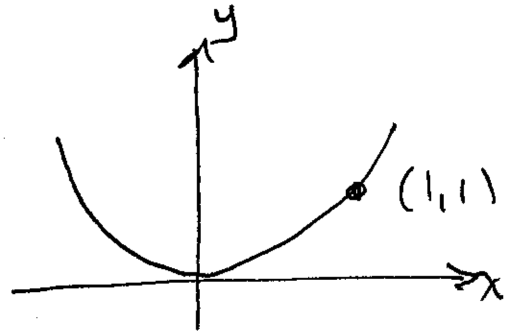
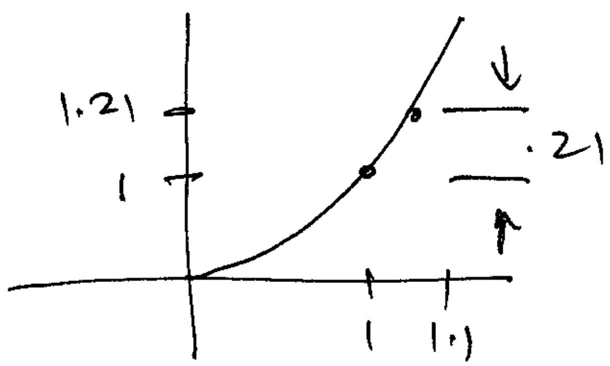
Math 1496 – Calc I

Consider $f(x) = x^2$ and the pt $(1,1)$



If we move from $x=1$ to $x=1.1$ can we find the change in y – Yes!

$$f(1.1) - f(1) = (1.1)^2 - 1 = 1.21 - 1 = .21$$



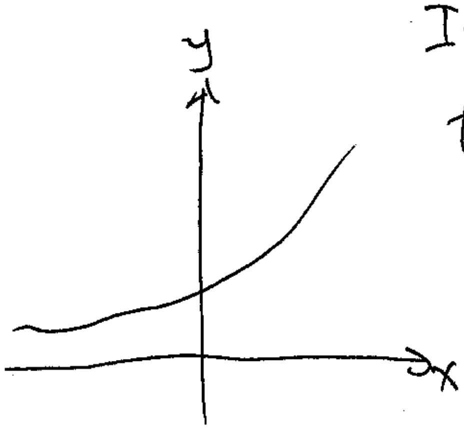
So given the change in x

$$\Delta x = .1$$

then the change in y is

$$\Delta y = .21$$

Consider $f(x) = e^x$ and the point $(0,1)$

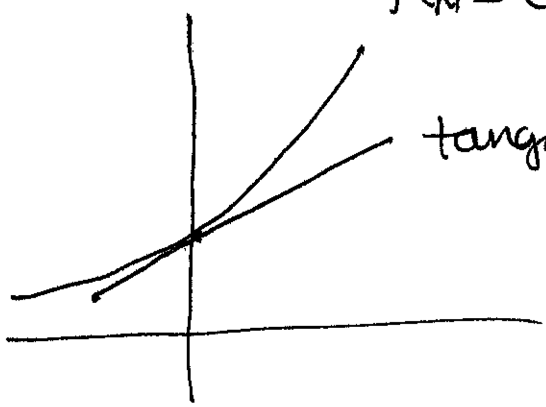


If we change from $x=0 \Rightarrow x=.1$ the change in y Δy

$$\Delta y = e^{.1} - e^0 = 1.1052 - 1$$

$$= .1052$$

Maybe there's a way to approximate the change in y. Here we will use the tangent line.



