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Scheduling Problem in a Job-shop with Common Due Date to Minimize Cost and Makespan: Modeling Approach

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Abstract

In a job shop, number of jobs are scheduled on a machine in such a way that the sequence responses to minimum cost as well as minimum completion time. A fundamental scheduling model is considered in search of understandings and interactions that suggests procedure for minimizing not only the total cost but also the overall makespan. Common due date is considered with equal earliness and tardiness cost. In this research work, an algorithm is developed which initially checks all the possible sequences which will give the minimum cost as the primary objective. The secondary objective of the algorithm is to find the sequence which will give the smallest makespan. The algorithm also finds the solution to reduce the makespan further if the due date is allowed to be tighter at the same minimum cost. The model also works for unequal cost of early and tardy jobs. Numerical illustrations are also solved with respect to all the stated cases. The model is developed in python which helps in testing all possible sequences and gives optimal sequence with least cost and makespan. The model is tested with different numerical consisting of processing time, common due date, and earliness and tardiness cost with consistent results.

Keywords: Earliness; Tardiness; Common Due Date; Scheduling.

1. Introduction

In engineering industries, jobs are scheduled to machine in a sequence that reduces the overall completion time. Most of the scheduling literature deals with the performance metrics like mean lateness, flow-time and tardiness. However, completing the jobs on due dates is also one of the vital goals. In traditional scheduling techniques, due-dates are assumed to be given as exogenous conclusions. Still, they are found by considering the system's capacity to match the given distribution dates. Therefore, in many research it has been noticed that due-date allotment is a part of the scheduling process. Association of due-dates with the performance metrics can be broadly studied by two approaches. In the first case, due-dates are known, and a performance metrics such as total tardiness represents the usefulness of schedule at meeting the known due-dates. Here, some jobs can be rejected to enhance performance. In another case, due-dates are internal choices. These due-dates are frequently settled with clients or selected by production control department to monitor or to speed up the development of work. It is evident that the performance metrics may comprise measures of due-date tightness.

A typical approach of judging conformance to due-date is considered to be mean tardiness; however, it oversees the effect of jobs finishing early. The concept of just-in-time production promotes the idea that not only tardiness but also earliness should be dejected. Costs associated with earliness and tardiness (E/T) are few of the general criteria considered to trace the performance of production. When the jobs are finished early, effects inventory carrying costs like storing and insurance costs. In contrast, jobs that finish later to their due-dates adversely effect in fines like late-dues, harm to client concern, damage to trades and poor responsiveness of the supply chains. After the introduction of the E/T by Sidney [1], a number of authors concentrated on the E/T problem, in which time of processing the jobs and due dates were known. Broad reviews can be seen in [2], [3] and [4]. After this the study of penalties due to E/T scheduling problems was studied extensively in the form of mathematical models.

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The main idea in sequencing, considering due-dates is generally to complete entire jobs on time. If due-dates are unrestricted, obviously, this intent may be achieved by permitting the loose due-dates. Still, in a situation, wherein due-dates can be carefully chosen, it is intended to allot due-dates to be as tight as probable. Tighter due-dates invite more clients than that of the situations where due-dates are loose in a market with large competition. Tighter due-dates also have a tendency to yield lesser inventory levels hence they are necessary in scheduling.

2. Literature Review

In this literature, an algorithm was developed which could find the minimum cost schedules. The algorithm was applicable either when all the tasks have the same length or the tasks are required to be executed in a given fixed sequence [5]. The researcher reviewed by considering the problem of scheduling 'n' jobs to reduce cost of earliness and tardiness [2]. The earliness plus tardiness scheduling problems were solved by taking into account the weighted deviance of finish times related to common due-date [6].

Two of the common assumptions in scheduling problems are that the processing times of the jobs are deterministic which are known beforehand and no machine breakdown occurs. Such type of assumptions give rise to deterministic type of problems. By relaxing these assumptions that is by making the processing time random, the problem becomes stochastic scheduling problem. Stochastic scheduling problem was formulated with E/T costs on a machine for static nature, wherein, the processing time of jobs are stochastic and due-dates are different and deterministic [7]. A heuristic was proposed for the stochastic one machine problem with the goal of determining the sequence of the jobs and the due dates which minimizes the total cost of earliness and tardiness [8], [9]. A branch and bound algorithm was also developed to search the optimal solution to this problem [9]. A one machine problem with uncertain processing time [10] was considered and a heuristic method was proposed for sequencing of jobs and due-date allotment which utilizes average and variances of processing time of jobs.

