Early Numeracy

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Say hello.

Describe the mathematics you support.

November 2022

Early Numeracy

- Counting principles
- Connecting number
- Comparison of numbers
- Addition and subtraction concepts

January 2023

Addition and Subtraction

- Addition computation
- Subtraction computation
- Addition and subtraction fluency
- Addition and subtraction word problems

March 2023

Place value and money

- Understanding tens and ones
- Representing thousands, hundreds, tens, and ones
- Money

April 2023

Geometry

- Identification of shapes
- Composing and decomposing shapes

Instructional Platform

MATH

Early Numeracy

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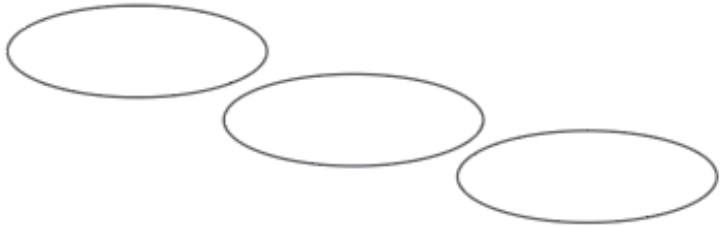
Instructional Platform

Instructional Delivery



Instructional Strategies

MATH

Instructional Platform

INSTRUCTIONAL DELIVERY

Explicit instruction

Precise language

Multiple representations

INSTRUCTIONAL STRATEGIES

Fluency building

Problem solving instruction

MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

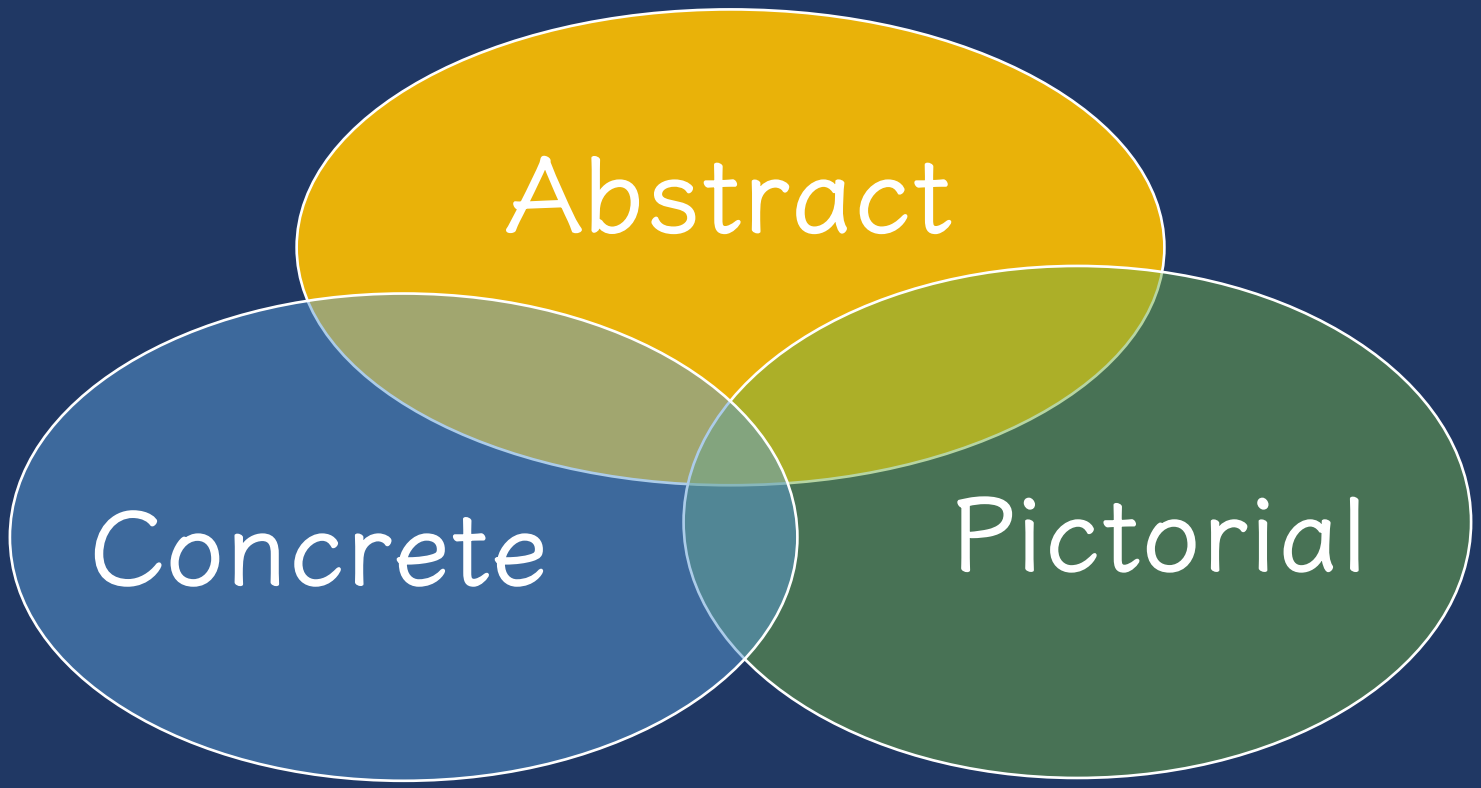
What math content do you model?

How do you engage students in guided practice?

Use formal math language

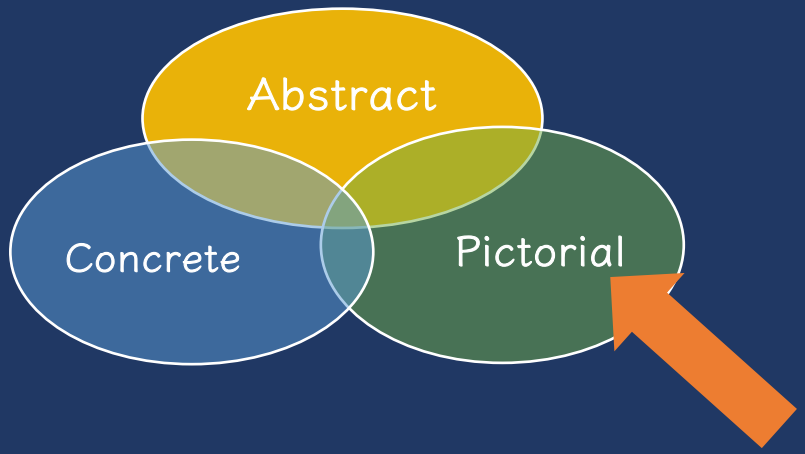
Use terms precisely

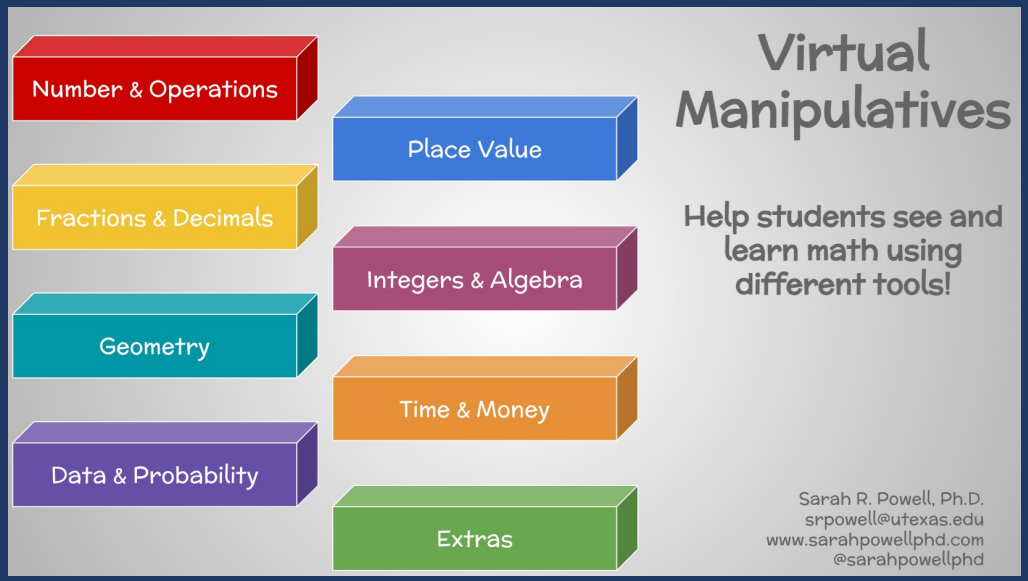
What's one way you support the math vocabulary of students?



What's a hands-on tool you use in your teaching?

What's a virtual manipulative you use?





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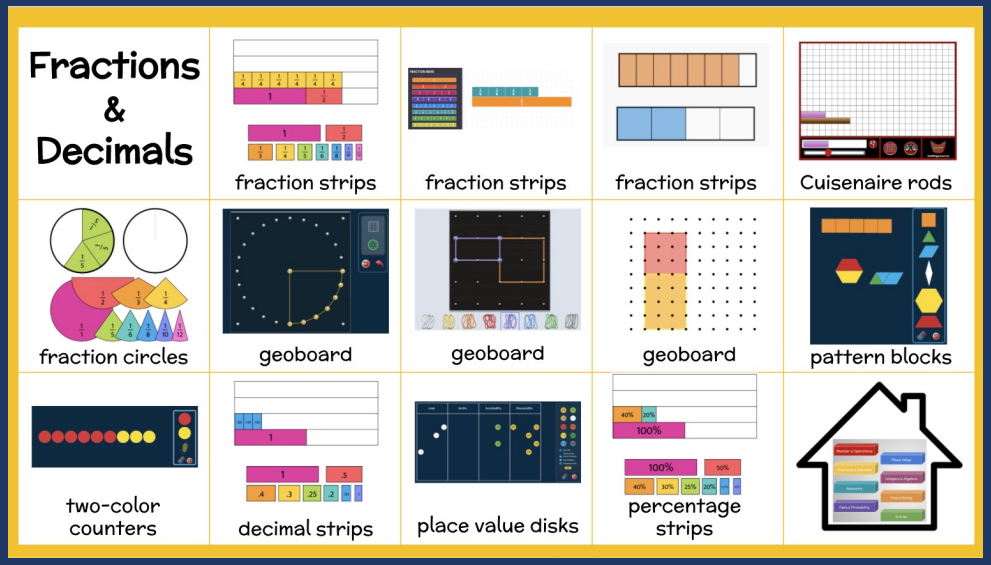


Addition	Subtraction
Multiplication	Division

Counting

Comparing numbers

Counting coins

Telling time

Identifying equivalent fractions

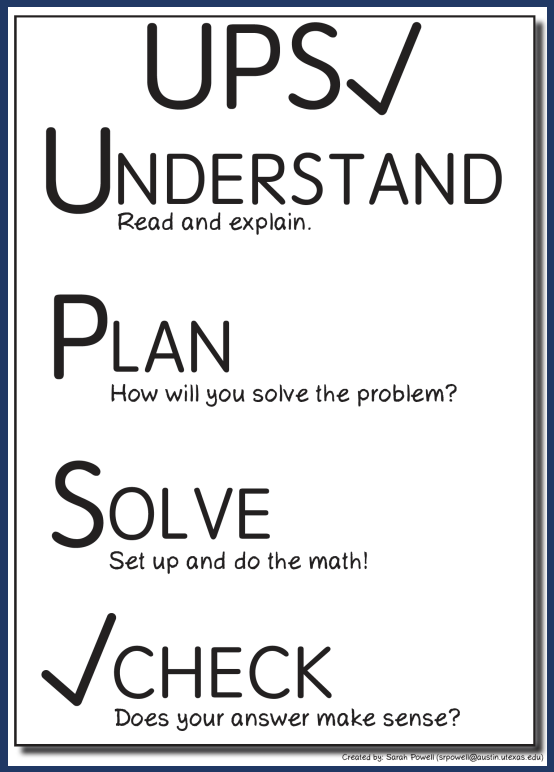
Knowing multiples

Identifying shapes

Knowing formulas

Addition	Subtraction
Multiplication	Division

How do you support students with fact fluency?



Total

Difference

Change

Equal Groups

Comparison

Ratios/Proportions

Counting

Counting Principles

Stable Order	
One-to-One Correspondence	
Cardinality	
Abstraction	
Order Irrelevance	

What are the difficulties your students have with counting?

Five Counting Principles

Stable order

One-to-one correspondence

Cardinality

Abstraction

Order irrelevance

Stable order

Saying the number words in order

"One, two, three, four, five…"

Stable order

Teacher modeling with echoing

One-minute timings

- *Count to 12 as many times as you can*

Songs

- www.youtube.com/watch?v=g9EgE_JtEAw
- www.youtube.com/watch?v=F5QLp9Wxrrg
- www.youtube.com/watch?v=dk9Yt1PqQiw
- www.youtube.com/watch?v=uxPfPyYp84E

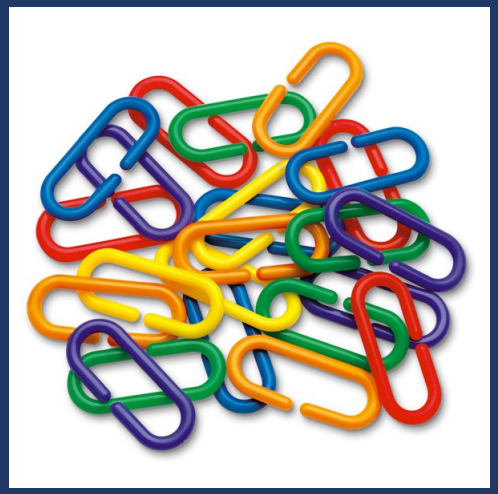
Books

Stable order

How do you model and practice stable order?

