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Frame Fields for the Higgs Sector: Goldstone Generalities

I. Introduction

This note is devoted to exploration of a certain class of Higgs potentials, which seem to be attractive candidates for application to the idea of large Higgs representations of the family group. The motivation came from examining how an electroweak-doublet, family-nonet multiplet of Higgs bosons might fit into the Big Picture at the grand unification level (GUT) level of consideration. Surprisingly to me, it appears to be not too easy to find an economical scenario, even when the family symmetry is considered to be an enlargement of the GUT symmetry of the standard model.

What I take as a working hypothesis is the idea that the family symmetry should be part of an extension of the generic O(10) GUT theory (without supersymmetry). As a first exploratory attempt, I have chosen to stay within the sequence of orthogonal groups and to go up to O(16) for both practical and esthetic reasons. Extending the rank of the group from 5 to 8 seems to be about the right size for accommodating three more families. And O(16) is a maximal subgroup of E(8). So from the point of view of string-theory esthetics, it also feels like a politically correct choice.

If we view this O(16) in terms of the obvious O(6) x O(10) subgroup, we see that the 120 gauge bosons inhabiting the O(16) adjoint representation break down into 15 "dark gluons" , 45 "visible" gauge bosons, only 12 of which are experimentally seen, and a remaining 6 x 10 = 60 coset bosons. It seems necessary to give all the coset bosons mass. Together with the 36 massive gauge bosons within the 45 of O(10), this means that at least 96 of the 120 members of the adjoint representation of O(16) must be massive. Presumably, these nonvanishing masses are the consequence of a generalized Higgs mechanism at work. This means that in the gaugeless limit, where we temporarily set the GUT gauge coupling constant to zero, the Higgs sector must contain at least 96 Goldstone modes.

This result seems to require, first, that the Higgs sector has to be large. But it also implies that in some sense this large Higgs sector has to be well-organized. If the O(16) symmetry were to be shattered into tiny pieces, why should there be so many Goldstone modes? In fact, the more we eliminate the large number of massless gauge bosons via the Higgs mechanism, the more we enlarge the Goldstone-boson portion of the Higgs sector. It would seem that either the Higgs sector is rich in massless modes, or otherwise the gauge sector is largely unbroken.

This inference does, however, depend upon application of Occam's razor. If one allows a limitless number of Higgs representations into the game just to give the sundry gauge bosons their masses, there is no limit to the complications that can in principle be introduced. So the name of this game will be to search for the simplest and smallest Higgs sector necessary to do the job.

This note is devoted to such a candidate description. The game will be to extend the symmetry to O(16) x O(16), with the new O(16) ungauged, and to place the Higgs representation into a 256 = (16, 16). It will turn out that as long as the gauged O(16) is not explicitly broken, one can apply quite a lot of explicit symmetry-breaking mutilation to the ungauged O(16) without losing Goldstone modes.

This kind of structure is not unfamiliar in the context of general relativity. The Einstein-Cartan first-order formalism is an O(3,1) gauge theory living in Minkowski spacetime. The degrees of freedom are 24 connection variables ω_μ^{AB} (gauge potentials), supplemented by 16 vierbein variables e_μ^A. It is these vierbein variables, or frame fields, which are the prototype for this Higgs structure (arXiv 1212.0585).

I find further encouragement for this choice from the work of Hong-Mo Chan and his collaborators (arXiv 1206.0199). They have produced a phenomenologically successful description of quark and lepton masses and mixings, conceptually based on this kind of frame-field hypothesis for their Higgs sector. However, despite their very attractive set of initial hypotheses, they have not yet succeeded in providing a detailed realization of the Higgs sector.

There are additional potential benefits present in this basic architecture. It is natural to presume that the 15 members of the adjoint representation of the gauge-sector O(6) are colorless, electrically neutral, and electroweak singlet. They can therefore serve as building blocks for the dark-matter sector of the standard model. Indeed, it is even tempting to leave some or all of these 15 members as massless and confined. This "dark gauge group" could be as large as the full O(6) = SU(4) itself, or as small as an O(3) = SU(2) subgroup thereto. If such "dark gluons" exist, a dark-confinement scale Λ_{dark} should be anticipated. It would be awkward to place such a confinement scale Λ_{dark} at a mass scale lower than the QCD confinement scale $\Lambda_{QCD} \sim 200$ MeV. But it seems prudent to at least consider $\Lambda_{dark} \gtrsim \Lambda_{QCD}$.

There is another positive byproduct of this "dark-gluon" hypothesis. The electroweak-doublet, family-nonet of Higgs bosons we have introduced in previous notes does not easily fit into the 256-plet of Higgs' considered above. This problem is mitigated if one assumes that this nonet is in fact a composite of a family triplet with its anti-triplet, bound together via exchange of the "dark gluons". There is a historical precedent for this kind of thing—in particular the 0^+, 0^-, and 1^- nonets containing, e.g. σ, π, and ρ respectively.

The existence of such an analogy invites its extension. Our O(16) gauge group breaks down into a "dark O(6) ", a "visible (Pati-Salam) O(6)", linked by an "electroweak O(4)". Perhaps the "dark O(6)" should be viewed in a way that is parallel to how the "visible O(6)" is viewed. In particular, suppose the dark-confinement scale is no larger than, say, 1 TeV. Above this scale, the description will be in terms of "dark partons". Below this scale the description will be in terms of "dark hadrons". In the visible sector, the corresponding description is very complex at distance scales large compared to the confinement scale. While it is not hopelessly difficult to anticipate that there will be a rich spectrum of hadrons, it is much harder to anticipate the existence of nuclear matter with the very small binding energy of 8 MeV per nucleon. Still more difficult is to anticipate the existence of a single charged-lepton degree of freedom with a mass of 0.5 MeV. But the consequences of such minor details within the Grand Scheme are, needless to say, profound. Given that there is 6 times as much dark matter as visible matter in the universe, it would seem prudent to be prepared for essential complications to also emerge in the infrared limit of the dark-sector description.

II. O(2N) Generalities

The most straightforward way of approaching this general problem seems to be to explicitly introduce the O(2N) gauge-bosons, in particular their couplings to the Higgs sector. Once the choice of Higgs representations has been made, a certain pattern of vevs will be assumed. Then, just from the structure of the gauged kinetic energy term for the Higgs sector, it is straightforward to determine which gauge bosons are massive and which remain massless. In addition, the linear combinations of Higgs fields which are Goldstone can also be read off reasonably easily. Having done this, the gauge coupling constant can be set to zero, leaving behind a putative set of Goldstone modes—one for each massive gauge-boson degree of freedom. The final task is then to construct a Higgs potential which generates this pattern. It is of course this last step which is trickiest.

