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The Uncovered Set and the Limits of Legislative Action

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We present a simulation technique for sorting out the size, shape, and location of the uncovered set to estimate the set of enactable outcomes in "real-world" social choice situations, such as the contemporary Congress. The uncovered set is a well-known but underexploited solution concept in the literature on spatial voting games and collective choice mechanisms. We explain this solution concept in nontechnical terms, submit some theoretical observations to improve our theoretical grasp of it, and provide a simulation technique that makes it possible to estimate this set and thus enable a series of tests of its empirical relevance.

1 Introduction

This paper offers a new technique for estimating the set of enactable proposals in majority-rule settings such as legislative bodies and committees. Our aim is to specify a fundamental constraint on legislative action: given the preferences of decision makers, reflecting personal taste and pressures ranging from constituent demands to lobbying from party leaders, which outcomes can emerge from majority-rule decision making? We operationalize enactability using the *uncovered set* (Miller 1980; McKelvey 1986), a solution concept devised for abstract voting games and commonly interpreted to capture enactability in real-world settings (e.g., Shepsle and Weingast 1984; Calvert 1985; Grofman et al. 1987). Until now, the uncovered set has not been applied to real-world settings because it has defied general characterization.[1] We offer a grid-search

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[1]Austen-Smith and Banks (2001). For special cases, see Miller (1980, 2002); Hartley and Kilgour (1987); Epstein (1997). De Donder (2000) and unpublished research by Rick Wilson use techniques similar to ours.

computational method for estimating the size, shape, and location of the uncovered set for any profile of Euclidean preferences on a two-dimensional space.

Our work touches on three contemporary debates. First, we consider the debate over the nature and causes of final outcomes in legislatures and other decision-making settings (Shepsle 1986). Are the regularities observed in majority-rule settings the product of institutional restrictions, e.g., committees with gatekeeping power, or are they due to the fact that only a few roughly similar outcomes are enactable? Second, our analysis is relevant to the debate over the power of legislative parties (Aldrich 1995; Aldrich and Rohde 2001; Krehbiel 1999, 2000). In particular, does a shift in outcomes toward majority-party preferences constitute evidence for conditional party government, or is it what we would expect majority-rule procedures to lead to, given open agendas and no agenda setting by party leaders?

Finally, our work is a response to complaints that the theoretic sophistication of formal models has not been matched by a willingness to test their predictions (Friedman 1997). We address this concern for a concept that embodies accepted notions of rational action under majority rule. Our research is a first step at evaluating these intuitions by offering a technique that translates them into behavioral predictions. Even for scholars who doubt the explanatory power of the uncovered set, our work is attractive in that it moves toward framing the debate in terms of empirics rather than intuition.

The next two sections review the known properties of the uncovered set. Section 4 presents our grid-search procedure and illustrates its results. Section 5 analyzes uncovered sets for six postwar House sessions to illustrate the applicability of our procedure. Section 6 uses preference and outcome data for the 91st and 96th House to evaluate the predictive power of the uncovered set. Finally, Appendices A and B report technical details of the theoretical argument and the grid-search procedure.

2 The Uncovered Set: Theoretical Background and Empirical Relevance

Seminal works in formal theory suggest that stable equilibria rarely exist in multidimensional majority-rule games (McKelvey 1976, 1979; Schofield 1978; McKelvey and Schofield 1987), implying that outcomes are sensitive to agendas, voting rules, and other institutional constraints (Shepsle 1979, 1986). The so-called chaos theorems (McKelvey 1976; Schofield 1978; McKelvey 1979; McKelvey and Schofield 1987) state that majority-based decision making, unchecked by institutions, can go “from anywhere to anywhere,” rendering the ultimate outcome of legislative action, absent institutional constraints, indeterminate.

Further work refined these results, showing that if voters consider the ultimate consequences of their behavior, rather than choosing myopically between alternatives presented at each point, outcomes of social choice situations will lie in the uncovered set (Miller 1980; Shepsle and Weingast 1984; McKelvey 1986; Feld et al. 1989). Uncovered set outcomes are not necessarily Condorcet winners—they need not be majority preferred to all other outcomes.[2] Yet regardless of at what “status quo point” a voting process begins, when decision makers vote using majority rule, there exists a simple two-step agenda that yields some point in the uncovered set as its final outcome (Shepsle and Weingast 1984).[3] Thus supporters of outcomes in the uncovered set can secure these

[2]If a Condorcet winner exists, the uncovered set consists of that single outcome.

[3]With notable two exceptions: first, when constituent demands (position taking) constrain legislators’ voting behavior (Bianco 1994); second, when legislators operate within (exogenous) institutions they cannot change, e.g., when supermajorities are required to enact proposals or to bring them to a vote.

outcomes using relatively simple agendas and, moreover, can defend them against attempts to overturn them by opponents who propose outcomes outside the uncovered set.

The significance of the uncovered set lies in its potential to specify the set of possible majority-rule voting outcomes in legislatures and elsewhere. The uncovered set captures the fundamental forces driving outcomes in the legislative process: legislators' underlying policy preferences, their ability to foresee the consequences of their actions, and their ability to select agendas. Previous work suggests that the uncovered set describes the set of enactable outcomes in many legislative and other majority-rule decision-making environments.

> If one accepts the ... assumption that candidates will not adopt a spatial strategy Y if there is another available strategy X which is at least as good as Y against any strategy the opponent might take, and is better against some of the opponent's possible strategies, then one can conclude that candidates will confine themselves to strategies in the uncovered set. (Cox 1987, p. 419)[4]

While Cox's argument focuses on candidates and electoral politics, its logic applies equally to legislatures and legislation: outcomes that lie outside the uncovered set are unlikely to be seriously considered by sophisticated decision makers, who know that such proposals are unlikely to survive whatever voting procedures are used. Thus, if we know which outcomes are in the uncovered set, we know what is possible in a legislative setting—what might happen when proposals are offered and voted on.[5]

A characterization of enactable outcomes in a legislative setting would help address some central issues in legislative studies. In particular, what does the set of enactable outcomes look like? Is it large or small? Is it sensitive to small changes in the distribution of legislators' ideal points? A quotation from a recent article captures the conventional wisdom:

> In multidimensional deterministic [electoral] competition ... research on the uncovered set suggests that candidates will cycle within a circumscribed policy space near the center of the voter distribution. (Adams and Merrill 2003, pp. 161–162).[6]

While this intuition has considerable appeal, it is completely untested. Moreover, it is unclear whether it holds for all settings. In particular, what if legislators' (voters') preferences are polarized across party lines or other policy-related factors, as they appear to be in the contemporary U.S. Congress?

The size and shape of the uncovered set are also of interest. If we find that uncovered sets in a legislative body are generally small and stable (not sensitive to turnover, for example), this finding suggests that the observed stability of legislative policy outcomes results not from institutions or behavioral norms or low turnover, but because only a few outcomes are enactable regardless of these constraints. If uncovered sets are large, however, stability must be explained by auxiliary institutional mechanisms (Shepsle 1979, 1986).

Our technique for locating the uncovered set would also contribute to the debate over theories of party government in the modern U.S. Congress (Aldrich 1995; Krehbiel 1999, 2000; Aldrich and Rohde 2001). Krehbiel (1999, p. 35) argues that "parties are said to be strong exactly when, viewed through a simple spatial model, they are superfluous." That

[4]See also Shepsle and Weingast (1984, 1994); Calvert (1985); McKelvey (1986); Grofman et al. (1987); Ordeshook and Schwartz (1987); Banks et al. (2002).

[5]Subject, of course, to caveats about exogenous, unchangeable, rules such as limitations on which proposal can be brought to a vote, the use of supermajority rules to enact some proposals, germaneness requirements, and so forth.

[6]See also Calvert (1985); Grofman et al. (1987); Ingberman and Villani (1993).

is, when legislative parties are polarized in a spatial setting, outcomes will shift toward the median of a homogeneous majority party simply because the party is a majority, not because of anything that party leaders do. Here again, the question is, is the uncovered set located in the center of legislators' preferences or skewed toward the ideal points of legislators in the majority party?[7] Answering this question for a multidimensional setting requires a way of determining the size, shape, and location of the uncovered set given real-world legislators' preferences.