Scheduling requires decision making in sequencing and resource distribution. When there is only single a resource meaning a single machine, the distribution of that resource is totally decided by sequencing decision. However, there are models with more than one machine called as multi machine models which can be classified as parallel machine (the machines are similar and the jobs are distinct), flow shop (the machines are arranged in series) and job shop system (the flow of job is not unidirectional). A heuristic approach was developed [4] where the intent was to find an optimum sequence which reduces the entire E/T costs. Multiple machine scheduling problems were surveyed considering a common due-date mainly focusing on shop scheduling problems. A mathematical programming model with heuristics and meta-heuristics to reduce the sum of due-date assignment and earliness and tardiness penalties on parallel machines was developed [11], while a Memetic algorithm for dissimilar parallel machines scheduling problem for the same objective was also studied [12].

There are cases where the provider is given some concession with reference to distribution period and it is not limited to an exact due date. This means that the jobs can be completed and accepted with no fine within a time interval, this time interval is known as the due window. Several researchers studied the concept of due-windows, where a time interval is presumed, so that the jobs finished in this time interval are not fined [13] [14]. A problem on production scheduling and single machine common due-window assignment was studied [15]. An algorithm of polynomial time solution to reduce the total cost considered as a weighted additive function of earliness and tardiness, starting time of due-window, and size of due-window was provided. A due window may imitate an assembly atmosphere where the parts of the product should be completed in a time interval to evade production lateness. However, it is but obvious that a widespread due-window rises the supplier's manufacturing and supply flexibility. Conversely, a huge due-window and postponing job accomplishment weaken the supplier's competitiveness and quality of service to the client [16].

In the last decade, authors mainly focused on heuristic and meta-heuristics computational procedures that find an optimum solution by iteratively trying to develop a solution with respect to a known measure of quality, but don't guarantee optimality. Heuristic are deliberated for a particular application while meta-heuristics are aimed for more general purpose applications. An algorithm of discrete harmony search (DHS) was proposed to solve problem of flexible job-shop scheduling (FJSP) to minimize the largest of the makespan and the average of earliness and tardiness [17]. The authors concluded that their algorithm imposes fewer mathematical requirements.

An idea in problems of E/T scheduling was introduced [18] wherein non-execution of the task is allowed and accordingly are fined for every non executed job. Authors developed an algorithm considering the non-execution penalties. The problems were solved on 2 single machine scheduling with tardiness fines and due-date allotment [19]. In the first situation the goal was to minimize cost function that comprised fines suffered because of due-date allotment, earliness and tardiness. In second situation the goal was to reduce cost which included fines incurred because of due-date allotment and tardy jobs. The optimal schedule of the jobs and their due dates considered by authors [20]. They studied three different methods of due date assignments - common, slack and unrestricted due date. They proved that the processing time of job is based on its location in a sequence, its beginning time, and its allocation of non-renewable resource.

Researchers implemented the E/T model considering various case studies. The concept of due date was applied for the system consisting of semiconductor wafer fabrication [21] which has a re-entrant processing flow meaning that jobs return more than one time on the same machine. The authors used mixed integer linear programming to reduce the total earliness and tardiness for job dependent time windows. Authors proposed a solution with a model to the scheduling problem of allotment of batteries and vehicles as resources to material handling jobs [22].

3. E/T Model and the Heuristic Procedure

In a job shop, jobs are manufactured for further assembly into final product. The due-dates for jobs are reliant on the assembly schedule of the final product. If job order is tardy, then the assembly of final product will be late. This will not only affect in terms of loss of assembly productivity but also influences on customer relationship. Alternatively, if the job is finished early, then it has to wait in stocks until the assembly period. This is undesirable as it build-up job inventory. So, the solution should be such that all the jobs need to accomplish just on their due-dates. Let, E_j and T_j denote earliness and tardiness respectively, while $\propto_j$ and β_j earliness cost and tardiness cost respectively. Assuming cost functions to be linear, the basic objective function for E/T of schedule S for n jobs is as follows:

$$f(S) = \sum_{j=1}^{n} \propto_j E_j + \beta_j T_j \tag{1}$$

In basic E/T problem, assuming $\propto_j = 1$, $\beta_j = 1$ and due-date which is common, the objective function can be given as:

$$f(S) = \sum_{j=1}^{n} E_j + T_j \tag{2}$$

Ideally, a schedule should be constructed such that due-date *d* is at the mid of jobs. If *d* is very tight, it will not be feasible to have adequate jobs ahead of *d*. This becomes restricted version. If *d* is not very tight, then it becomes unrestricted version. Here, the research focuses on unrestricted version where the intent is to reduce the cost and the makespan. The restricted version is discussed by the authors in their earlier research [23]

Considering the unrestricted version for E/T problem, following properties are proposed [24],

1) Schedules not having idle time among consecutive jobs consists a dominant set.
2) Jobs those finish ahead of the due-date can be arranged in longest processing time (LPT), first sequence. Whereas, the jobs which begin later the due-date can be arranged in shortest processing time (SPT), first sequence.
3) In an optimum schedule where one job ends just at the due-date.
   This job can be considered as job [*b*], where *b* = [*n/2*]

The following steps are considered while developing an algorithm for the unrestricted version,

1) Divide the jobs in two lists like list A, and list B, starting with assigning the longest job to list B.
2) Find the next two longest jobs and give one of them to list A and the other to list B.
3) Iterate the above step 2 until no job is left, or if one job is left assign the same to list A or list B.
4) Apply LPT to arrange the jobs in list B and SPT for the jobs in list A.
5) Merge two lists, list B and list A to get the sequence.

The total number of optimal schedule can be given by 2^m, where m = n/2 if the number of jobs are even and m = (n-1)/2 if the number of jobs are odd. It is presumed, the processing time for all the jobs are distinct or else if they are same, the number of optimal schedule will increase. Thus, there will be many optimal schedules which will yield minimum cost in the basic E/T problem. With more than one optimal schedule, intent is to see if any of the sequence can achieve any other secondary objective. If the next intent is to reduce the total processing time in list B, then assign the job to list B, every time step 2 is performed. If n is even give the shortest job to list A in step 3.
The complete processing time (*PT*) in list B can then be written as follows:

$$\Delta_1 = PT_n + PT_{n-2} + PT_{n-4} + \cdots \tag{3}$$

If the next intent is to reduce makespan, which will be required to keep machine ready for the next set of jobs then this can be achieved by giving the longer job to list B, every time step 2 is performed. If n is even giving the shortest job to list B in step 3.

The complete processing time (*PT*) in list B can be shown as follows:

$$\Delta_2 = PT_n + PT_{n-1} + PT_{n-3} + \cdots \tag{4}$$

The begin time is calculated as the due date (DD) minus total processing time of jobs in list B. Importance of Δ_1 and Δ_2 tells the definition of the restricted and unrestricted version. If $d \geq \Delta_1, \Delta_2$ then it is unrestricted else, it represents restricted version. If $\Delta_1 < d < \Delta_2$, the E/T problem is unrestricted but reducing the other intent will be non-deterministic polynomial-time hard.

In many situations due dates are known and given by the clients. But in other due dates can be internal choices set up by the production control department or can be settled with the customers. If the due dates can be made tighter, it will attract more customers and will yield lesser inventory levels. With this discussion the delay in starting the jobs can be eliminated and the jobs can be started on the very first day. In both the above mentioned cases the objective of cost minimization still holds good and it is same. In fact, the total completion time that is the makespan also is reduced in both the cases. In the first case the due date which can be set is less than that in the second case as far as unrestricted version is considered but the number of jobs completed afore the due date in the first case may be less as compared to the second one.

Figure 1 shows process for finding minimum cost and makespan in a scheduling problem through common due date. It first takes input data as number of jobs considered for scheduling, processing time for each job and due date for the jobs. Later jobs are settled in declining sequence of processing time. Ordered list of jobs is used to find the optimal sequence for minimizing cost using the steps mentioned in section 3 for unrestricted version. Find the processing time of jobs using equation 3 and 4 and denote it as Δ_1, Δ_2 respectively. Compare d with Δ_1 and Δ_2, if d is greater than Δ_1 and Δ_2, then delay is there in processing of jobs. Find the cost and makespan of sequence corresponding to Δ_2. If client is ready to make due date tighter choose sequence corresponding to Δ_1 to find the minimum cost and makespan with no delay in schedule.