The Higgs multiplets which will turn out to be relevant to the problem at hand appear to be O(2N) vectors and antisymmetric second rank tensors (adjoints). This set of multiplets may or may not suffice to create Yukawa couplings to the fermions. But, no matter what, we will in this note set the issue of the fermion masses aside.

A single adjoint representation can go a long way in giving the gauge bosons masses, and we begin the detailed considerations by introducing the adjoint multiplet of gauge bosons W, plus one adjoint representation of Higgs' Φ. Our notation is as follows:

$$W_\mu = \begin{pmatrix} \omega_0+\omega_3 & \omega_1 - i\omega_2 \\ \omega_1 + i\omega_2 & \omega_0-\omega_3 \end{pmatrix}_\mu \equiv \frac{(\omega_0 + \vec{\tau}\cdot\vec{\omega})_\mu}{\sqrt{2}} \qquad \Phi = \frac{1}{\sqrt{2}}\begin{pmatrix} \phi_0+\phi_3 & \phi_1 - i\phi_2 \\ \phi_1 + i\phi_2 & \phi_0-\phi_3 \end{pmatrix} \equiv \frac{(\phi_0 + \vec{\tau}\cdot\vec{\phi})}{\sqrt{2}}$$

Because W_{ij} and Φ_{ij} are real antisymmetric matrices, the quantities ω_i and ϕ_i (i = 0, 1, and 3) are N x N real antisymmetric matrices, while ω_2 and ϕ_2 are pure imaginary N x N symmetric matrices:

$$\left.\begin{aligned} \omega_i^T &= -\omega_i \\ \phi_i^T &= -\phi_i \end{aligned}\right\} i = 0,1,3 \qquad\qquad \begin{aligned} \omega_2 &= \omega_2^T = -\omega_2^* \\ \phi_2 &= \phi_2^T = -\phi_2^* \end{aligned}$$

This means that all of these ω_i's and ϕ_i 's are antihermitian.

$$\omega_i^\dagger = -\omega_i \qquad \phi_i^\dagger = -\phi_i \qquad (i = 0,1,2,3)$$

Now assume that

$$\langle\Phi\rangle = \begin{pmatrix} 0 & V \\ -V & 0 \end{pmatrix} \qquad V = \begin{pmatrix} v_1 & & & \\ & v_2 & & \\ & & \ddots & \\ & & & v_N \end{pmatrix} \qquad \begin{aligned} &\langle\phi_i\rangle = 0 \quad (i = 0,1,3) \\ &\langle\phi_2\rangle = iV \end{aligned}$$

Given this hypothesis, we look at the gauged kinetic term for the adjoint Higgs representation;

$$\mathcal{L}_\Phi = -\frac{1}{4}\sum_{i,j=1}^{2N}\left[\partial_\mu \Phi_{ij} - \frac{ig}{\sqrt{2}}(W_\mu)_{ik}\Phi_{kj} - \frac{ig}{\sqrt{2}}(W_\mu)_{jk}\Phi_{ik}\right]^2$$

$$\Rightarrow \mathcal{L}_{mass} = +\frac{g^2}{8}\sum_{i,j=1}^{2N}[W_\mu, \langle\Phi\rangle]_{ij}[W^\mu, \langle\Phi\rangle]_{ij} = -\frac{g^2}{8}\mathrm{Tr}[W,\langle\Phi\rangle]^2$$

Write

$$[W, \langle\Phi\rangle] = -\frac{i}{2}\left[(\omega_0 + \vec{\tau}\cdot\vec{\omega}), V\tau_2\right]$$

It follows that

$$2[W,\langle\Phi\rangle] = -i\tau_2[\omega_0, V] + \tau_3\{\omega_1, V\} - i[\omega_2, V] - \tau_1\{\omega_3, V\}$$

Therefore, after taking the trace over the 2×2 τ- matrices, the mass term for the gauge-boson sector becomes

$$\mathcal{L}_{mass} = -\frac{g^2}{16} tr\left(-[\omega_0, v]^2 + \{\omega_1, v\}^2 - [\omega_2, v]^2 + \{\omega_3, v\}^2\right)$$

The trace instruction tr evidently refers to the trace over the $N \times N$ matrices. Upon doing the trace, we obtain

$$\mathcal{L}_{mass} = -\frac{g^2}{16}\sum_{i,j=1}^{N}\left\{\left[|(\omega_0)_{ij}|^2 + |(\omega_2)_{ij}|^2\right](v_i - v_j)^2 + \left[|(\omega_1)_{ij}|^2 + |(\omega_3)_{ij}|^2\right](v_i + v_j)^2\right\}$$

The sundry masses fall into two categories;

$$(m^2)_{ij} = \frac{g^2}{4}(v_i \pm v_j)^2$$

If the $N \times N$ matrix of vevs v is a multiple of the unit matrix, evidently the w_0 and w_2 multiplets remain massless. The number of degrees of freedom contained therein is

$$N_{tot} = \frac{N(N-1)}{2} + \frac{N(N+1)}{2} = (N^2-1) + 1$$

Evidently the corresponding gauge group will be $SU(N) \times U(1)$.

If the vev v breaks up into blocks of unit matrices of size

$$N_1, N_2, \cdots N_k$$

evidently the number of massless gauge bosons will be

$$N_{tot} = \frac{N_1(N_1-1)}{2} + \frac{N_1(N_1+1)}{2} + \cdots = N_1^2 + N_2^2 + \cdots N_k^2$$

The gauge group in this case will be

$$SU(N_1) \times SU(N_2) \times \cdots SU(N_k) \times U(1)^k$$

Note that no matter what the pattern of such vevs, there will be at the minimum an unbroken $U(1)^N$ symmetry which remains.

However, it seems that this scenario is a very promising one, provided the appropriate vevs can be created via some choice of Higgs potential. This in turn will be facilitated by the introduction of the frame fields.