Finally, numerous scholars have argued that decision makers (whether in legislatures or elsewhere) lack the cognitive abilities or the information to make the rational vote decisions and agenda choices that underlie the uncovered set, or that legislators' preferences are too complex to be represented using a small or even finite number of policy dimensions (e.g., Friedman 1997). If any of these criticisms is true, then uncovered sets estimated using two-dimensional ideal point data should not be a good predictor of legislative outcomes. Conversely, if such uncovered sets capture a high fraction of legislative outcomes, then we gain confidence in precepts that underlie many rational choice and New Institutional analyses (Shepsle and Weingast 1994).

Before turning to the heart of the matter, four objections deserve attention. First, can the uncovered set be used to describe the set of enactable outcomes without specifying the procedures used to achieve these outcomes, such as rules governing who can offer amendments and the order in which they are voted on? The earlier discussion suggests that the uncovered set characterizes the set of enactable outcomes in many majority-rule choice environments, implying that its predictions hold regardless of procedural variation. In any case, this objection is an empirical question, one that this article touches on and our future work will address in detail. A second, related objection is that our work involves computational methods rather than analytical characterizations and yields only an approximation of the uncovered set. Our response is that the uncovered set has defied characterization for over 20 years. It is time to consider alternate approaches.

A third objection stems from the fact that the uncovered set is a cooperative game theoretic concept, inconsistent with the current stress on noncooperative game theory in analyses of the legislative process (e.g., Baron and Ferejohn 1989; Baron 1994, 2000). Our response is couched in practical terms. The uncovered set is one of the most articulated products of the last two generations of research into majority rule. It therefore seems plausible to investigate its empirical bite.

A final objection is that our technique is only as good as the preference data used to estimate uncovered sets. The results in this article are derived largely from two-dimensional NOMINATE scores (Poole and Rosenthal 2000), with some additional data from Groseclose and Snyder (2000) and Jackman (2001).[8] One concern is that even if these two dimensions are salient for all legislators, other dimensions might be relevant for some decisions or individuals. Alternatively, party leaders or interest groups might offer inducements or threats to override legislators' policy preferences or constituent pressures on a particular issue. Our response is that the high accuracy of predictions derived from NOMINATE and similar estimates—including studies of vote trading and strategic voting (Poole and Rosenthal 2000)—suggests that the two-dimensional presentation provides

[7]This conjecture is sufficiently complicated that we address it in a separate paper (Bianco and Sened 2003).

[8]While we could estimate uncovered sets for additional dimensions given sufficient computational resources, our decision to focus on the two-dimensional case rests on data availability. Even so, analyzing enactability using a two-dimensional model is a significant advance over many contemporary analyses, and we know of no well-cited analysis of the legislative process that utilizes a spatial model with more than two dimensions.

a good analytical tool of the forces driving politics in the U.S. House, implying that two-dimensional uncovered sets may yield an adequate characterization of what is enactable.[9] In any case, this concern is a matter for an empirical test, such as that offered later in this article. It should also be noted that our technique is easily generalized to higher dimensions.

3 Known Properties of the Uncovered Set

The next two sections offer a plain-language presentation of the technical properties of the uncovered set as well as some original results that guide our work. Appendices A and B supplement these sections with the customary mathematical notations and formal proofs.

Let N be the set of n voters or legislators. We assume n is odd. For any agent, $i \in N$, preferences are defined by an *ideal point* ρ_i. Let x,y,z be elements of the set X of all possible outcomes. A point x *beats* another point y by majority rule if it is closer than y to more than half of the ideal points.[10] A point x is *covered* by y if y beats x and any point that beats y beats x. The uncovered set includes all points not covered by other points.

The attractiveness of the uncovered set as a solution concept lies in the fact that if y covers x, y dominates x as an outcome of a majority-rule voting game (McKelvey 1986; Ordeshook 1986, pp. 184–185); inasmuch as y defeats x, any outcome that ties y defeats or ties x and any outcome that defeats y also defeats x. Strategic legislators should therefore eliminate covered points from voting agendas. Instead of promoting outcomes that are bound to be defeated later in the game, sophisticated legislators should promote points in the uncovered set that may survive the voting process. This logic suggests that the enactable set that may be implemented by legislative bodies is restricted to the uncovered set.

We now state five relevant properties known about the uncovered set.

1. The uncovered set is never empty (McKelvey 1986, p. 290, Theorem 1).
2. The majority core (or *Condorcet winner*) is a point that beats all other points in X. If the core is not empty, the uncovered set coincides with the core (Miller 1980, p. 74, Theorem 1; McKelvey 1986, p. 285).
3. A point x is *unanimously preferred* to a point y if x is closer than y to all ideal points. The Pareto set is the set of points such that there is no point that is unanimously preferred to any point in the Pareto set. The uncovered set is a subset of the Pareto set. (Miller 1980, p. 80, Theorem 4; Shepsle and Weingast 1984, p. 65, Proposition 3).
4. A *median hyperplane* is a hyperplane (in two dimensions, hyperplanes are straight lines that cut the space in two) that passes through k ideal points, $k \geq 1$, so that there are at least $(n + 1)/2 - \text{k}$ ideal points on each side of it. Thus, if n is odd, at least one ideal point must lie on any median line/hyperplane, with an equal number of points

[9]One may wonder whether these ideal point estimates are biased by strategic behavior by legislators. The answer depends both on the frequency of such behavior and on its manifestation. For example, in Weingast and Marshall (1987), committee power arises because committees act as gatekeepers that report only proposals they prefer to the status quo that are in the win set of the status quo and can be enacted without requiring anyone to vote against their preferences. Under these conditions, committee deference would not bias the estimates.

[10]Throughout the paper, we assume that preferences are Euclidian (Type 1).

on each side. Let Y be the smallest ball that intersects all median hyperplanes and Y4 a ball centered on Y's center with a radius, 4r, equal to four times the radius, r, of the ball Y. The uncovered set is contained within Y4 (McKelvey 1986, p. 304). Y is referred to as the *yolk.*[11]

5. **Theorem 1.** Let B be any subset of A. B is the uncovered set of A if and only if:[12] Every point outside of B is covered by a point within B. No point within B is covered by a point inside B.

The motivation behind our research is that these five known properties of the uncovered set do not establish the shape, location, or size of the uncovered set. In particular, properties 3–5, the best analytic estimates for nonspecific cases of the uncovered set, are very imprecise, rendering the uncovered set useless as a predictive tool.[13]

To appreciate the problem, consider Fig. 1, which gives Poole-Rosenthal NOMINATE ideal points for legislators in the 106th U.S. House, the yolk,[14] the 4r circle that the uncovered set lies within, and the uncovered set computed by our procedure (we present details on the procedure in the next section).

In this figure, legislators' ideal points are dots. We calculate the yolk using a computational procedure whereby we draw candidate median lines through each ideal point at one-degree intervals (that is, 360 lines through each ideal point), retaining the lines that have the same number of legislators on each side. The resulting median lines are the dark solid lines. We then add the yolk as the smallest possible circle that touches all the median lines (shaded circle) and add the 4r circle that contains the uncovered set (dashed circle). We then add the uncovered set as computed by our grid-search procedure.

As Fig. 1 indicates, the 4r circle containing the uncovered set is quite large relative to the uncovered set computed by our procedure. Moreover, existing theory gives us no indication of how much of the 4Y ball within the 4r circle is taken up by the uncovered set, or whether the set is centered or skewed to one side or up or down. The only other known bound on the size, shape, and location of the uncovered set, i.e., that it lies within the Pareto set (here the convex hull of legislators' ideal points), supplies even less information.