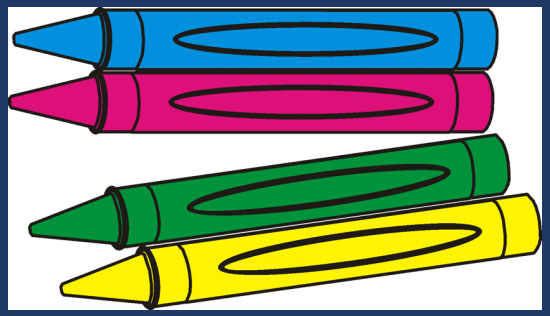
One-to-One Correspondence

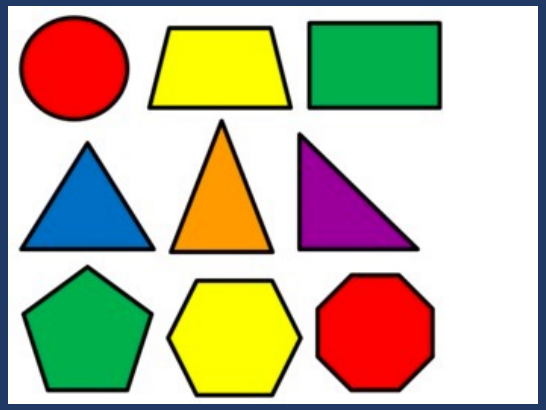
Ability to match number words to objects

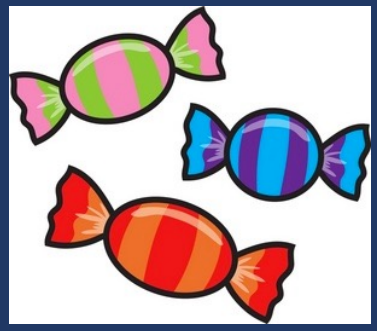


One-to-One Correspondence

Ability to match number words to objects



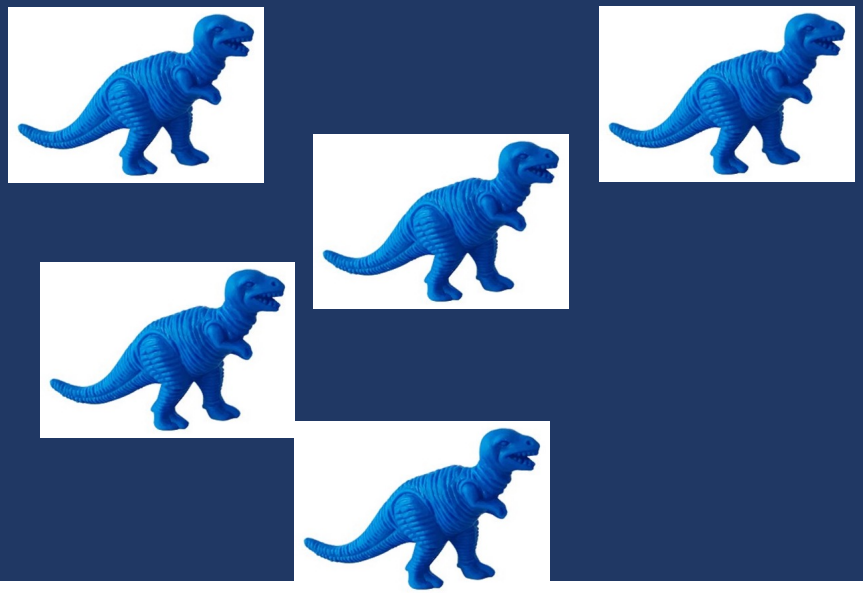




One-to-One Correspondence

Partitioning and tagging

- Transferred from the "to-be-counted" category to the "already-counted" category
- A distinct numeral word is assigned and not to be used again in the counting sequence



Stable Order AND One-to-One Correspondence

Model:

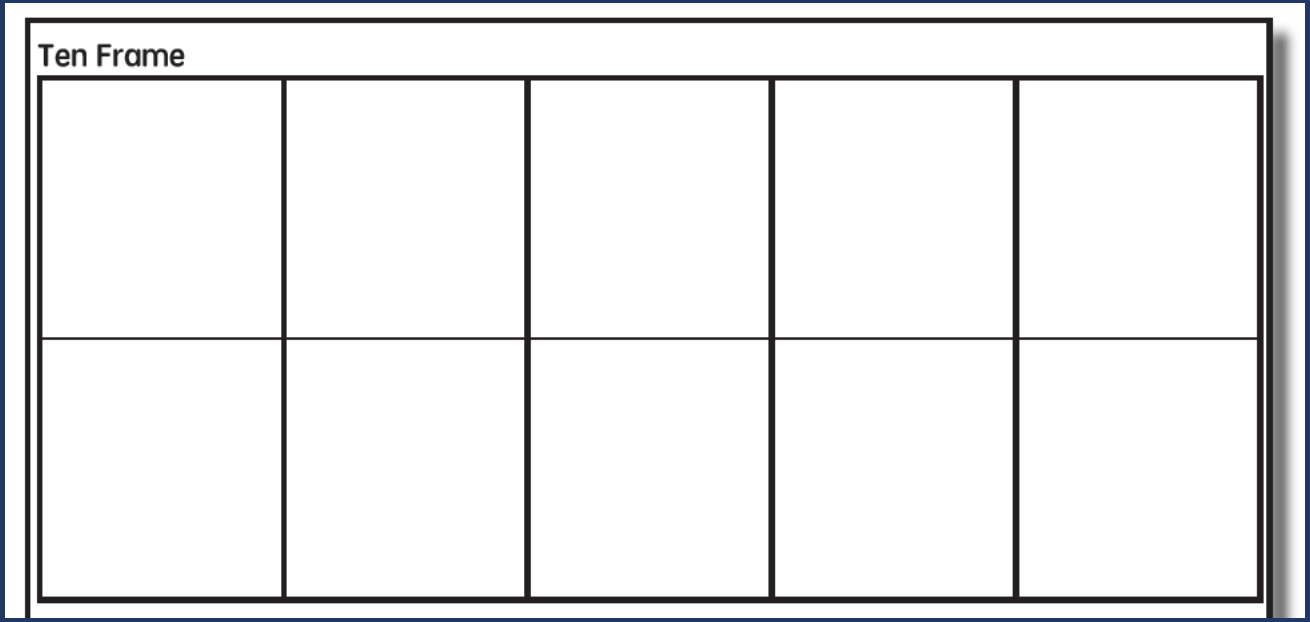
Count to 4.

Count to 7.

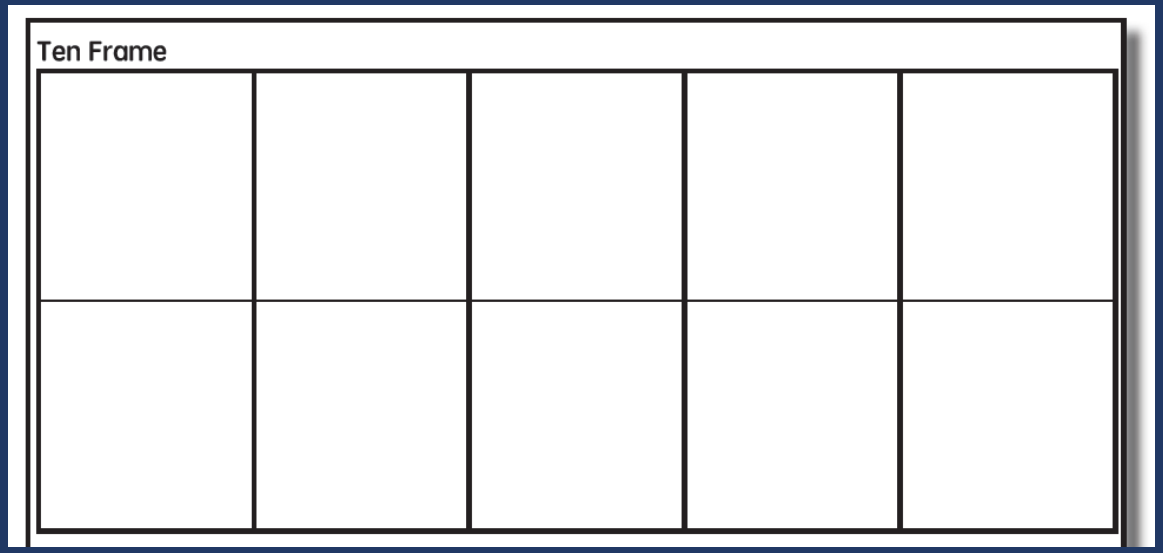
Ten Frame

Counting Boards

Stable Order AND One-to-One Correspondence



Stable Order AND One-to-One Correspondence



Model:

Count to 8.

Count to 5.

Cardinality

The number tag used for the last object in a count symbolizes the total number of objects in a set

- Students **must coordinate** the stable order and one-to-one correspondence

Cardinality

Teacher asking, "How many?"

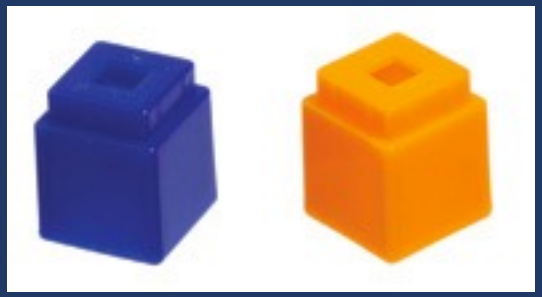
Model:

Count to 11.

Count to 5.

Abstraction

Any types of objects can be counted together in a set



Order Irrelevance

The order in which objects are counted does not matter as long as none of the other counting principles are violated

When teaching counting to "inefficient" counters, however, you should teach a strategy – like partitioning and tagging, working left to right, or using a work mat.

Five Counting Principles

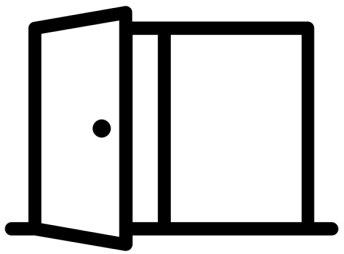
Stable order

One-to-one correspondence

Cardinality

Abstraction

Order irrelevance



(1) Describe how you teach the three essential counting principles.

(2) Discuss whether you will teach the two additional counting principles.

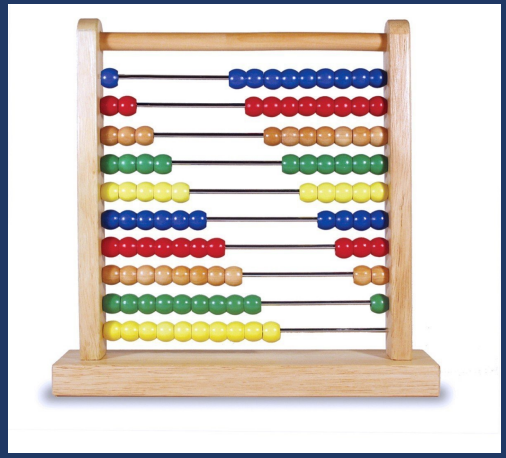
(3) Provide an example of your counting instruction.