4. Numerical Illustrations and Working Model

4.1. Numerical Illustration 1

A following numerical illustration is being solved with respect to above E/T model and algorithm using input data as represented in table 1. The processing time (PT) and common due date (DD) in days are given for six jobs. The intent is to discover the optimal sequence of jobs which gives minimum cost.

Table 1. Input Data

Jobs	1	2	3	4	5	6
PT	2	3	5	7	9	10
DD (Common)	30					

Initially, the jobs are arranged in descending order of their processing time as 6-5-4-3-2-1. Then it is checked whether the problem is unrestricted by applying the procedure of unrestricted version as listed in section 3 and shown in table 2.

Table 2. Sequences for equal earliness and tardiness cost

List B jobs	List A jobs	Sequence	Time for List B	Start time
6--5--3--1	2--4	6--5--3--1--2--4	26	4
6--5--3	1--2--4	6--5--3--1--2--4	24	6
6--5--2--1	3--4	6--5--2--1--3--4	24	6
6--5--2	1--3--4	6--5--2--1--3--4	22	8
6--4--3--1	2--5	6--4--3--1--2--5	24	6
6--4--3	1--2--5	6--4--3--1--2--5	22	8
6--4--2--1	3--5	6--4--2--1--3--5	22	8
6--4--2	1--3--5	6--4--2--1--3--5	20	10