4 Estimating the Uncovered Set

Our technique for estimating the uncovered set treats the policy space as a collection of discrete potential outcomes rather than as a continuous space. Thus it recovers only an approximation of the actual uncovered set—an approximation that, as stated in Theorem 2 below, converges to the interior of the uncovered set as the resolution of the grid goes to infinity. For the cases treated here, the ideal points and outcomes are located in a two-

[11]Feld et al. (1987, p. 138, Theorem 7) proved that at least for the two-dimensional case, "the uncovered set is contained within a circle of radius 3.7r around the center of the yolk."

[12]Necessity was proved by McKelvey (1986, p. 291, Proposition 4.2). We complete the result by proving sufficiency. The formal statements of Theorems 1 and 2 with the proofs of both theorems and all propositions appear in Appendix A.

[13]The depth of this problem was not apparent in previous work due to at least two factors. First, lacking computational methods, the yolk was drawn for a relatively small number of ideal points and, moreover, ideal points were distributed in a roughly circular pattern around an empty center, a configuration that yielded a relatively small yolk. Second, owing to the formula for calculating the area of a circle, the area included in the 4r circle is 16 times, not 4 times, the area of the yolk.

[14]While this figure is ancillary to our analysis, it should be noted that as far as we have been able to determine, this is the first time the yolk has been calculated for a significant number of ideal points.

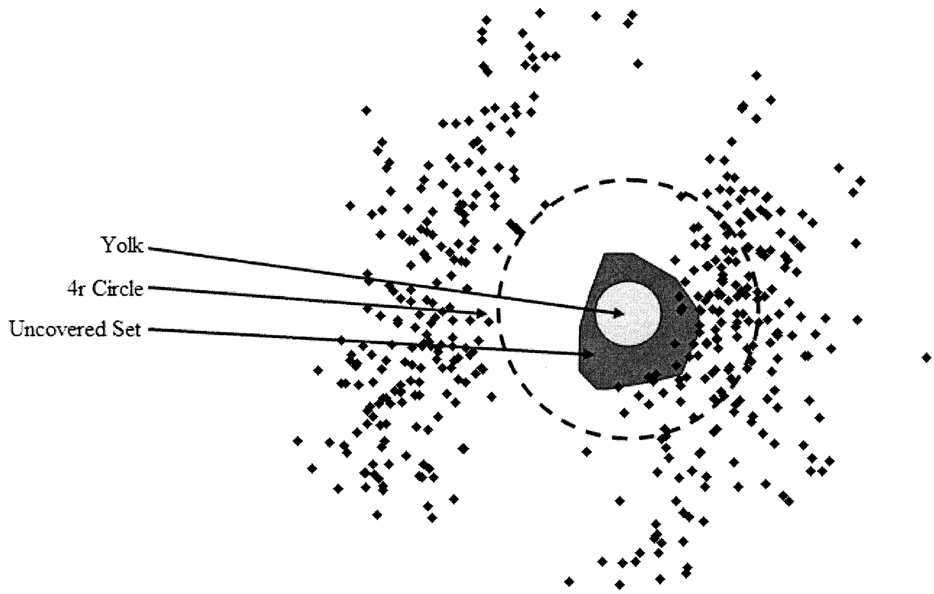

Fig. 1 Ideal points, median lines, and yolk, 106th House.

dimensional space. This approach follows McKelvey (1986, p. 27): "proposition 4.1 gives a potential 'brute force' [iterative search] method for computing [the uncovered set] up to any desired degree of accuracy" (see also Miller 1980, p. 93).[15]

In order to use McKelvey's intuition, we need to know two things: First, is the test in McKelvey's proposition 4.1 sufficient? Second, can we approximate the UC by looking at fine enough grids? Theorem 1 cited above took care of the first concern. Propositions 1 and 2 resolve the second.

> **Proposition 1.** If x is covered by a set with a nonempty interior,[16] it will eventually appear as covered on a fine enough grid.
> **Proposition 2.** If x is in the interior of the uncovered set, then on a fine enough grid x, or a point arbitrarily close to x, it will appear as an element of the uncovered set provided by the grid procedure.

[15]De Donder (2000) uses a similar approach to compare the predictions of the uncovered set with those of the bipartisan set and the minmax set in a model of purely redistributive taxation. De Donder's focus is on whether any point is more or less likely to be in any of the three sets, given repeated sampling from a bivariate log-normal distribution of ideal points (p. 611). Unlike us, De Donder does not report the exact simulation procedure he uses, but personal communication indicates that it is a grid-search procedure similar to ours. However, the grid used by De Donder is of a considerably lower resolution. More important, the use of repeated sampling from a bivariate log-normal distribution leads him (2000, p. 625) to conclude that the uncovered set is both "selective (... selecting between 1% and 7% of the feasible options)" and "not too sensitive to slight modifications of preference profiles." We show that neither conclusion holds under more realistic specifications of legislators' preference profiles; the same is true for the common intuition that the uncovered set is small and centrally located.

[16]Thus, if the set that covers x has an empty interior, x may appear as uncovered even though it is covered. By the nature of the uncovered set this loss of generality does not pose a major problem because any x will almost always be covered by a set with a nonempty interior if it is covered at all. It does, however, explain why our technique typically slightly overestimates the size of the uncovered set.

Propositions 1 and 2 yield a general analytical rationale for our project. Together they state that at a high enough resolution, any point outside the uncovered set will disappear from the uncovered set produced by the grid procedure, and for every point in the uncovered set there will be a point as close to it as we want, in the uncovered set produced by the grid estimation procedure. We state this conclusion as Theorem 2.

> **Theorem 2.** Our grid procedure estimate of the uncovered set converges to the interior of the uncovered set. If the uncovered set has a nonempty interior, or is a union of sets, each of which has a nonempty interior,[17] then the uncovered set estimated by an increasingly fine grid converges[18] to the true uncovered set.

Theorem 2 provides a theoretical asymptotic rationale to our grid procedure estimate of the uncovered set, stating that in the limit, the uncovered set delineated by the grid procedure will converge to the continuous uncovered set. It should be emphasized that in the discrete case, our procedure is not an approximation but actually computes the exact uncovered set in the set of discrete points under investigation.

Our algorithm is specified as a two-step process using *Gauss* software (details are in Appendix B). We start with two-dimensional preferences data and compare points across a coarse grid to determine the general location of the uncovered set. Using this information, we search this general location for a more precise specification of the uncovered set using a higher resolution grid. In the version used here, each dimension varies from -1 to 1 and the discrete outcomes compared are 0.02 apart.[19]

The difficulty in assessing the reliability of our estimates is that there exists no analytic characterization of the UC, thus there is nothing to compare to any given estimate from our algorithm. Comparing our estimates to Pareto sets and 4Y balls established that our estimates always lie within these constraints. We also compared our estimates to a few uncovered sets derived in previous work (Miller 1980; Hartley and Kilgour 1987) and to unpublished examples (Miller 2002). Again, our estimates closely match these results. For example, Fig. 2 shows an application of our technique in a game with five legislators. The small diamonds are legislators' ideal points; the shaded area is the uncovered set. As the plot shows, when applied to Plott's (1967) equilibrium distribution of ideal points (upper left plot), our algorithm yields, as expected, the (0,0) point as the only point in the uncovered set.[20] The next five plots show how the uncovered set expands given changes in the location of one legislator's ideal point. Across the five figures, this ideal point moves

[17]An anonymous referee pointed out that our proof in the technical appendix does not cover the case in which the uncovered set is a union of a set with a nonempty interior and a finite number of isolated points. This conditional corrects this problem, but we must emphasize that this eventuality is extremely unlikely given what we know of the uncovered set. In fact, we have every reason to believe that if the number of decision makers is odd, the uncovered set is connected, in which case this problem never arises.