III. Frame Field Generalities

We introduce 2N vector fields Ψ_i^I, with a gauge-invariant kinetic-energy term as follows:

$$\mathcal{L}_\Psi = -\frac{1}{2}\sum_{I,i=1}^{2N}\left[\partial_\mu \Psi_i^I - i\frac{g}{\sqrt{2}}(W_\mu)_{ij}\,\Psi_j^I\right]^2$$

We assume that the vevs of these fields follow the same pattern as that for the adjoint fields. In general, some of them might vanish. But in this section we assume the most general case

$$\langle \Psi_i^I \rangle = \frac{1}{\sqrt{2}}\left(\begin{array}{c|c} 0 & \begin{matrix}\tilde{v}_1 & & \\ & \tilde{v}_2 \ddots & \\ & & \tilde{v}_N\end{matrix} \\ \hline \begin{matrix}-\tilde{v}_1 & & \\ & -\tilde{v}_2 \ddots & \\ & & -\tilde{v}_N\end{matrix} & 0 \end{array}\right)$$

$$\Psi_i^I \Rightarrow \frac{1}{\sqrt{2}}\left(\psi_0 + \vec{\tau}\cdot\vec{\psi} + i\tau_2\tilde{v}\right)$$

This leads to a mass term of the gauge bosons that can be written as

$$\mathcal{L}_{Mass} = +\frac{g^2}{8}\mathrm{Tr}\, W_\mu\langle\Psi\rangle\langle\Psi^T\rangle W^\mu = +\frac{g^2}{8}\mathrm{Tr}\, W_\mu \tilde{v}^2 W^\mu$$
$$= -\frac{g^2}{8}\mathrm{tr}\left[\left(\omega_0^2 + \vec{\omega}\cdot\vec{\omega}\right)\tilde{v}^2\right] = -\frac{g^2}{8}\sum_{i,j=1}^{N}\sum_{a=0}^{3}\left|(\omega_a)_{ij}\right|^2\tilde{v}_i^2$$

Consequently, none of the gauge bosons remain massless. In order to reduce the gauge symmetry from O(2N) to O(2N – 2n), the simplest scenario for implementing this will be a set of 2N independent potentials, one for each O(2N) frame vector, but with only 2n of them Mexican-hat. If this is done, it turns out that there are more Goldstone modes than those eaten by the gauge bosons. Straightforward arithmetic gives the number of excess uneaten Goldstone bosons as n(2n – 1). We will return to this issue in Section V.

IV. The Higgs Potential for the O(2N) Adjoint Field

Write the adjoint field as follows:

$$\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix} \phi_0 + \phi_3 & v + \phi_1 - i\phi_2 \\ -v + \phi_1 + i\phi_2 & \phi_0 - \phi_3 \end{pmatrix} \qquad \langle \Phi \rangle = \frac{1}{\sqrt{2}}\left(\begin{array}{cccc|cccc} & & & & v_1 & & & \\ & 0 & & & & v_2 & & \\ & & & & & & \ddots & \\ & & & & & & & v_N \\ \hline -v_1 & & & & & & & \\ & -v_2 & & & & & 0 & \\ & & \ddots & & & & & \\ & & & -v_N & & & & \end{array}\right)$$

$$= \frac{1}{\sqrt{2}}\left(\phi_0 + \vec{\tau}\cdot\vec{\phi} + i\tau_2 v\right)$$

Recall that the symmetry properties of the adjoint field are:

$$\left.\begin{array}{l} \phi_i^* = \phi_i \\ \phi_i^T = -\phi_i \end{array}\right\} i = 0,1,3 \qquad \begin{array}{l} \phi_2^* = -\phi_2 \\ \phi_2^T = \phi_2 \end{array} \qquad \phi_i^\dagger = -\phi_i \quad (i = 0,1,2,3)$$

In particular,

$$\mathrm{Tr}\,\phi_i^2 < 0 \qquad (i = 0,1,2,3)$$

The potential has the form

$$U_{adj} = \frac{\mu^2}{2}\mathrm{Tr}\,\Phi^2 + \frac{\lambda}{4}\left(\mathrm{Tr}\,\Phi^2\right)^2 + \frac{\lambda'}{4}\mathrm{Tr}\,\Phi^4$$

Now expand the potential out as usual to quadratic order in the dynamical fields

$$\begin{aligned} U_{adj} = & -\frac{\mu^2}{2}\mathrm{tr}\,v^2 + \frac{\lambda}{4}\left(\mathrm{tr}\,v^2\right)^2 + \frac{\lambda'}{8}\left(\mathrm{tr}\,v^4\right) \\ & + \frac{i\mu^2}{2}\mathrm{tr}\{v, \phi_2\} - \frac{i\lambda}{2}\left(\mathrm{tr}\,v^2\right)\left(\mathrm{tr}\{v,\phi_2\}\right) - \frac{i\lambda'}{2}\left(\mathrm{tr}\,v^3\phi_2\right) \\ & + \frac{\mu^2}{2}\mathrm{tr}\left(\phi_0^2 + \vec{\phi}\cdot\vec{\phi}\right) - \frac{\lambda}{2}\left(\mathrm{tr}\,v^2\right)\left[\mathrm{tr}\left(\phi_0^2 + \vec{\phi}\cdot\vec{\phi}\right)\right] - \frac{\lambda}{4}\left[\mathrm{tr}\{v,\phi_2\}\right]^2 \\ & - \frac{\lambda'}{2}\left[\mathrm{tr}\,v^2\left(\phi_0^2 + \vec{\phi}\cdot\vec{\phi}\right)\right] - \frac{\lambda'}{4}\left[\mathrm{tr}\left(v\phi_0 v\phi_0 - v\vec{\phi}\cdot v\vec{\phi}^*\right)\right] \end{aligned}$$

The minimum conditions are

$$-\mu^2 v_i + \lambda(\mathrm{tr}\, v^2) v_i + \frac{\lambda'}{2} v_i^3 = 0$$

This implies that the squared vevs must be equal:

$$\lambda'(v_i^2 - v_j^2) = 0 \quad i \neq j \qquad\qquad v_i^2 = \frac{\mu^2}{(N\lambda + \frac{\lambda'}{2})} \equiv v^2$$