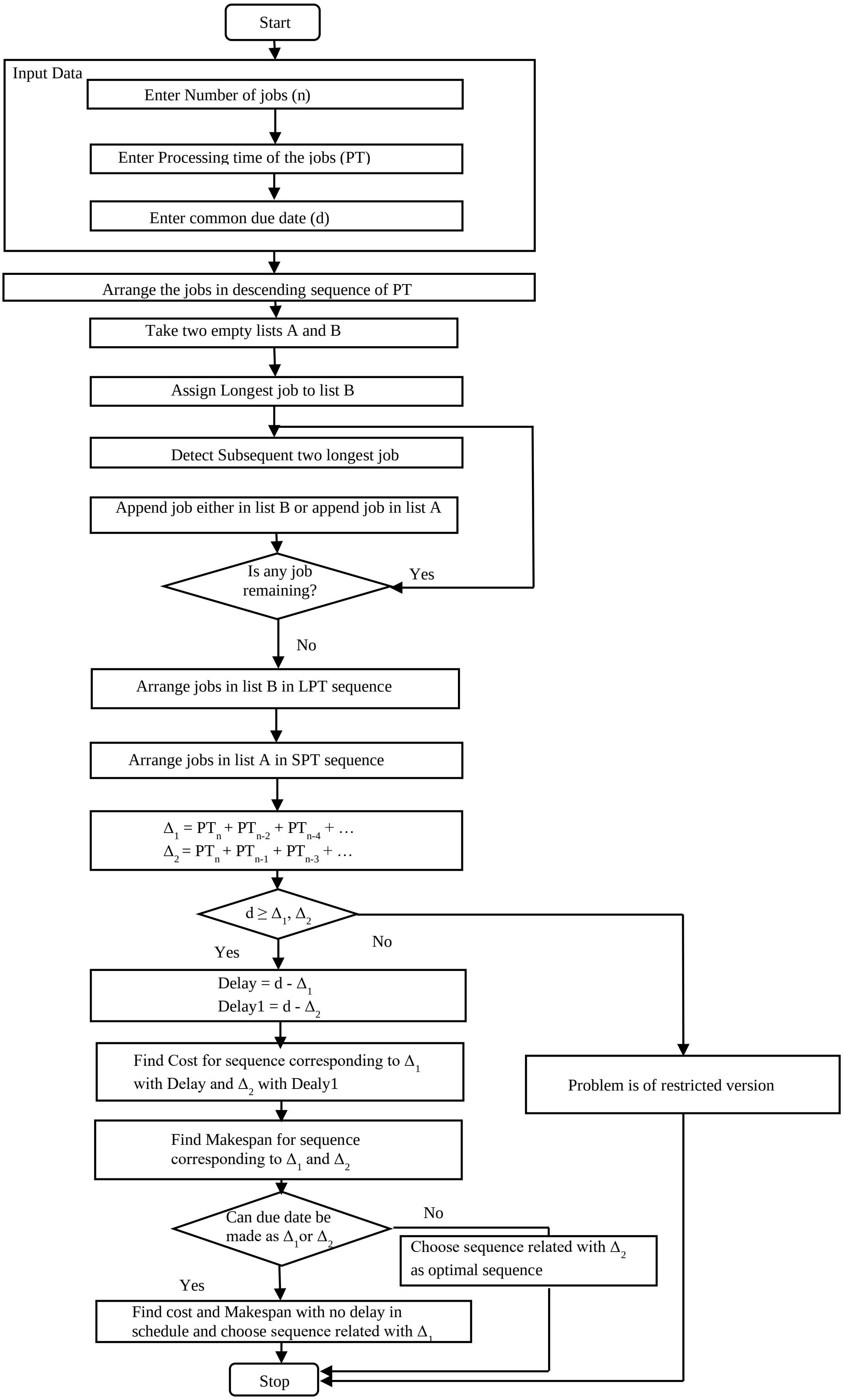

Fig. 1 Flowchart for finding cost and makespan when earliness and tardiness cost is equal

Since time for jobs in list B is less than the due date, this is unrestricted version and the jobs can be delayed by 4 to 10 days; as can be conformed from the last column of table 2. Consider the two extreme cases (first and last row in table 2), for finding the objectives. The sequence 6-4-2-1-3-5 with a delay in the start time of 10 days and showing the time line in figure 2.

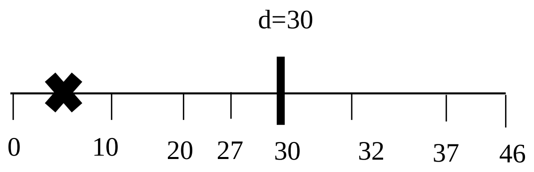

Fig. 2. Time line for sequence 6-4-2-1-3-5

Cost = 10 + 3 + 0 + 2 + 7 + 16 = 38

So the optimal sequence of jobs is 6-4-2-1-3-5 which gives minimum cost of 38 units and makespan as 46 days by delaying the jobs by 10 days.

Consider the sequence 6-5-3-1-2-4 with a delay in the start time of 4 days and showing the time line in figure 3 which has a minimum cost of 38 and makespan as 40 days.

Cost = 16 + 7 + 2 + 0 + 3 + 10 = 38

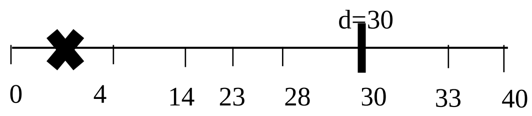

Fig. 3. Time line for sequence 6-5-3-1-2-4

If the due date is fixed and cannot be varied, then considering the above two cases it can be seen that in both the cases the cost is same and these sequences give minimum cost which is the primary objective to be considered. Now considering the secondary objective of minimizing the makespan it is observed that in the second case of sequence i.e. 6-5-3-1-2-4, it is 40 which is less than that of the first case where it is 46. Therefore, it is better to choose this sequence which will have minimum cost as well as minimum makespan.

In the above two cases the due date was known and fixed. If the due date is negotiable with the customers and can be made still tighter, it will attract more customers and there will be no need to delay in the starting of the jobs. With this consideration, the above two sequences are followed but without any delay in the start time. In the first case of sequence 6-4-2-1-3-5 the entire schedule is shifted earlier by 10 days. The total cost is 38 and the makespan is 36 days. The time line for this is shown in figure 4.

Cost = 10 + 3 + 0 + 2 + 7 + 16 = 38

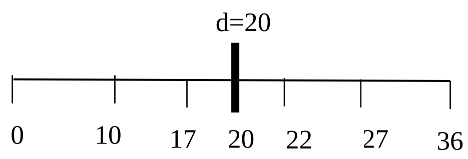

Fig. 4. Time line for sequence 6-4-2-1-3-5

In the second case where the sequence is 6-5-3-1-2-4, the entire schedule is shifted earlier by 4 days as shown in figure 5. The total cost is obtained to be 38 and the makespan is 36 days.

