Math 2471 - Calc 3

Vector Functions

$$\bar{r}(t) = \langle f(t), g(t) \rangle$$

consider the following

$$\bar{r}(t) = \langle t+1, 2t-1 \rangle$$

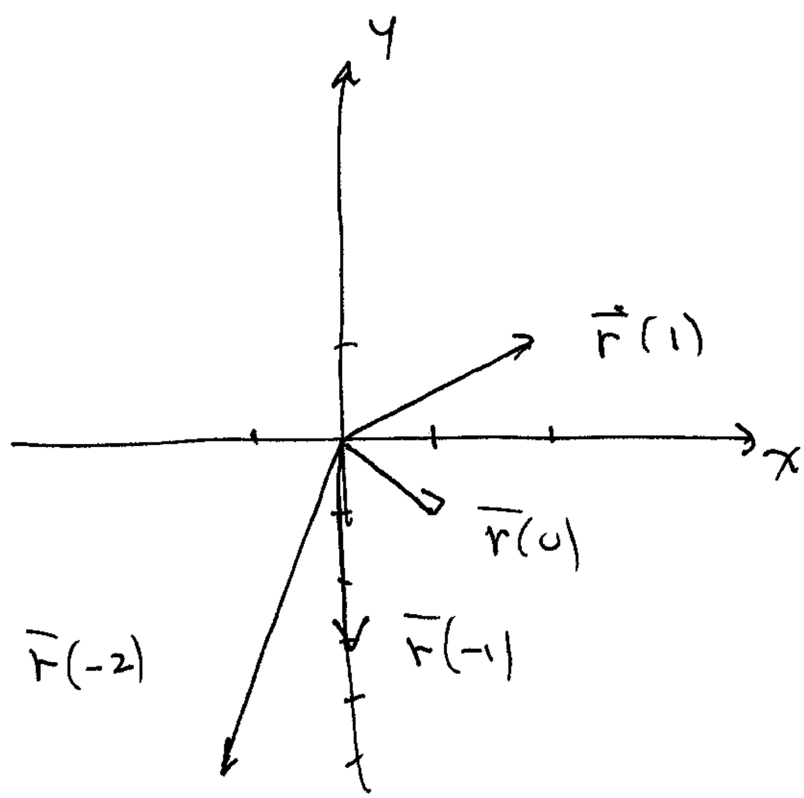
consider $\bar{r}(-2)$, $\bar{r}(-1)$, $\bar{r}(0)$ $\bar{r}(1)$