However, these vevs will be split when they are mixed with the frame-field vevs. However, we first have a look at the nature of the spontaneous symmetry breaking before this complication is introduced. Imposition of the minimum condition eliminates the terms linear in the dynamical fields, and the remaining terms of quadratic order reduce to the following:

$$\begin{aligned} U_{adj} &= \cdots + \frac{1}{2}(\mu^2 - \lambda\, \mathrm{tr}\, v^2)\left[\mathrm{tr}(\phi_0^2 + \vec{\phi}\cdot\vec{\phi})\right] - \frac{\lambda}{4}\left[\mathrm{tr}\{v,\phi_2\}\right]^2 \\ &\quad - \frac{\lambda'}{2} v^2\, \mathrm{tr}(\phi_0^2 + \vec{\phi}\cdot\vec{\phi}) - \frac{\lambda'}{4} v^2\, \mathrm{tr}(\phi_0^2 - \vec{\phi}\cdot\vec{\phi}^*) + \cdots \\ &= \cdots + \frac{1}{2}\left\{\mathrm{tr}\left[\mu^2 - \lambda(\mathrm{tr}\, v^2) - \frac{\lambda'}{2} v^2\right](\phi_0^2 + \vec{\phi}\cdot\vec{\phi})\right\} - \frac{\lambda}{4}\left[\mathrm{tr}\{v,\phi_2\}\right]^2 \\ &\quad - \frac{\lambda'}{4} v^2\left[\mathrm{tr}(\phi_0^2 + \phi_1^2 + \phi_2^2 + \phi_3^2) + \mathrm{tr}(\phi_0^2 - \phi_1^2 + \phi_2^2 - \phi_3^2)\right] + \cdots \\ &= \cdots - \lambda(\mathrm{tr}\, v\phi_2)^2 - \frac{\lambda'}{2} v^2\left[\mathrm{tr}(\phi_0^2 + \phi_2^2)\right] + \cdots \end{aligned}$$

We see that the ϕ_1 and ϕ_3 fields are indeed Goldstone. The remaining Higgs fields have a common mass, with one exception:

$$mass^2(\phi_0) = mass^2(\phi_2 \neq \hat{\phi}_2) = \lambda' v^2 = 2\mu^2\left(\frac{\lambda'}{\lambda' + 2N\lambda}\right)$$

The exceptional degree of freedom is the unit element of ϕ_2, denoted as $\hat{\phi}_2$:

$$(\phi_2)_{ij} = \hat{\phi}_2 \frac{\delta_{ij}}{\sqrt{N}} + \cdots$$

The mass for this state is

$$mass^2(\hat{\phi}_2) = v^2(\lambda' + 2N\lambda) = 2\mu^2$$

V. Nature of the Goldstone Modes

At this point we have specified the Higgs representations which will be used, as well as the pattern of vevs which are assumed. This has allowed us to ascertain which gauge bosons become massive, and which remain massless. This in turn tells us the number of Goldstone bosons which will be "eaten" in order for this to happen.

However, there is more information which we may glean from the gauged kinetic-energy terms for the Higgs sector. This comes from the cross term, linear in the gauge-boson field and quadratic in the Higgs fields. If we replace one of the two Higgs-field factors by its vev, the resulting structure is, after an integration by parts, a coupling of the divergence of the gauge field to a linear combination of the Higgs fields. According to the Goldstone theorem, this linear combination will indeed be a Goldstone mode.

Our strategy here will be to assume the maximum possible number of vevs. This means that all 2N frame vector fields are Mexican-hat and have distinct vevs. In addition, we will assume, as above, the same kind of structure for the adjoint Higgs fields.

The general structure of the cross term then takes the form

$$\mathcal{L}_{Kin} = \cdots + \frac{ig}{2\sqrt{2}} \sum_{i,j,k=1}^{2N} (\partial_\mu \Phi_{ij}) \left[W^\mu_{ik} \langle \Phi_{kj} \rangle + W^\mu_{jk} \langle \Phi_{ik} \rangle \right] + \frac{ig}{\sqrt{2}} \sum_{I,i,j=1}^{2N} (\partial_\mu \Psi^I_i) W^\mu_{ij} \langle \Psi^I_j \rangle$$

$$\Rightarrow \sum_{i,j,k=1}^{2N} \Phi_{ij} D_{ik} \langle \Phi_{kj} \rangle + \sum_{I,i,k=1}^{2N} \Psi^I_i D_{ik} \langle \Psi^I_k \rangle$$

$$= \cdots - \sum_{i,k=1}^{N} \begin{cases} + \left[-i(d_0)_{ik}(\phi_2)_{ik} + (d_1)_{ik}(\phi_3)_{ik} - i(d_2)_{ik}(\phi_0)_{ik} + (d_3)_{ik}(\phi_1)_{ik} \right] v_k \\ + \left[-i(d_0)_{ik}(\psi_2)^k_i + (d_1)_{ik}(\psi_3)^k_i - i(d_2)_{ik}(\psi_0)^k_i + (d_3)_{ik}(\psi_1)^k_i \right] \tilde{v}_k \end{cases}$$

We here see that the divergence of the gauge-boson fields

$$D_{ik} \equiv \frac{ig}{\sqrt{2}} (\partial_\mu W^\mu)_{ik} \equiv (d_0 + \vec{\tau} \cdot \vec{d})_{ik}$$

indeed mix with the Goldstone modes.

In this fully-broken scenario, we see that the Goldstone modes break into threefold linear combinations, and that contributions of the different d_a (a = 0, 1, 2, 3) do not mix. Coefficients of d_0, d_1, and d_3 are N x N antisymmetric Goldstone fields, while the coefficient of d_2 is an N x N symmetric array of Goldstone modes. Upon focusing in on any one of these threefold linear combinations, two orthogonal-complement modes can be constructed which are non-Goldstone. One of these is the linear combination of $(\psi_a)_i^k$ and $(\psi_a)_k^i$ which is orthogonal to the Goldstone mode. It is always a symmetric N x N matrix. On the other hand, the remaining linear combination is easily seen to be antisymmetric. Therefore, for each choice of a = 0, 1, 2, 3 we find N^2 non-Goldstone degrees of freedom, or $(2N)^2$ in all. So in this limit all Higgs degrees of freedom are accounted for.

To obtain fewer Goldstone modes and more massless gauge bosons, we must arrange that some of the adjoint vevs v_i are equal to each other, and that this feature is shared in the appropriate way by the frame vevs $\tilde{v}_i$.

