2)/J^2}\right)}$$

HORSEPOWER AND BLADE ANGLE OF ATTACK OF POWER-GENERATING, WINDMILLING PROPELLERS

The horsepower output of fluid-driven propellers is absorbed by bearing friction and useful horsepower. In the case of free-windmilling propellers, the power output is completely absorbed by the bearing friction, so that the net useful torque is zero. Therefore it is seen that the useful power output of a windmilling propeller is proportional to $(K - K_B)$ where K_B is the bearing friction torque. The useful horsepower output is

$$\boxed{HP = \frac{(K - K_B)(\omega + \dot{\phi})}{550}}$$

where $\omega + \dot{\phi}$ is the net rotational speed. Since energy is obtained from the fluid, $\dot{\phi}$ in all cases will be negative. The term $(K - K_B) = K_s + K_D - K_B$ and is calculated as in the previous section.

SOLUTION OF EQUATIONS

The method of solution is actually simpler than it might appear. It is seen from the equations that the basic parameters in all the equations are ω, V_o, ρ, c, R, and N. The advance ratio, J, is $2\pi V_o/\omega d$, and C_d is assumed to equal 0.015.

Knowing these parameters for a given propeller design, D_d, K_d, and $K_{s\dot{\phi}}$ can be computed from the following equations:

$$D_d = \frac{NC_dcRq_o\pi}{2J}\left[\sqrt{1 + \frac{J^2}{\pi^2}} + \frac{J^2}{\pi^2}\log\left(\frac{\pi}{J} + \sqrt{\frac{\pi^2}{J^2} + 1}\right)\right]$$

$$K_d = -\frac{NC_dcR^2q_o\pi^2}{8J^2}\left[2\left(1 + \frac{J^2}{\pi^2}\right)^{3/2} - \frac{J^2}{\pi^2}\sqrt{1 + \frac{J^2}{\pi^2}} - \frac{J^4}{\pi^4}\log\left(\frac{\pi}{J} + \sqrt{\frac{\pi^2}{J^2} + 1}\right)\right]$$

$$K_{s\dot{\phi}} = \frac{K_s}{\dot{\phi}} = -\frac{\rho\pi NR^4V_o}{8\left(\sqrt{\frac{R^2}{4c^2} + 1} + 1\right)\sqrt{1 + \frac{\pi^2}{2J^2}}}$$

$$K = K_d + K_{s\dot{\phi}}\dot{\phi}$$

$$D = D_d + \frac{\pi}{JR}K_s$$

$$\alpha_x = \frac{\dot{\phi}}{\omega}\left(\frac{X\pi/J}{1 + (X^2\pi^2)/J^2}\right)$$

If the propeller is windmilling, the net torque, K, is set equal to the bearing torque, K_B, so that the slip $\dot{\phi}$ can be found. Substituting this $\dot{\phi}$ value into the drag equation, the net propeller drag, D, is obtained. The angle of attack, α_x, at each blade section can then be calculated since α_x is a function of $\dot{\phi}$, ω, and J for each Station X.

If the propeller is used to provide useful power, the required power output is set equal to horsepower, HP. Knowing the propeller parameters, and the bearing-friction torque, $K - K_B$ can be expressed as a function of $\dot{\phi}$ as follows:

$$K - K_B = K_d - K_B + K_{so\dot{\phi}}\dot{\phi}$$

The horsepower output is then

$$HP = \frac{(K_d - K_B + K_{so\dot{\phi}}\dot{\phi})(\omega + \dot{\phi})}{550}$$

Simplifying,

$$\dot{\phi}^2 + \left(\frac{K_d - K_b + K_{so\dot{\phi}}\omega}{K_{so\dot{\phi}}}\right)\dot{\phi} - \frac{550\ HP}{K_{so\dot{\phi}}} = 0$$

This equation can be solved for $\dot{\phi}$, and in a manner similar to the preceding case, the propeller drag, D, and the local blade angles of attack can be obtained.

To illustrate the use of this theory, a special case may be considered where it is desired to know the propeller drag, slip, and local blade angles of attack as a function of the bearing friction. This example is a water-tunnel model which is to be fitted with four different windmilling propellers having the following characteristics:

N = four blades
J = 2 and 3
c = 1/2 inch and 1 inch
d = 2 inches
V_o = 30 fps
C_d = 0.015

Several types of bearings can be used, each having a different friction torque. This study is made to determine whether the propellers will windmill adequately, and if so, which bearing is best to adopt.

The free-stream dynamic pressure is $q_o = 1/2\rho V_o^2 = 900$ psf, since $\rho \approx 2.0$.