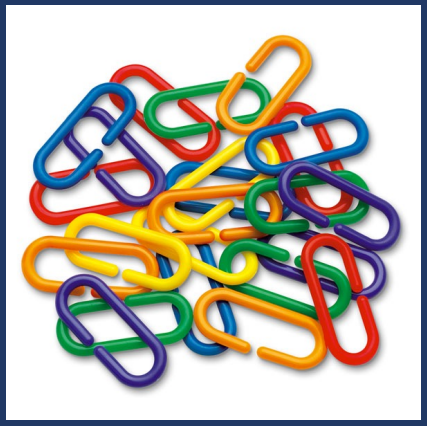
More Counting

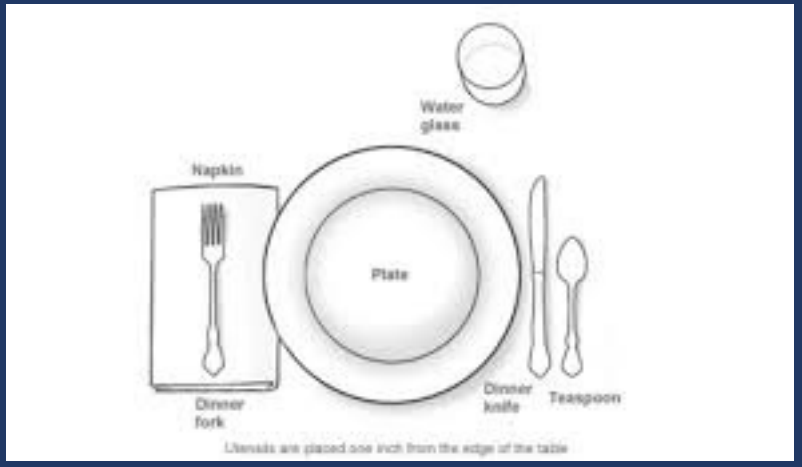
Counting objects/pictures

- Bears, cubes, clips, pencils
- Abacus
- Table setting
- Passing out papers

Counting with storyboards

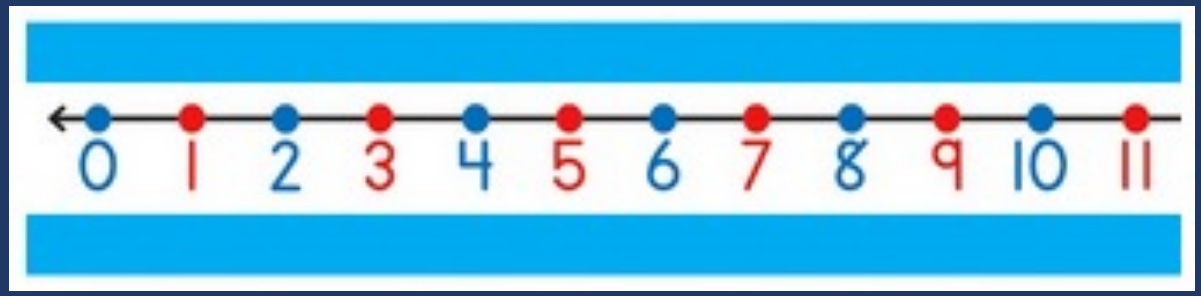






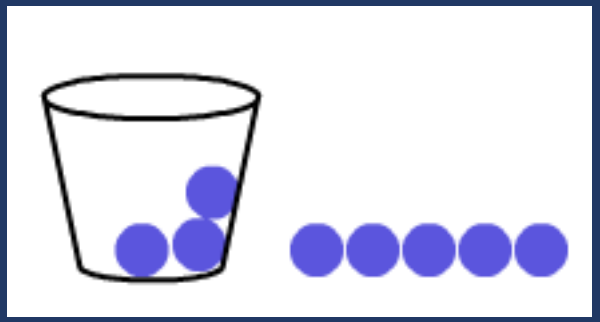
Counting

Counting with number lines



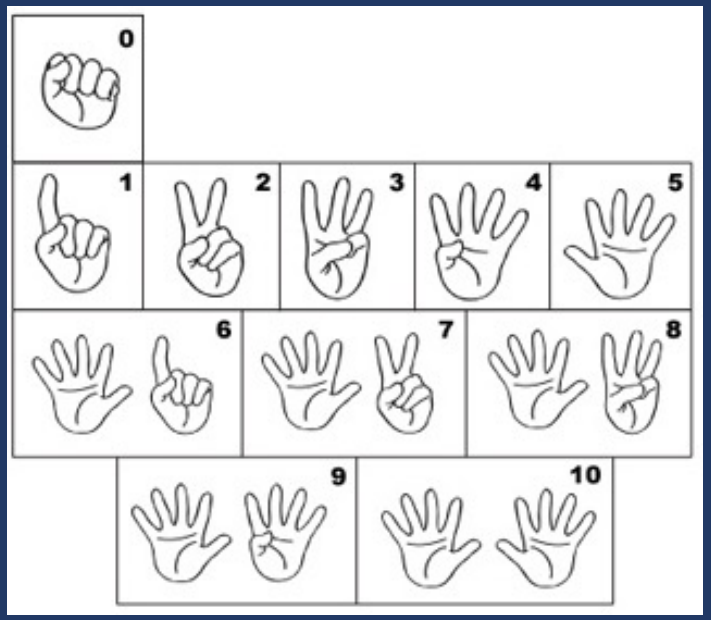
Counting on

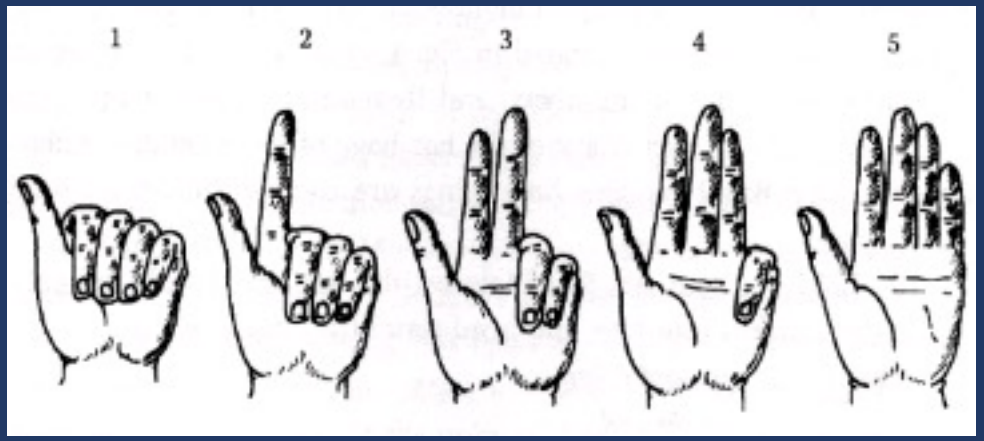
- *I'm hiding three. Count, starting from the hidden counters.*

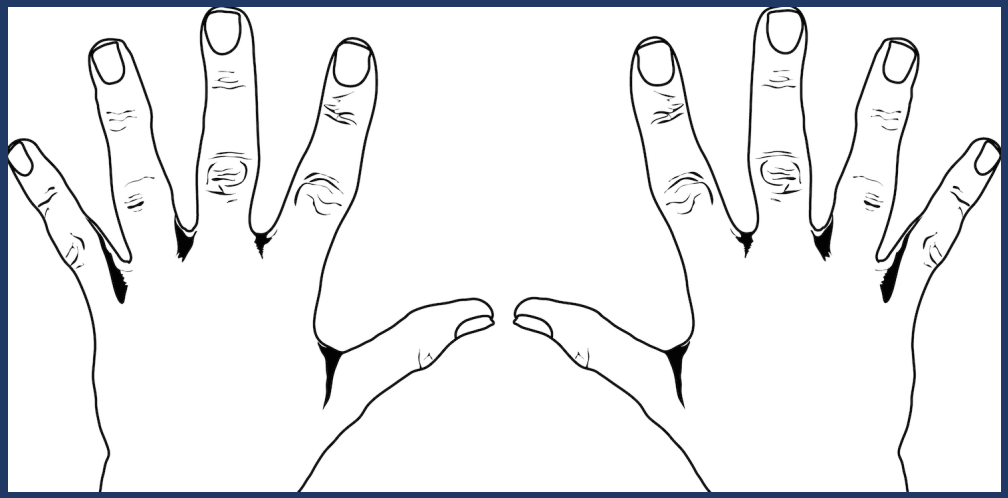


Counting

Finger counting







Counting

In virtual settings, how can you help students with finger counting?

Ordinal Counting

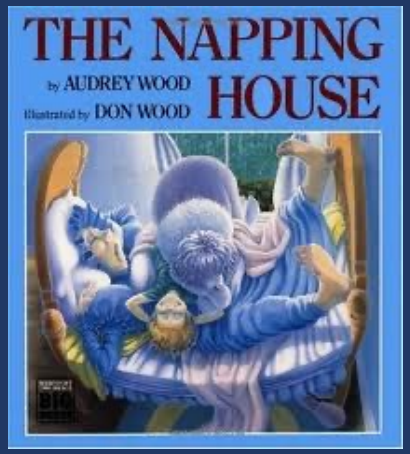
Numbers relative to their position in time or space

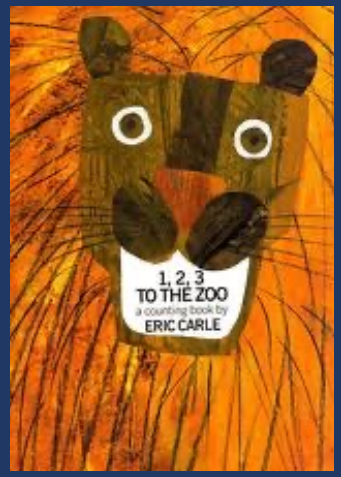
- First, second, third, fourth, fifth…

Ordinal Counting

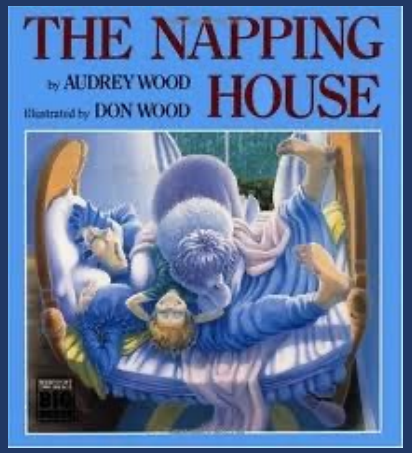
Modeling and activities for counting can be used plus:

- Lining up in classroom
- Running a race and determining place
- Following steps in a recipe
- Calendar dates
- Chapters in a book





Ordinal Counting



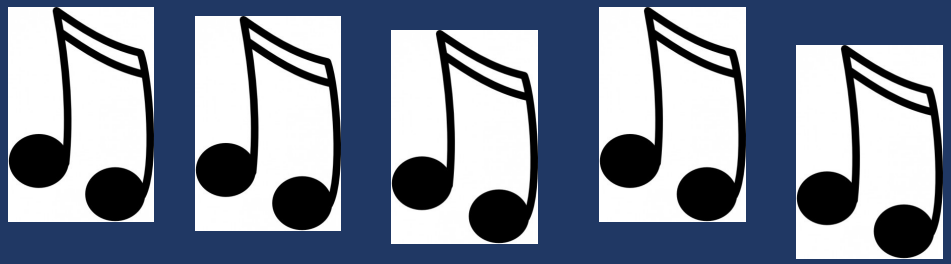
Ordinal Numbers

The Napping House

First (1st)	Second (2nd)	Third (3rd)	Fourth (4th)
Fifth (5th)	Sixth (6th)	Seventh (7th)	Eighth (8

Skip Counting

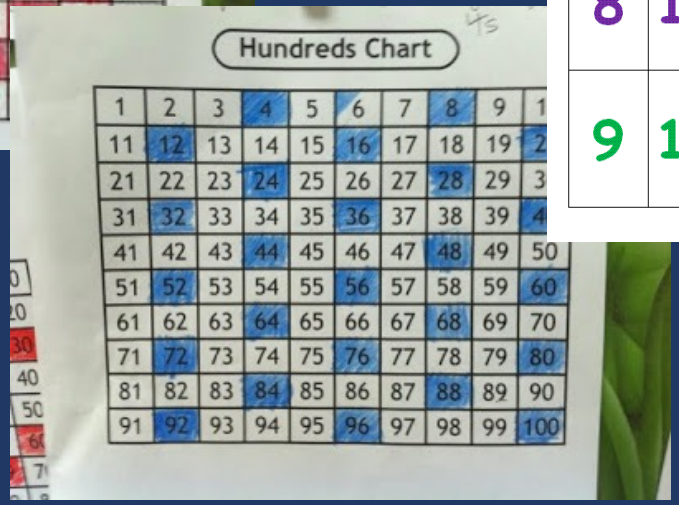
Counting by 2s, 5s, and 10s



Hundred Chart									
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Skip Counting

Counting by 3s, 4s, 6s, 7s, 8s, 9s



Skip Counting By 6's, 7's, 8's, and 9's Chart

6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90

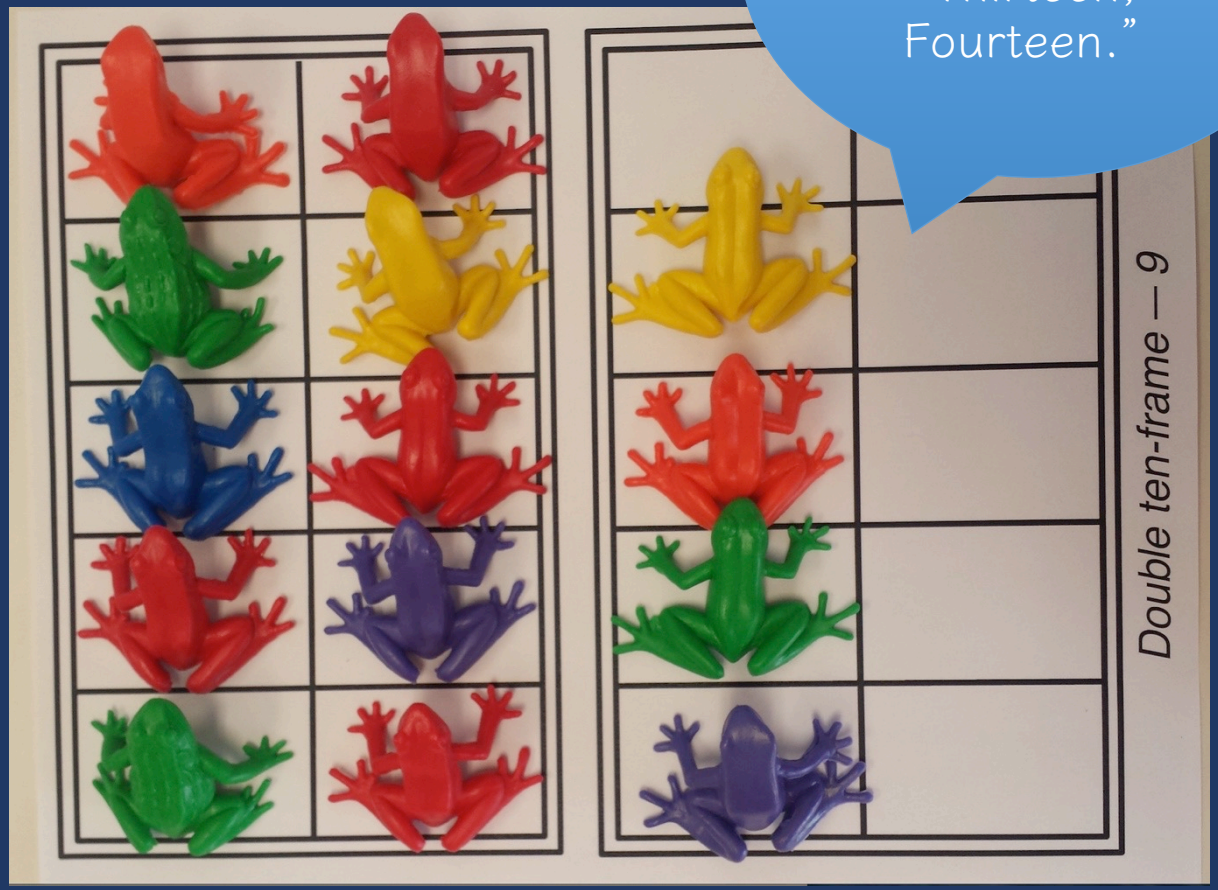
Counting

Describe your skip counting activities.

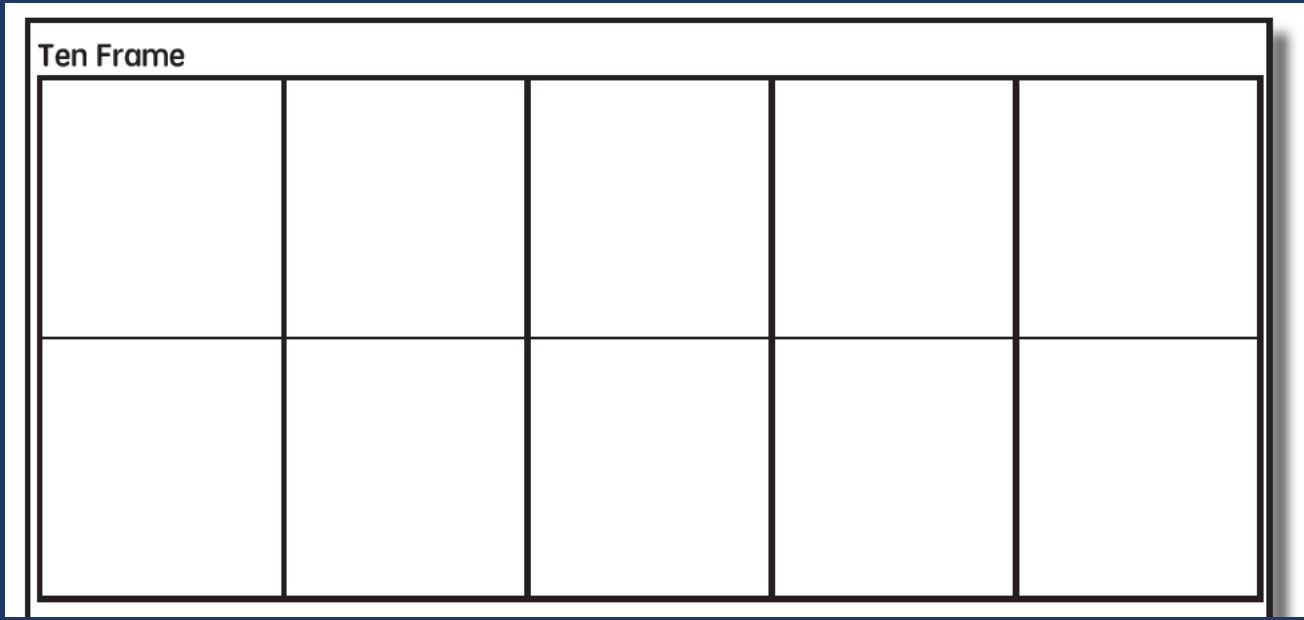
Place Value

Ten Frame

Place Value



Place Value



Model:

Count to 11.