We close this section with a listing of the Goldstone modes in a notation more convenient for the material which follows.

$$\mathcal{L}_{Kin} = \cdots - \frac{1}{2}\sum_{i,k=1}^{N} \begin{cases} (d_0)_{ik}\left[+(v_i - v_k)(i\phi_2)_{ik} + \tilde{v}_i (i\psi_2)_k^i - \tilde{v}_k (i\psi_2)_i^k\right] \\ (d_1)_{ik}\left[+(v_i + v_k)(\phi_3)_{ik} - \tilde{v}_i (\psi_3)_k^i + \tilde{v}_k (\psi_3)_i^k\right] \\ (id_2)_{ik}\left[+(v_i - v_k)(\phi_0)_{ik} - \tilde{v}_i (\psi_0)_k^i - \tilde{v}_k (\psi_0)_i^k\right] \\ (d_3)_{ik}\left[+(v_i + v_k)(\phi_1)_{ik} - \tilde{v}_i (\psi_1)_k^i + \tilde{v}_k (\psi_1)_i^k\right] \end{cases}$$

VI. The Frame-Field Higgs Potential

We reintroduce the 2N x 2N frame fields Φ^I_i and again assume a vev structure which to large extent mirrors that used for the adjoint field:

$$\langle \Psi^I_i \rangle = \frac{1}{\sqrt{2}} \left(\begin{array}{cc:cc} \multicolumn{2}{c:}{\multirow{2}{*}{0}} & \begin{matrix} \tilde{v}_1 & & \\ & \tilde{v}_2 & \\ & & \ddots \; \tilde{v}_n \end{matrix} & 0 \\ \cdashline{3-4} & & 0 & 0 \\ \hdashline \begin{matrix} -\tilde{v}_1 & & \\ & -\tilde{v}_2 & \\ & & \ddots \; -\tilde{v}_n \end{matrix} & 0 & \multicolumn{2}{c}{\multirow{2}{*}{0}} \\ \cdashline{1-2} 0 & 0 & & \end{array} \right)$$

This form represents a design choice, which clearly can be generalized. It is motivated by the results of Section III. Only the first n of the above vevs $\tilde{v}$ are assumed to be nonvanishing. Consequently we would expect the SU(N) symmetry obtained in Section IV to be reduced down to SU(N – n) or something close to it by this construction.

As before, write

$$\Psi = \frac{1}{\sqrt{2}} \begin{pmatrix} \psi_0 + \psi_3 & \tilde{v} + \psi_1 - i\psi_2 \\ -\tilde{v} + \psi_1 + i\psi_2 & \psi_0 - \psi_3 \end{pmatrix} = \frac{1}{\sqrt{2}} \left(\psi_0 + \vec{\tau} \cdot \vec{\psi} + i \tau_2 \tilde{v} \right)$$

The piece of the Higgs potential belonging to Ψ can be chosen as a sum over 2N independent potentials. The first n of these are assumed to be Mexican hat. The remaining components are passive and represented by mass terms:

$$U_{frame} = \sum_{I=1}^{2N} U_I \left(\sum_{i=1}^{2N} \Psi^I_i \Psi^I_i \right)$$

$$U_I(\rho_I^2) = U_{I+N}(\rho_I^2) \begin{cases} -\dfrac{m_I^2}{2} \rho_I^2 + \dfrac{\lambda_I}{4} \rho_I^4 & I \le n \\ + \dfrac{M_I^2}{2} \rho_I^2 & N \ge I > n \end{cases}$$

The assumption $U_I = U_{I+N}$ is made in order to enforce the symmetry structure of the vevs.

Expansion of the ρ_I^2 in the new variables yields the following expression:

$$\rho_I^2 = \sum_{i=1}^{N} \sum_{a=0}^{3} \left|(\psi_a)_i^I\right|^2 + \tilde{v}_I^2 - 2i\tilde{v}_I(\psi_2)_I^I$$

Expansion of U_{frame} through quadratic order then yields

$$U_{frame} = \sum_{I=1}^{n} \begin{cases} U_I(\tilde{v}_I^2) - 2i\tilde{v}_I U_I'(\tilde{v}_I^2)(\psi_2)_I^I \\ + U_I'(\tilde{v}_I^2)\left[\sum_{i=1}^{N}\sum_{a=0}^{3}\left|(\psi_a)_i^I\right|^2\right] \\ + 2U''(\tilde{v}_I^2)\tilde{v}_I^2\left|(\psi_2)_I^I\right|^2 \end{cases}$$

$$+ \sum_{I=n+1}^{N}\left[\frac{M_I^2}{2}\sum_{i=1}^{N}\sum_{a=0}^{3}\left|(\psi_a)_i^I\right|^2\right.$$

At this level, the minimum condition, $U' = 0$, leads to ψ_0 and ψ_3 each providing nN Goldstone modes. In addition to these, ψ_1 and ψ_2 each provide $n(N-1)$ goldstones, leading to a grand total of $2n(2N-1)$. The gauge degrees of freedom eat up N_{gauge} of these, where N_{gauge} is given by

$$N_{gauge} = N(2N-1) - (N-n)(2N-2n-1) = n[4N-2n-1]$$

This, as previously noted, leaves behind an excess of $n(2n-1)$ frame Goldstone modes at this level. Of course these will be influenced by the coupling of the frame fields to the adjoint fields. We now turn to this issue.

VII. Coupling the Frame Fields to the Adjoint Fields

We are finally ready to introduce a Yukawa-like coupling of the frame fields to the adjoint field. This will evidently break the symmetry down in a way dictated by the vev structure of the frame fields. The Yukawa coupling has the form

$$U_3 = \sum_{I,J,i,j=1}^{2N} F_{IJ}\, \Psi_i^{I}\, \Phi_{ij}\, \Psi_j^{J}$$

A general analysis appears to be cumbersome. We tentatively choose the same antisymmetric-diagonal structure used for the vevs to describe the coupling constants F :

$$F = \frac{1}{\sqrt{2}} \left(\begin{array}{c|c} 0 & \begin{array}{ccc|c} f_1 & & & \\ & f_2 \ddots & & 0 \\ & & f_n & \\ \hline & 0 & & 0 \end{array} \\ \hline \begin{array}{ccc|c} -f_1 & & & \\ & -f_2 \ddots & & 0 \\ & & -f_n & \\ \hline & 0 & & 0 \end{array} & 0 \end{array} \right)$$

Expansion of U_3 to quadratic order in the dynamical fields then yields a reasonably simple result:

$$U_3 = -\frac{1}{2}\sum_{I=1}^{n} f_I V_I \tilde{V}_I^2 + \frac{i}{2}\sum_{i=1}^{n} f_i \left[2V_i \tilde{V}_i^2 (\psi_2)_i^i + \tilde{V}_i^2 (\phi_2)_{ii} \right]$$

$$+ \sum_{I=1}^{n} f_I \tilde{V}_I \left[\sum_{a=0}^{3} \sum_{i=1}^{N} (\psi_a)_i^I (\phi_a)_{iI} \right]$$

$$- \frac{1}{2}\sum_{I,j=1}^{n} f_I V_j \sum_{a=0}^{3} (-)^a \left| (\psi_a)_j^I \right|^2$$

Note that there is no mixing between fields with different values of a = 0, 1, 2, 3 .