[18]Convergence is formally defined as follows: Let $V = \{V_1, V_2, \ldots, V_\omega, \ldots\}$ be an infinite series of grids with $\lim r(V_w)_{w\to\infty} \to 0$ and $\forall w \in N: V_w \subseteq V_{w+1}, \forall x \in V_w$ such that the set that covers x has an interior $\exists\delta \in N: k > \delta \Rightarrow (x \notin UC(X) \Rightarrow x \notin UC(V_k))$ for any neighborhood of x, A(x) $\exists\delta > 0 : k > \delta \Rightarrow \exists y \in UC(V_k) \cap A(x)$, i.e., for any x in the UC(X) there exists a resolution that will depict a point as close to x as one would want as being in the uncovered set. For any point y not in the uncovered set, if it is covered by a set with a nonempty interior, there exists a resolution that will eliminate it from the uncovered set obtained by the grid procedure (see technical appendices for details).

[19]If stretched across the entire space the fine grid contains 101^2 or 10,201 discrete points. We experimented with much finer grids. They do not significantly alter the size, shape, or location of the uncovered sets shown here or in many other examples that we looked at. The computational load, however, increases exponentially with resolution, because determination of whether a point is covered requires binary comparisons between each point and all other points. Our grid size strikes a balance between accuracy and tractability.

[20]Given this configuration, the uncovered set is also the core and the Condorcet winner.

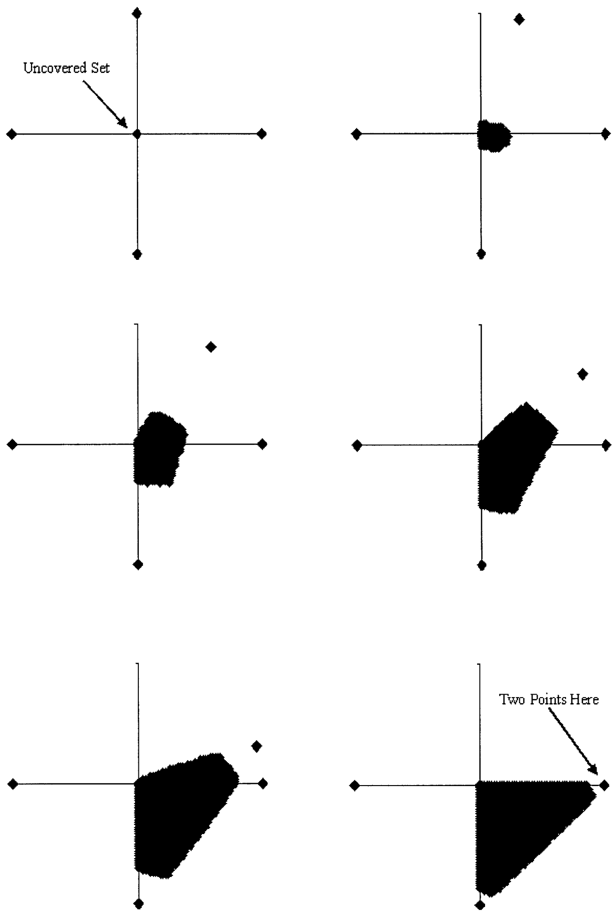

Fig. 2 The uncovered set and near-Plott conditions.

from the vertical position clockwise a total of 90° until it overlaps, in the final plot, with the eastward voter. It is easy to see that as the deviation from the Plott configuration becomes larger, the uncovered set expands. The results in Fig. 2 also suggest that, contrary to De Donder (2000), the uncovered set can be quite big. Moreover, its size and location are extremely sensitive to the location of legislators' ideal points.

Figure 3 describes variation in the uncovered set across the six plots in Fig. 2. The bars in Fig. 3 show the area of each uncovered set, measured as a percentage of the total outcome space (the square bounded by -1 and 1 in both dimensions) and the Pareto set. As the plot shows, the size of the uncovered set varies from only one point in the upper left-hand figure to almost a quarter of the space in the last plot, shown at the lower left of Fig. 1. Moreover, the change in the size of the uncovered set as the fifth ideal point shifts from location to location is clearly nonlinear: note that the difference between the intermediate plots is relatively large, while the difference between the last two plots is relatively small. Thus, contrary to the conventional wisdom discussed earlier, under some conditions small changes in preferences can lead to dramatic (but not discontinuous) changes in the set of enactable outcomes, while in other cases the change is quite small.

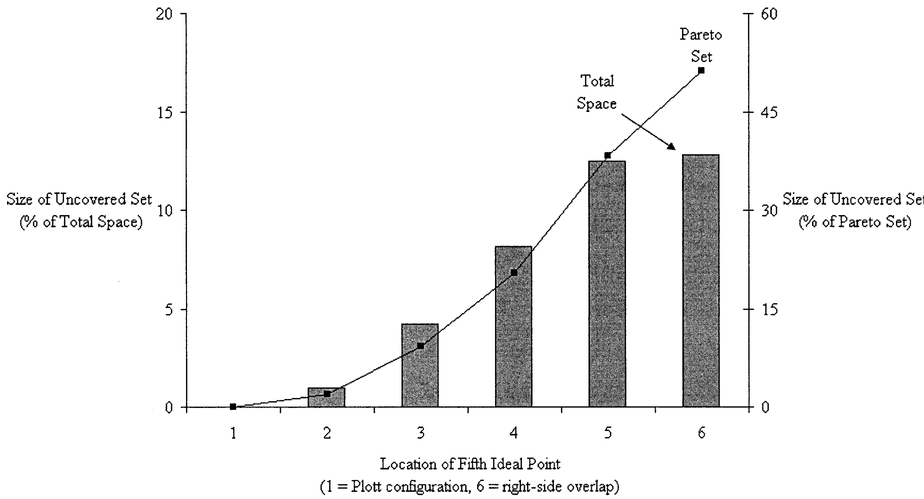

Fig. 3 Variation in the uncovered set.

Figures 2 and 3 also reveal the relationship of the uncovered set to the Pareto set. In a spatial voting game, the Pareto set is the convex hall of all ideal points and is a natural first-cut candidate for the set of enactable outcomes. As stated earlier, the uncovered set is a subset of the Pareto set (Miller 1980). The line in Fig. 3 shows the percentage of the Pareto set that is occupied by the uncovered set in each plot. It demonstrates that the amount of the Pareto set that is occupied by the uncovered set can vary substantially given small changes in legislators' preferences. Thus, while the Pareto set is easily determined for most situations, it clearly does not give us as much insight into the size, shape, and location of the uncovered set as previously argued (Epstein 1997). This result tells us that using the Pareto set as a proxy to determine what is enactable in a legislature is misleading, both in the number of potential outcomes and in their substantive description.

Finally, Fig. 3 also suggests that the locations of the uncovered set tend to shift toward groupings of ideal points. In particular, as the fifth voter's ideal point moves along the arc toward the eastmost ideal point, the uncovered set moves toward these ideal points. The point is not that like-minded decision makers can collude to produce outcomes they favor; rather, this pattern suggests that majority rule conveys some decision makers with a natural advantage given the similarity of their preferences.

5 Uncovered Sets in the U.S. House

In this section, we use our technique to estimate uncovered sets for the contemporary U.S. House of Representatives—more specifically, a total of six House sessions in the last 50 years: the 81st (1949–1950), 86th (1959–1960), 91st (1969–1970), 96th (1979–1980), 101st (1989–1990), and 106th (1999–2000), with particular emphasis on the 106th. We use a combination of data sets. In most cases, our estimates are based on common-space NOMINATE scores (Poole and Rosenthal 2000) that define ideal points in terms of two ideological dimensions: a north-south civil rights dimension and a west-east (liberal-conservative) economic dimension. However, in Fig. 4, for the analysis of the 106th