Count to 14.

Place Value

Place Value

Model:

Count to 17.

Count to 22.

Instructional Platform

INSTRUCTIONAL DELIVERY

Explicit instruction

Precise language

Multiple representations

INSTRUCTIONAL STRATEGIES

Fluency building

Problem solving instruction

MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

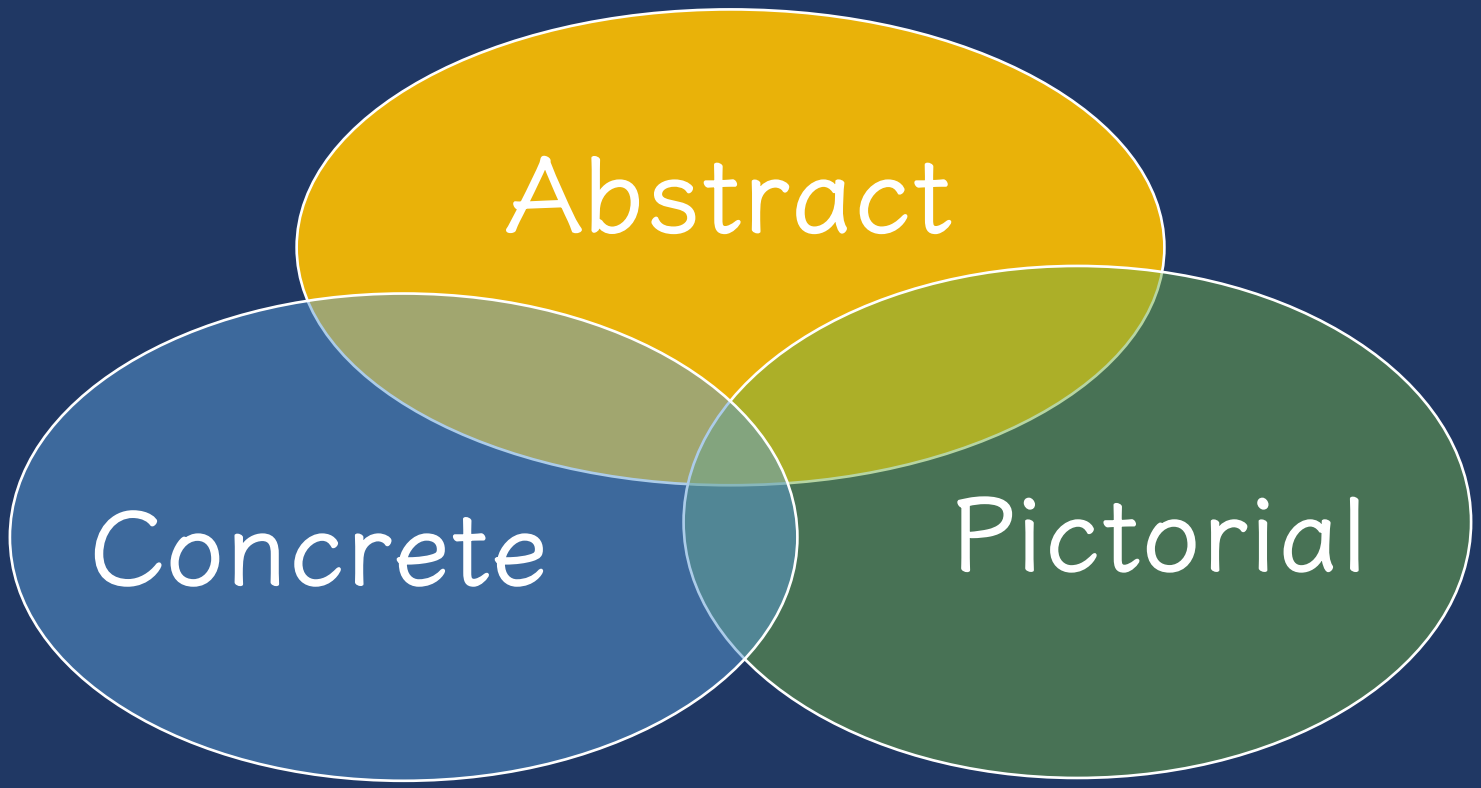
What are your strengths with modeling counting?

What are your opportunities for growth?

Use formal math language

Use terms precisely

What are five essential math vocabulary for counting?

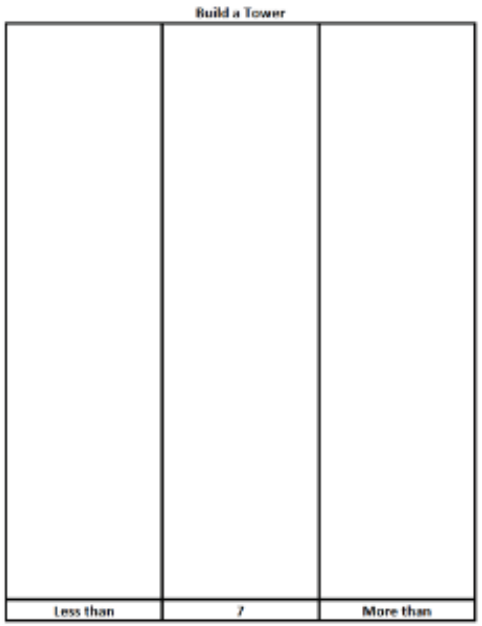


What are the representations you'll use to teach counting?

Connecting Number

Three Representations of Number

Comparing Numbers

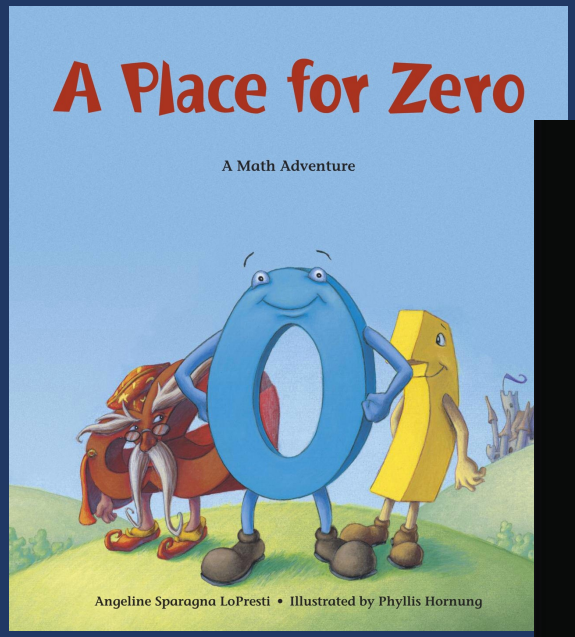


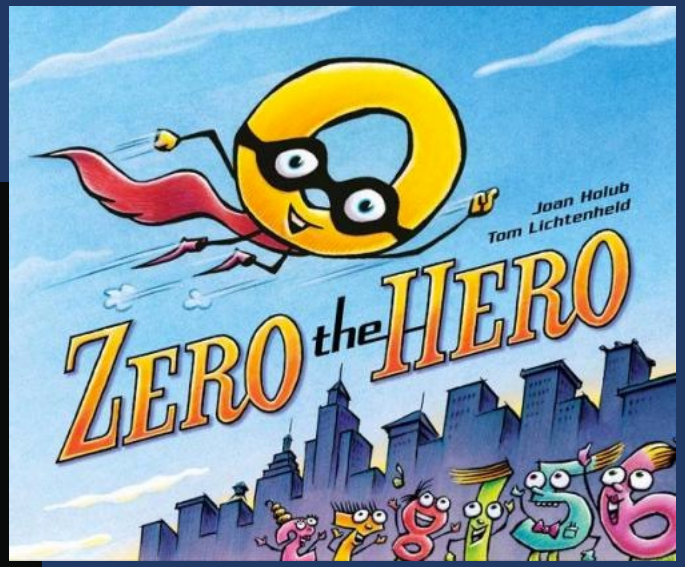
What are the difficulties your students have connecting numerals, number words, and quantity?

Zero

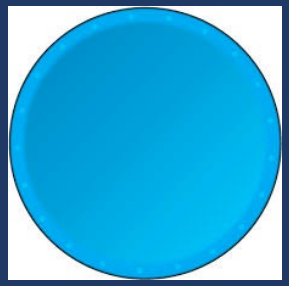
Most important digit in Base-10 system

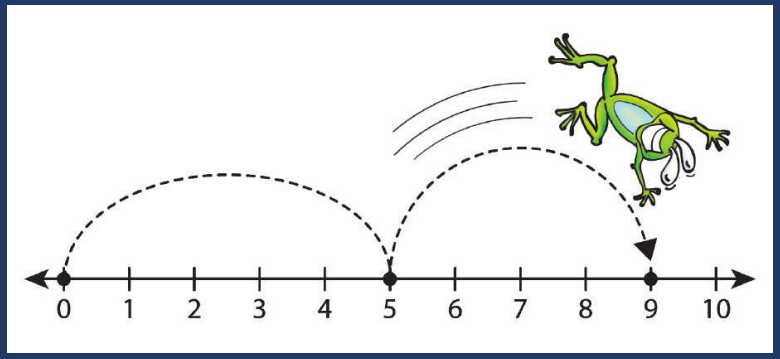
Typically introduced after 1, but should be introduced alongside 1





Zero





Zero

Describe activities to help students understand zero.

Three Representations of Number

7

seven

Numerals and Number Words

Numerals

- 0, 1, 2, 3, 4, 5, 6, 7, 8, 9…

Number words (cardinal)

- one, two, three, four, five…

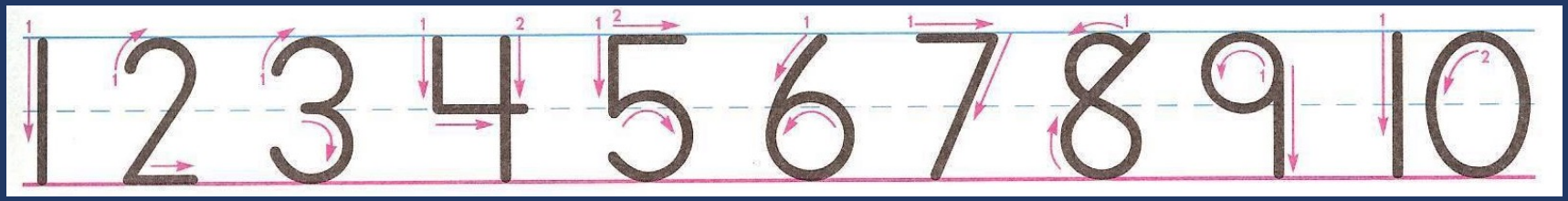
Ordinal numerals

- 1st, 2nd, 3rd, 4th, 5th…

Ordinal number words

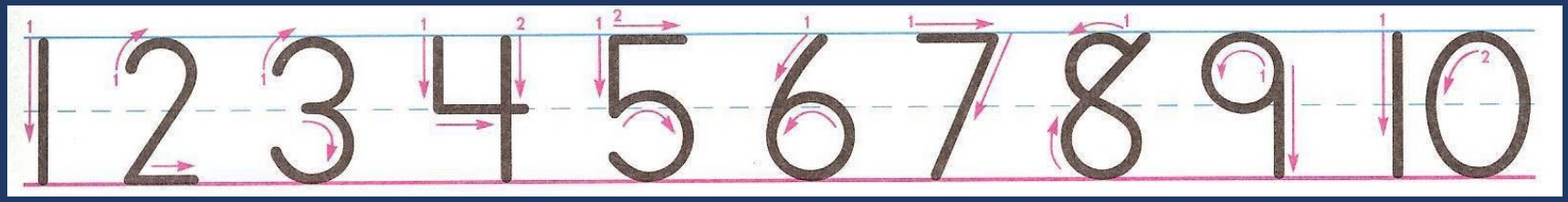
- first, second, third, fourth, fifth…

Numerals and Number Words



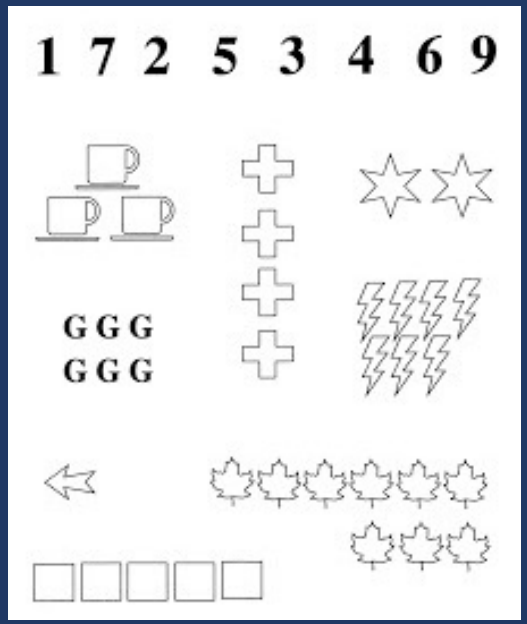
1	Straight down and then you're done. That's the way to make a one!
2	Around and back on the railroad track. Two! Two! Two!
3	Around the tree, around the tree. That's the way to make a three!
4	Down and over. Down some more. That's the way to make a four!
5	Across the top, then take a dive. Make a big round tummy, now that's a five!
6	Make a loop, then make a hoop! Six! Six! Six!
7	Across the top, down for the win. That's the way to make seven!
8	Make an "S," but do not wait. Go back up to make an eight!
9	Make a hoop and then a line. That's the way to make a nine!
0	Around, around, around you go. That's the way to make zero!

Numerals

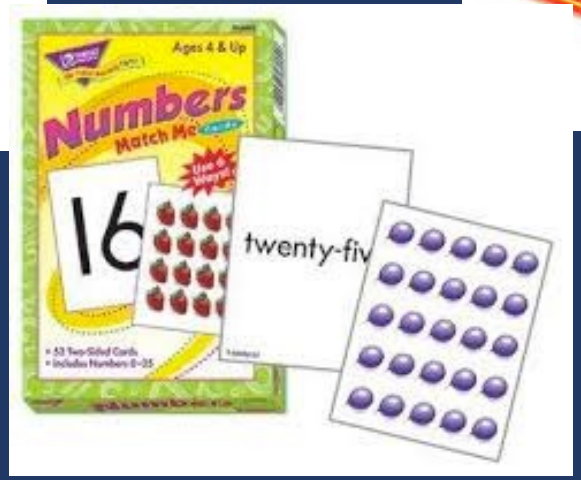


Describe activities to help students write their numerals.