So $y - f(a) = f'(a)(x-a)$

$f(x) = e^x \quad a = 0$

so $f'(x) = e^x \quad f(a) = e^0 = 1$

$f'(a) = e^0 = 1$

tangent

$y - 1 = 1(x-1)$

$y = 1 + (x-1)$

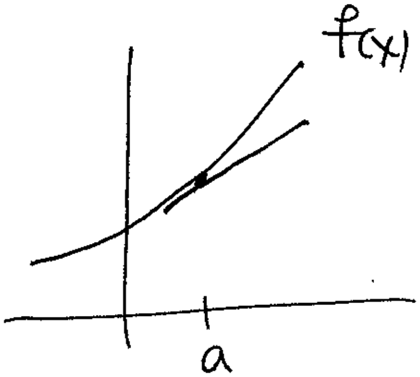
if we pick $x = 1.1$ then $y = 1 + (1.1 - 1)$

$= 1 + .1$

$= 1.1$

which is close to 1.1052 10% error

Can we do this in general



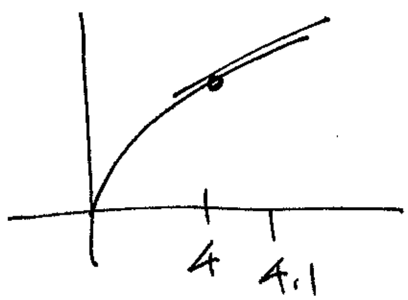
$\Delta y = f(a+h) - f(a)$ Exact change

tangent

$y - f(a) = f'(a)(x-a)$

sub $x = a+h$ $\quad y = f(a) + \underbrace{f'(a)\cdot h}_{\text{approx change}}$

Approximate $\sqrt{4.1}$



$f(x) = \sqrt{x} \quad f'(x) = \frac{1}{2\sqrt{x}}$

$f'(4) = \frac{1}{2\sqrt{4}} = \frac{1}{4}$

$\sqrt{4.1} = f(4) + f(4)(.1)$

$= 2 + \frac{1}{4}\cdot .1$

$= 2 + .025$

Abs Error

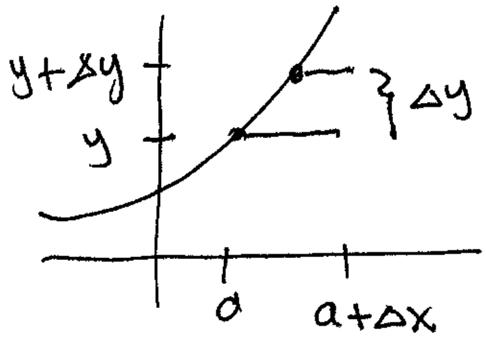
.000154

Rel Error

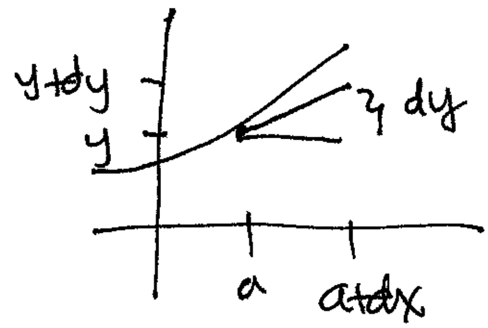
.0076%

Exc 2.024845

Exact



Approximate



So exact change in y $\quad \Delta y = f(a+\Delta x) - f(a)$

Approx. change following tangent

$$y - f(a) = f'(a)(x-a)$$

$y = f(a) + dy \quad x = a + dx$

So $\quad \cancel{f(a)} + dy - \cancel{f(a)} = f'(a)(\cancel{a} + dx - \cancel{a})$

$$\Rightarrow \quad dy = f'(a)\,dx$$

If dx & dy are "differentials"

then we define

$$dy = f'(x)\,dx$$

Ex $y = x^2$ $dy = 2x\,dx$

ex $y = \tan x$ $dy = \sec^2 x\,dx$

Ex $y = \sqrt{x^2+1}$ $dy = \frac{\cancel{2}x}{\cancel{2}\sqrt{x^2+1}}\,dx$

Ex If we measure the sides of a square at 6" ± .1" what is the approx. error in measuring the area

Sol^n $A = x^2$

$dA = 2x\,dx = 2(6)(.1) = 1.2$ sq inches

Now we compare with the exact

$$5.9^2 \le A \le 6.1^2$$

$$34.81 \le A \le 37.21$$

$$-1.19 \quad \Delta A \le 1.21$$

so the approx is very close to the exact.