We must now put together the various pieces of the potential, namely U_{adj}, U_{frame} and U_3. It seems to be best to use a very explicit notation. We therefore rewrite the frame potential contributions as follows:

$$
\begin{aligned}
U_{frame} = {} & \sum_{I=1}^{n} U_I(\tilde{v}_I^2) - 2i\sum_{I=1}^{n} U_I'(\tilde{v}_I^2)\,\tilde{v}_I\,(\psi_2)_I^I \\
& + \sum_{I=1}^{n} U_I'(\tilde{v}_I^2)\left[\sum_{a=0}^{3}\sum_{i=1}^{N} |(\psi_a)_i^I|^2\right] \\
& + \sum_{i=n+1}^{N} \frac{M_I^2}{2}\left[\sum_{a=0}^{3}\sum_{i=1}^{N} |(\psi_a)_i^I|^2\right] \\
& + 2\sum_{I=1}^{n} U_I''(\tilde{v}_I^2)\,|(\psi_2)_I^I|^2\,\tilde{v}_I^2
\end{aligned}
$$

The adjoint potential contributions are

$$
\begin{aligned}
U_{adj} = {} & -\frac{\mu^2}{2}\sum_{i=1}^{N} v_i^2 + \frac{\lambda}{4}\left(\sum_{i=1}^{N} v_i^2\right)^2 + \frac{\lambda'}{8}\left(\sum_{i=1}^{n} v_i^4\right) \\
& + i\sum_{i=1}^{N} (\phi_2)_{ii}\left[\mu^2 v_i - \lambda\left(\sum_{k=1}^{N} v_k^2\right) v_i - \frac{\lambda'}{2} v_i^3\right] \\
& + \sum_{a=0}^{3}\sum_{i,j=1}^{N}\left[-\frac{\mu^2}{2} + \frac{\lambda}{2}\left(\sum_{k=1}^{N} v_k^2\right) + \frac{\lambda'}{2} v_i^2\right]|(\phi_a)_{ij}|^2 - \lambda\left[\sum_{i=1}^{N} v_i(\phi_2)_{ii}\right]^2 \\
& + \frac{\lambda'}{4}\sum_{i,j=1}^{N} v_i v_j\left[\sum_{a=0,2} |(\phi_a)_{ij}|^2 - \sum_{a=1,3} |(\phi_a)_{ij}|^2\right]
\end{aligned}
$$

The form of the vacuum potential is

$$
U_{vac} = \sum_{I=1}^{n} U_I(\tilde{v}_I^2) + \left[-\frac{\mu^2}{2}\sum_{i=1}^{N} v_i^2 + \frac{\lambda}{4}\left(\sum_{i=1}^{N} v_i^2\right)^2 + \frac{\lambda'}{16}\sum_{i=1}^{N} v_i^4\right] - \frac{1}{2}\sum_{I=1}^{n} f_I v_I \tilde{v}_I^2
$$

Variation with respect to the adjoint vevs v_i yields

$$
-\mu^2 v_i + \lambda\left(\sum_{k=1}^{N} v_k^2\right) v_i + \frac{\lambda'}{2} v_i^3 = \begin{cases} \frac{1}{2} f_i \tilde{v}_i^2 & (i \le n) \\ 0 & (i > n) \end{cases}
$$

Variation with respect to the frame vevs $\tilde{v}_I$ yields

$$
2U_I'\tilde{v}_I = f_I v_I \tilde{v}_I \qquad (I \le n)
$$

It follows that

$$U_i'(\tilde{v}_i^2) = \frac{f_i v_i}{2}$$

Consequently

$$\left[-\mu^2 + \lambda\left(\sum_{k=1}^{N} v_k^2\right) + \frac{\lambda'}{2} v_i^2\right] = \begin{cases} \frac{f_i}{2}\left(\frac{\tilde{v}_i^2}{v_i}\right) = U_i'\left(\frac{\tilde{v}_i}{v_i}\right)^2 & (i \le n) \\ 0 & (i > n) \end{cases}$$

In this form we can see how the frame-field contribution drives the adjoint-field vevs to new values, which can of course include symmetry breaking.

Upon compiling the terms linear in the dynamical fields, it is easy to determine that they indeed sum to zero:

$$\begin{aligned} U_{linear} &= i\sum_{I=1}^{n} (\psi_2)^I_I \left[-2U_I'\tilde{v}_I + f_I v_I \tilde{v}_I\right] \\ &\quad + i\sum_{i=1}^{N} (\phi_2)_{ii}\left\{\left[\mu^2 v_i - \lambda\left(\sum_{k=1}^{N} v_k^2\right)v_i - \frac{\lambda'}{2} v_i^3\right] + \frac{f_i \tilde{v}_i^2}{2}\right\} \\ &= 0 \end{aligned}$$

The terms quadratic in the dynamical fields are

$$\begin{aligned} U = \cdots &+ \sum_{I=1}^{n} U_I'\left[\sum_{a=0}^{3}\sum_{i=1}^{N} |(\psi_a)^I_i|^2\right] + \sum_{I=n+1}^{N} \frac{M_I^2}{2}\left[\sum_{a=0}^{3}\sum_{i=1}^{N} |(\psi_a)^I_i|^2\right] \\ &+ 2\sum_{I=1}^{n} U_I''\left[|(\psi_2)^I_I|^2\right]\tilde{v}_I^2 \\ &+ \frac{1}{4}\sum_{a=0}^{3}\sum_{j=1}^{N}\left\{\sum_{i=1}^{n}\left[\lambda' v_i^2 + \frac{f_i\tilde{v}_i^2}{v_i}\right]|(\phi_a)_{ij}|^2 + \sum_{i=n+1}^{N} \lambda' v_i^2 |(\phi_a)_{ij}|^2\right\} \\ &+ \lambda\left(\sum_{i=1}^{N} v_i |(\phi_2)_{ii}|\right)^2 + \frac{\lambda'}{4}\sum_{i,j=1}^{N} v_i v_j \left[\sum_{a=0}^{3}(-)^a |(\phi_a)_{ij}|^2\right] \\ &+ \sum_{I=1}^{n} f_I \tilde{v}_I\left[\sum_{a=0}^{3}\sum_{i=1}^{N} (\psi_a)^I_i (\phi_a)_{iI}\right] - \frac{1}{2}\sum_{I=1}^{n}\sum_{j=1}^{N} f_I v_j\left[\sum_{a=0}^{3}(-)^a |(\psi_a)^I_j|^2\right] \end{aligned}$$

Note that we have already used the minimum condition to simplify the expression.