Numeral, Number Word, and Quantity

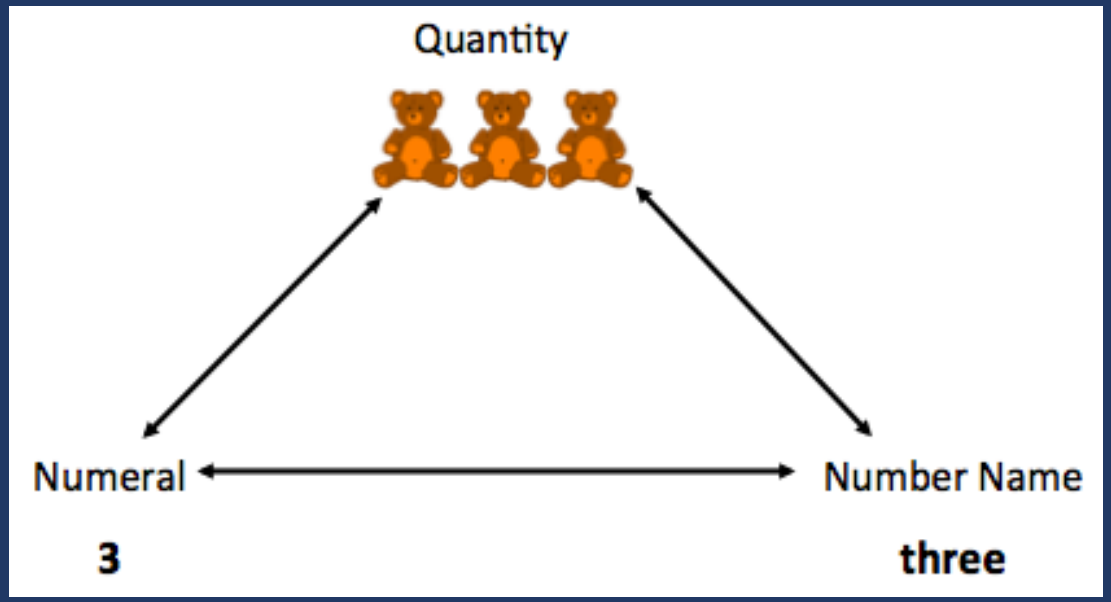


Numeral, Number Word, and Quantity



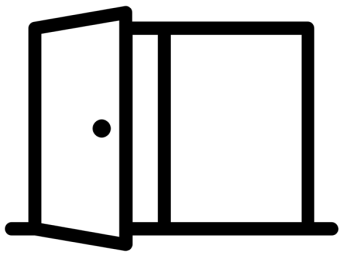
Literature

Literature should have three representations of number



Literature should be accurate

Objects should be easy to count



(1) Describe how you connect numerals, number words, and quantities.

(2) Provide an example for 5, five, and *****.

Instructional Platform

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Precise language

Multiple representations

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Eliciting frequent responses

Providing affirmative and corrective feedback

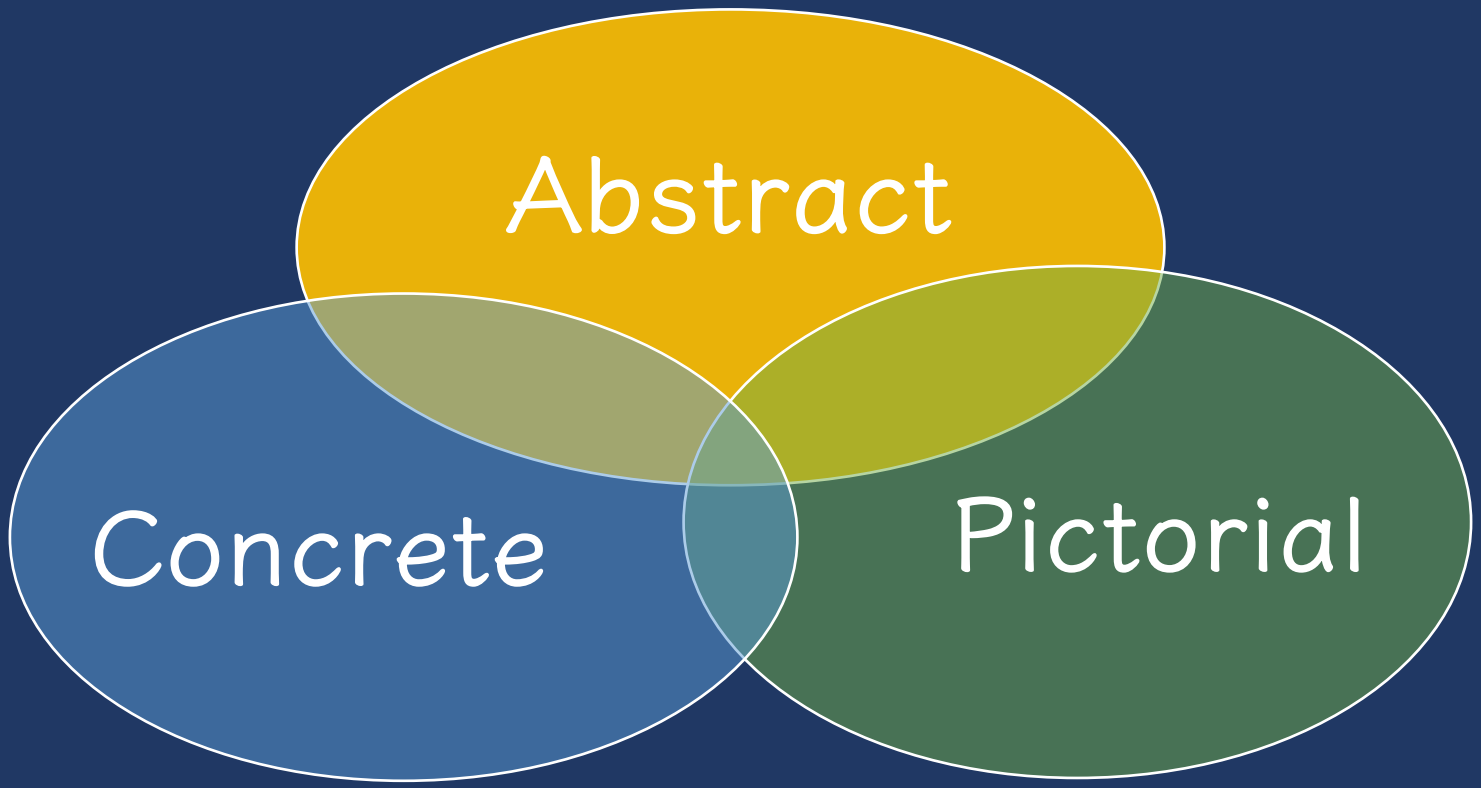
What are your strengths with modeling connecting number?

What are your opportunities for growth?

Use formal math language

Use terms precisely

What are five essential math vocabulary for connecting number?

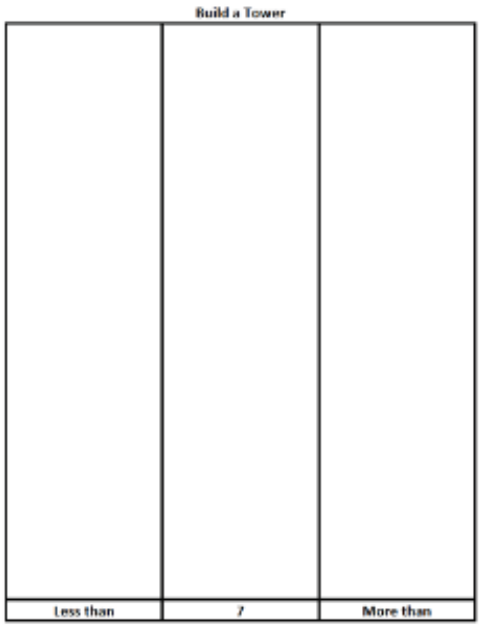


What are the representations you'll use to teach connecting number?

Comparison of Numbers

Three Representations of Number

Comparing Numbers



What are the difficulties your students have with comparison?

Quantity Comparison

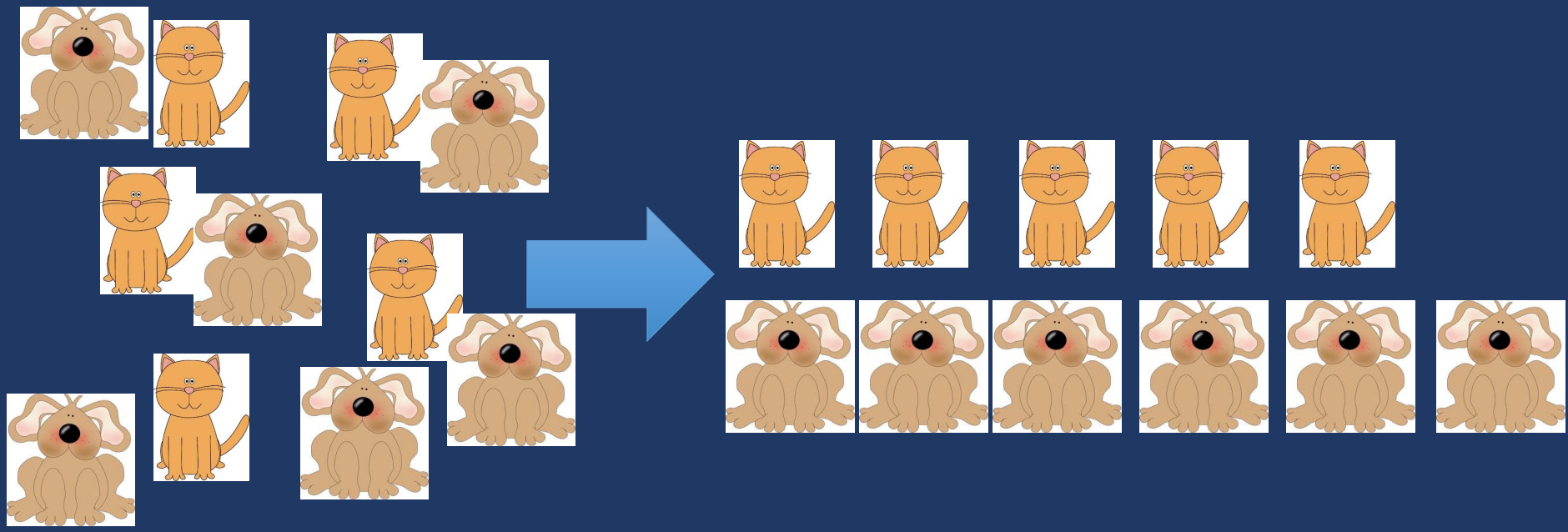
Vocabulary:

- More, greater, bigger
- Less, smaller, fewer
- Same, as many as

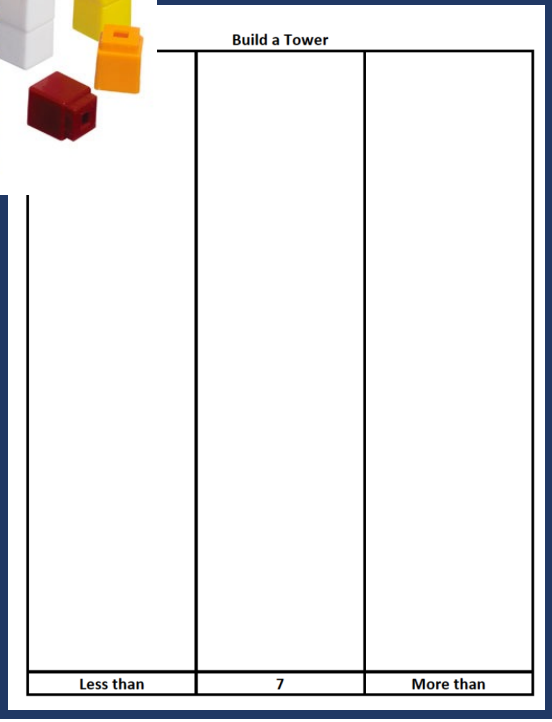
Quantity Comparison

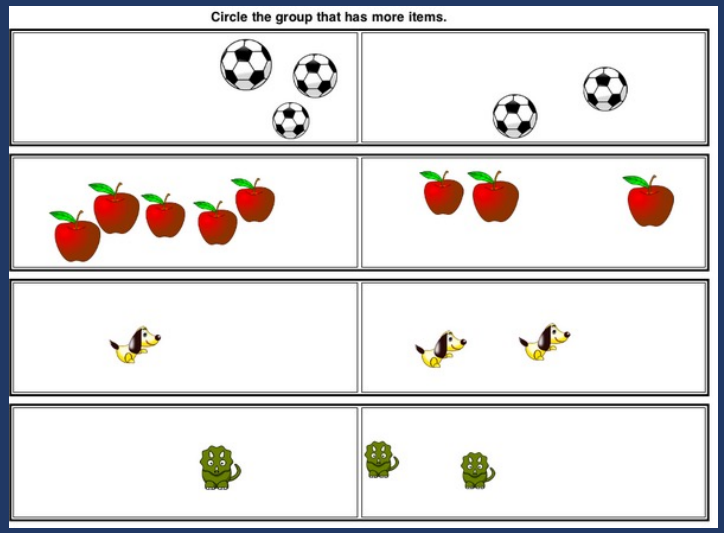
Teacher modeling

- Find the pairs
- Finish when one group runs out of items
- Figure out more than, less than, or equal



Quantity Comparison



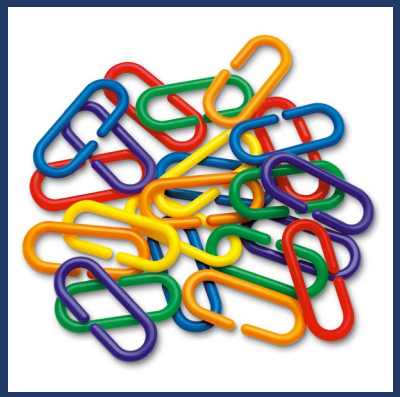


Quantity Comparison

Make chains

Bear compare

Counting cars



Comparison

Describe activities to help students with comparison.