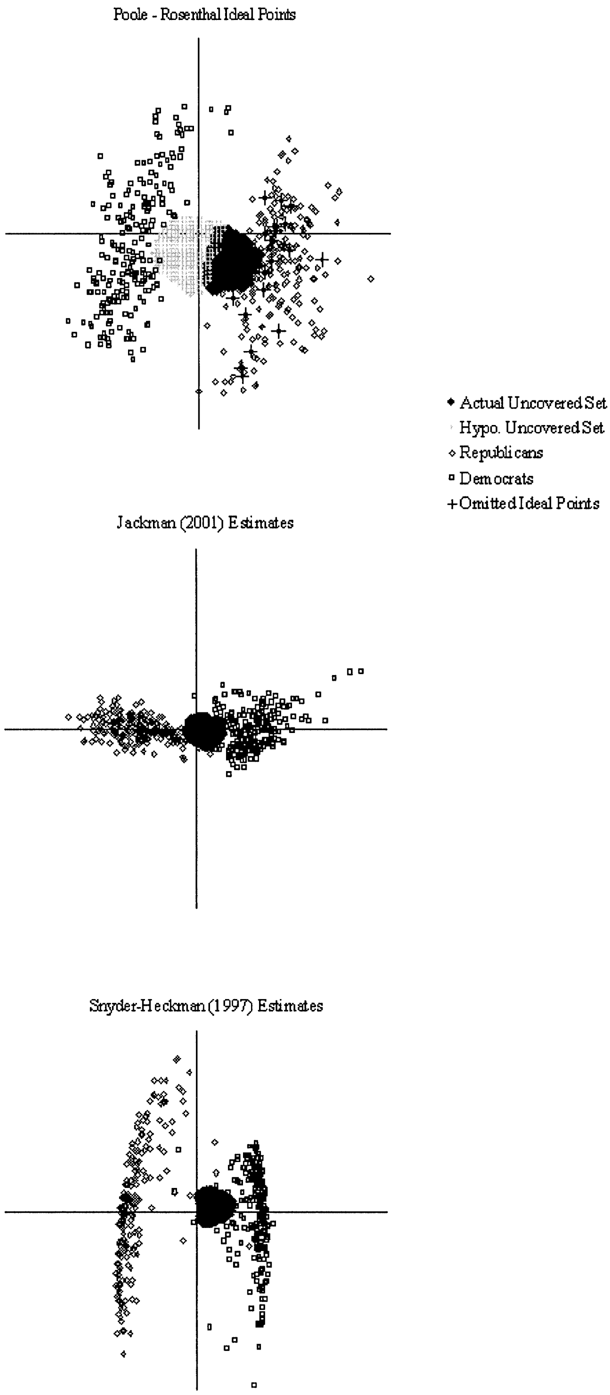

Fig. 4 Ideal points and uncovered sets, 106th House.

House, we compare uncovered sets based on NOMINATES with uncovered sets calculated from two other sets of ideal points: Groseclose and Snyder (2000) and Jackman (2001). In all plots, Democrats constitute the northwest/west cloud of ideal points, while Republicans are clustered in the east/southeast.[21] All plots replicate the essential nature of the results presented earlier. While the shape of the uncovered sets varies, all three are relatively large, and all are shifted toward the majority Republicans.

We use the top plot in Fig. 4, which shows the 106th House uncovered set using NOMINATE data (shown earlier in Fig. 1), to illustrate the impact of small shifts in ideal points—in particular, shifts that switch majority control in a legislature where preferences are polarized across parties. To do this, we take the 106th House ideal points calculated by NOMINATE and subtract every 20th Republican (omitted ideal points are denoted by "+"), creating a narrow Democratic majority. The uncovered set of this hypothetical House is the leftward shaded region in the top plot. Note that by removing less than 5% of the U.S. House of Representatives, we have radically transformed the set of enactable outcomes.

Next, we estimate uncovered sets for five additional House sessions using NOMINATE data. All six uncovered sets with associated ideal points are in Fig. 5.[22] These plots confirm and extend our analysis of the 106th House. While the size of the uncovered sets varies considerably across sessions, the uncovered sets in all cases are substantial and skewed toward the majority.[23] We offer an interpretation of these findings below.

6 The Predictive Power of the Uncovered Set

All of the work presented up to now is subject to an important caveat: does the uncovered set have any predictive power? Even if our technique allows us to locate the uncovered set given real-world preference data, this innovation is useless if the set's intuitive appeal is not matched by its ability to capture real-world outcomes.

In this section we use the Poole-Rosenthal uncovered sets for the 91st and 96th House as shown in the previous section, combined with data on the location of winning outcomes in these legislatures (Poole and Rosenthal 2000), to address a simple question: what percentage of winning outcomes lie in the uncovered set?[24] While it is not obvious what percentage constitutes strong evidence for the uncovered set as a predictor, higher levels clearly support this claim more than lower levels. To see how this test works, consider Fig. 6, which shows a hypothetical uncovered set and information on two hypothetical votes. The lines in the figure reflect information provided by NOMINATE. For each vote, NOMINATE calculates a cutting line that divides legislators into predicted yea and nay votes. The program also calculates predicted locations for the winning and losing outcomes that are consistent with the cutting line. For technical reasons, the predicted locations of winning and losing outcomes from NOMINATE are relatively

[21]Note the differences in the distribution of ideal points across the three plots. These differences are the product of the estimation techniques used to produce each set of ideal points—most notably, in the Jackman data, the first (horizontal) dimension is assumed to be more important in legislators' decisions than the second (vertical) dimension.

[22]The dimensions in earlier figures range from −1 to 1, but those in Fig. 5 are −.5 to .5.

[23]We use NOMINATE common space scores, so the uncovered sets should be comparable. The only caveat is that the interpretation of the policy dimensions might vary over time, although such changes are unlikely between adjacent or nearby Congresses.

[24]As we discuss below, the nature of the NOMINATE outcome data necessitate a slightly different test; however, we retain this specification here for clarity.