$$\bar{r}(-2) = \langle -1, -5 \rangle$$

$$\bar{r}(-1) = \langle 0, -3 \rangle$$

$$\bar{r}(0) = \langle 1, -1 \rangle$$

$$\bar{r}(1) = \langle 2, 1 \rangle$$

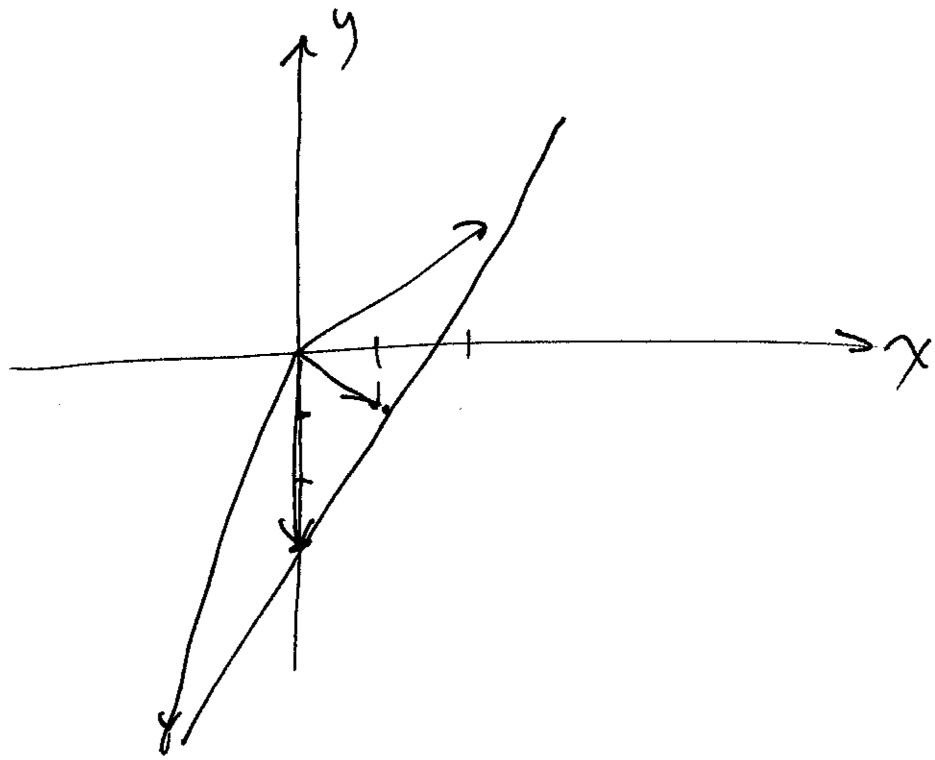


So we see the vector changes as t change note: This tip of the vector is located

at $x = t+1, \; y = 2t-1$

& eliminating t gives $y = 2(x-1)-1$

a line



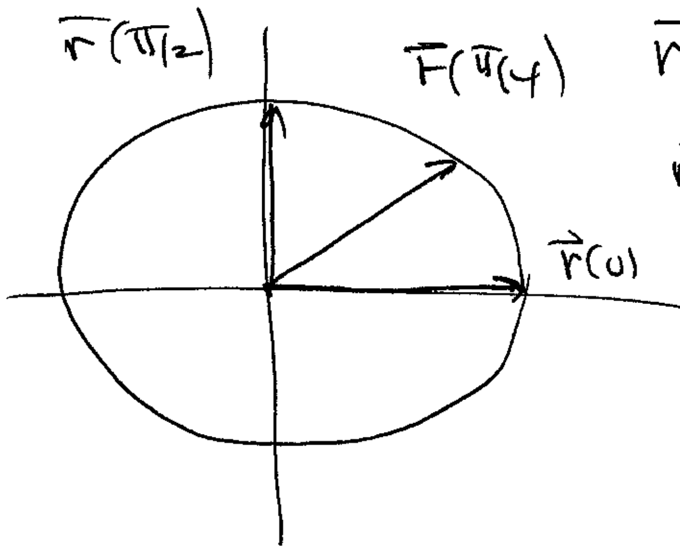
in general

$\bar{r}(t) = \langle f(t), g(t) \rangle$

tip vector touches space curve $x = f(t), y = g(t)$

Ex2 $\vec{r}(t) = \langle \cos t, \sin t \rangle$

space curve $x = \cos t$, $y = \sin t$ (circle)



$\vec{r}(0) = \langle 1, 0 \rangle$

$\vec{r}(\pi/4) = \langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \rangle$

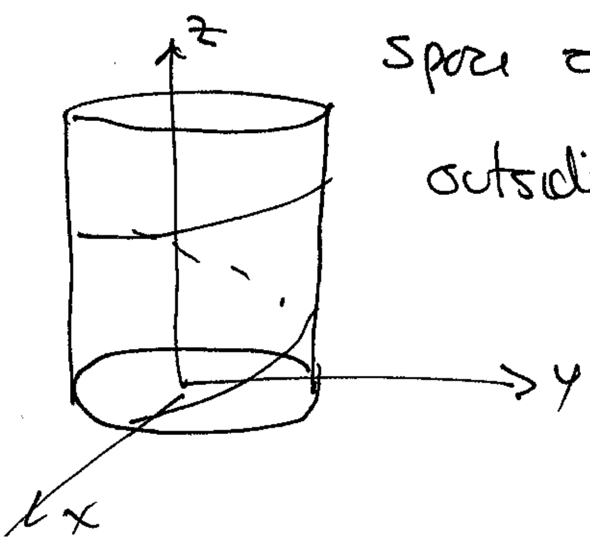
How about 3-D

Ex $\vec{r}(t) = \langle \cos t, \sin t, t \rangle$

$x = \cos t$, $y = \sin t$, $z = t$

space curve is a helix

outside cylinder $x^2 + y^2 = 1$

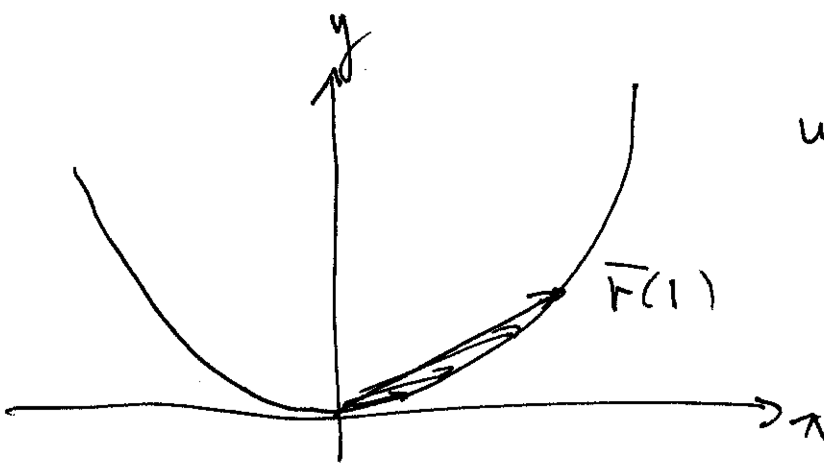


so all of things we learned in Calc 1
Can be applied to vector functions

(1) Limits

$\lim_{t \to a} \vec{r}(t)$ when $\bar{r}(t) = \langle t, t^2 \rangle$

so $\lim_{t \to 1} \langle t, t^2 \rangle = \langle \lim_{t \to 1} t, \lim_{t \to 1} t^2 \rangle = \langle 1, 1 \rangle$