Subitizing

Subitizing

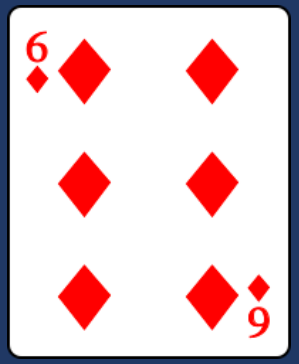
Instantly seeing how many

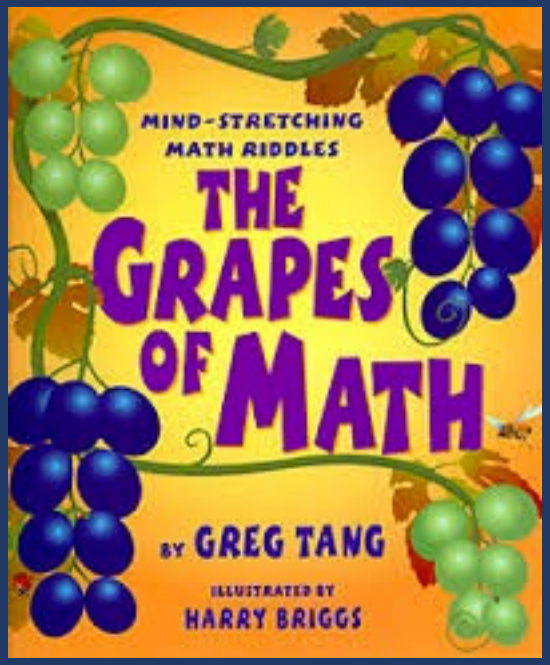
Young students can subitize sets of 1, 2, or 3 without counting (perceptual subitizing)

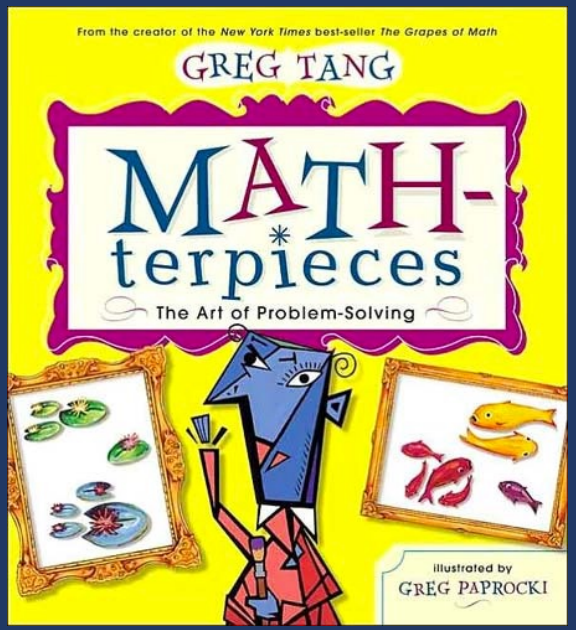
- 4 or 5 is the maximum subitizing amount

Students can subitize larger amounts by combining smaller amounts (conceptual subitizing)

Subitizing Instruction







Subitizing

Is it important to practice subitizing?

How do you practice subitizing?

Instructional Platform

INSTRUCTIONAL DELIVERY

Explicit instruction

Precise language

Multiple representations

INSTRUCTIONAL STRATEGIES

Fluency building

Problem solving instruction

MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

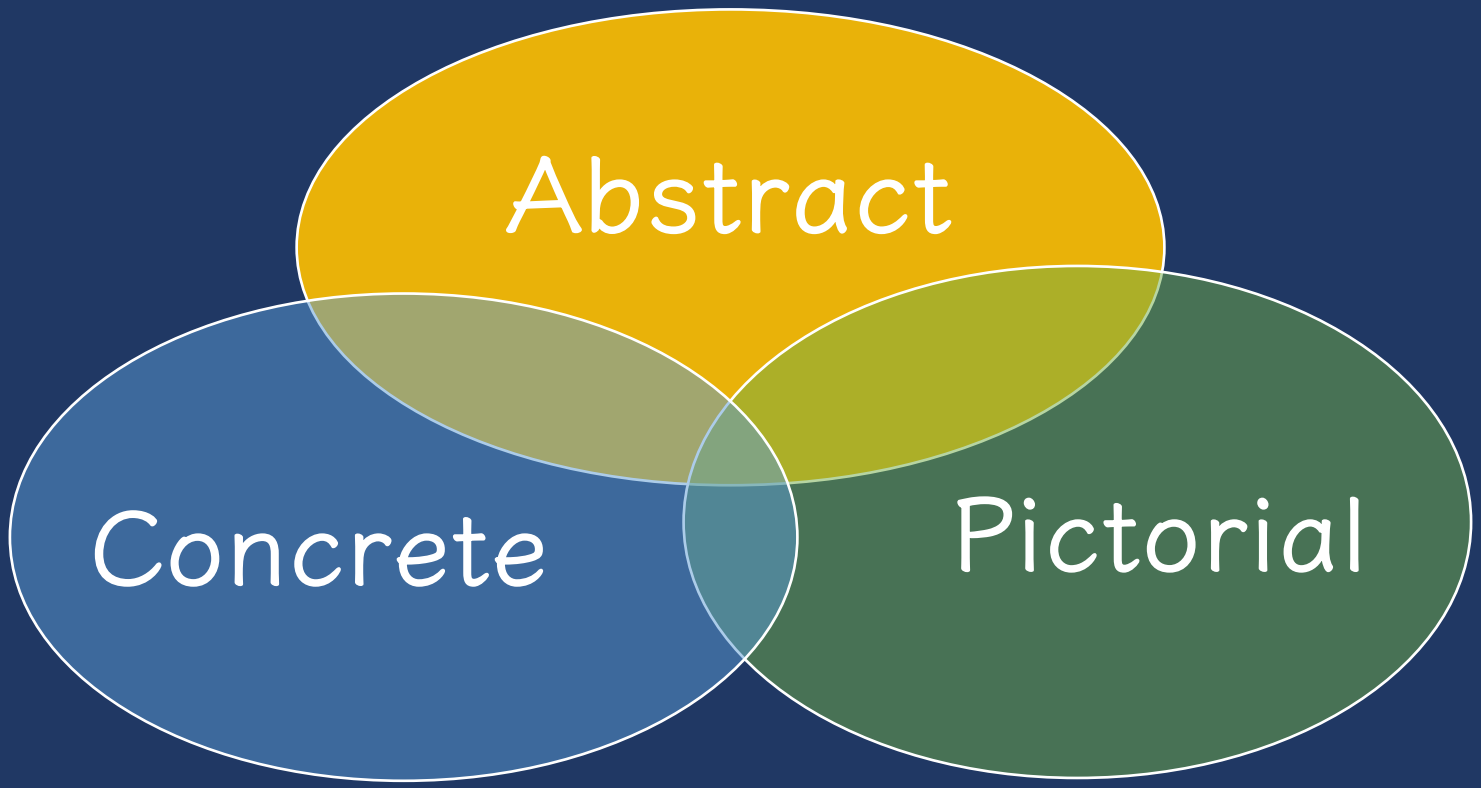
What are your strengths with modeling comparison?

What are your opportunities for growth?

Use formal math language

Use terms precisely

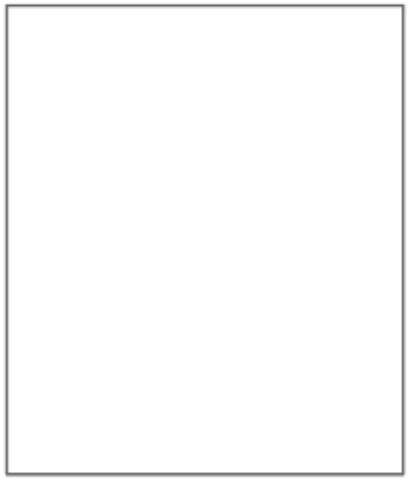
What are five essential math vocabulary for comparison?

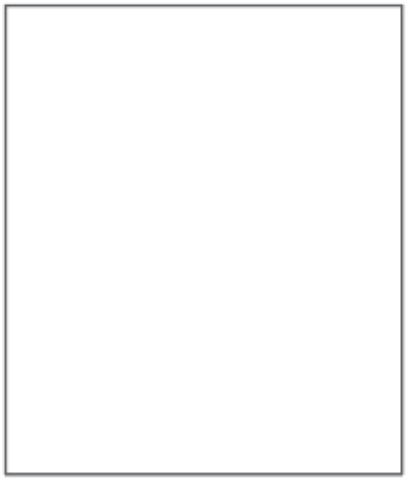


What are the representations you'll use to teach comparison?

Addition and Subtraction Concepts

Addition





What are the difficulties your students have with comparison?

Addition	Subtraction
Multiplication	Division

Addition

100 addition facts

Single-digit addends sum to a single- or double-digit number

$$\begin{array}{rl} 5 & \text{(addend)} \\ +\ 4 & \text{(addend)} \\ \hline 9 & \text{(sum)} \end{array}$$

Total

Addition

Count one set, count another set, put sets together, count sum

$$2 + 3 = 5$$

Total

Addition

Count one set, count another set, put sets together, count sum

Model:

4 + 5

9 + 3

Change

Addition

Start with a set, add the other set, count sum

$$2 + 3 = 5$$

Change

Addition

Start with a set, add the other set, count sum

Model:

4 + 5

9 + 3

Total

Addition

Parts put together into a total

Karly saw 4 cardinals and 5 blue jays. How many birds did Karly see?

Total

Addition

Parts put together into a total

Write a total story.

Change

Addition

An amount that **increases** or decreases

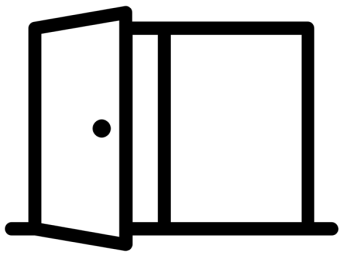
Premila had $4. Then they earned $5 for cleaning their room. How much money does Premila have now?

Change

Addition

An amount that **increases** or decreases

Write a change (increase) story.

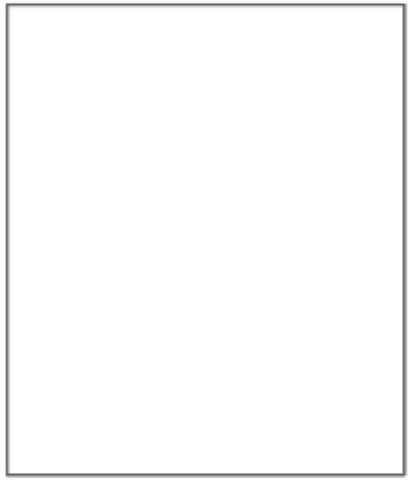


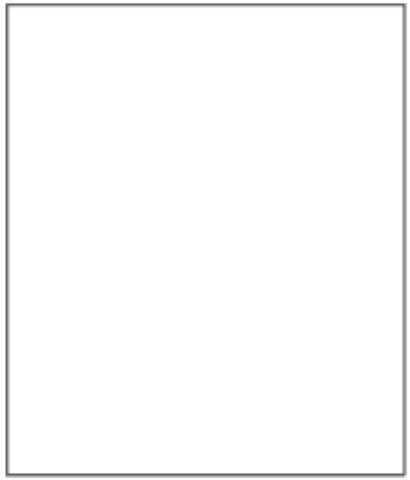
(1) Model 3 + 9 as a total problem.

(2) Model 3 + 9 as a change problem.

(3) Discuss how to distinguish between total and change.

Subtraction





Subtraction

100 subtraction facts

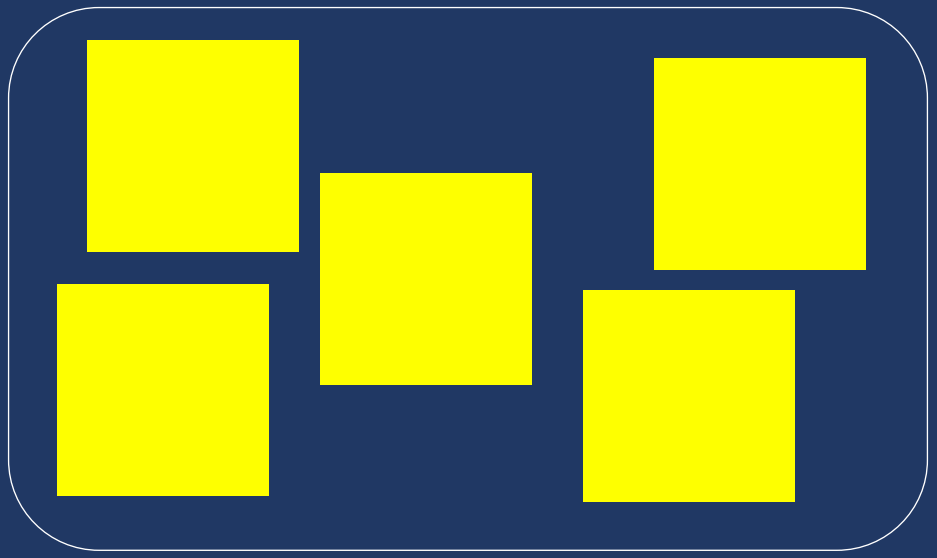
Subtrahend and difference are single-digit numbers and minuend is single- or double-digit number

$$\begin{array}{rl} 16 & \text{(minuend)} \\ -\ \ 8 & \text{(subtrahend)} \\ \hline 8 & \text{(difference)} \end{array}$$

Change

Subtraction

Start with a set, take away from that set, count difference

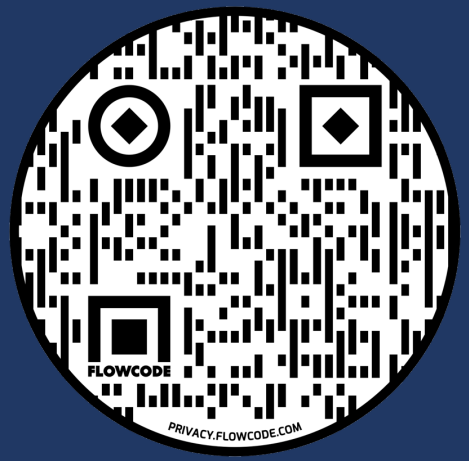


$$5 - 3 = 2$$

Change

Subtraction

Start with a set, take away from that set, count difference



Model:

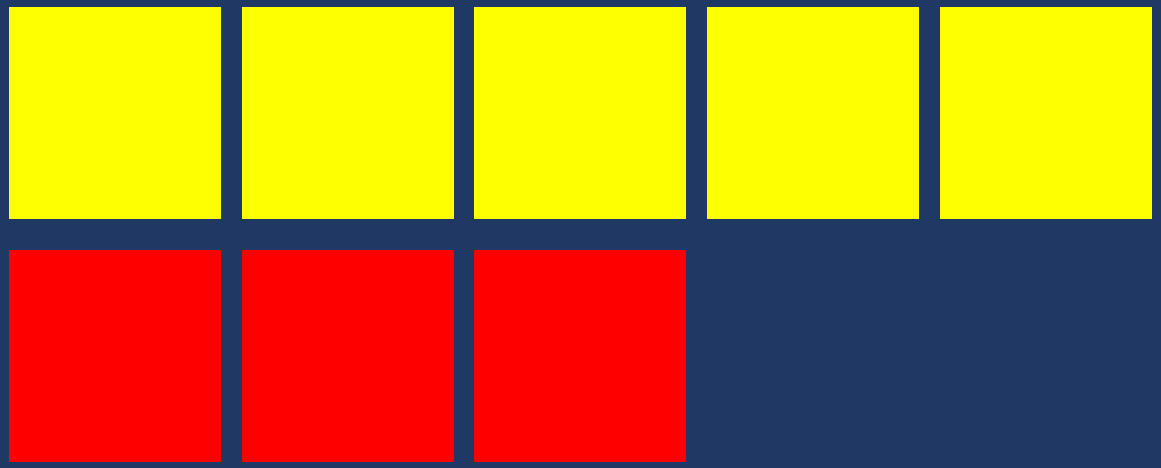
$9 - 3$

$11 - 7$

Difference

Subtraction

Compare two sets, count difference



$$5 - 3 = 2$$

Difference

Subtraction

Compare two sets, count difference

Model:

$9 - 3$

$11 - 7$

Change

Subtraction

An amount that increases or **decreases**

Bronwyn had 9 cookies. Then they ate 2 of the cookies. How many cookies does Bronwyn have now?