The rotational speed ω is

$$\omega = \frac{\pi V_o}{JR} = \begin{bmatrix} 565 \text{ rad/sec } (J = 2) \\ 376 \text{ rad/sec } (J = 3) \end{bmatrix}$$

The frictional propeller drag component is

$$D_d = \frac{NC_d cRq_o \pi}{2J}\left[\sqrt{1 + \frac{J^2}{\pi^2}} + \frac{J^2}{\pi^2}\log\left(\frac{\pi}{J} + \sqrt{\frac{\pi^2}{J^2} + 1}\right)\right]$$

$$D_d = \frac{4 \cdot 0.015 \cdot 1/12 \cdot 900\pi}{2} \cdot \frac{c}{J}\left[g(J)\right]$$

$$D_d = 7.05\frac{c}{J}\begin{bmatrix} 1.69 \ (J = 2) \\ 2.21 \ (J = 3) \end{bmatrix}$$

$$D_d = \begin{bmatrix} 0.25 \text{ lb } (J = 2.0 \quad c = 1/2 \text{ inch}) \\ 0.22 \text{ lb } (J = 3.0 \quad c = 1/2 \text{ inch}) \\ 0.50 \text{ lb } (J = 2.0 \quad c = 1 \text{ inch}) \\ 0.43 \text{ lb } (J = 3.0 \quad c = 1 \text{ inch}) \end{bmatrix}$$

The frictional torque component is

$$K_d = -\frac{NC_d cR^2 q_o \pi^2}{8J^2}\left[2\left(1 + \frac{J^2}{\pi^2}\right)^{3/2} - \frac{J^2}{\pi^2}\sqrt{1 + \frac{J^2}{\pi^2}} - \frac{J^4}{\pi^4}\log\left(\frac{\pi}{J} + \sqrt{\frac{\pi^2}{J^2} + 1}\right)\right]$$

Substituting,

$$K_d = -0.462\frac{c}{J^2}\left[f(J)\right]$$

where

$$f(2.0) = 2.65 \text{ and } f(3.0) = 3.26$$

Therefore,

$$K_d = \begin{bmatrix} -0.0127 \text{ ft-lb } (J = 2.0 \quad c = 1/2 \text{ inch}) \\ -0.0070 \text{ ft-lb } (J = 3.0 \quad c = 1/2 \text{ inch}) \\ -0.0254 \text{ ft-lb } (J = 2.0 \quad c = 1 \text{ inch}) \\ -0.0139 \text{ ft-lb } (J = 3.0 \quad c = 1 \text{ inch}) \end{bmatrix}$$

The basic slip torque coefficient is

$$K_{so\dot{\phi}} = - \frac{\rho \pi N R^4 V_o}{8\left(\sqrt{\frac{R^2}{4c^2} + 1} + 1\right)}$$

Substituting,

$$K_{so\dot{\phi}} = \begin{bmatrix} -0.00187 \ (c = 1/2 \text{ inch}) \\ -0.00213 \ (c = 1 \text{ inch}) \end{bmatrix}$$

$$\sin \beta_o = \frac{1}{\sqrt{1 + (4.90)/J^2}} = \begin{bmatrix} 0.67 \ (J = 2.0) \\ 0.81 \ (J = 3.0) \end{bmatrix}$$

The slip torque coefficient is

$$K_{s\dot{\phi}} = K_{so\dot{\phi}} \sin \beta_o$$

Substituting,

$$K_{s\dot{\phi}} = \begin{bmatrix} -0.00125 \ (J = 2, \quad c = 1/2 \text{ inch}) \text{ (case 1)} \\ -0.00152 \ (J = 3, \quad c = 1/2 \text{ inch}) \text{ (case 2)} \\ -0.00142 \ (J = 2, \quad c = 1 \text{ inch}) \text{ (case 3)} \\ -0.00173 \ (J = 3, \quad c = 1 \text{ inch}) \text{ (case 4)} \end{bmatrix}$$

The slip drag is

$$D_{s\dot{\phi}} = \frac{\pi}{JR} K_{s\dot{\phi}} = \frac{37.6}{J} K_{s\dot{\phi}}$$

Substituting,

$$D_s\dot{\emptyset} = \begin{bmatrix} -0.0236 \text{ (case 1)} \\ -0.0190 \text{ (case 2)} \\ -0.0268 \text{ (case 3)} \\ -0.0217 \text{ (case 4)} \end{bmatrix}$$

Summarizing the equations and setting $K = K_b = K_d + K_{s\dot{\emptyset}}\dot{\emptyset}$ as follows:

$$K_b = \begin{bmatrix} -0.0127 - 0.00125\ \dot{\emptyset} \text{ (case 1)} \\ -0.0070 - 0.00152\ \dot{\emptyset} \text{ (case 2)} \\ -0.0254 - 0.00142\ \dot{\emptyset} \text{ (case 3)} \\ -0.0139 - 0.00173\ \dot{\emptyset} \text{ (case 4)} \end{bmatrix}$$