in the limit
the vectors approach
$\langle 1, 1 \rangle$

easily extends to 3D

$$\lim_{t \to 0} \left\langle t, \frac{\sin t}{t}, \frac{t^2-1}{t-1} \right\rangle = \left\langle \lim_{t \to 0} t, \lim_{t \to 0} \frac{\sin t}{t}, \lim_{t \to 0} \frac{t^2-1}{t-1} \right\rangle$$

$$= \langle 0, 1, 1 \rangle$$

Derivative

if $\bar{r}(t) = \langle f(t), g(t) \rangle$

$$\bar{r}'(t) = \lim_{h \to 0} \frac{\bar{r}(t+h) - \bar{r}(t)}{h}$$

$$= \lim_{h \to 0} \frac{\langle f(t+h), g(t+h) \rangle - \langle f(t), g(t) \rangle}{h}$$

$$= \left\langle \lim_{h \to 0} \frac{f(t+h) - f(t)}{h}, \lim_{h \to 0} \frac{g(t+h) - g(t)}{h} \right\rangle$$

$$= \langle f'(t), g'(t) \rangle$$

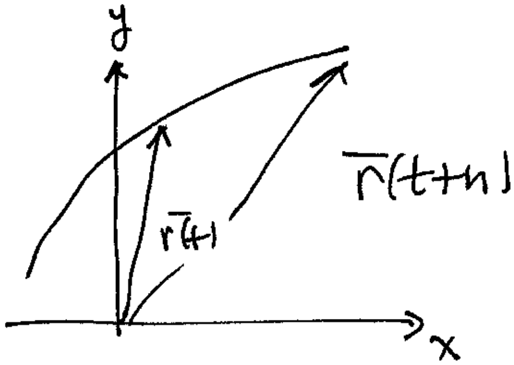
ex $\bar{r}(t) = \langle t, t^2 \rangle$

$\bar{r}'(t) = \langle 1, 2t \rangle$ easy calculation

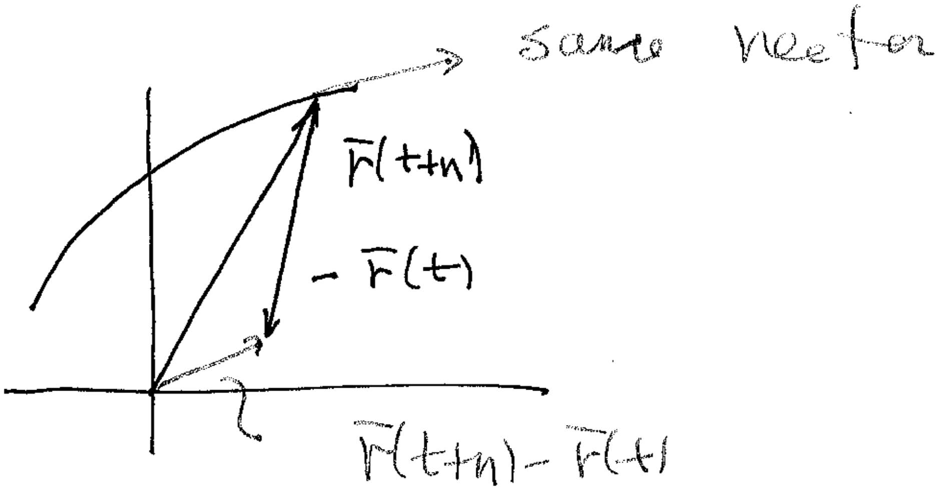
ex $\bar{r}(t) = \langle \cos t, \sin t, t \rangle$

$\bar{r}'(t) = \langle -\sin t, \cos t, 1 \rangle$

So what does this mean geometrically



so



$$\lim_{h\to 0} \frac{\bar{r}(t+h)-\bar{r}(t)}{h} = \bar{r}'(t)$$ is a vector tangent to the curve

HW pg 806-7,
 # 29, 31, 41, 43

pg 814
 # 7, 9, 11, 13, 15, 17, 19, 21, 23, 25