Cost = 16 + 7 + 2 + 0 + 3 + 10 = 38

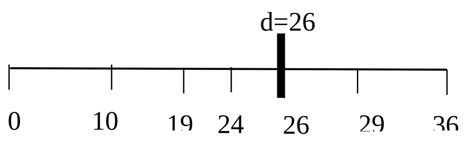

Fig. 5. Time line for sequence 6-5-3-1-2-4

It can be noted from the above two cases that the primary objective of cost minimization and secondary objective of makespan minimization is same when the jobs are started on the very first day without any delay. But the due date in the first case is 20 which is minimum as compared to that in the second case which is 26. Therefore, if the due date is allowed to be tighter, it will be better to choose the sequence 6-4-2-1-3-5; although only three jobs are completed on or before its due date as compared to four jobs in the latter case.

4.2. Numerical Illustration 2

In the above example it was assumed that $\propto_j = \beta_j = 1$. But these costs are expected to be distinct as α tends to be calculated by internal factors whereas β tends to be external. Hence the model is further extended to $\propto \neq \beta$. Continuing with the same input data shown in table 1 and considering $\propto = 5$ and $\beta = 2$. Here, the steps considered in section 3 while developing an algorithm for the unrestricted version are followed except the initial condition where list B and list A are empty and if $\propto * |B| < \beta * (1 + |A|)$ then allocate the next job to list B, else allocate the next job to list A.

Table 3. Sequence of jobs when earliness and tardiness cost is unequal

$\|B\|$	$\|A\|$	$\propto$	β	$\propto * \|B\|$	$\beta * (1 + \|A\|)$	Assignment
0	0	5	2	0	2	6-B
	0	5	2	5	2	5-A
1	1	5	2	5	4	4-A
1	2	5	2	5	6	3-B
2	2	5	2	10	6	2-A
2	3	5	2	10	8	1-A

From last column (Assignment) of table 3, the jobs in list B are arranged in LPT order while those in list A are arranged in SPT order. This gives sequence as 6-3-1-2-4-5 The total cost is 105 and the makespan is 51 days as shown in figure 6.
Cost = 5*5 + 0*5 + 2*2 + 5*2 + 12*2 +21*2 = 105.

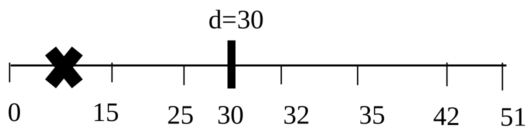

Fig. 6. Time line for sequence 6-3-1-2-4-5

In this case also of $\propto \neq \beta$. Now, if the due date is negotiable with the customers and can be made tighter, then the jobs can be started without any delay. In that case the cost will be same but the makespan will be reduced to 36 days as shown in figure 7.

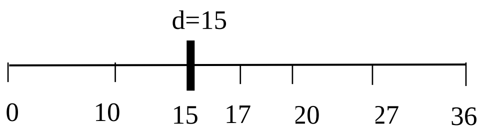

Fig. 7. Time line for sequence 6-3-1-2-4-5

4.3. Working Model

A model is developed using python programming language on personal computer with configuration of 1TB hard disk, 8 GB RAM and Windows 10 operating system. Python is scripting language, it works with all version of Windows and Linux operating system. No other superior requirements for hardware and software are needed. Figure 8 represents the working model for minimizing cost and makespan in a scheduling problem through common due date. Fig 8 (a) presents data entry page with job numbers, common due date and processing time of jobs. Once the data is entered, window pop ups with query for user to make due date tighter as shown in figure 8(b). If user chooses 'No' then, window pop-ups which is shown in figure 8(c), where model shows optimal sequence with delay in processing time, minimum cost and makespan of optimal sequence. Else window pop-ups as displayed in figure 8(d), where model shows optimal sequence with no delay in processing time, minimum cost and makespan of optimal sequence. Snapshots for unequal earliness and tardiness cost with optimal sequence and makespan are shown in figure 8(e) and 8(f) respectively.