Solving for $\dot{\emptyset}$,

$$\dot{\emptyset} = \begin{bmatrix} -10.2 - 800\ K_b \text{ (case 1)} \\ -4.6 - 657\ K_b \text{ (case 2)} \\ -17.9 - 704\ K_b \text{ (case 3)} \\ -8.0 - 578\ K_b \text{ (case 4)} \end{bmatrix}$$

So

$$\dot{\emptyset}/\omega = \begin{bmatrix} -0.0181 - 1.42\ K_b \text{ (case 1)} \\ -0.0122 - 1.75\ K_b \text{ (case 2)} \\ -0.0317 - 1.24\ K_b \text{ (case 3)} \\ -0.0213 - 1.54\ K_b \text{ (case 4)} \end{bmatrix}$$

The net propeller drag is

$$D = D_d + D_{s\dot{\emptyset}}\dot{\emptyset}$$

$$D = \begin{bmatrix} 0.25 - 0.0236\ \dot{\emptyset} \text{ (case 1)} \\ 0.22 - 0.0190\ \dot{\emptyset} \text{ (case 2)} \\ 0.50 - 0.0268\ \dot{\emptyset} \text{ (case 3)} \\ 0.43 - 0.0217\ \dot{\emptyset} \text{ (case 4)} \end{bmatrix}$$

Substituting for $\dot{\emptyset}$

$$D = \begin{bmatrix} 0.49 + 18.9\ K_b \text{ (case 1)} \\ 0.31 + 12.5\ K_b \text{ (case 2)} \\ 0.98 + 18.9\ K_b \text{ (case 3)} \\ 0.60 + 12.5\ K_b \text{ (case 4)} \end{bmatrix}$$

The angle of attack at X = 0.7, is

$$\alpha_x \text{ deg} = \frac{\dot{\phi}}{\omega} \frac{0.7 \frac{\pi}{J} (57.3)}{1 + \frac{\pi^2}{2J^2}} = \frac{\dot{\phi}}{\omega} \begin{bmatrix} 28.2 \; (J = 2.0) \\ 27.2 \; (J = 3.0) \end{bmatrix}$$

Then

$$\alpha_x \text{ deg} = \begin{bmatrix} -0.51 - 40.0\, K_b \; (\text{case } 1) \\ -0.33 - 47.6\, K_b \; (\text{case } 2) \\ -0.90 - 35.0\, K_b \; (\text{case } 3) \\ -0.58 - 41.9\, K_b \; (\text{case } 4) \end{bmatrix}$$

The characteristics, D, $\alpha_{0.7 \text{ deg}}$, and $\dot{\phi}/\omega$ are plotted as functions of K_b in Fig. 3 and 4.

It can be seen that the bearing friction torque, K_b, should be less than 1 inch-pound if the slip ratio $\dot{\phi}/\omega$ and blade angles of attack are to be reasonable. The drag is plotted so that its effect on the bearing friction torque can be obtained.

This illustrative example is necessarily lengthy because four different propellers were used. However, if the characteristics of a single propeller were desired, the calculations would be much simpler and only one quarter as long. The example has merit, however, since the relative effects of advance ratio and chord length on propeller drag, slip ratio, and blade angles of attack can be obtained.