Change

Subtraction

An amount that increases or **decreases**

Write a change (decrease) story.

Difference

Subtraction

Greater and lesser amounts compared for a difference

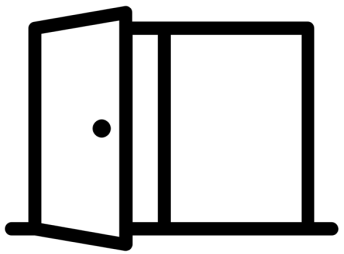
Rachel has 9 apples. Jodie has 2 apples. How many more apples does Rachel have? (How many fewer does Jodie have?)

Difference

Subtraction

Greater and lesser amounts compared for a difference

Write a difference story.



(1) Model 12 – 5 as a change problem.

(2) Model 12 – 5 as a difference problem.

(3) Discuss how to distinguish between change and difference.

Building Fluency

Fluency is doing mathematics *easily and accurately.*

Fluency in mathematics *makes mathematics easier.*

Fluency provides less stress on working memory.

Fluency helps students build confidence with mathematics.

With fluency, it is important to emphasize both *conceptual learning and procedural learning.*

Addition	Subtraction
Multiplication	Division

Counting

Comparing numbers

Counting coins

Telling time

Identifying equivalent fractions

Knowing multiples

Identifying shapes

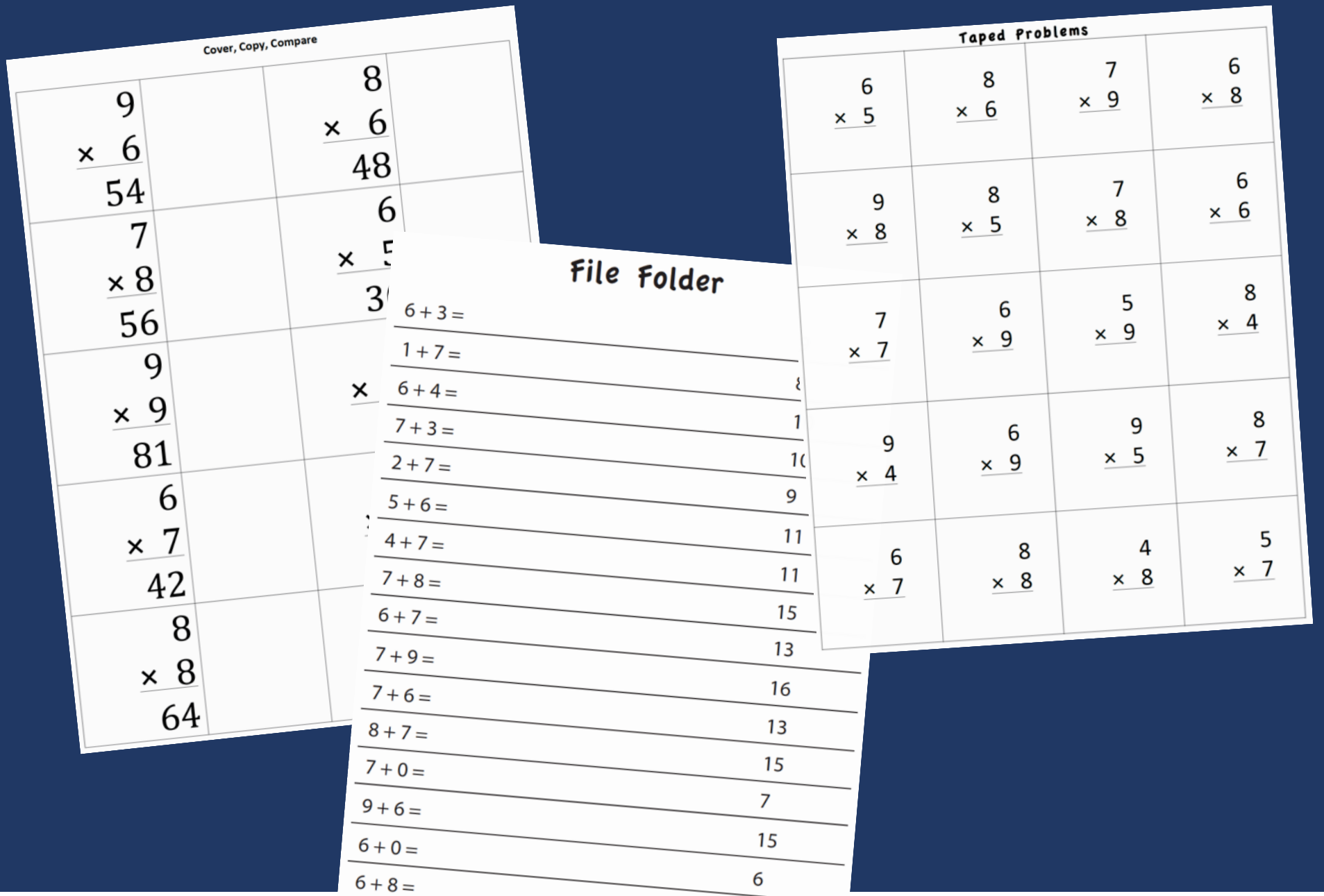
Knowing formulas

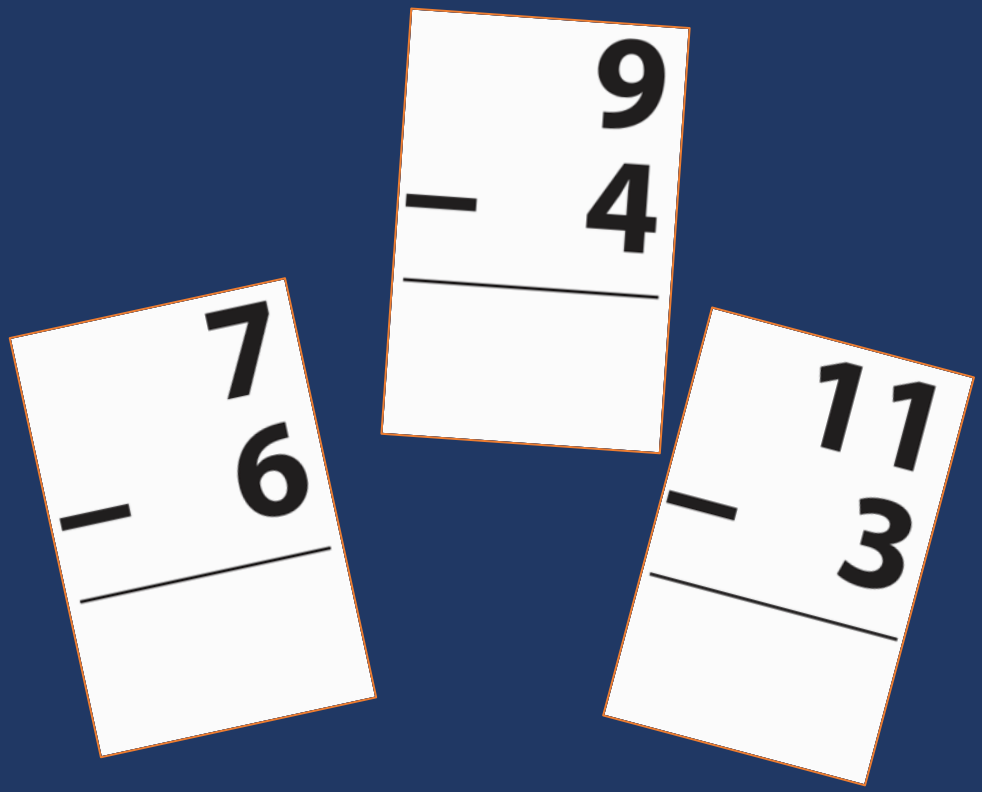
Addition	Subtraction
Multiplication	Division

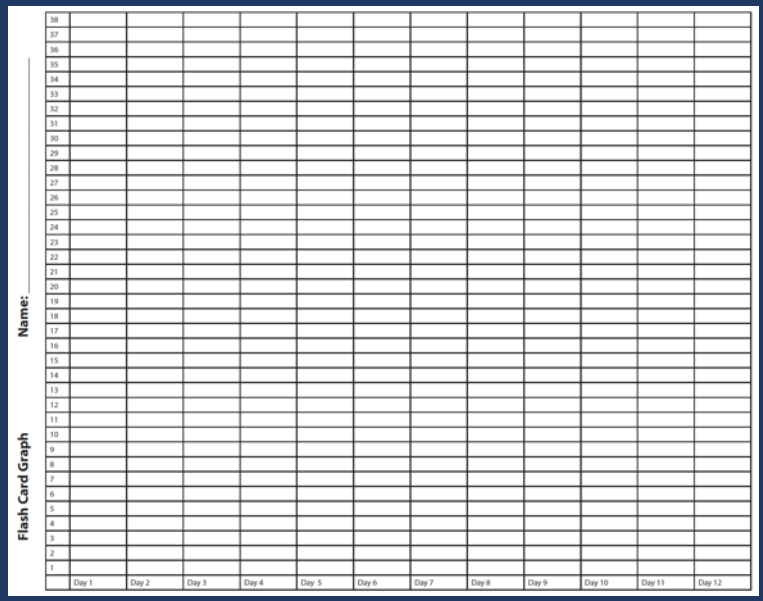
Build fluency with math facts.

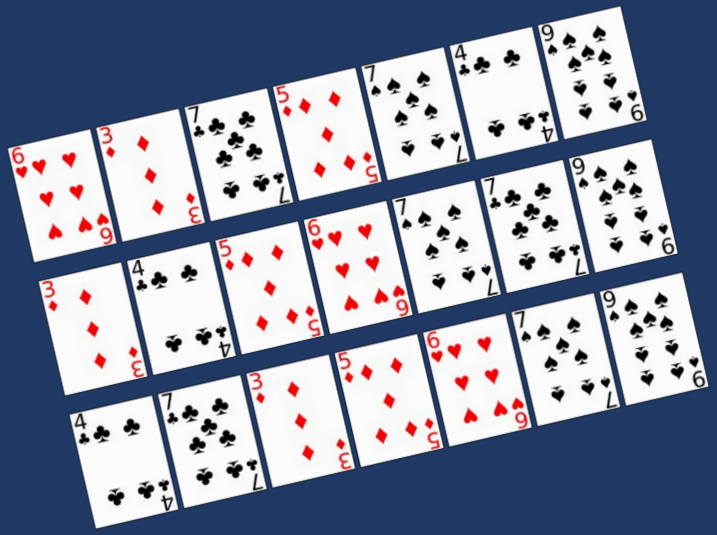
- Addition: single-digit addends
- Subtraction: single-digit subtrahend
- Multiplication: single-digit factors
- Division: single-digit divisor

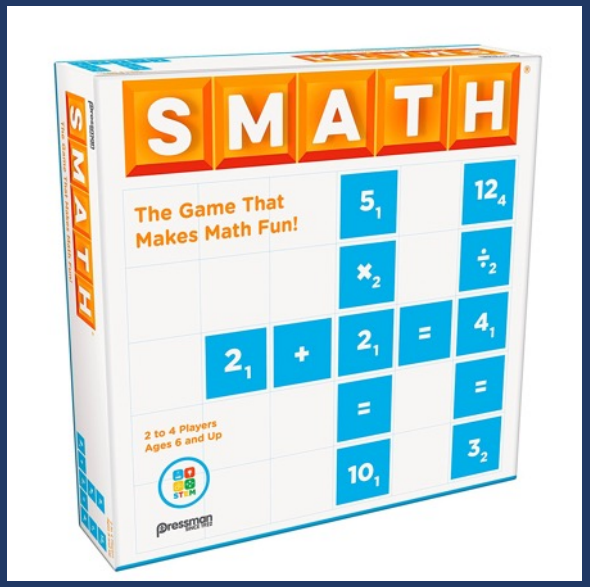
$$\begin{array}{r} 5 \\ +\ 8 \\ \hline \end{array} \qquad \begin{array}{r} 9 \\ -\ 4 \\ \hline \end{array} \qquad \begin{array}{r} 6 \\ \times\ 7 \\ \hline \end{array} \qquad \begin{array}{r} 56 \\ \div\ 8 \\ \hline \end{array}$$

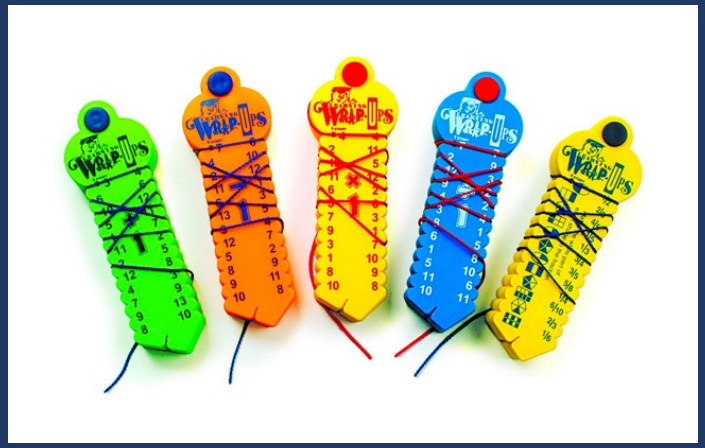


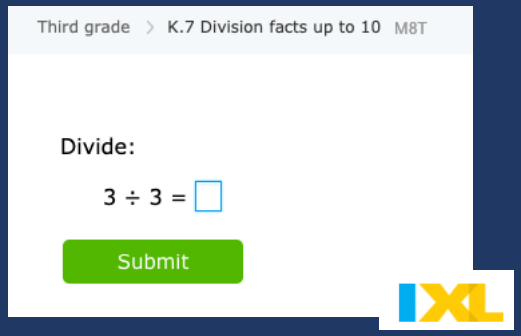


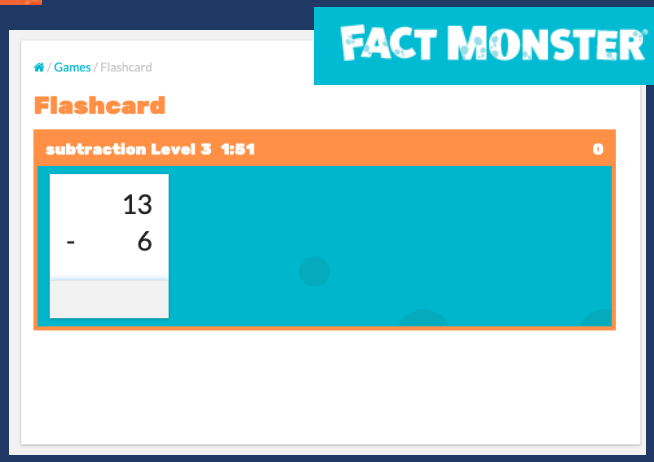












DAILY and BRIEF

Instructional Platform

INSTRUCTIONAL DELIVERY

Explicit instruction

Precise language

Multiple representations

INSTRUCTIONAL STRATEGIES

Fluency building

Problem solving instruction

Addition	Subtraction
Multiplication	Division

Describe three activities to help students with fact fluency.

MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

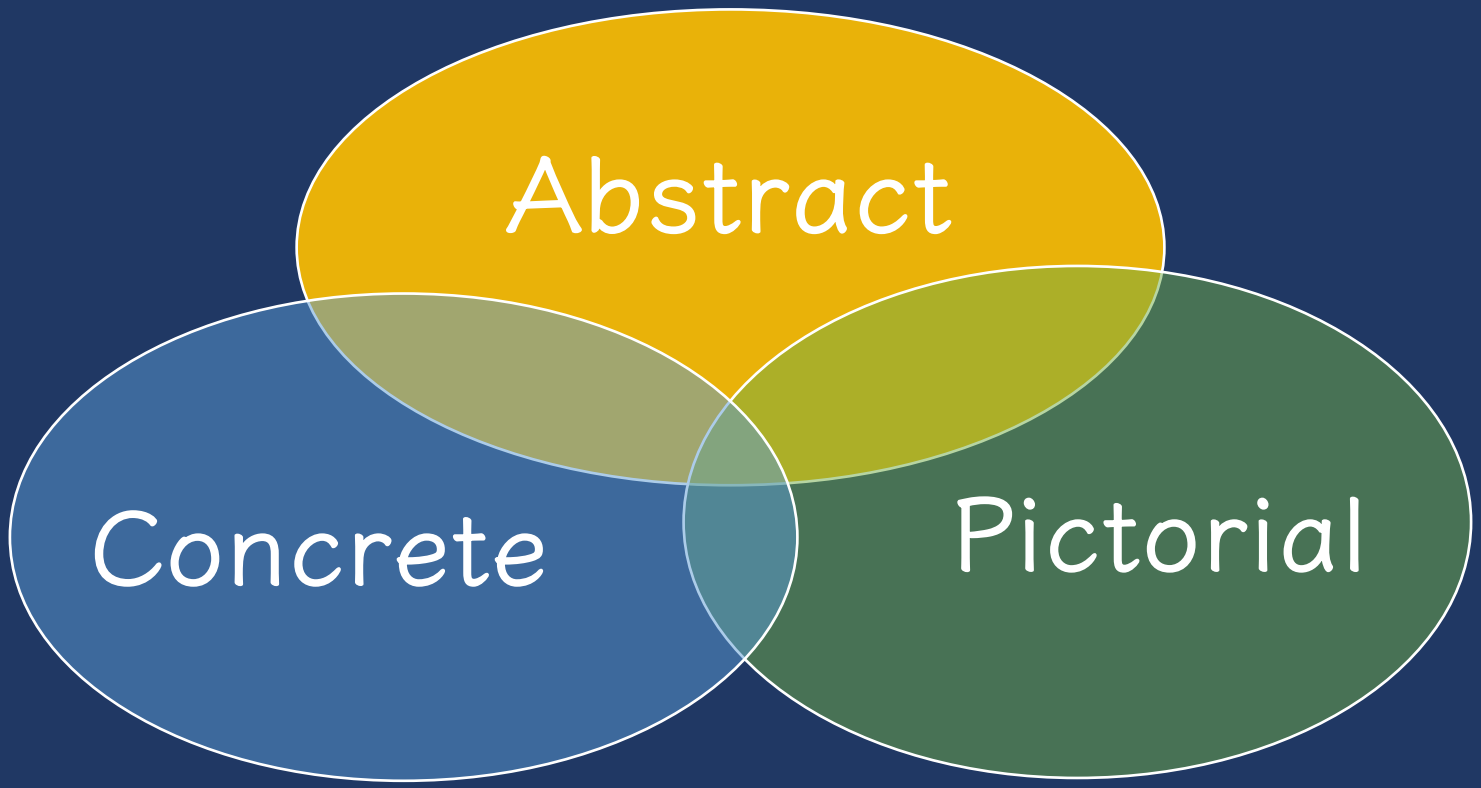
What are your strengths with modeling addition and subtraction?

What are your opportunities for growth?

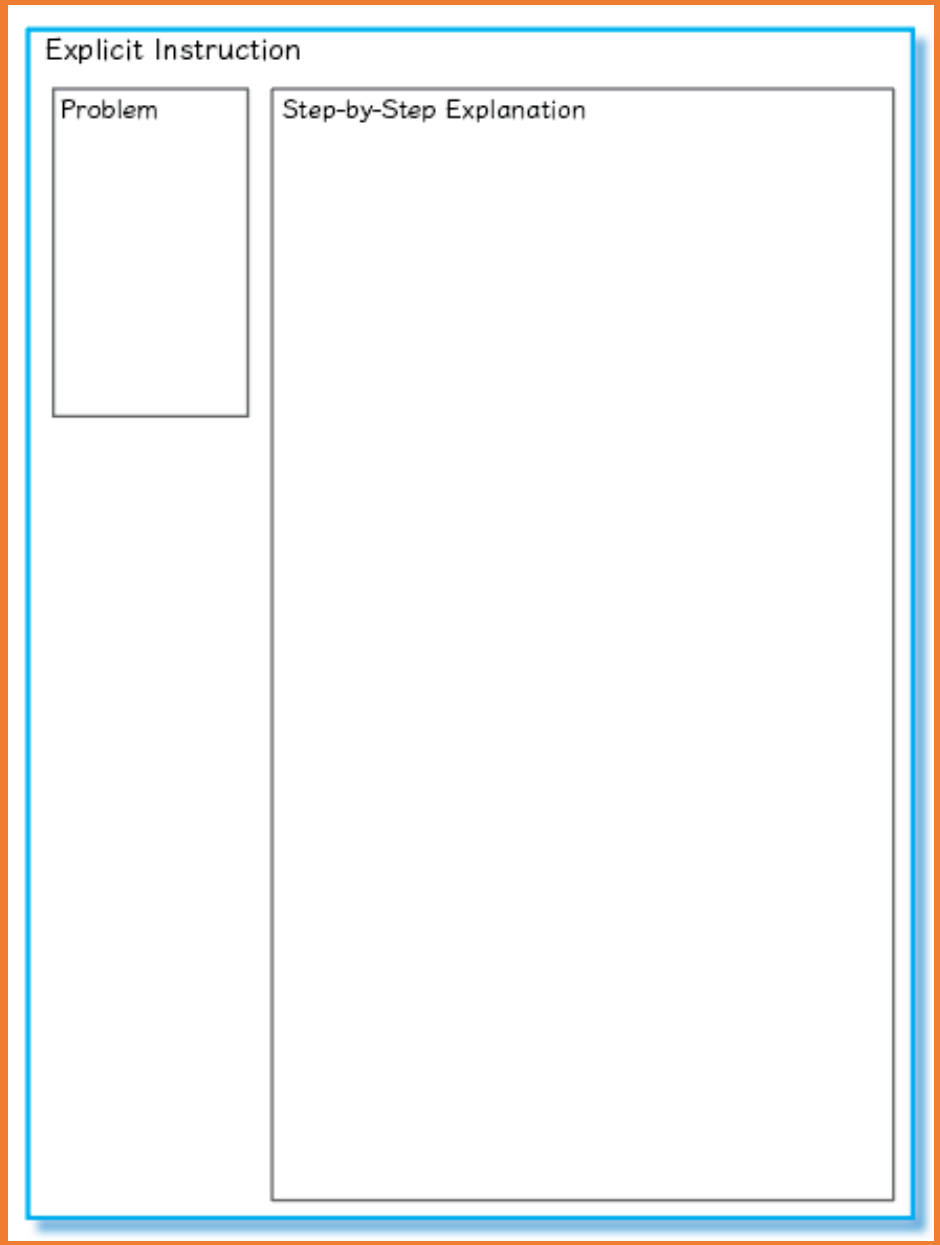
Use formal math language

Use terms precisely

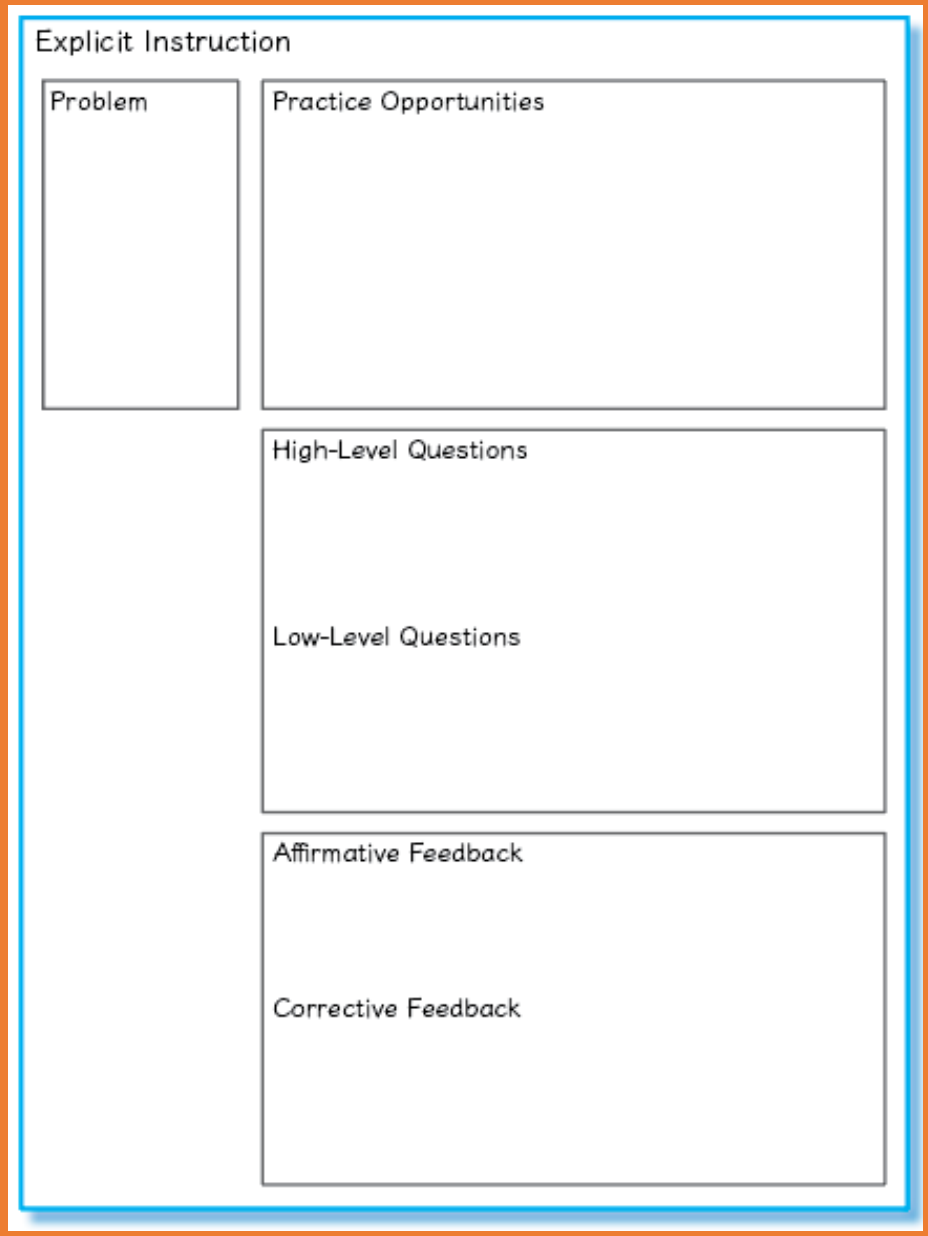
What are five essential math vocabulary for addition and subtraction?



What are the representations you'll use to teach addition and subtraction?



1. Choose a math problem.
2. Write a step-by-step explanation. Focus on the language of math in your explanation. Consider the representations you will use.



1. Describe the practice opportunities you will use.
2. Write 3 high-level questions.
3. Write 3 low-level questions.
4. Write 2 ways to provide affirmative feedback.
5. Write 2 ways to provide corrective feedback.

Explicit Instruction

Problem	Step-by-Step Explanation

1. Teach your problem.

What were your strengths with your teaching?

What are your opportunities for growth?

November 2022

Early Numeracy

- Counting principles
- Connecting number
- Comparison of numbers
- Addition and subtraction concepts

January 2023

Addition and Subtraction

- Addition computation
- Subtraction computation
- Addition and subtraction fluency
- Addition and subtraction word problems

March 2023

Place value and money

- Understanding tens and ones
- Representing thousands, hundreds, tens, and ones
- Money

April 2023

Geometry

- Identification of shapes
- Composing and decomposing shapes

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