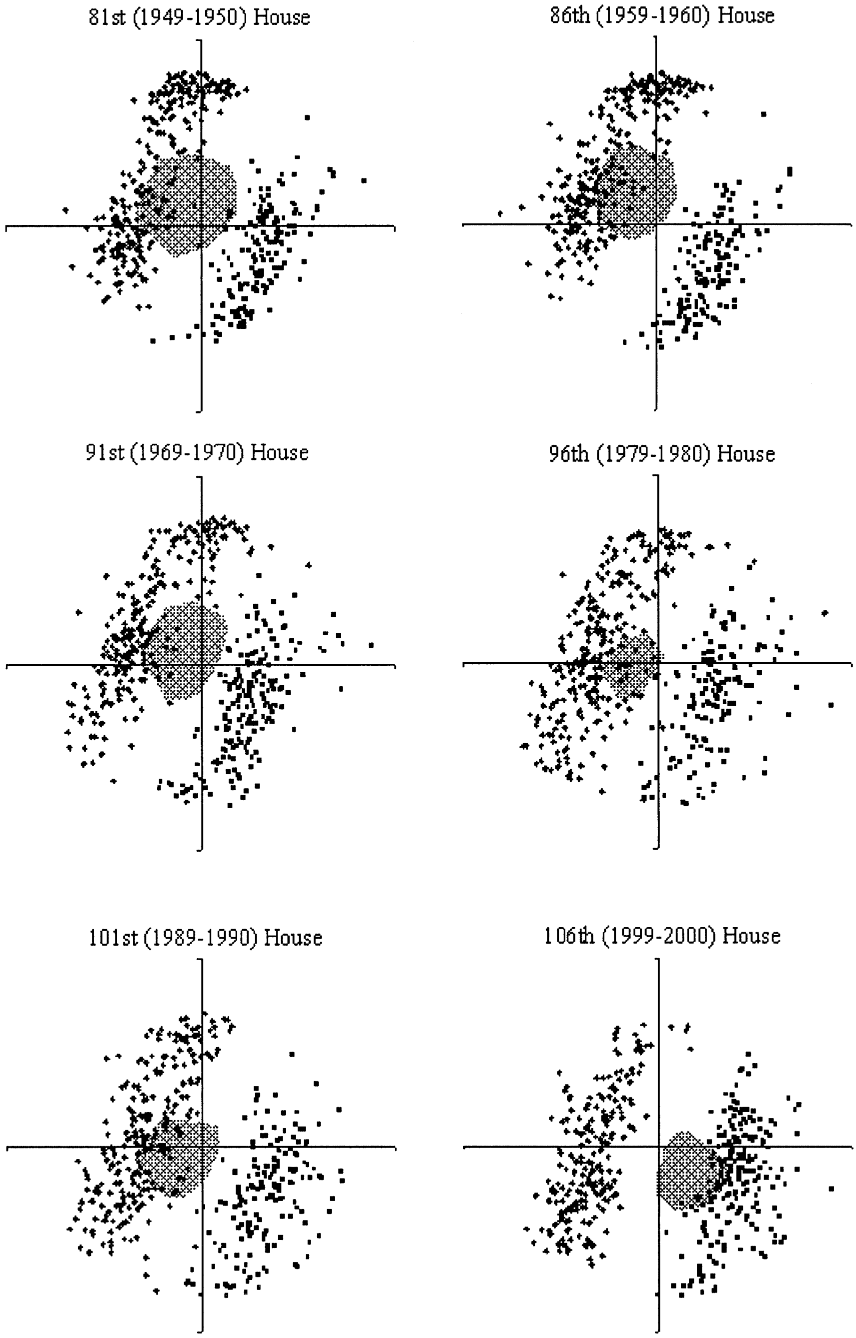

Fig. 5 Uncovered sets, 81st–106th Houses.

imprecise, but the locations of the cutting line and the line on which the winning outcome lies are generally estimated with relative precision (Poole, personal communication). These peculiarities require a change in our test: rather than calculating the percentage of winning outcomes in the uncovered set, we compute the percentage of winning outcome

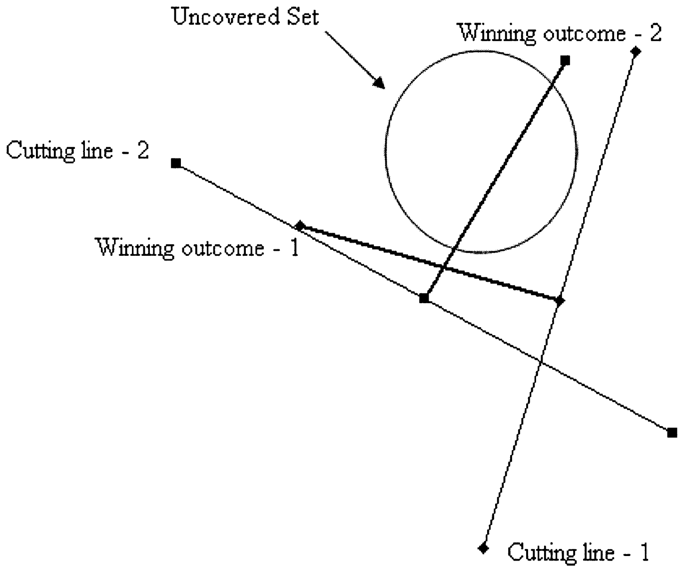

Fig. 6 Testing the uncovered set: logic.

lines that pass through the uncovered set. In Fig. 6, for example, the first winning outcome line passes through the uncovered set, but the second does not.

Our analysis here is based on cutting lines, winning outcome lines, and preference data calculated by NOMINATE for the 91st and 96th House sessions. The uncovered sets used for the analysis are those shown in Fig. 5. Table 1 gives the results of our analysis, showing the percentage of winning outcome lines that pass through the actual uncovered set in both Congresses. As a way of accounting for uncertainties in the location of the winning outcome lines, we present two results: the percentage of winning outcome lines that actually pass through our estimated uncovered set and the percentage that pass through the uncovered set or are "close misses."[25] Consider the top row, which gives results for the 91st House. The actual percentage of winning outcomes that are consistent with our estimated uncovered set was 78.6%. If we include close outcome lines (within one standard deviation), the percentage increases to 88.6%. The bottom lines give similar results for the 96th House.

While this test is clearly a first step in establishing the predictive power of the uncovered set in real-world outcomes, the findings suggest that the uncovered set's intuitive appeal is matched by its empirical bite. The uncovered set as estimated for these House sessions is consistent with more than three-fourths of the winning outcomes—an impressive result given the complexity of the data. These results are consistent with the theory that in a world of sophisticated decision makers, majority-rule voting procedures will converge on the uncovered set. Further work is needed to evaluate whether there is any evidence of party-based agenda setting in these results (e.g., are some areas of the

[25]Our definition of a "close miss" is designed to account for errors in the estimation of the winning outcome line. These errors are the result of errors in the estimation of the cutting line (i.e., the midpoint and slope of the line). Even small cutting line errors can translate into significant errors in the winning outcome line, particularly as distance from the cutting line increases. Poole (personal communication) suggests that .10 is a reasonable estimate for the standard deviation of the outcome line; we use this estimate in our analysis to determine whether a winning outcome line that does not intersect the uncovered set is a close miss.

Table 1 The predictive power of the uncovered set

	Outcomes consistent with uncovered set	
	Actual	*Including close misses*
91st House (210)	78.6% (165)	88.6% (186)
96th House (496)	74.0% (367)	84.7% (420)

uncovered set more likely to contain outcomes than others?), as well as to extend the analysis to other legislatures and decision-making venues.

7 Discussion

Our analysis of the size, shape, and location of the uncovered set in abstract social choice situations as well as in the contemporary U.S. House yields four tentative insights. First, the uncovered set can be much larger than our expectations based on conventional wisdom and previous work. Sometimes, given symmetric configurations of ideal points, the uncovered set is small. But given other distributions of ideal points, the uncovered set occupies a much larger portion of the outcome space. Moreover, analysis of U.S. House data suggests that the uncovered set does not necessarily collapse to a point or a small region as the number of ideal points increases.

These findings suggest a new focus for studies of sophisticated behavior in legislatures. At first glance, the uncovered set dramatically reduces the potential for strategic behavior, as it moves us from the notion that anything can happen to saying that possible results for legislative action must be contained in the uncovered set. However, if uncovered sets in real-world settings such as the U.S. House are relatively large, as our results suggest, then the task for would-be agenda setters and sophisticated legislators is to devise institutions and tactics that generate their preferred outcome within the uncovered set. Conversely, our results suggest a new possible explanation for the absence of apparent sophisticated behavior in a legislature or other majority-rule setting: sometimes the uncovered set is sufficiently small that there is little to gain from such behavior.

Second, our analysis has shown that in situations in which legislators' preferences are polarized—party caucuses are relatively homogenous internally but relatively heterogeneous externally, as in the contemporary House—the uncovered set does not lie in the center of the distribution of legislators' ideal points but is skewed toward the majority caucus.[26] There are two ways to interpret this finding. One is that it reveals a natural advantage for the majority party in polarized legislatures, as argued by Krehbiel (1999, 2000). That is, even if party leaders make no attempt to use agendas or lobbying to influence legislative outcomes, the set of enactable outcomes will naturally be close to the ideal points of their party's legislators, and the majority caucus will look powerful, even though it is inactive. A second possibility is that the polarization of ideal point estimates is endogenous, reflecting successful efforts by party leaders to get their caucuses to vote together on important proposals.[27] To decide between these two alternative explanations is an empirical question that the data presented here do not allow us to adjudicate. In work in progress we (Bianco and Sened 2003) use ideal

[26]In work presented elsewhere (Bianco and Sened 2003), we find that the degree to which the uncovered set is skewed depends on the degree to which preferences are polarized.
[27]We are grateful to an anonymous referee for making this suggestion.

point estimates that "subtract out" party leaders' influence (Groseclose and Snyder 2000) to provide a critical test between these two alternative explanations.

Third, our analysis has shown that the size, shape, and location of the uncovered set are very sensitive to the distribution of ideal points. In Fig. 2, relatively small changes in the distribution of legislators' ideal points produced a major change in our estimated uncovered set. Switching majority in the 106th House data showed that the impact of changes in ideal points on the location of the uncovered set is especially profound when associated with a shift in majority control. These findings suggest the need to reconsider the impact of legislator turnover on legislative outcomes. The notion that small turnover (replacement of legislators due to retirement or defeat) has a relatively small impact on what is enactable in a legislature has considerable appeal.[28] Our results contradict this intuition. While it would come as no surprise to see large policy shifts given a change in majority control, our results suggest that these changes can simply reflect a new set of enactable outcomes, not a change in who controls legislative committees or floor proceedings.