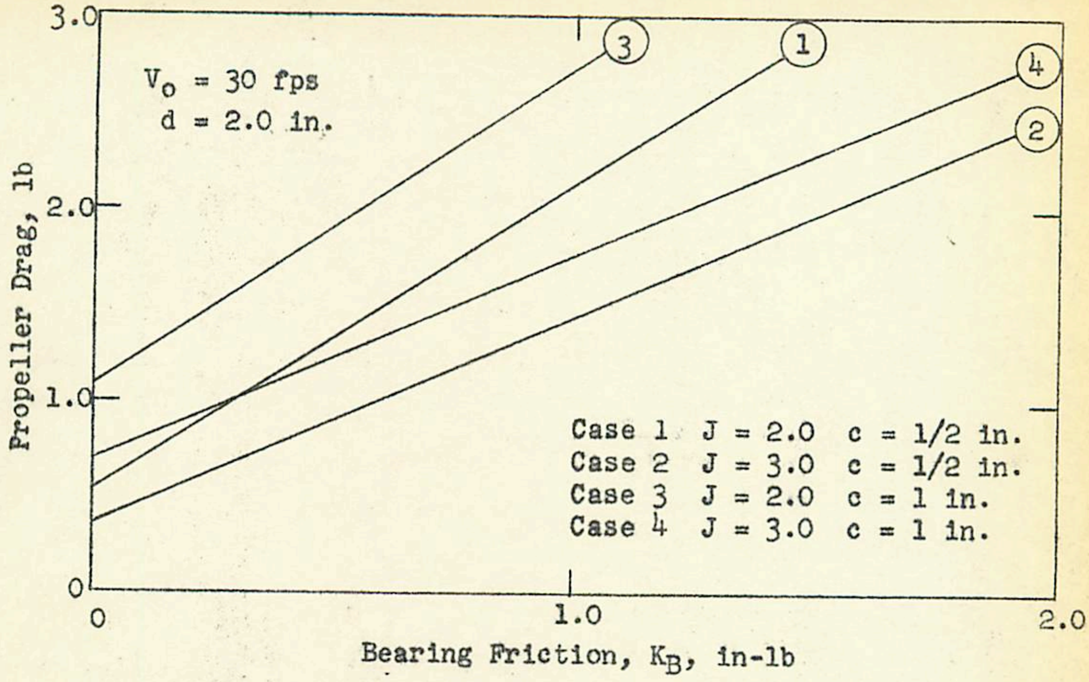

FIG. 3. Drag of One Windmilling Propeller.

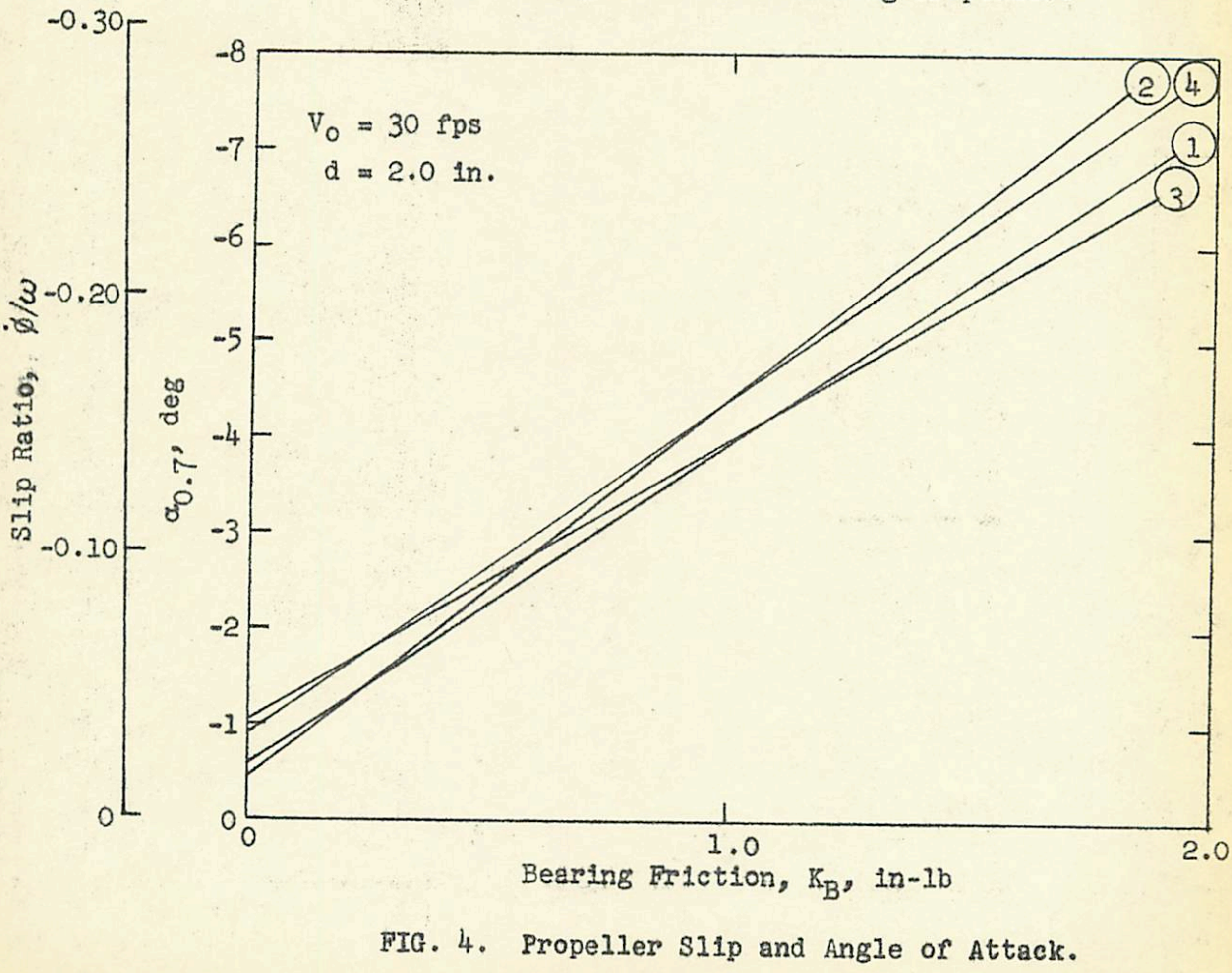

FIG. 4. Propeller Slip and Angle of Attack.

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