With one exception, the only mixings are three-way, namely $(\phi_a)_{ij}$ can mix with $(\psi_a)^j_i$ and $(\psi_a)^i_j$. The exception occurs for the diagonal elements of ϕ_2. In what follows we will be mainly interested in identifying the Goldstone modes that survive, and matching them with those expected from the considerations in Section V. In fact the only candidates with diagonal indices which might contribute a Goldstone mode are $(\psi_0)^i_i$ and $(\psi_3)^i_i$. The relevant terms in the potential are (for $i \le n$)

$$U_{Higgs} = \cdots + U_i' \left[|(\psi_0)^i_i|^2 + |(\psi_3)^i_i|^2 \right] - \frac{f_i v_i}{2} \left[|(\psi_0)^i_i|^2 - |(\psi_3)^i_i|^2 \right] + \cdots$$
$$= \cdots + f_i v_i |(\psi_3)^i_i|^2 + \cdots \qquad (i \le n)$$

Indeed we do find a Goldstone mode $(\psi_0)^i_i$ does exist, and that it matches what was anticipated in Section V.

The next easiest case is when the indices are different and both larger than n. Under these circumstances, it follows from the vacuum minimum conditions that the relevant vevs v_i and v_j are equal to a common value, which we denote as v. The relevant terms in the potential for specific choices of $i > j$ are

$$U_{Higgs} = \cdots + \frac{1}{2}\sum_{a=0}^{3} \left[M_i^2 |(\psi_a)^i_j|^2 + M_j^2 |(\psi_a)^j_i|^2 \right]$$
$$+ \frac{\lambda' v^2}{2} \sum_{a=0}^{3} \left[|(\phi_a)_{ij}|^2 + (-)^a |(\phi_a)_{ij}|^2 \right] + \cdots$$
$$= \cdots + \frac{1}{2}\sum_{a=0}^{3} \left[M_i^2 |(\psi_a)^i_j|^2 + M_j^2 |(\psi_a)^j_i|^2 + \lambda' v^2 \sum_{a=0,2} |(\phi_a)_{ij}|^2 + \cdots \right.$$

Consequently $(\phi_1)_{ij}$ and $(\phi_3)_{ij}$ ($i > n$; $j > n$) are Goldstone modes. This again matches the expectations from Section V.

When $i > n$ and $j < n$, the relevant terms in the potential are

$$U_{Higgs} = \cdots + U_j' \sum_{a=0}^{3} |(\psi_a)^j_i|^2 + \frac{M_i^2}{2} \sum_{a=0}^{3} |(\psi_a)^i_j|^2$$
$$+ \frac{1}{4}\sum_{a=0}^{3} \left[\lambda' v_j^2 + \frac{f_j \tilde{v}_j^2}{v_j} \right] |(\phi_a)_{ij}|^2 + \frac{1}{4}\sum_{a=0}^{3} \lambda' v^2 |(\phi_a)_{ij}|^2$$
$$+ \frac{1}{2}\lambda' v_j v \sum_{a=0}^{3} (-)^a |(\phi_a)_{ij}|^2$$
$$+ f_j \tilde{v}_j \sum_{a=0}^{3} (\psi_a)^j_i (\phi_a)_{ij} - \frac{f_j v}{2} \sum_{a=0}^{3} (-)^a |(\psi_a)^j_i|^2 + \cdots$$

We use the vacuum minimum conditions to express all relevant terms in terms of f_j and the vevs:

$$U'_j = \frac{f_j v_j}{2}$$

$$\lambda'(v_j^2 - v^2) = \frac{f_j \tilde{v}_j^2}{v_j} \quad \Longrightarrow \quad \lambda' = \frac{f_j \tilde{v}_j^2}{v_j (v_j^2 - v^2)}$$

When inserted into the expression for the potential, one finds—after a moderate amount of algebra—that indeed the anticipated Goldstone modes are there. Only the orthogonal combinations get mass:

$$U_{Higgs} = \cdots + \frac{1}{2}\sum_{a=0}^{3} \frac{f_j}{[v_j - (-)^a v]} \left| \tilde{v}_j (\phi_a)_{ij} + [v_j - (-)^a v](\psi_a)^j_i \right|^2 + \cdots$$

This leaves only the case $n > i > j$. The relevant terms of the potential, quadratic in the dynamical fields, are as follows:

$$\begin{aligned} U_{Higgs} = \cdots &+ \sum_{a=0}^{3} \left\{ U'_i |(\psi_a)^i_j|^2 + U'_j |(\psi_a)^j_i|^2 - \frac{1}{2}(-)^a \left[f_j v_i |(\psi_a)^j_i|^2 + f_i v_j |(\psi_a)^i_j|^2 \right] \right\} \\ &- \sum_{a=0,1,3} (\phi_a)_{ij} \left[f_j \tilde{v}_j (\psi_a)^j_i - f_i \tilde{v}_i (\psi_a)^i_j \right] - (\phi_2)_{ij} \left[f_j \tilde{v}_j (\psi_2)^j_i + f_i \tilde{v}_i (\psi_2)^i_j \right] \\ &+ \sum_{a=0}^{3} \frac{1}{4} |(\phi_a)_{ij}|^2 \left[\lambda'(v_i^2 + v_j^2) + \frac{f_i \tilde{v}_i^2}{v_i} + \frac{f_j \tilde{v}_j^2}{v_j} + 2\lambda'(-)^a v_i v_j \right] \end{aligned}$$

We again eliminate U' and λ' in terms of the f's and the vevs. This time, the expression for the coupling parameter λ' is

$$\lambda' = \frac{1}{(v_i^2 - v_j^2)} \left[\frac{f_i \tilde{v}_i^2}{v_i} - \frac{f_j \tilde{v}_j^2}{v_j} \right]$$