Finally, our analysis of outcome data suggests that the uncovered set has considerable explanatory power: at least in the contemporary House, legislators appear to resemble the picture of sophisticated decision makers that underlies the definition of the uncovered set and the larger research program into the properties of majority rule. These results also support our finding of an off-center uncovered set, in that this location is the best predictor of real-world outcomes, at least in the two Congresses examined here. Future work must consider other sources of outcome data in order to further corroborate these findings.

Appendix A: Technical Appendix

Let X be a convex policy space, $X \in R^m$. $\rho = \{1, \ldots, n\}$ is a group of n agents (n odd), and the preferences of each agent are represented by a weak ordering $R_i \in X \times X$. $R_i(x) = \{y \in X : yR_ix\}$. $P_i(x)$ and $I_i(x)$ are, respectively, the interior and boundary of $R_i(x)$. For majority preference, $xPy \Leftrightarrow |\{i : xP_iy\}| > |\{i : yP_ix\}|$; $W(x) = \{y \in X : yPx\}$; $W^{-1}(x) = \{y \in X : xPy\}$.

We use two different sets of assumptions. Set A is applicable to general, theoretical results about the uncovered set. Set B applies to our estimation system, which is enacted on a grid.[29]

A.1. R_i is a continuous for all i. Therefore $R_i(x)$ is closed.

A.2. Strict quasi concavity $-\forall i \in \rho$, $x, y \in X$, $y \in R_i(x)$ and if $z = ty + (1 - t)x$, $0 < t < 1$, then $z \in P_i(x)$

A.3. $R_i(x)$ is compact, that is, $R_i(x)$ is a closed and bounded set.

A.4. All agents have Euclidean preferences. That is, every agent has an ideal point $\rho_i \in X$, and $xR_iy \Leftrightarrow \|y - \rho_i\| \geq \|x - \rho_i\|$.[30]

Assumption set B is introduced for the discrete, grid case.

B.1.(A.3.) $R_i(x)$ is compact, that is, $R_i(x)$ is a closed and bounded set.

B.2.(A.4). All agents have Euclidean preferences. That is, every agent has an ideal point $\rho_i \in X$, and $xR_iy \Leftrightarrow \|y - \rho_i\| \geq \|x - \rho_i\|$.

B.3. We will call $G \subset X$ a *grid* if G is a finite set and $\exists\, r(G)$ such that $\forall x \in G\ \exists y \in G$: $\|x - y\| \leq r$. Note that r does not define the grid but is a feature of the grid. In computer-generated grids points in G belong to Q^m, the m-dimensional set of rational numbers.

[28]For example, in a single-dimension spatial setting, small changes do not produce large shifts in the location of the median voter.

[29]We follow the notations and assumptions of Shepsle and Weingast (1984) and McKelvey (1986).

[30]It is well known that A1–A3 are implied by A4, but since this may not be trivial for all our readers we specify A1–A3 separately to highlight these features of the assumption of Euclidean preferences.

Let $V = \{V_1, V_2, \ldots, V_\omega, \ldots\}$ be an infinite series of grids with $\lim r(V_\omega)_{\omega \to \infty} = 0$ and $\forall \omega \in N : V_\omega \subseteq V_{\omega+1}$.

We now define the covering relation. For any compact set A ⊆ X (in particular, G is compact):

Definition 1. $yCx \Leftrightarrow yPx,\ W(y) \subseteq W(x)$.

Definition 2. $\mathrm{UC(A)} = \{y \in A : \forall x \in A - xCy\}$.

Under assumption set A six results are known about the uncovered set as discussed in the text.

1. $UC(A) \neq \phi$ (McKelvey 1986, p. 290, Theorem 1): The uncovered set is never empty.
2. Let $C(A) = \{x \in A : xPy\ \forall y \in A,\ y \neq x\}$ be the majority core, or Condorcet winner. If $C(A) \neq \phi$, C(A)=UC(A) (Miller 1980, p. 74, Theorem 1; McKelvey 1986, p. 285).
3. The uncovered set characterizes the set of feasible or enactable outcomes in many legislative and other majority rule decision-making procedures (Miller 1980; Shepsle and Weingast 1984, 1994; McKelvey 1986; Ordeshook and Schwartz 1987).
4. $UC(A) \subseteq PO(A)$, where $PO(A) = \{x \in A : yP_i x \forall i$ for no $y \in A\}$ is the Pareto set of A (Miller 1980, p. 80, Theorem 4; Shepsle and Weingast 1984, p. 65, Proposition 3).
5. Let B(y,t) be a ball centered on y with a radius t and $B(\bar{y}, \bar{t})$ be the ball that intersects all of the median hyperplanes with minimum radius. Then $UC(A) \subseteq B(\bar{y}, 4\bar{t})$ (McKelvey 1986, p. 304, Theorem 5). $B(\bar{y}, \bar{t})$ is referred to in the literature as the yolk.
6. Theorem 1 states that UC(A) is the uncovered set of A ***if and only if*** any point in UC(A) is not covered in UC(A) and any point in its compliment is covered in UC(A).

Theorem 1. $A \supseteq B = UC(A) \Leftrightarrow \forall x \notin B\ \exists y \in B : yCx$ and $\forall x \in B\ \neg\exists y \in B : yCb$.

Proof: McKelvey (1986, p. 291, Proposition 4.2) proved necessity, i.e., if B = UC(A) then $\forall x \notin B\ \exists y \in B : yCx$ and $\forall x \in B\ \neg\exists y \in B : yCb$. We need to prove sufficiency, i.e., if $\forall x \notin B\ \exists y \in B : yCx$ and $\forall x \in B\ \neg\exists y \in B : yCb$ then B = UC(A).
For $B \supseteq UC(A)$. Let x ∉ B. Then $\exists y \in B s.t. yCx$ so x ∉ UC(A).
For B ⊆ UC(A). Suppose x ∈ B but x ∉ UC(A). Then $\exists y \in UC(A) s.t. yCx$. By (1), $y \in B$, which is a contradiction. □

Proofs of Propositions and Theorem 2 (under assumption set B):

Proposition 1. $\forall x \in V_\omega$ such that the set that covers x has an nonempty interior, $\exists \delta \in N : k > \delta \Rightarrow (x \notin UC(X) \Rightarrow x \notin UC(V_k))$.

Proof: If $x \notin UC(X)$ then $\exists z \in X : zCx$ so that z is in the interior of the set that covers x. Then there is an r-neighborhood of z, N(z), so that $\forall y \in N(z)\ yCx$. Then by Lemma 1 always $yC_{vk}x$. Let the radius of N(z) be r. Then because $\lim r(V_\omega)_{\omega \to \infty} = 0\ \exists \delta \in N : k > \delta \Rightarrow r(V_k) < r$, so that $\exists y \in V_k \cap N(z) \Rightarrow x \notin UC(V_k)$ and x is covered on V_k. □

Lemma 1. $\forall x, y \in V_\omega\ \exists \delta \in N : k > \delta \Rightarrow yC_{vk}\, x \Leftrightarrow yCx$

Proof: For any k, if yCx, $W(y) \cap A \subseteq W(y) \subseteq W(x)$ and yPx. Now let us assume that x is uncovered by y. Then $\exists t \in X,\ tPy,\ xRt$. Now we want a $s \in X$ so that sPy and xPs. If xPt then s = t; otherwise xIt. Because W(y) is open [as the interior of R(y)] there is a neighborhood of t, N(t), so that W(y) ⊇ N(t). Then, because xIt, by the thin tie set rule (Shepsle and Weingast 1984, p. 52, Lemma T), $\exists s \in N(t) : xPs,\ sPy$. Then because W(y) and $W^{-1}(x)$ are open there is a neighborhood of s, N(s), so that $N(s) \subseteq W(y) \cap W^{-1}(x)$. Let the radius of N(s) be r. Since lim $r(V_\omega)_{\omega\to\infty} = 0\ \exists\delta \in N : k > \delta \Rightarrow r(V_k) < r$, so that $\exists z \in V_k \cap N(s) \Rightarrow z \in W(y) \cap V_k, z \notin W(x)$, x is uncovered on V_k. □

Proposition 2. Let x be in the interior of UC(X); then for any neighborhood of x, A(x), $\exists\delta > 0 : k > \delta \Rightarrow \exists y \in UC(Vk) \cap A(x)$.