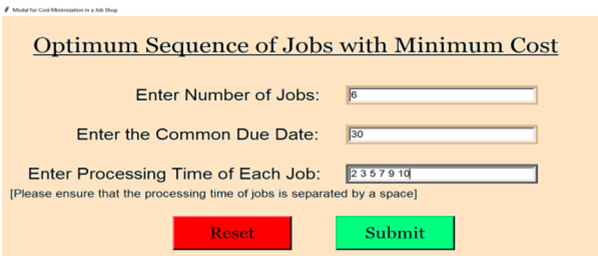

Fig. 8 (a) Data entry

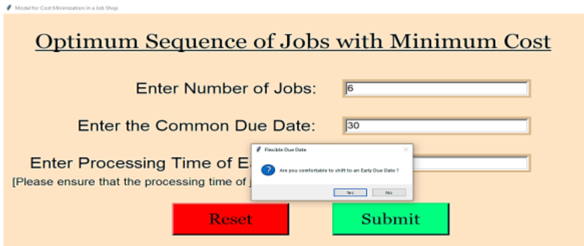

Fig. 8 (b) Choose option to make due date tighter

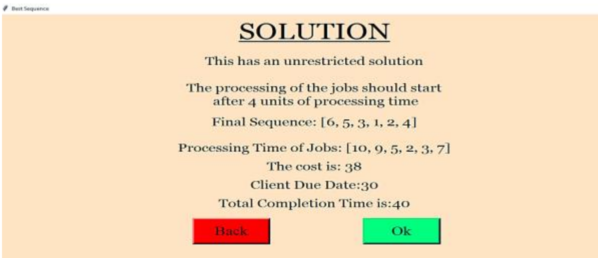

Fig. 8 (c) Cost and makespan with delay in processing time

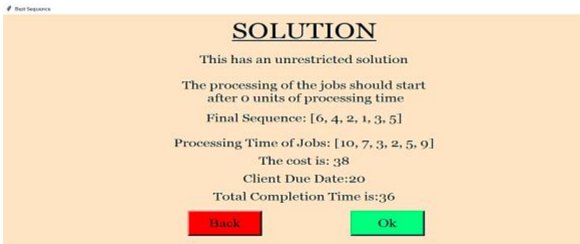

Fig. 8 (d) Cost and makespan without delay in processing time

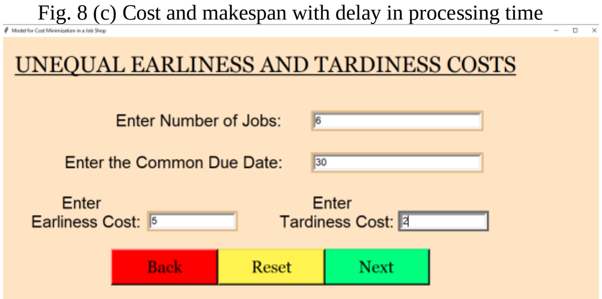

Fig. 8 (e) Unequal earliness and tardiness cost

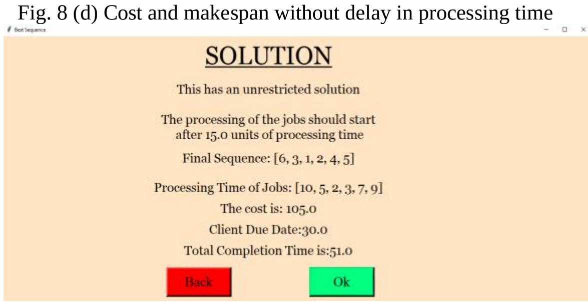

Fig. 8 (f) Cost and makespan for unequal earliness and tardiness cost

Fig. 8 Working model developed using Python

5. Conclusions

In this paper, unrestricted version of scheduling problem is studied wherein number of jobs are to be scheduled on a machine. The processing times of the jobs are distinct using a common due date. An algorithm is constructed to discover an optimum schedule of sequencing the jobs that incurs minimum cost considering the earliness and tardiness of jobs as main objective and minimizing the makespan as the next objective. The algorithm is verified by executing various numerical through program which is developed in Python programming language and found to be accurate. The algorithm also checks for an optimum sequence if due date is allowed to made tighter and gives a choice to the user whether the due date can be made early or not. The implemented model also handles unequal earliness and tardiness cost. The work can further be developed by considering the problems of distinct due dates and distinct earliness and tardiness cost for jobs.

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