We find, after more algebra:

$$\begin{aligned} U_{Higgs} = \cdots &+ \sum_{a=0}^{3} \left\{ \frac{f_i}{2}[v_i - (-)^a v_j] |(\psi_a)^i_j|^2 + \frac{f_j}{2}[v_j - (-)^a v_i] |(\psi_a)^j_i|^2 \right\} \\ &- \sum_{a=0,1,3} (\phi_a)_{ij} \left[f_j \tilde{v}_j (\psi_a)^j_i - f_i \tilde{v}_i (\psi_a)^i_j \right] - (\phi_2)_{ij} \left[f_j \tilde{v}_j (\psi_2)^j_i + f_i \tilde{v}_i (\psi_2)^i_j \right] \\ &+ \sum_{a=0}^{3} \frac{[f_i \tilde{v}_i^2 - (-)^a f_j \tilde{v}_j^2]}{2[v_i - (-)^a v_j]} |(\phi_a)_{ij}|^2 \end{aligned}$$

Upon forming the relevant 3 x 3 mass matrix, one easily finds that its determinant does vanish.

VIII. Summary

The next—and final—page of this note consists of a summary page containing expressions for the masses and/or mass matrices of the sundry gauge and Higgs bosons we have considered. The only entries therein which are not easily reconstructed from the text above have to do with the "diagonal" Higgs fields $(\psi_2)^i_i$ and $(\phi_2)_{ii}$. These are all massive modes, even in the absence of mixing due to the Yukawa couplings f_i. We will not delve further into the properties of these states in this note, and only record here the relevant terms in the Higgs potential:

$$U_{Higgs} = \cdots + \sum_{i=1}^{n} 2\tilde{V}_i^2 U_i'' \left|(\psi_2)^i_i\right|^2 + \sum_{i=n+1}^{N} \frac{M_i^2}{2} \left|(\psi_2)^i_i\right|^2$$

$$+ \frac{1}{2}\sum_{i=1}^{n} \left(\lambda' V_i^2 + \frac{f_i \tilde{V}_i^2}{2V_i}\right) \left|(\phi_2)_{ii}\right|^2 + \frac{1}{2}\sum_{i=n+1}^{N} \lambda' V^2 \left|(\phi_2)_{ii}\right|^2$$

$$+ \lambda \left| \sum_{i=1}^{n} V_i (\phi_2)_{ii} + \sum_{i=n+1}^{N} V (\phi_2)_{ii} \right|^2 + \sum_{i=1}^{n} f_i \tilde{V}_i (\psi_2)^i_i (\phi_2)^i_i$$

Going beyond the above expression and/or the contents of the table on the next page is to enter the world of model building. This includes, in particular, the choice of the parameter set determining the sundry vevs, as well as which orthogonal groups are under consideration. This is beyond the scope of this, already rather lengthy, note. I expect to revisit these issues in the near future.

TABLE OF SQUARED MASSES

		$a=0$	$a=1,3$	$a=2$
$i,j>n$	ω_{ij}	0	g^2v^2	0
	ψ^i_j	M_i^2	M_i^2	M_i^2
	ϕ_{ij}	$2\lambda' v^2$	$[0]$ (Goldstone)	$i\neq j: 2\lambda' v^2$ $\;$ $i=j$: see text
$i>n\geq j$	ω_{ij}	$\frac{g^2}{4}\left[(v-v_j)^2+\tilde{v}_j^2\right]$	$\frac{g^2}{4}\left[(v+v_j)^2+\tilde{v}_j^2\right]$	$\frac{g^2}{4}\left[(v-v_j)^2+\tilde{v}_j^2\right]$
	ψ^i_j	M_i^2	M_i^2	M_i^2
	$\begin{pmatrix}\psi^j_i\\ \phi_{ij}\end{pmatrix}$	$f_j\begin{pmatrix}(v_j-v) & \tilde{v}_j\\ \tilde{v}_j & \frac{\tilde{v}_j^2}{(v_j-v)}\end{pmatrix}$	$f_j\begin{pmatrix}(v_j+v) & \tilde{v}_j\\ \tilde{v}_j & \frac{\tilde{v}_j^2}{(v_j+v)}\end{pmatrix}$	$f_j\begin{pmatrix}(v_j-v) & \tilde{v}_j\\ \tilde{v}_j & \frac{\tilde{v}_j^2}{(v_j-v)}\end{pmatrix}$
$n\geq i>j$	ω_{ij}	$\frac{g^2}{4}\left[(v_i-v_j)^2+\tilde{v}_i^2+\tilde{v}_j^2\right]$	$\frac{g^2}{4}\left[(v_i+v_j)^2+\tilde{v}_i^2+\tilde{v}_j^2\right]$	$\frac{g^2}{4}\left[(v_i-v_j)^2+\tilde{v}_i^2+\tilde{v}_j^2\right]$
	$\begin{pmatrix}\psi^i_j\\ \psi^j_i\\ \phi_{ij}\end{pmatrix}$	$\begin{pmatrix}f_i(v_i-v_j) & 0 & f_i\tilde{v}_i\\ 0 & -f_j(v_i-v_j) & -f_j\tilde{v}_j\\ f_i\tilde{v}_i & -f_j\tilde{v}_j & \frac{(f_i\tilde{v}_i^2-f_j\tilde{v}_j^2)}{(v_i-v_j)}\end{pmatrix}$	$\begin{pmatrix}f_i(v_i+v_j) & 0 & f_i\tilde{v}_i\\ 0 & f_j(v_i+v_j) & -f_j\tilde{v}_j\\ f_i\tilde{v}_i & -f_j\tilde{v}_j & \frac{(f_i\tilde{v}_i^2+f_j\tilde{v}_j^2)}{(v_i+v_j)}\end{pmatrix}$	$\begin{pmatrix}f_i(v_i-v_j) & 0 & -f_i\tilde{v}_i\\ 0 & -f_j(v_i-v_j) & -f_j\tilde{v}_j\\ -f_i\tilde{v}_i & -f_j\tilde{v}_j & \frac{f_i\tilde{v}_i^2-f_j\tilde{v}_j^2}{(v_i-v_j)}\end{pmatrix}$
$n\geq i=j$	ψ^i_i	$[0]$ Goldstone	$2f_iv_i$	see text
	ϕ_{ii}	—	—	