Proof: Assume x is in the interior of the uncovered set. So there is a neighborhood of x, B(x), so that B(x) ⊆ UC(x). Define N(x) = $A(x) \cap B(x)$. Let r be the radius of N(x). Since lim $r(V_\omega)_{\omega\to\infty} = 0\ \exists\ \delta_1 \in N : k > \delta_1 \Rightarrow r(V_k) < r$, so that $\exists y \in V_k \cap N(x)$. Then y is uncovered on X. Now assume z covers y on V_k. So by Lemma 1 there is an appropriate δ_z so that y is uncovered on z. There is a minimal δ_z as all z are part of the finite grid, so there is a finite number of possible points z, and $\delta = \min_{z\in V_k:zC_{V_k}y}(\delta_z,\delta_1)$. If x is on the boundary of the uncovered set then there is a point y in A(x) that is in the interior of the uncovered set, and there is a neighborhood of y A(y) so that $A(y) \subseteq A(x)$. Then $\exists\ \delta > 0 : k > \delta \Rightarrow \exists y \in UC(Vk) \cap A(y) \subseteq UC(Vk) \cap A(x)$. □

Theorem 2. If the UC(X) has a nonempty interior or is union of sets all of which have nonempty interiors, then as the resolution of the grid goes to infinity, the uncovered set delineated by our grid procedure converges to the continuous uncovered set, i.e., lim $UC(V_\omega)_{\omega\to\infty} = UC(X)$.

Proof: By Proposition 2, for any resolution small enough we can get as close as we want to any point in UC(X) with a point in lim $UC(V_\omega)_{\omega\to\infty}$, so lim $UC(V_\omega)_{\omega\to\infty} \supseteq UC(X)$. Now if x is covered by a set with an interior, then by proposition 1 it is covered on some fine enough grid. Otherwise, because by Theorem 1 if x is covered it is covered by a point in UC(X) and since lim $UC(V_\omega)_{\omega\to\infty} \supseteq UC(X)$, x is covered on lim $UC(V_\omega)_{\omega\to\infty}$. Thus lim $UC(V_\omega)_{\omega\to\infty} \subseteq UC(X)$. □

Appendix B: Computational Algorithm

Let $X \subset R^2$ be a bounded set of policy outcomes and let N be an odd number of legislators. Each legislator is characterized by an ideal point $p_i \in X$, i = 1, . . . ,N. Point x is said to beat point y under majority rule if x is closer than y to more than half of the ideal points $\{p_i\}$.

Definition 1. A point $x \in X$ is *covered* by a point $y \in X$ if y beats x and any point that beats y also beats x. The *uncovered set* $U \subseteq X$ is the set of points that are not covered in X.
This definition suggests a direct method for computing the uncovered set. Our estimation technique relies on a discretization of the policy space into a finite collection of potential outcomes taken to lie on a grid in R^2. Let **G** be the $T \times 2$ matrix of grid point coordinates whose tth row, **G**[t,.], contains the coordinate vector for the tth point in the grid. The algorithm below compares the points in **G** to determine the ones that are not covered.

Algorithm 1. Determining the uncovered set

```
for t = 1, . . . , T
   a = G[t,.];
   unc = 1;   /* initialize */
   for i = 1, . . . , T, i ≠ t;
      b = G[i,.];
      if b beats a;
         cvr = 1;/* initialize */
         for j = 1, . . . , T, j ≠ i;
```

```
            c = G[j,.];
            if c beats b;
               if j ≠ t;   /*
            c ≠ a */
                  cvr = c
               beats a; /*
               binary indicator
               */
               endif;
            endif;
            if cvr < 1;
               break; /* terminate j
            loop */
            endif;
         endfor;
         unc = unc − cvr;
      endif;
      if unc < 1;
         break; /* terminate i loop */
      endif;
   endfor;
   if unc = 1;
      store a;
   endif;
endfor;
```

Efficiency Enhancements

Algorithm 1 reveals that the number of operations necessary to determine the uncovered set grows exponentially with T. For this reason we suggest three approaches to decreasing the computational demands arising with the application of finer grids. These approaches can be used individually or in various combinations depending on the setting being considered.

The first approach relies on a repeated application of Algorithm 1 under two different resolutions—a coarser grid $\mathbf{G}_1$ with a low number of points T_1 and a finer grid $\mathbf{G}_2$ with a larger number of points T_2. The first application of the algorithm serves as a pilot run that roughly identifies the general location and size of the uncovered set, helping to focus the search during the second run of the algorithm. More specifically, suppose that applying Algorithm 1 with $\mathbf{G}_1$ results in a $u_1 \times 2$ matrix $\mathbf{U}_1$ of uncovered points. The points in $\mathbf{U}_1$ can then be used to select an appropriate subset from $\mathbf{G}_2$ for which the outmost loop of Algorithm 1 is to be applied in the second run. For example, defining the 2-vectors of columnwise extrema $\boldsymbol{\alpha} = \min \mathbf{U}_1$ and $\boldsymbol{\beta} = \max \mathbf{U}_1$, in the second run one might choose to restrict attention to a rectangle containing $\mathbf{U}_1$:

$$S = \{\mathbf{g} = \mathbf{G}_2[t,.]\colon \boldsymbol{\beta}[1] + b > \mathbf{g}[1] > \boldsymbol{\alpha}[1] - b,\ \boldsymbol{\beta}[2] + b > \mathbf{g}[2] > \boldsymbol{\alpha}[2] - \mathrm{b}\},$$

where $b \geq 0$ is an appropriately chosen buffer. Hence, in the second application of Algorithm 1, one has the following outmost loop:

for $t = 1, \ldots, T$, and t s.t. $\mathbf{G}_2[t,.] \in S$;

...
endfor;

We emphasize that in the second run $\mathbf{G}_2$ is not truncated in any way and that all internal loops operate with the full number of points T_2.

The repeated application of the algorithm can save substantial amounts of time when the pilot run is effective at eliminating from consideration a large number of points in the outmost loop. Because the computational loads are exponential in T, the savings from eliminating points in the second run usually outweigh by a large margin the additional computational loads associated with the pilot run.

The second approach to reducing the computational loads can be combined with the above approach very easily. As pointed out by a reviewer, centrally located points (in and around the uncovered set) tend to cover a large proportion of the points in the space. For this reason, one can simply rearrange the points in $\mathbf{G}_2$ after the pilot run, placing points of $\mathbf{G}_2$ that are close to points in $\mathbf{U}_1$ at the beginning of $\mathbf{G}_2$, making it more likely that one can establish a covering relation much sooner in the loops in Algorithm 1. Our practical experiments suggest that this can result in dramatic improvements in running times.

The third shortcut is applicable to problems with a small number of legislators N when their ideal points are clustered together in the policy space. Since it is known that the uncovered set is a subset of the Pareto set, one can compute the Pareto set and restrict the search for uncovered points only to elements of the Pareto set. This method is theoretically sound and is quite useful when N is small and the ideal points are close together. However, as N grows, the Pareto set is likely to expand while the uncovered set is likely to shrink, resulting in a greater mismatch between the two. For this reason, when the ideal points are spread out the Pareto set may fail to eliminate from consideration a sufficient number of points from $\mathbf{G}$ and may thus fail to justify the additional computational loads associated with its determination. In such cases, using the above two approaches will be much more valuable.

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