Math 1496 Calc I

3.4 Chain Rule

Consider $f(x) = (x^2+1)^2$

what is $f'(x)$

well $f(x) = (x^2+1)(x^2+1)$
$= x^4+2x^2+1$

$f'(x) = 4x^3+4x$ ← which I rewrite as
$= 2(x^2+1)(2x)$

How about $f(x) = \sin^2 x$
$= \sin x \cdot \sin x$

product rule $f' = \sin x \cdot \cos x + \cos x \cdot \sin x$

$= 2\sin x \cdot \cos x$

In both example we have

$$\frac{d}{dx}\, g^2(x) = 2g(x)\,\frac{dg}{dx}$$

and this is the chain rule

In other words —

$$\boxed{\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx}}$$

Ex $y = (x^2+1)^2$

let $u = x^2+1$ so $y = u^2$

$$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx}$$

$$\frac{dy}{du} = 2u \qquad \frac{du}{dx} = 2x$$

$$\frac{dy}{dx} = 2u\cdot 2x = 2(x^2+1)\cdot 2x$$ which is what we saw last page.

Ex 2 $\frac{d}{dx}(\sin x)^2$

let $u = \sin x$ so $y = u^2$

$\frac{du}{dx} = \cos x \qquad \frac{dy}{du} = 2u$

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = 2u \cdot \cos x$

$= 2\sin x \cdot \cos x$

Ex 3

$y = e^{\tan x}$ find $\frac{dy}{dx}$

let $u = \tan x$ so $y = e^u$

$\frac{du}{dx} = \sec^2 x \qquad \frac{dy}{du} = e^u$

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = e^u \sec^2 x$

$= e^{\tan x} \cdot \sec^2 x$

Ex 4 $\frac{d}{dx} \sec(3x)$

let $u = 3x$ so $y = \sec u$

$\frac{du}{dx} = 3 \quad \frac{dy}{du} = \sec u \tan u$

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \sec u \tan u \cdot 3$

$= 3 \sec(3x) \tan(3x)$

Ex 5 $y = x^2 \sqrt{16 - x^2}$ #26 pg 164

This is a product rule & chain rule togeth

$\frac{dy}{dx} = 2x \sqrt{16 - x^2} + x^2 \frac{d}{dx} \sqrt{16 - x^2}$

$\cdot\, y = \sqrt{16 - x^2}$ let $u = 16 - x^2$ $y = \sqrt{u} = u^{1/2}$

$\frac{du}{dx} = -2x \quad \frac{dy}{du} = \frac{1}{2} u^{-1/2} = \frac{1}{2\sqrt{u}}$

so $\frac{d}{dx}\sqrt{16-x^2} = -2x \cdot \frac{1}{2\sqrt{u}} = \frac{-x}{\sqrt{16-x^2}}$

so if $y = x^2\sqrt{16-x^2}$

$$\frac{dy}{dx} = 2x\sqrt{16-x^2} + x^2\left[\frac{-x}{\sqrt{16-x^2}}\right]$$

we could simplify but won't

ex pg 164 # 32

$g(x) = \left[2+(x^2+1)^4\right]^3$ 2 step chain Rule

let $u = x^2+1$ so $g = \left[2+u^4\right]^3$

let $v = 2+u^4$ so $g = v^3$

$$\frac{dg}{dx} = \frac{dg}{dv}\cdot\frac{dv}{du}\cdot\frac{du}{dx}$$

$= 3v^2 \cdot 4u^3 \cdot 2x$ ← back sub

$= 3(2+u^4)^2 \cdot 4u^3 \cdot 2x$

$$= 3\left[a+(x^2+1)^4\right]^2 4(x^2+1)^3 \cdot 2x$$

Ex $y = (1+\sin^2 x)^{-1}$

$u = \sin x$ so $y = (1+u^2)^{-1}$

$v = 1+u^2$ so $y = v^{-1}$

$$\frac{dy}{dx} = \frac{dy}{dv} \cdot \frac{dv}{du} \cdot \frac{du}{dx}$$

$= -1v^{-2} \cdot 2u \cdot \cos x$ ← back sub

$= -(1+u^2)^{-2} \cdot 2u \cos x$

$= -(1+\sin^2 x)^{-2} \cdot 2\sin x \cos x$